Band touching from real space topology

Doron Bergman
Congjun Wu
LB

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Band Touching

- Spaghetti Diagram
- When do they touch?
  - Level repulsion argument
- Must tune 3 parameters for a touching at a generic wavevector - get "accidental" touchings at points in 3d.
Graphene

- Sometimes 2d bands do touch!
Stability

• Common reason: irreducible representation of Little group has dim > 1.
• these touchings are very sensitive to symmetry.
• But sometimes they are more stable...
Topological stability

• Dirac spinor: \(2\pi\) rotation \(\psi \rightarrow -\psi\)
• More generally:
  • Berry gauge field \(\vec{A} = \text{Im} \langle u|\vec{\nabla}_k u\rangle\)
  • Flux \(\oint d\vec{k} \cdot \vec{A} = \int d^2 k \, B(k) = \pi\)
• \(T+I:\) \(B(k) = 0\)
  • Singularity must be preserved!
This talk

- A different kind of topological band touching
- Real space topology instead of momentum space
Frustrated Hopping Models

- Certain lattice hopping Hamiltonians display flat bands
- These are interesting because they offer prospects for strong interaction physics (c.f. FQHE)

\[ H_{\text{eff}} = \hat{P}V\hat{P} \]

if \( V \) is small compared to the gap to the next band
Optical lattices

Theoretical proposals from various atomic theory groups (Lewenstein, Demler/Lukin, Zoller)
High field antiferromagnets

Single magnon excitations governed by frustrated hopping Hamiltonian

c.f. Tsunetsugu and others
Kagome lattice
Kagome lattice

- Flat band
- Band touchings
- Dirac points and touching of flat band
Kagome lattice

- Flat band
- Band touchings
- Dirac points \textit{and} touching of flat band

\textit{no} Berry phase here!
Honeycomb p-bands

- Graphene: 2p\(_z\) -orbital band; Dirac cone; isotropic and non-degenerate.

- In optical lattices, p\(_x\) and p\(_y\) -orbital bands are well separated from s.

- Even more interesting physics in the p\(_x\), p\(_y\) -orbital bands.

- If \(\sigma\)-bonding is included, the flat bands acquire small width at the order of \(t\). Flat band + Dirac cone.

\(\sigma\)-bonding
Honeycomb p-bands

Honeycomb lattice: a surge of research interest

- Graphene: $2p_z$-orbital band; Dirac cone; isotropic and non-degenerate.

However, in graphene, $2p_x, 2p_y$-orbital bands hybridize with $2s$.

In optical lattices, $p_x$ and $p_y$-orbital bands are well separated from $s$.

Even more interesting physics in the $p_x, p_y$-orbital bands.

If $\sigma$-bonding is included, the flat bands acquire small width at the order of $t$.

Flat bands in the entire Brillouin zone!

- Flat band + Dirac cone.
- Localized eigenstates.

$\sigma$-bonding
Pyrochlore lattice

- Spin S=3/2
- No orbital degeneracy
- Isotropic
- Spins form pyrochlore lattice
- Cubic Fd3m
- Antiferromagnetic interactions
- CW = -390K, -70K, -32K for A=Zn, Cd, Hg

Takagi group
Pyrochlore bands

Band touching
Why all this touching?

• Touching is *troublesome* for strong interaction physics

• projection into flat band problematic because there is no gap

• Can we keep the flat band but remove the touching?
Why flat bands?

- Wannier states are eigenstates
- Localized states with finite support
- Reason: interference
Why flat bands?

- Wannier states are eigenstates
- Localized states with finite support
- Reason: interference
Similar in other lattices

- If $t$-bonding is included, the flat bands acquire small width at the order of $t$.
- Flat bands in the entire Brillouin zone.

• Flat band + Dirac cone.
• Localized eigenstates.
Flatness is *not* robust

- Interference condition violated by most additional hoppings
Flatness is *not* robust

- Interference condition violated by most additional hoppings
A sort of protection

• As long as the flat band remains flat, the touching *always* remains
• (somewhat) bad news for “LLL” projection
• Reason: real space topology
Counting

• Flat band = localized states but...
• How many (linearly independent) localized states are there?
• Flat band (with periodic B.C.’s)
  • 1 state per unit cell
Elementary Hexagons

One per unit cell?
Elementary Hexagons

One per unit cell?
Elementary Hexagons

One per unit cell?
Superposition
Superposition
Superposition
Superposition
Superposition

Sum of all elementary hexagons = 0 with PBCs!
Problem

• On torus with N unit cells, find N-1 linearly independent states

• Where is the missing state?
Loops on torus
Loops on torus

Non-trivial loop
Non-trivial Loops

- Two non-contractible loops can be formed on the torus
- The difference between any two loops with the same topology is a sum of elementary hexagons

Two more linearly independent states!
Counting

• Elementary hexagons: $N-1$ states
• Non-contractible loops: 2 states
• Total states: $N+1$ states
  • 1 more state than the flat band!
• This requires another band to touch the flat band.
Summary

• Band touchings in most frustrated hopping hamiltonians are “protected” in this way

• kagome, dice, pyrochlore, honeycomb p-orbital models