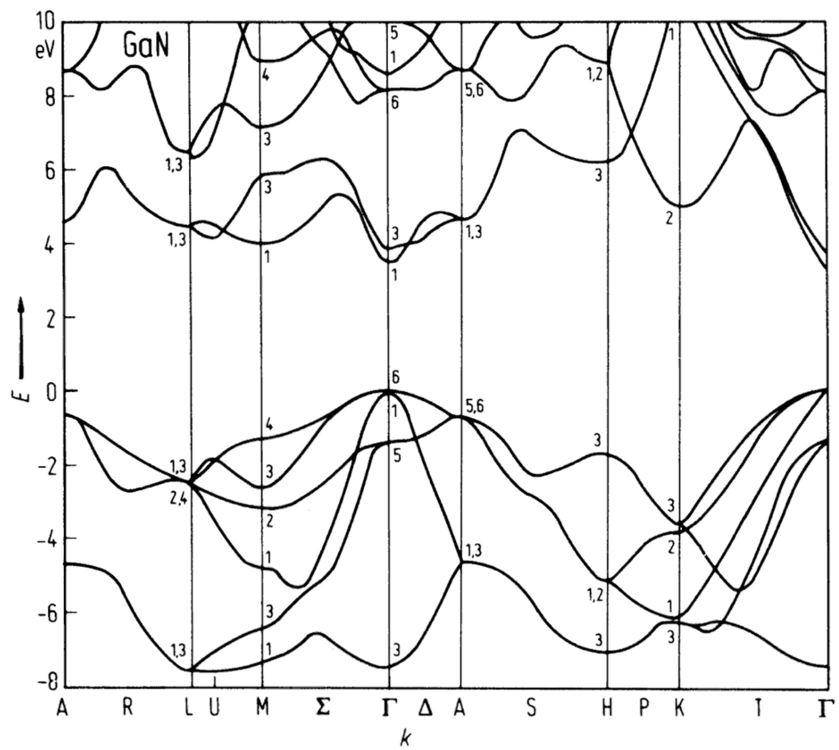
Band touching from real space topology

Doron Bergman
Congjun Wu
LB

TASSP, Kyoto June 2008

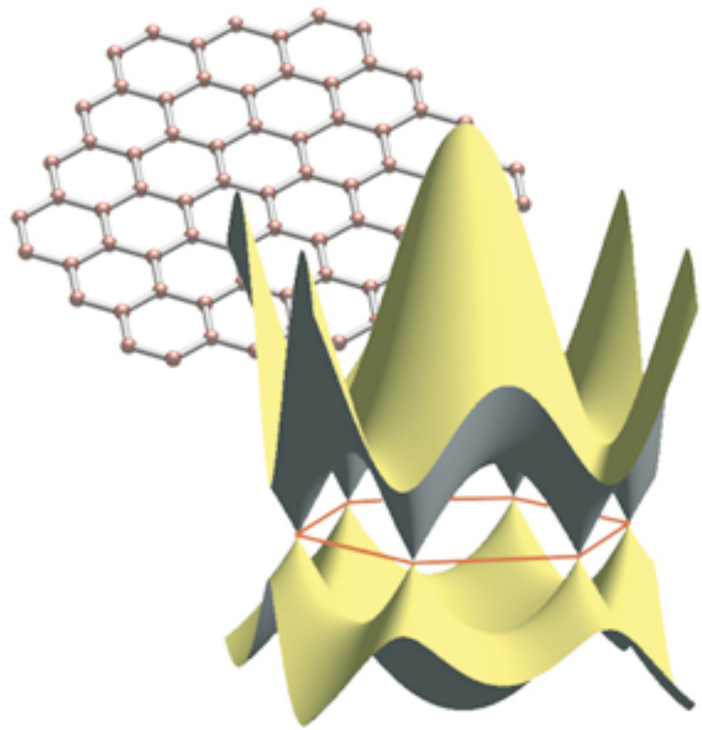
Band Touching

- Spaghetti Diagram
- When do they touch?
  - Level repulsion argument
  - Must tune 3 parameters for a touching at a generic wavevector - get “accidental” touchings at points in 3d.



Graphene

- Sometimes 2d bands do touch!



Stability

- Common reason: irreducible representation of Little group has dim>1.
  - these touchings are very sensitive to symmetry.
- But sometimes they are more stable...

Topological stability

- Dirac spinor: $2\pi \text{ rotation } \psi \to -\psi$
- More generally:
    - Berry gauge field $\vec{A} = \mathrm{Im}\, \langle u | \vec{\nabla}_k u \rangle$
    - Flux $\oint d\vec{k} \cdot \vec{A} = \int d^2k\, B(k) = \pi$
- T+I: $B(k) = 0$
    - Singularity must be preserved!

This talk

- A *different* kind of topological band touching
- Real space topology instead of momentum space

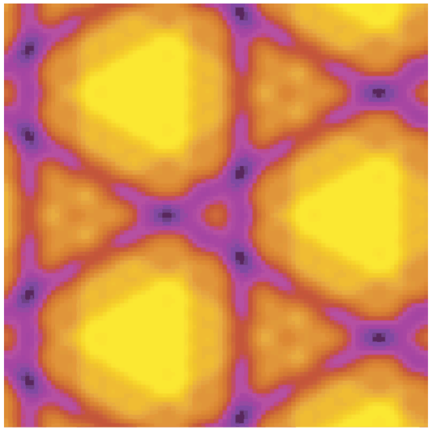
Frustrated Hopping Models

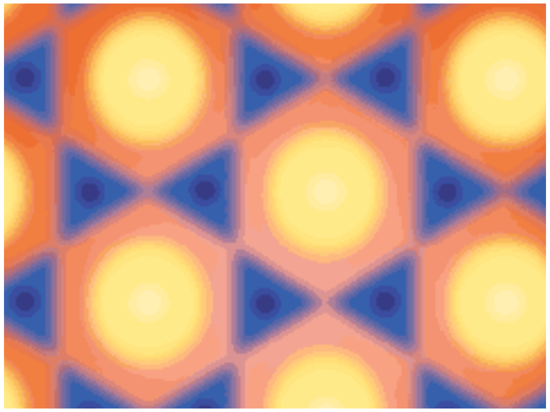
- Certain lattice hopping Hamiltonians display flat bands
- These are interesting because they offer prospects for strong interaction physics (c.f. FQHE)

$$H_{eff} = \hat{P} V \hat{P}$$

if V is small compared to the gap to the next band

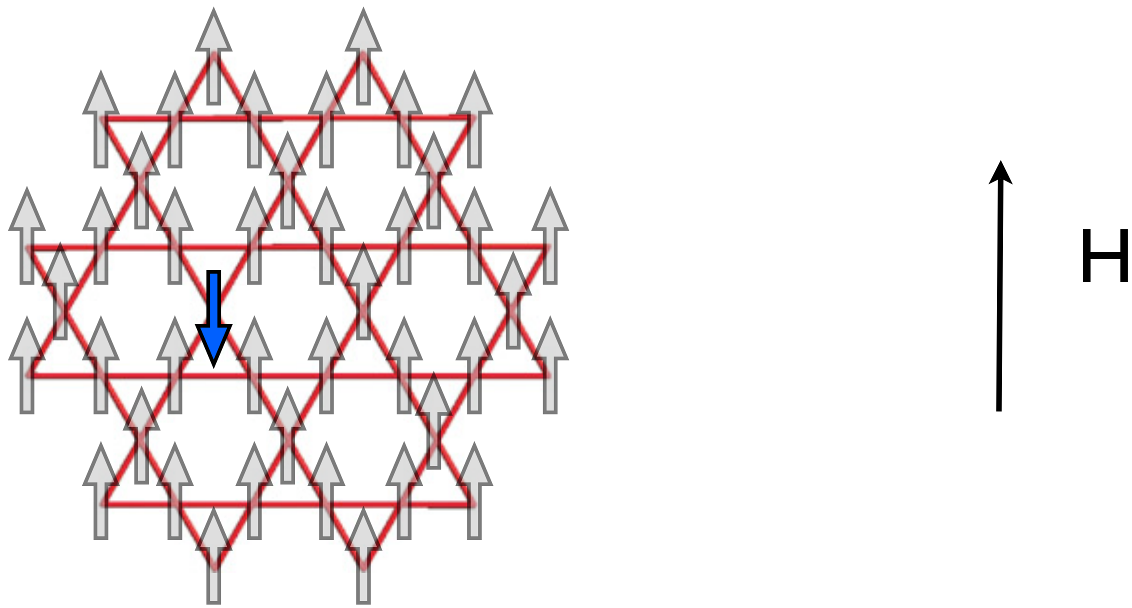
Optical lattices





Theoretical proposals from various atomic theory groups (Lewenstein, Demler/Lukin, Zoller)

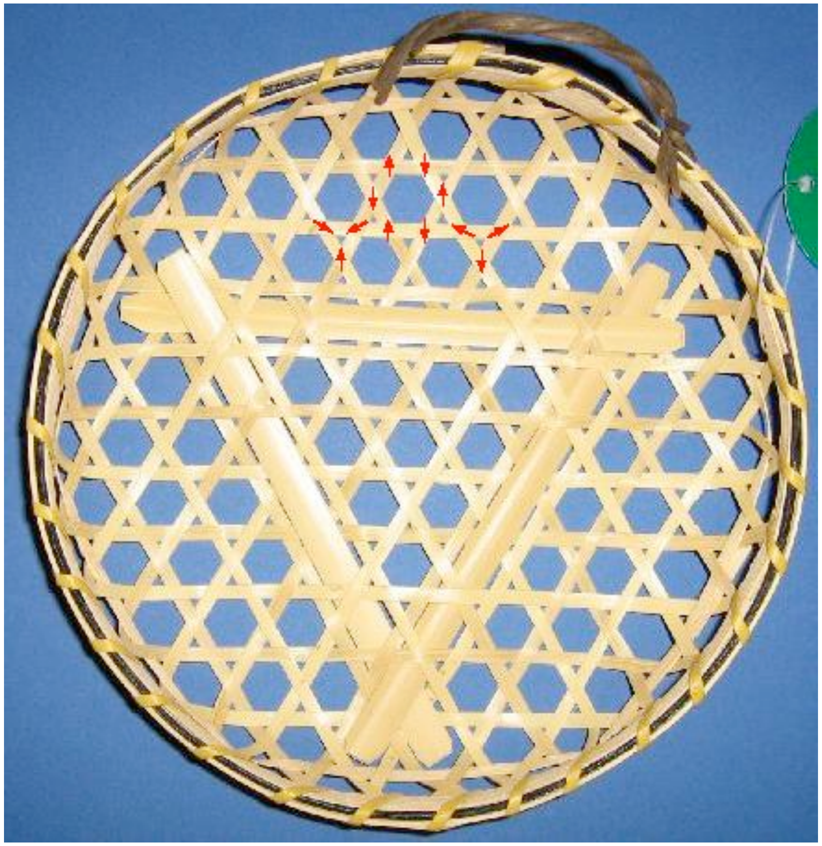
High field antiferromagnets



Single magnon excitations governed by frustrated hopping Hamiltonian

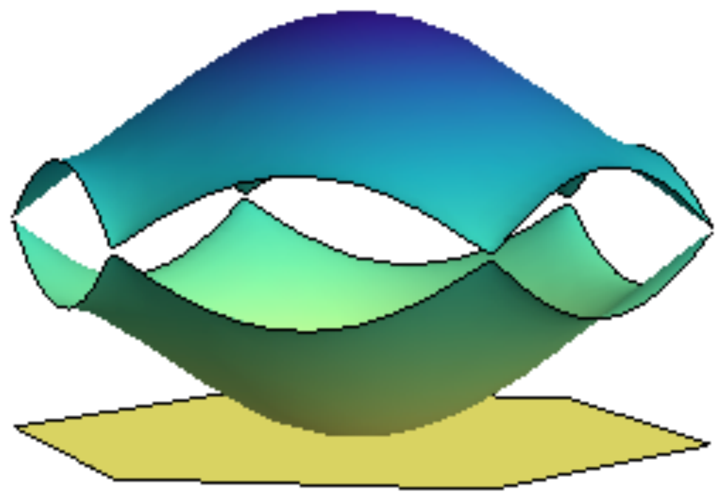
c.f. Tsunetsugu and others

Kagome lattice



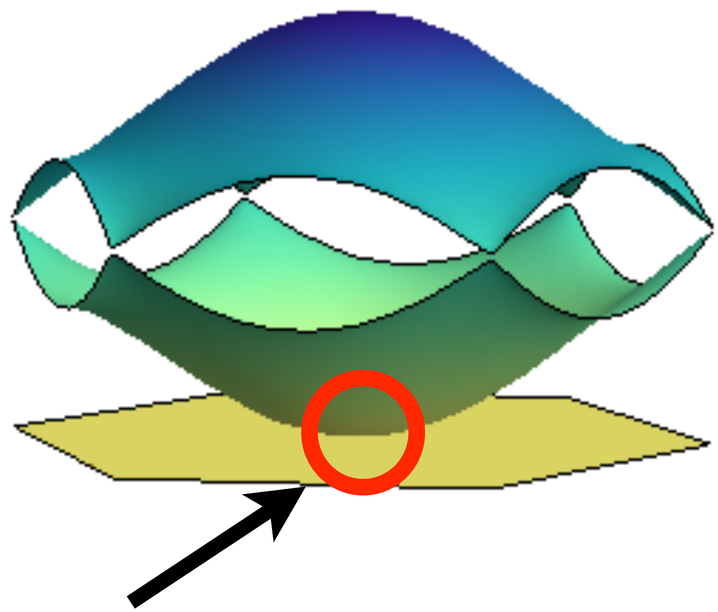
Kagome lattice

- Flat band
- Band touchings
    - Dirac points *and* touching of flat band



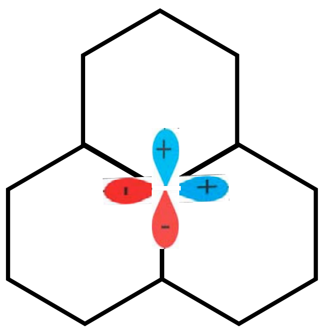
Kagome lattice

- Flat band
- Band touchings
  - Dirac points *and* touching of flat band

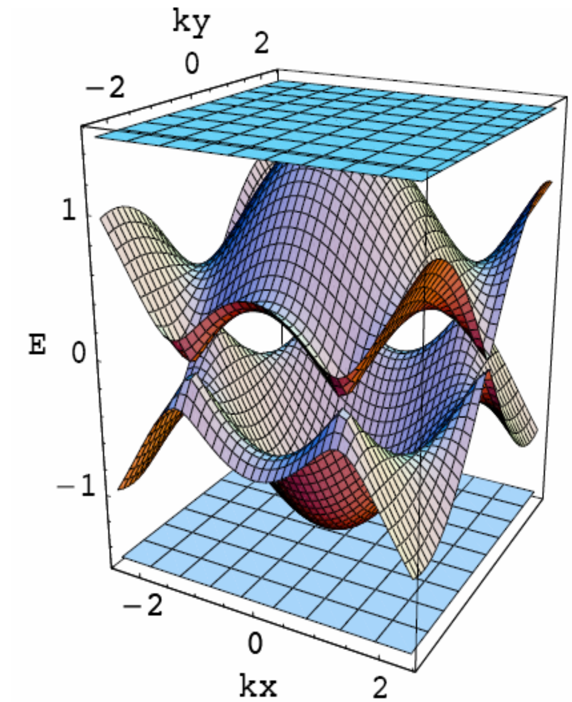


no Berry phase here!

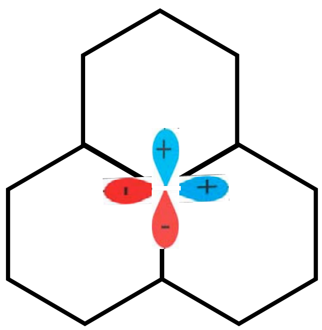
Honeycomb p-bands



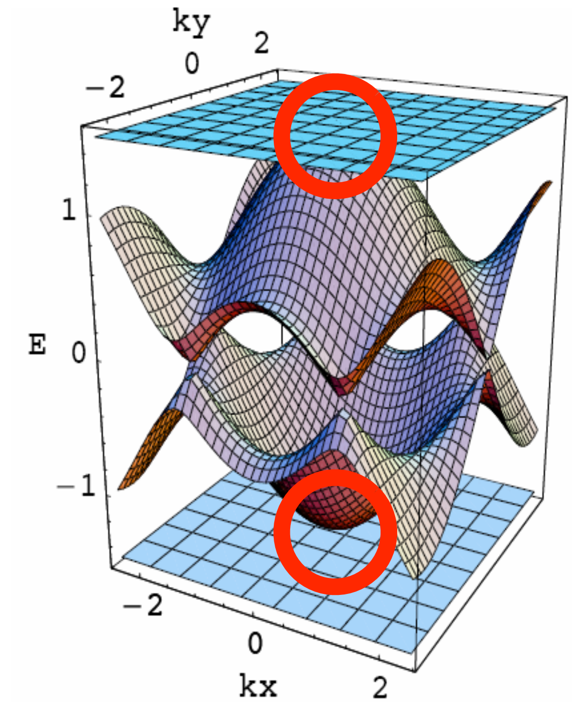
σ -bonding



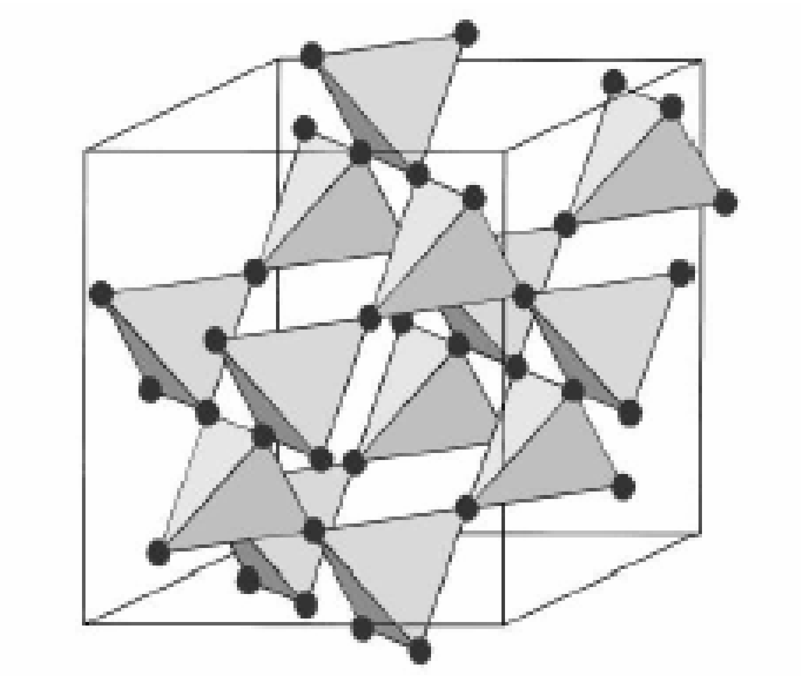
Honeycomb p-bands



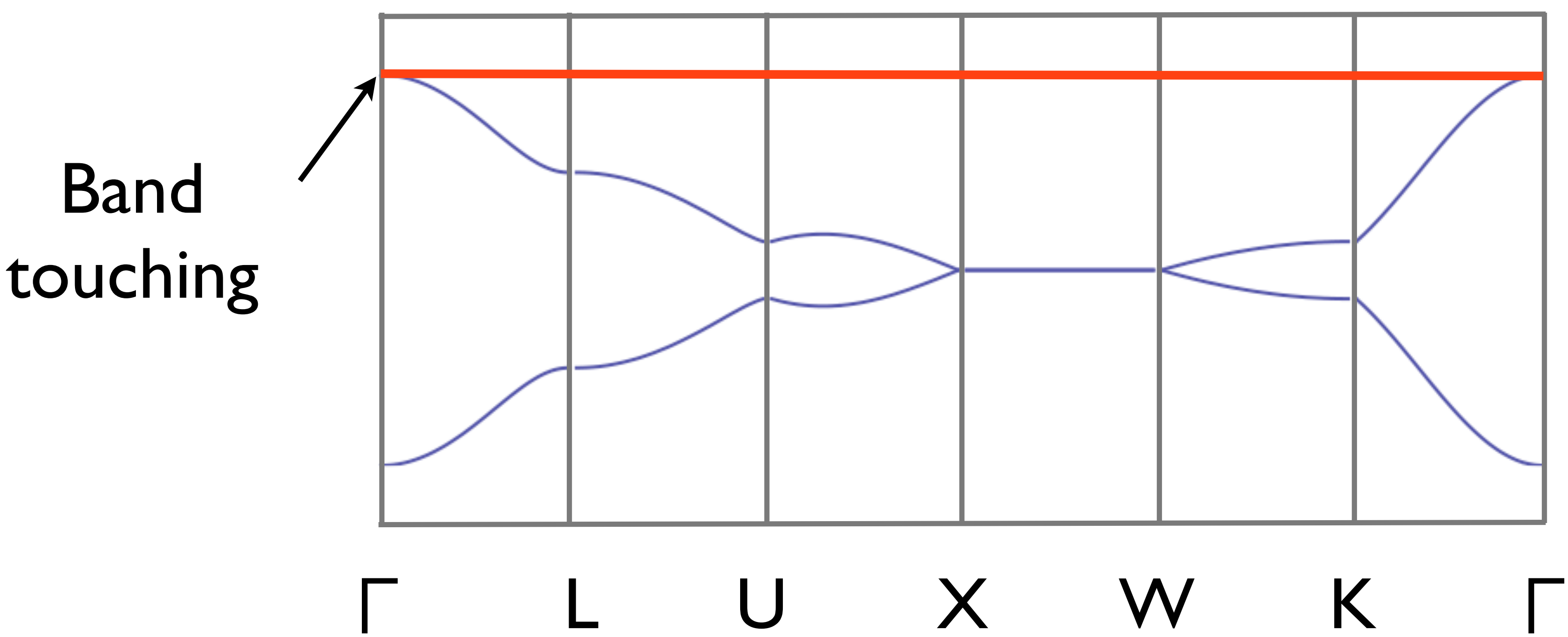
σ -bonding



Pyrochlore lattice



Pyrochlore bands

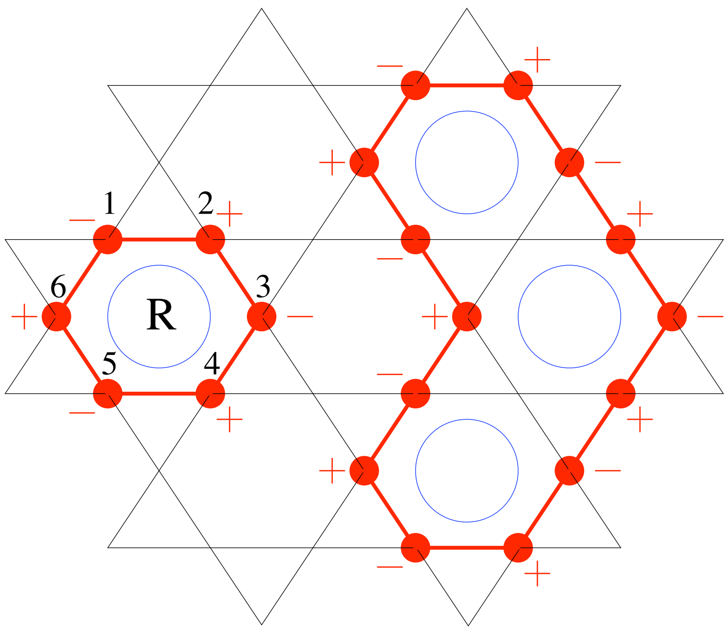


Why all this touching?

- Touching is *troublesome* for strong interaction physics
  - projection into flat band problematic because there is no gap
- Can we keep the flat band but remove the touching?

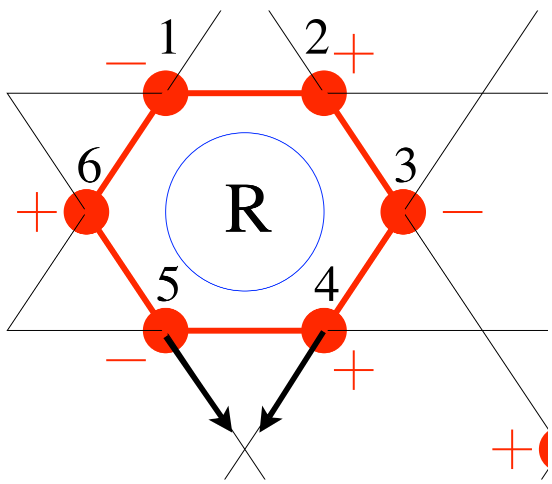
Why flat bands?

- Wannier states are eigenstates
  - localized states with *finite* support
  - reason: interference

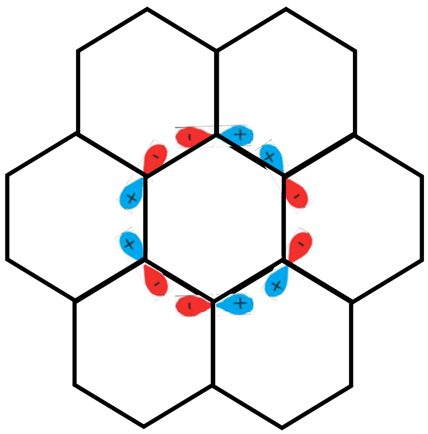


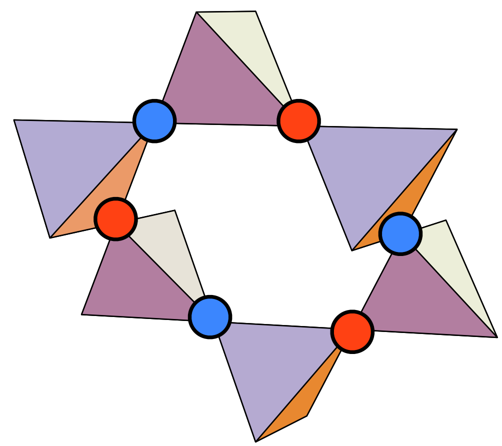
Why flat bands?

- Wannier states are eigenstates
  - localized states with *finite* support
  - reason: <u>interference</u>



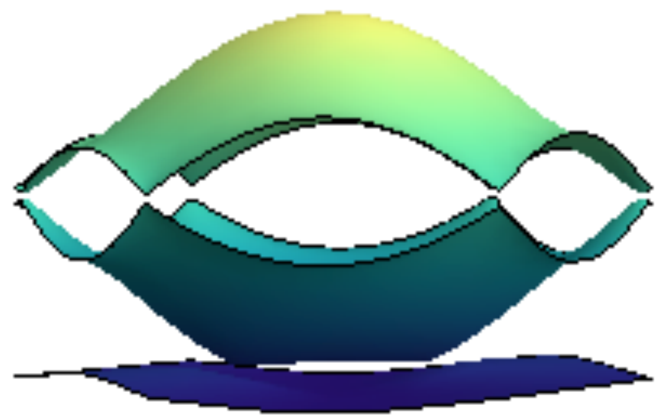
Similar in other lattices





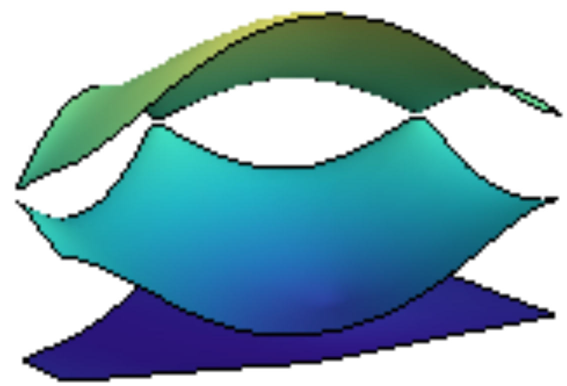
Flatness is *not* robust

- Interference condition violated by most additional hoppings



Flatness is *not* robust

- Interference condition violated by most additional hoppings



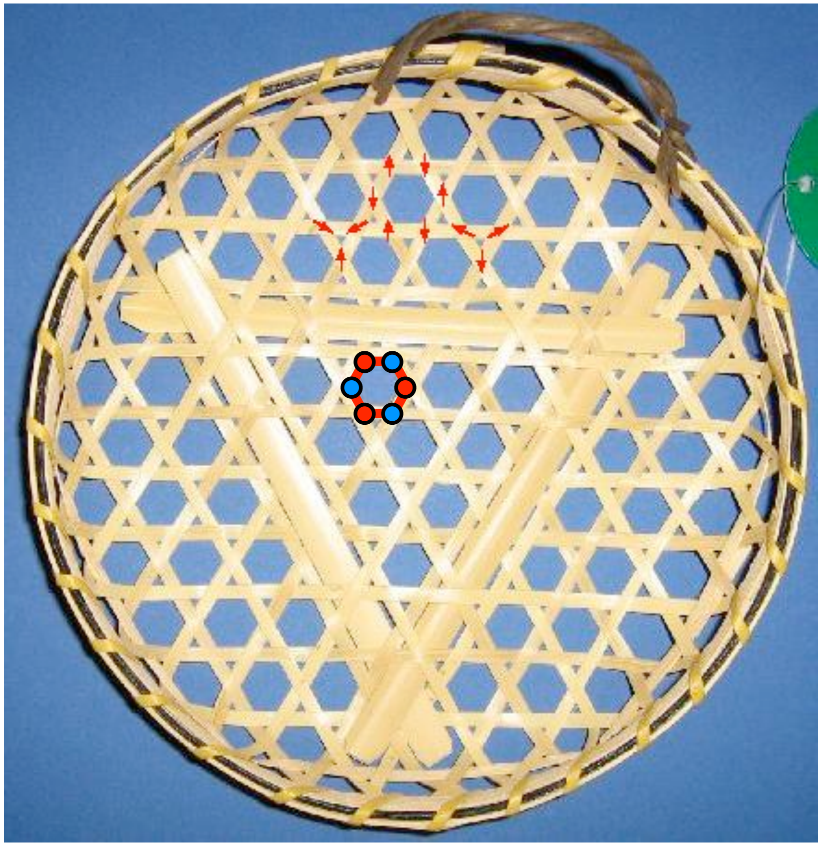
A sort of protection

- As long as the flat band remains flat, the touching *always* remains
  - (somewhat) bad news for “LLL” projection
- Reason: real space topology

Counting

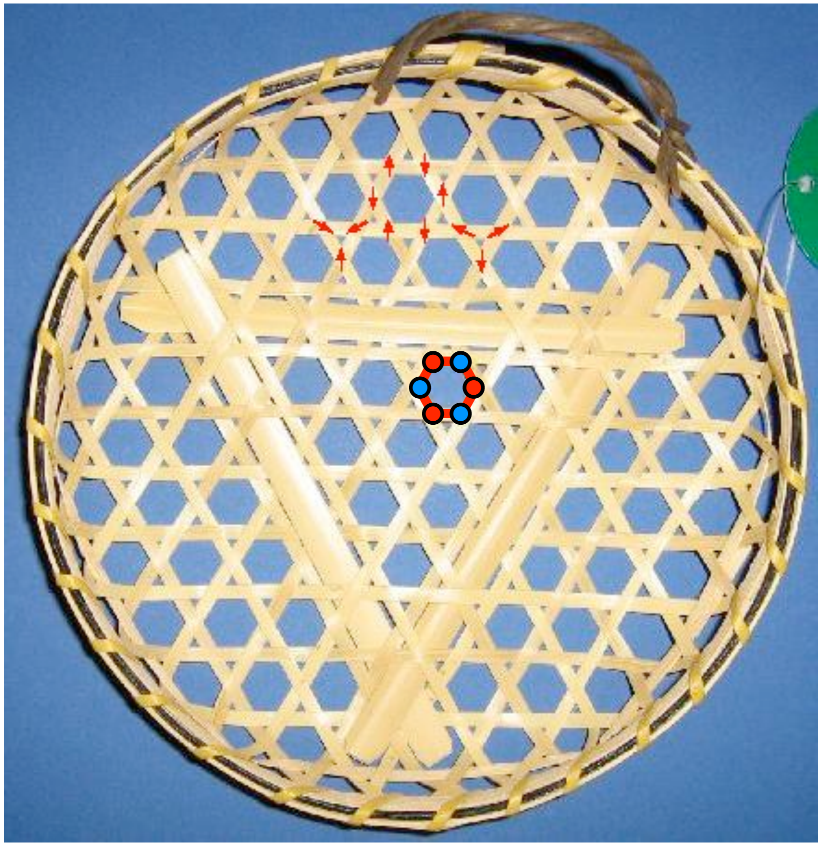
- Flat band = localized states *but...*
- How many (linearly independent) localized states are there?
- Flat band (with periodic B.C.'s)
  - 1 state per unit cell

Elementary Hexagons



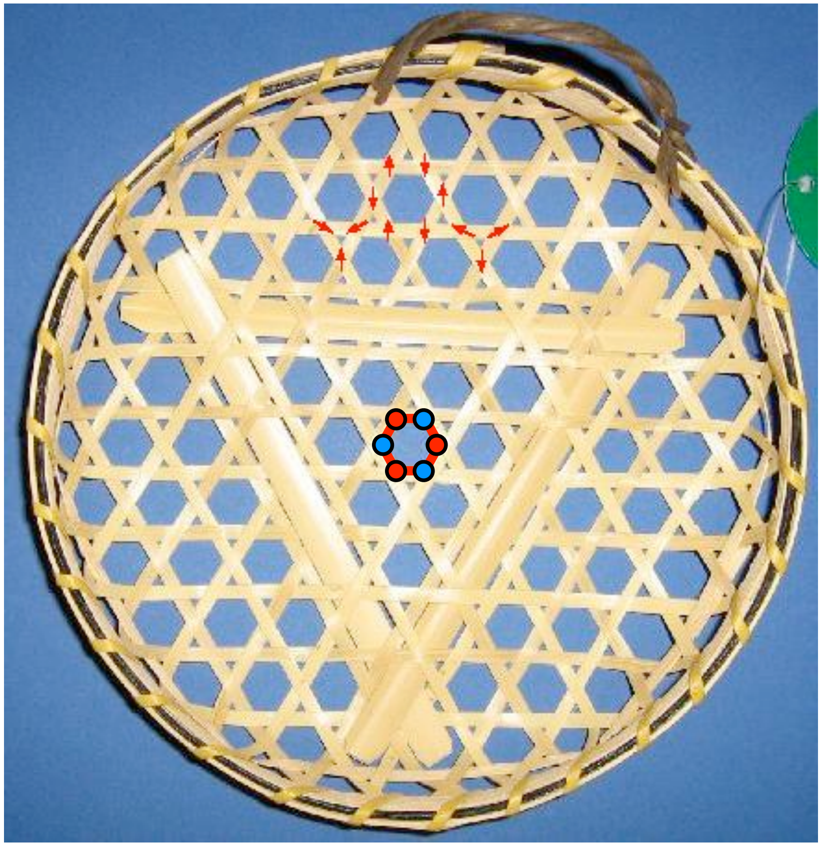
One per unit cell?

Elementary Hexagons



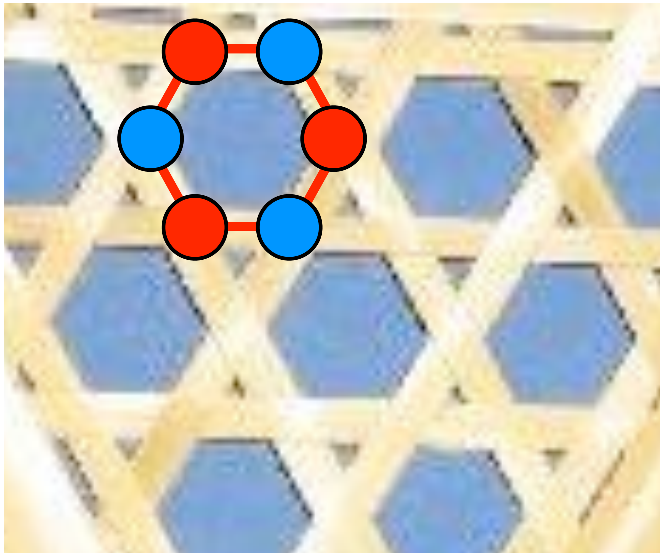
One per unit cell?

Elementary Hexagons

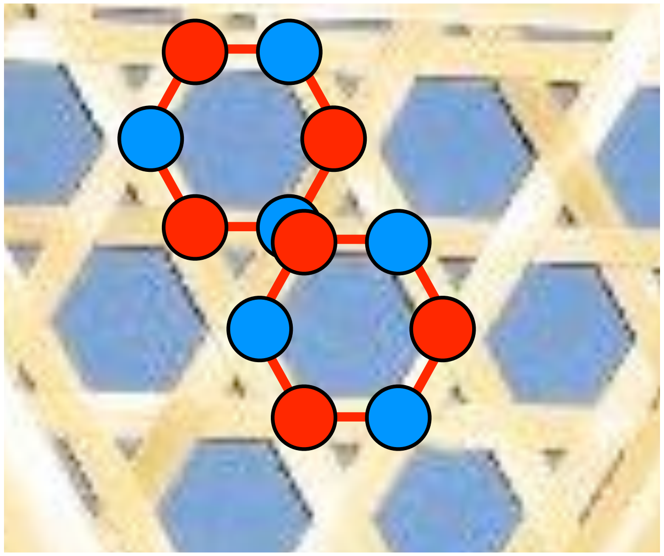


One per unit cell?

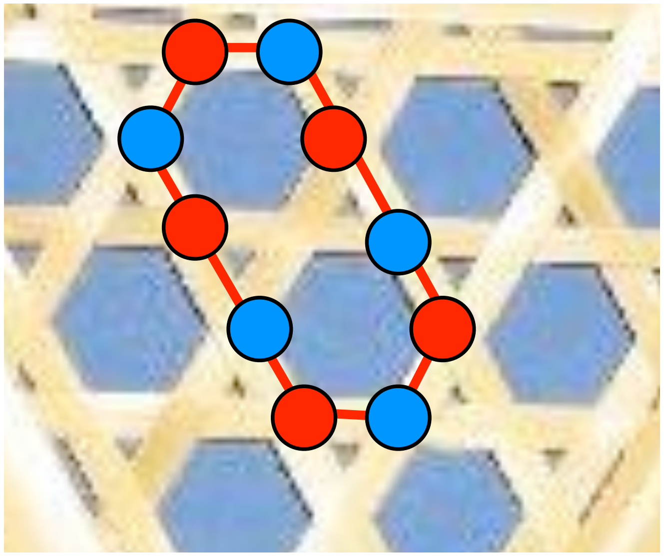
Superposition



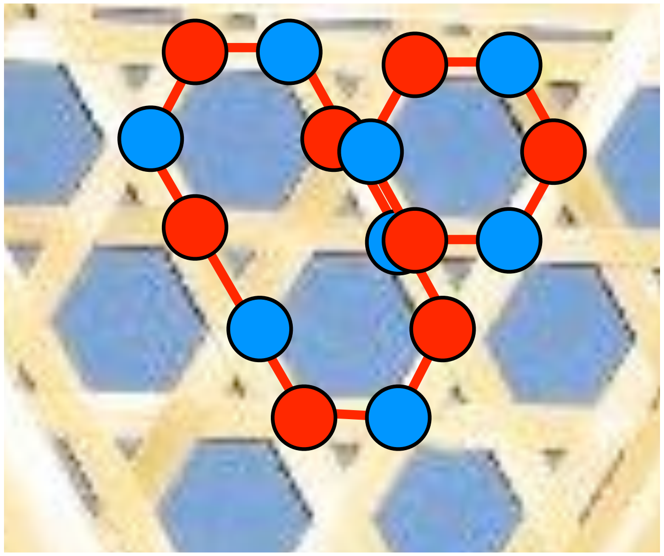
Superposition



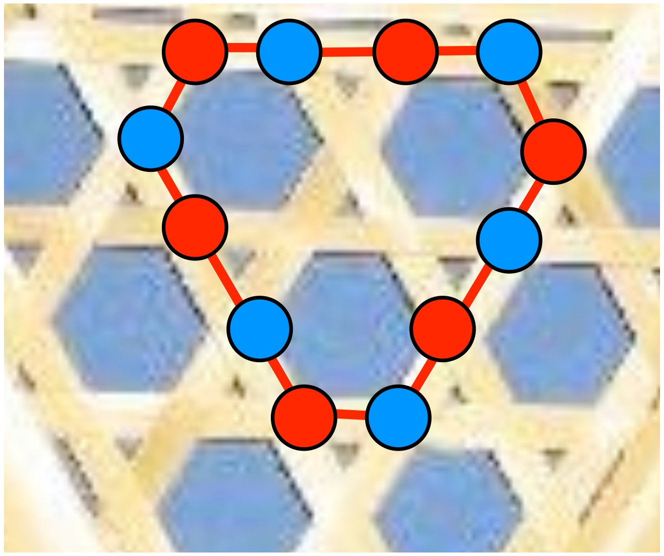
Superposition



Superposition



Superposition

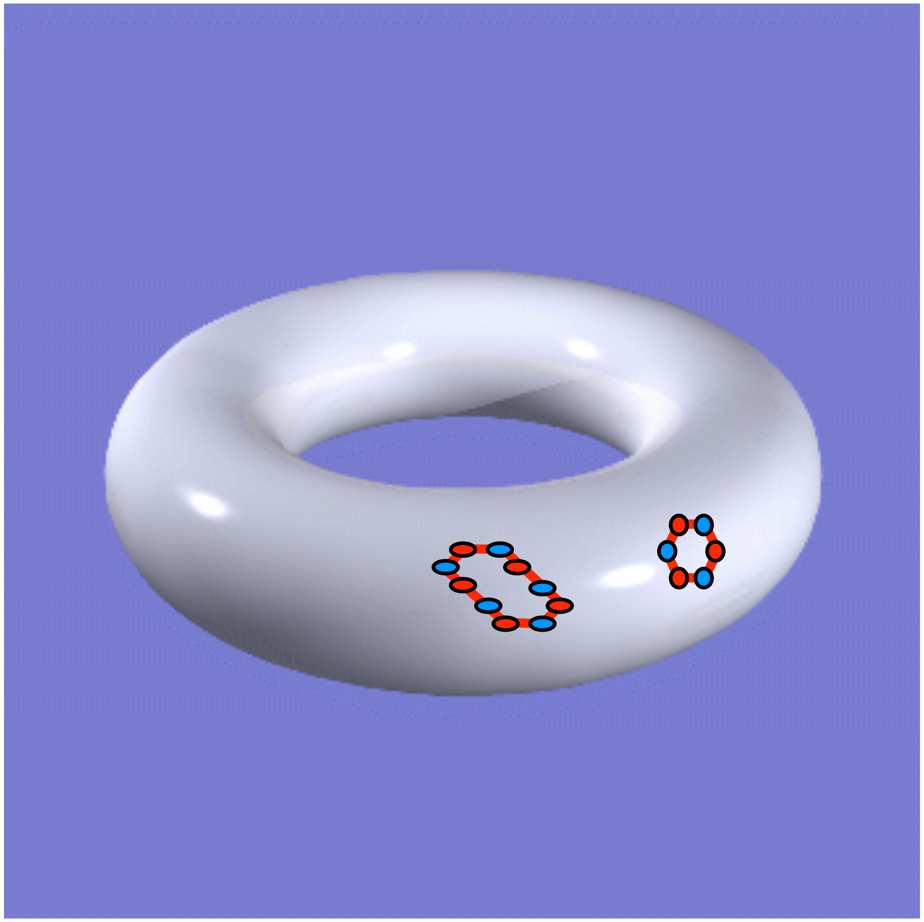


Sum of *all* elementary hexagons = 0 with PBCs!

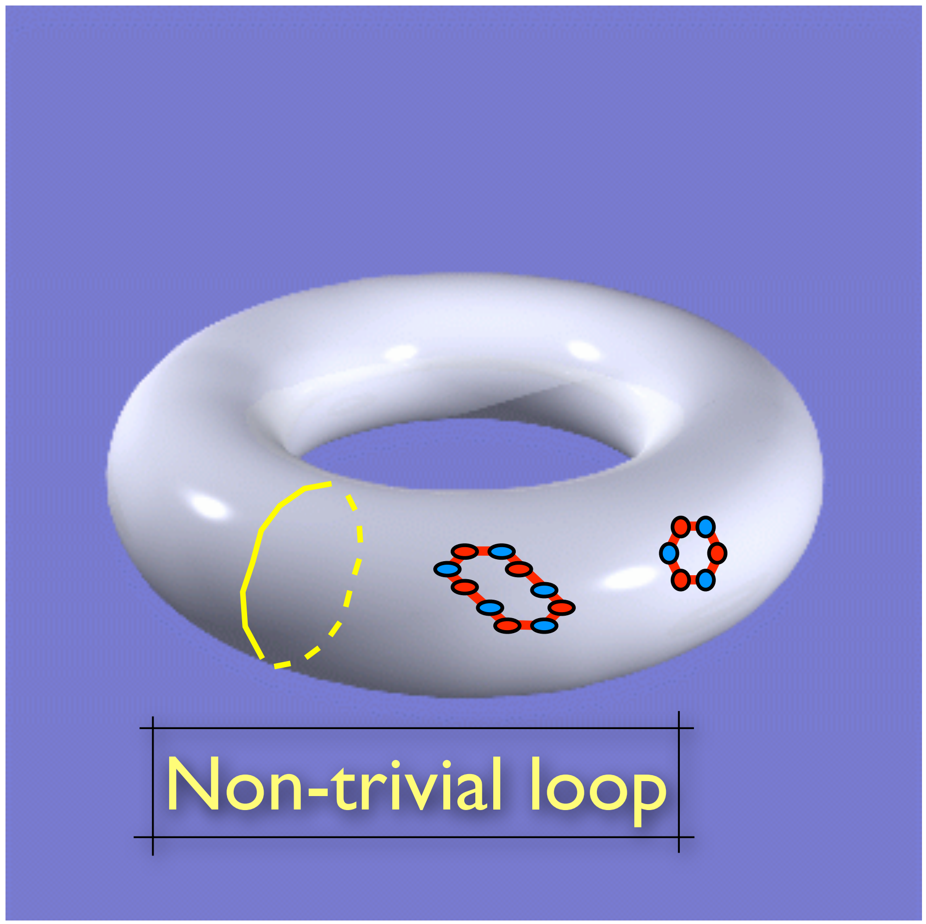
Problem

- On torus with N unit cells, find N-1 linearly independent states
- Where is the missing state?

Loops on torus

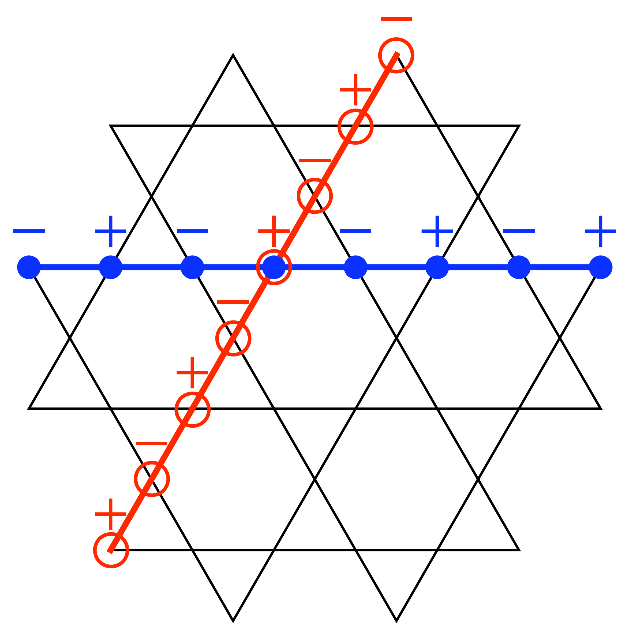


Loops on torus



Non-trivial Loops

- Two non-contractible loops can be formed on the torus
- The difference between any two loops with the same topology is a sum of elementary hexagons



Two more linearly independent states!

Counting

- Elementary hexagons: N-1 states
- Non-contractible loops: 2 states
- Total states: N+1 states
  - 1 *more* state than the flat band!
  - This requires another band to touch the flat band.

Summary

- Band touchings in most frustrated hopping hamiltonians are "protected" in this way
  - kagome, dice, pyrochlore, honeycomb p-orbital models