Disorder effects on 3-dimensional $Z_2$ spin Hall insulators / chiral metals

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3-dimensional $Z_2$ quantum spin Hall insulator (QSHI), originally proposed by Fu, Kane and Mele, supports a spin-selective edge state, forming a Dirac-cone like energy dispersion at its 2-dimensional surface boundary. Having no “$U(1)$ counterpart” into which this 3-d $Z_2$ QSHI can be adiabatically connected, this electronic phase is currently regarded as a new state of matter which goes beyond the quantum Hall paradigm (namely, c.f. 2-d $Z_2$ QSHI). In this note, we have studied the disorder effect (non-magnetic impurities) on this peculiar electronic phase, mainly focusing on the quantum critical point between the $Z_2$ QSHI and trivial band insulator:

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\mathcal{H} \equiv \int dr \psi^\dagger(r) \{ \mu \hat{1} + \gamma_\mu(-i\partial_\mu) + m \hat{\gamma}_5 \} \psi(r),
$$

$$
\hat{\gamma}_1 \equiv \sigma_y \otimes 1, \quad \hat{\gamma}_2 \equiv \sigma_z \otimes a_x, \quad \hat{\gamma}_3 \equiv \sigma_z \otimes s_y, \quad \hat{\gamma}_5 \equiv \sigma_x \otimes 1,
$$

where a finite mass term $m$ induces the phase transition between the nontrivial insulator and trivial one. Taking into account various type of “on-site” impurities, we first derive the phase diagram spanned by the mass-term $m$, chemical potential $\mu$ and strength of the disorder within the self-consistent Born approximation. Thereby, we found a finite density of state even at the zero-energy and at the phase transition point, i.e. $m = \mu = 0$, if the strength of the disorder potential exceeds some critical value. To uncover whether this bundle of states registered at the zero-energy are extended or localized, we next derive the self-consistent equation for the current relaxation kernel (i.e. inverse of the diffusion constant), only to discuss about the number of mobility edges and the criticality around them within the mode-mode coupling theory framework.