Interactions and Charge Fractionalization in an Electronic Hong-Ou-Mandel Interferometer

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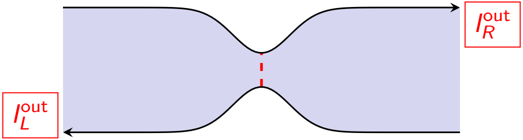


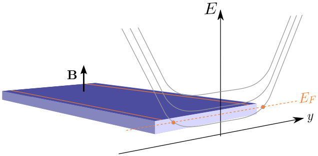
Electronic quantum optics in quantum Hall systems

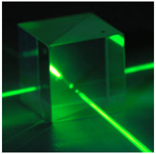
Quantum optics analogs with electrons, i.e. the controlled preparation, manipulation and measurement of single excitations in ballistic conductors

INGREDIENT LIST

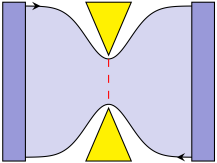
Photons
↓
Electrons

Light beam
↓
Chiral edge QHE





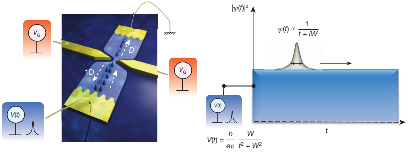
Beam-splitter
↓
Point contact



Coherent light source
↓
Single electron source

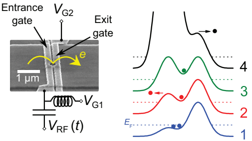
Single electron sources

Lorentzian voltage pulses



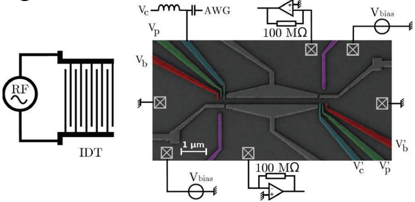
[Dubois et al., Nature ('13)]

Charge pumps, quantum turnstiles



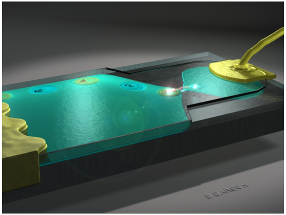
[Giblin et al., Nature Comm. 3, 930 ('12)]

Flying electrons on surface acoustic waves



[Hermelin et al., Nature 477, 435 ('11)]
[McNeil et al., Nature 477, 439 ('11)]

Mesoscopic capacitor

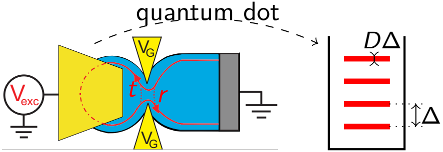


[Fève et al., Science 316, 1169 ('07)]

➡ opens the way to all sorts of interference experiments!

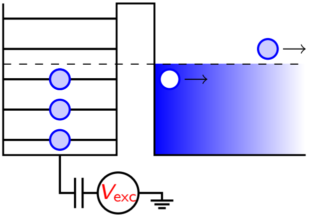
Single electron source: the LPA mesoscopic capacitor

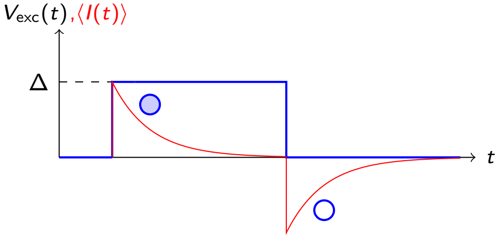
- Setup [Fève et al., Science 316, 1169 ('07); Mahé et al., PRB 82,201309 ('10)]



- quantum dot coupled to edge
- discrete levels spaced by Δ
- tunable dot transmission via V_g
  ➞ sets level broadening

- Operating the source
  - time-dependent excitation voltage $V_{\text{exc}}(t)$
    ➞ well described through Floquet scattering theory
  - emission of one electron + one hole per period

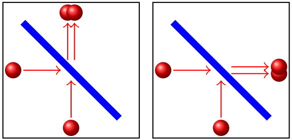




- Injected wave-packet ? ➞ exponential shape

Hong-Ou-Mandel interference experiment

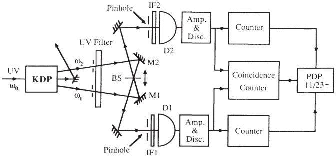
- Two-photon interferences



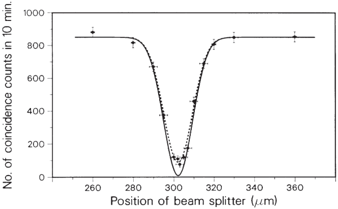
- two identical photons sent on a beam-splitter
- necessarily exit by the same output channel

➞ signature of bosonic statistics

- Interference experiment
  - [Hong, Ou and Mandel, PRL 59, 2044 ('87)]
  - non-linear crystal generates photon pairs
  - measure the coincidence rate



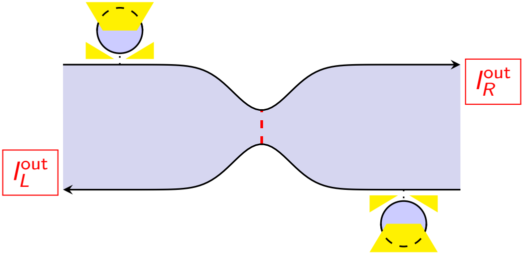
- Coincidence rate



- counts occurrences of photons present in the two output channels
- dip is observed when photons arrive at the same time
- signatures of incoming wave packets

HOM with electrons: general principle

- Setup
  - 2 single electron sources
  - counter-propagating channels coupled at QPC
  - measure output currents



- Zero-frequency cross-correlations of output currents

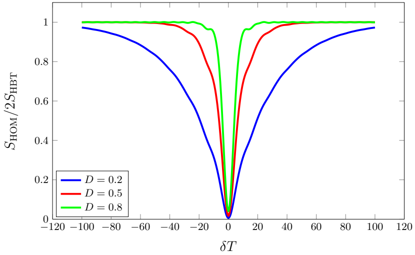
$$S_{RL}^{\text{out}} = \int dt dt' \left[\langle I_R^{\text{out}}(x,t) I_L^{\text{out}}(x',t') \rangle - \langle I_R^{\text{out}}(x,t) \rangle \langle I_L^{\text{out}}(x',t') \rangle \right]$$

- Differences with photons
  - they obey fermionic statistics
    ➞ existence of a Fermi sea, hole-like excitations, ...
  - thermal effects do matter
  - they interact via Coulomb interaction

HOM with electrons: theoretical results for $\nu = 1$

[Jonckheere et al. Phys. Rev. B 86, 125425 ('12)]

- Collision of identical wave-packets



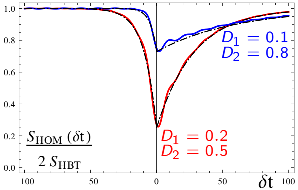
When electrons arrive independently

- $S_{12} < 0$: sum of the partition noise
- flat background contribution
  ➤ Hanbury-Brown and Twiss

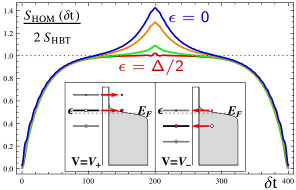
When electrons arrive simultaneously

- $S_{12} = 0$ ➤ HOM/Fermi/Pauli dip
- signatures of injected object

- More exotic situations



Different packets ➤ asymmetry

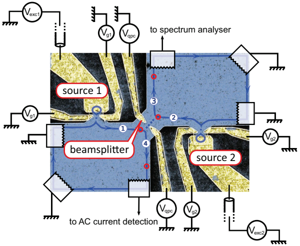


Electron-hole collision ➤ peak

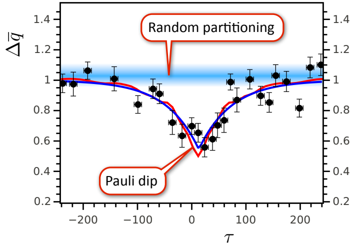
HOM with electrons: experimental results

- Experimental setup
  - two independent sources
  - synchronized electron emission, and collision at the beamsplitter
  - observe two-particle interferences?

  [Bocquillon et al., Science 339, 1054 ('13)]



- Main results



As expected

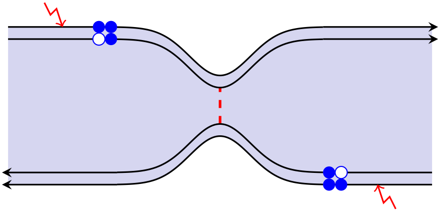
- Flat background contribution (random partiotioning)
- Pauli dip for simultaneous injection

But... How come it does not reach 0?
$\longrightarrow$ decoherence

Something special happens when we go beyond the simple $\nu = 1$ picture

Beyond $\nu = 1$: interactions!

- Different possible types of interactions
  - between counter-propagating channels, near the QPC
  - strong interactions within a channel: Fractional QHE
  - between co-propagating channels at filling $\nu = 2$



(this talk)

- Formalism and methods

	$\nu = 2$
• injection	prepared state $\vert\varphi\rangle$
• propagation	bosonized H (+ diagonalization)
• tunneling	scattering matrix

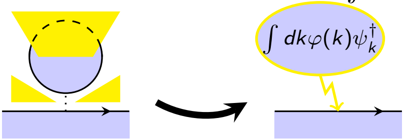
Injection

- Simplified model of injection: prepared state
  - injection in the past at $t = -T_0$: $|\varphi\rangle = \mathcal{O}^\dagger(-T_0)\,|0\rangle$
  - preparation operator $\mathcal{O}^\dagger = \mathcal{O}_R^\dagger \mathcal{O}_L^\dagger$ with

ground-state

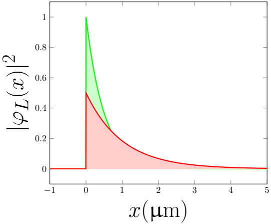
preparation operator

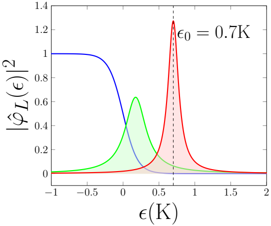
$$\mathcal{O}_{R,L}^\dagger = \int dk \varphi_{R,L}(k) \psi_{R,L}^\dagger(k; t = -T_0)$$



- True one shot injection of electron or hole
- Versatile: any wave-packet

- Exponential wave-packets $\varphi_{R,L}(x) = \sqrt{\frac{2\Gamma}{v_F}} e^{\pm(i\epsilon_0+\Gamma)x/v_F} \theta(\mp x)$





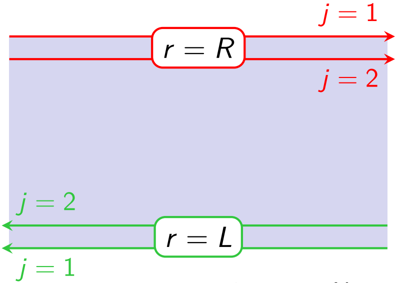
Tunable resolution $\gamma = \epsilon_0/\Gamma$

$\left.\begin{array}{l}\epsilon_0 = 0.175\text{K}\\ \Gamma = 0.175\text{K}\end{array}\right\} \longrightarrow \gamma = 1$

$\left.\begin{array}{l}\epsilon_0 = 0.7\text{K}\\ \Gamma = 0.0875\text{K}\end{array}\right\} \longrightarrow \gamma = 8$

Propagation

- $\nu = 2$ QHE ➙ Two channels: outer and inner ($j = 1, 2$)
- Bosonization identity: $\psi_{j,r}(x) = \frac{U_{j,r}}{\sqrt{2\pi a}} \exp\left(i\ \phi_{j,r}(x)\right)$
- Hamiltonian $H = H_0 + H_{\text{intra}} + H_{\text{inter}}$



Typically $0 \leq u \leq U$

$U \gtrsim v_F$

- Propagation along the edge

$$H_0 = \frac{\hbar}{\pi} \sum_{j=1,2} v_j^{(0)} \sum_{r=R,L} \int dx \left(\partial_x \phi_{j,r}\right)^2$$

- Intra-channel interaction

$$H_{\text{intra}} = \frac{\hbar}{\pi} U \sum_{j=1,2} \sum_{r=R,L} \int dx \left(\partial_x \phi_{j,r}\right)^2$$

- Inter-channel interaction

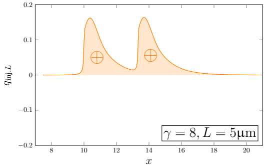
$$H_{\text{inter}} = 2\frac{\hbar}{\pi}\ u \sum_{r=R,L} \int dx \left(\partial_x \phi_{1,r}\right)\left(\partial_x \phi_{2,r}\right)$$

- Hamiltonian: $H = \frac{\hbar}{\pi} \sum_{r=R,L} \int dx \left[v_+ \left(\partial_x \phi_{+,r}\right)^2 + v_- \left(\partial_x \phi_{-,r}\right)^2 \right]$

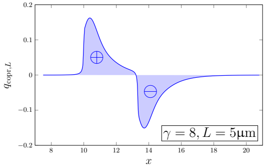
- Diagonalization ➞ mixing angle $\theta \mid \tan\theta = \frac{2u}{v_1 - v_2}$
  - Rotated fields and eigen-velocities

$$\begin{cases} \phi_1 = \cos\theta\ \phi_+ + \sin\theta\ \phi_- \\ \phi_2 = \sin\theta\ \phi_+ - \cos\theta\ \phi_- \end{cases} \quad \text{and} \quad v_\pm = \frac{v_1 + v_2}{2} \pm \sqrt{\left(\frac{v_1 - v_2}{2}\right)^2 + u^2}$$

- Propagating excitations



injection channel



co-propagating channel

- Average charge density

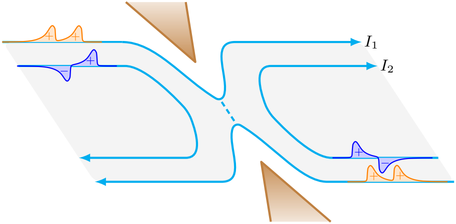
$$q_{s,r}(x,t) = \frac{e}{\pi} \langle \partial_x \phi_{s,r}(x,t) \rangle_\varphi$$

- Strong interaction: $\theta = \pi/4$
- Excitations characterized by the charge they carry $\oplus/\ominus$

Free propagation of two modes: fast charged ϕ_+ and slow neutral ϕ_-

Tunneling

- QPC couples counter-propagating channels ➜ two possibilities
- Two setups $s = 1, 2$



SETUP 1 SETUP 2

- Tunneling Hamiltonian $H_{\text{tun}} = \Gamma \left[\psi^\dagger_{s,R}(0)\psi_{s,L}(0) + \psi^\dagger_{s,L}(0)\psi_{s,R}(0) \right]$
- Scattering matrix:

$$\begin{pmatrix} \psi_{s,R} \\ \psi_{s,L} \end{pmatrix}^{\text{outgoing}} = \begin{pmatrix} \sqrt{\mathcal{T}} & i\sqrt{\mathcal{R}} \\ i\sqrt{\mathcal{R}} & \sqrt{\mathcal{T}} \end{pmatrix} \begin{pmatrix} \psi_{s,R} \\ \psi_{s,L} \end{pmatrix}^{\text{incoming}}$$

$\mathcal{T}$ is the transmission and $\mathcal{R}$ the reflexion probability

Performing the calculation

Zero-frequency crossed correlations of outgoing currents

$$S_{RL}^{\text{out}} = \int dt dt' \left[\langle I_{s,R}^{\text{out}}(x,t) I_{s,L}^{\text{out}}(x',t') \rangle_\varphi - \langle I_{s,R}^{\text{out}}(x,t) \rangle_\varphi \langle I_{s,L}^{\text{out}}(x',t') \rangle_\varphi \right]$$

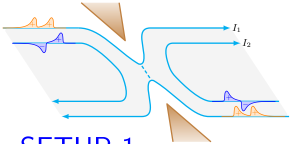
$I_{s,r}^{\text{out}}(x,t) \to I_{s,r}^{\text{out}}(0,t) \to \underset{\text{out}}{\psi_{j,r}}(0,t) \to \underset{\text{in}}{\psi_{j,r}}(0,t) \to \phi_{j,r}^{\text{in}}(0,t) \to \phi_{\pm,r}^{\text{in}}(0,t)$

- Linear dispersion ➞ $\int dt \; I_{j,r}^{\text{out}}(x,t) = \int dt \; I_{j,r}^{\text{out}}(0,t)$
- Electric current: $I_{j,r}^{\text{out}}(0,t) = -ev :\underset{\text{out}}{\psi_{j,r}^\dagger}(0,t) \underset{\text{out}}{\psi_{j,r}}(0,t):$
- Scattering matrix: $\begin{pmatrix} \psi_{j,R}(0,t) \\ \psi_{j,L}(0,t) \end{pmatrix}_{\text{out}} = \mathcal{S} \times \begin{pmatrix} \psi_{j,R}(0,t) \\ \psi_{j,L}(0,t) \end{pmatrix}_{\text{in}}$
- Bosonization: $\underset{\text{in}}{\psi_{j,r}}(0,t) = \dfrac{U_{j,r}}{\sqrt{2\pi a}} \exp\left(i\phi_{j,r}^{\text{in}}(0,t)\right)$
- Diagonalization: $\begin{cases} \phi_{1,r}^{\text{in}}(0,t) = \cos\theta \; \phi_{+,r}^{\text{in}}(0,t) + \sin\theta \; \phi_{-,r}^{\text{in}}(0,t) \\ \phi_{2,r}^{\text{in}}(0,t) = \sin\theta \; \phi_{+,r}^{\text{in}}(0,t) - \cos\theta \; \phi_{-,r}^{\text{in}}(0,t) \end{cases}$

Quantity of interest: $S_{RL}^{\text{out}}\left[I_{s,r}^{\text{out}}(x,t)\right] \longrightarrow S_{RL}^{\text{out}}\left[\phi_{\pm,r}^{\text{in}}(0,t)\right]$

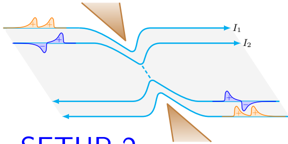
Performing the calculation: final expression

- Focus on the following situation



SETUP 1

- different setups $s = 1, 2$
- strong interaction $\theta = \frac{\pi}{4}$
- symmetric injection $\pm L$
- identical packets $\varphi(x)$
- time delay δT



SETUP 2

- Final expression of noise for the Hong-Ou-Mandel experiment

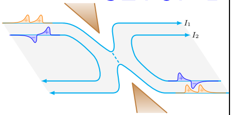
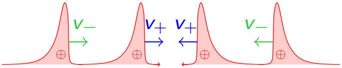
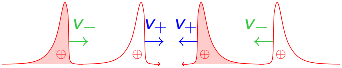
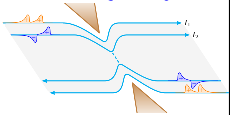
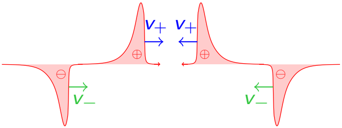
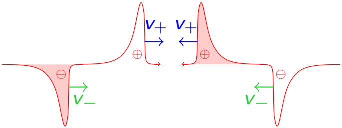
$$S_{\text{HOM}} = -2S_0 \mathrm{Re}\left\{ \int d\tau \mathrm{Re}\left[g(\tau,0)^2 \right] \right.$$

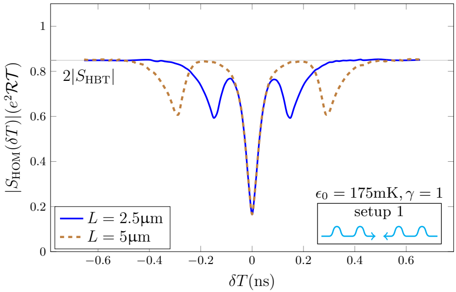
$$\times \int dy_R dz_R \frac{\varphi_R(y_R)\varphi_R^*(z_R)}{(2\pi a)^2 \mathcal{N}_R} g(0, y_R - z_R) \int dy_L dz_L \frac{\varphi_L(y_L)\varphi_L^*(z_L)}{(2\pi a)^2 \mathcal{N}_L} g(0, z_L - y_L)$$

$$\left. \times \int dt \left[\frac{h_s(t; y_L + L, z_L + L) h_s(t + \tau - \delta T; L - y_R, L - z_R)}{h_s(t + \tau; y_L + L, z_L + L) h_s(t - \delta T; L - y_R, L - z_R)} - 1 \right] \right\}$$

$$g(t,x) = \left[\frac{\sinh\left(i\frac{\pi a}{\beta v_+}\right) \sinh\left(i\frac{\pi a}{\beta v_-}\right)}{\sinh\left(\frac{ia + v_+ t - x}{\beta v_+/\pi}\right) \sinh\left(\frac{ia + v_- t - x}{\beta v_-/\pi}\right)} \right]^{\frac{1}{2}} \quad h_s(t; x, y) = \left[\frac{\sinh\left(\frac{ia - v_+ t + x}{\beta v_+/\pi}\right)}{\sinh\left(\frac{ia + v_+ t - y}{\beta v_+/\pi}\right)} \right]^{\frac{1}{2}} \left[\frac{\sinh\left(\frac{ia - v_- t + x}{\beta v_-/\pi}\right)}{\sinh\left(\frac{ia + v_- t - y}{\beta v_-/\pi}\right)} \right]^{s - \frac{3}{2}}$$

Interference pattern: expected structures

	time delay $\delta T = 0$ interference of excitations with same charge and velocity	time delay $\delta T = \pm L \frac{v_+ - v_-}{v_+ v_-}$ interference of excitations with different velocity and possibly different charge
SETUP 1 		
SETUP 2 		
	+ flat background contribution from non-interfering excitations	

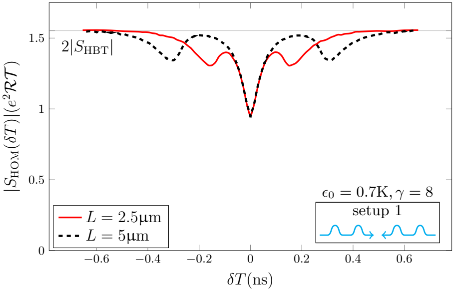


Central dip

- noise reduction ➔ destructive interference of ⊕/⊕ excitations
- loss of contrast due to interactions, strong dependence on resolution

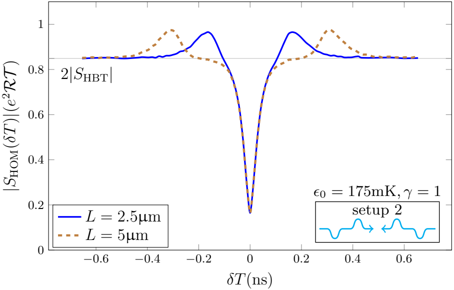
Side dips

- ⊕-excitations with different velocities
- destructive interference
- velocity mismatch : asymmetry + smaller than half central dip



3-dip structure + flat background contribution (no interference)

Results: setup 2



Central dip identical for the 2 setups

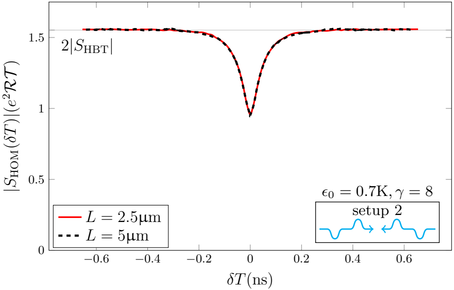
➡ interference independent of the charge carried by the excitations, both in sign and amplitude

Side peaks

- excitations with opposite charge

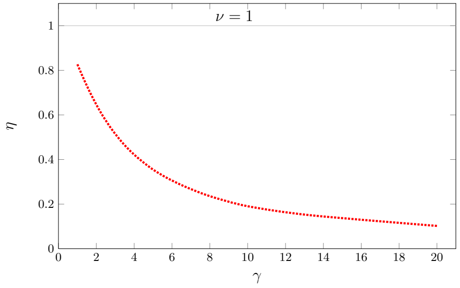
  ➡ constructive interference

- vanish as the resolution increases
- velocity mismatch: asymmetry



peak-dip-peak structure + flat background contribution (no interference)

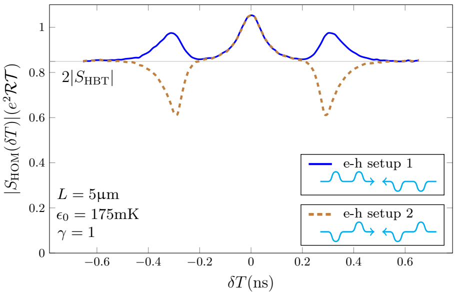
Results



- Central dip vs. packet broadening
  ➡ bottom of dip sinks deeper for wider packets
- Contrast $\eta = 1 - \frac{S_{\mathrm{HOM}}(\delta T=0)}{2S_{\mathrm{HBT}}}$
  ➡ dramatic reduction as resolution increases

Electron-hole collision

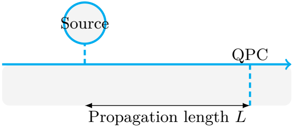
- confirms the pattern
  $\oplus/\oplus$ or $\ominus/\ominus$ $\longrightarrow$ destructive
  $\oplus/\ominus \longrightarrow$ constructive
- side peaks smaller than dips
  ➡ $\oplus/\ominus$ interference is weaker



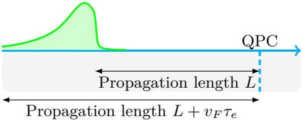
Refined model for the source

- More experimental results are now becoming available for setup 1
- Injection ➙ problematic for spatially extended packets

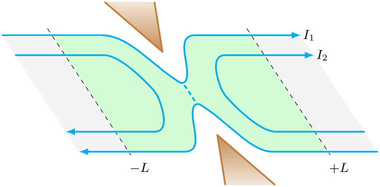
From the SES



In our model



- Refined setup and contrast



$$\eta(\epsilon_0, \tau_e, \beta, L, v_+, v_-)$$

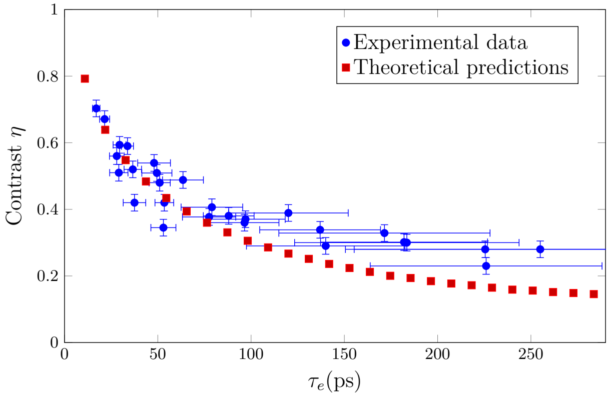
$$\downarrow$$

$$\eta(\epsilon_0, \tau_e, \beta, \tau_s)$$

- All these are given by the experiment ➙ No adjustable parameters!
  Allows for a more careful comparison with experimental results

Comparing with experimental results

- Contrast



Parameters:

$\epsilon_0 = 0.7$ K

$1/(k_B\beta) = 100$ mK

$\tau_s = 70$ ps

$f = 1$ GHz

- Possible sources of decoherence?
  - Differences between emitters
  - Environmental noise
  - Coulomb interaction

Only the last scenario can account for all observed data!

Conclusions

- Our interacting model recovers the main experimental features
  ➙ Detailed quantitative comparison is under way!
- Strong coupling between channels accounts for a sensible loss of contrast of the HOM central dip
- The contrast strongly depends on the energy resolution of the injected wave-packet
- Fast and slow modes interfere and produce, depending on the charge carried by the colliding excitations, smaller asymmetric dips or peaks

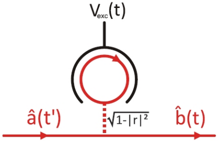
Interactions and charge fractionalization in an electronic HOM interferometer
Claire Wahl, Jérôme Rech, Thibaut Jonckheere, Thierry Martin
Phys. Rev. Lett. 112, 046802 (2014)

Floquet scattering theory

- Stationary scattering matrix $\hat{b}(\epsilon) = S(\epsilon)\hat{a}(\epsilon)$
  relates outgoing fermionic operators $\hat{b}(\epsilon)$ to incoming ones $\hat{a}(\epsilon)$:
- Dynamic scattering matrix $S(\epsilon) \longrightarrow S(\epsilon_1, \epsilon_2)$
  ➡ two energy arguments! The energy of incoming and scattered electron can be different
- Floquet scattering matrix extends this to time-periodic problems, by expressing the absorbtion/emission of a quantized number of energy quanta $\hbar\Omega$ (Ω is the frequency of the periodic potential):

$$\hat{b}(\epsilon) = \sum_m U_m(\epsilon)\hat{a}(\epsilon_m)$$

$$U_m(\epsilon) = \sum_n c_n c^*_{n+m} S(\epsilon_{-n})$$



with $\epsilon_{\pm m} = \epsilon \pm m\hbar\Omega$, and c_n the Fourier coeff. of the periodic potential

Back to TALK

Tunneling and refermionization

- Tunnel Hamiltonian $H_{\text{tun}} = \Gamma \left[\psi^\dagger_{1,R}(0)\psi_{1,L}(0) + \psi^\dagger_{1,L}(0)\psi_{1,R}(0) \right]$
- Refermionization

$$\Psi_{p\pm}(x) = \frac{U_{p\pm}}{\sqrt{2\pi a}} e^{i\phi_{p\pm}x} \quad \text{where} \quad \begin{aligned} \phi_{A\pm} &= \pm \frac{(\phi_{1,R} - \phi_{1,L}) \pm (\phi_{2,R} - \phi_{2,L})}{2} \\ \phi_{S\pm} &= \pm \frac{(\phi_{1,R} + \phi_{1,L}) \pm (\phi_{2,R} + \phi_{2,L})}{2} \end{aligned}$$

- Full Hamiltonian is now quadratic!

$$H = -i\hbar \sum_{p,\sigma} v_\sigma \int dx \Psi^\dagger_{p\sigma}(x) \partial_x \Psi_{p\sigma}(x) - \Gamma \Psi^\dagger_{A+}(0)\Psi_{A-}(0)$$

- Scattering matrix $r_0 = \cos\varphi$ and $t_0 = \sin\varphi$ with $\varphi = -\Gamma/(\hbar\sqrt{v_+ v_-})$

$$\begin{pmatrix} \Psi_{A+} \\ \Psi_{A-} \end{pmatrix}^{\text{outgoing}} = \begin{pmatrix} t_0 & -ir_0 \\ -ir_0 & t_0 \end{pmatrix} \begin{pmatrix} \Psi_{A+} \\ \Psi_{A-} \end{pmatrix}^{\text{incoming}}$$

- Outgoing current?

  ➡ exact same expression up to defining $r_0 = \sqrt{\mathcal{R}}$ and $t_0 = \sqrt{\mathcal{T}}$

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