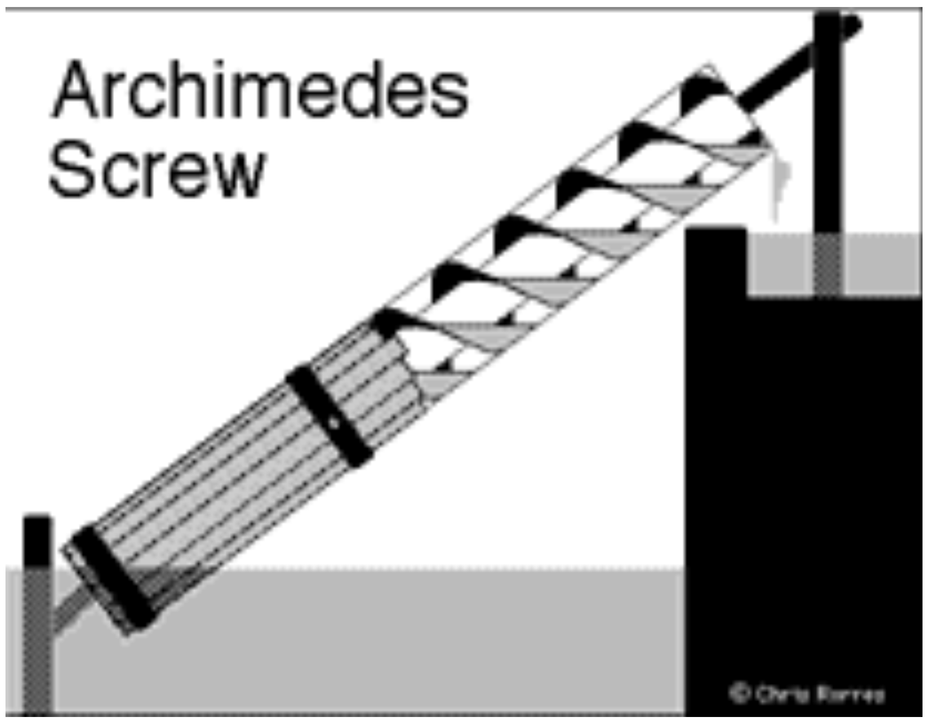
NPSMP seminar, 2015.6.3

Quantum pumping in mesoscopic systems

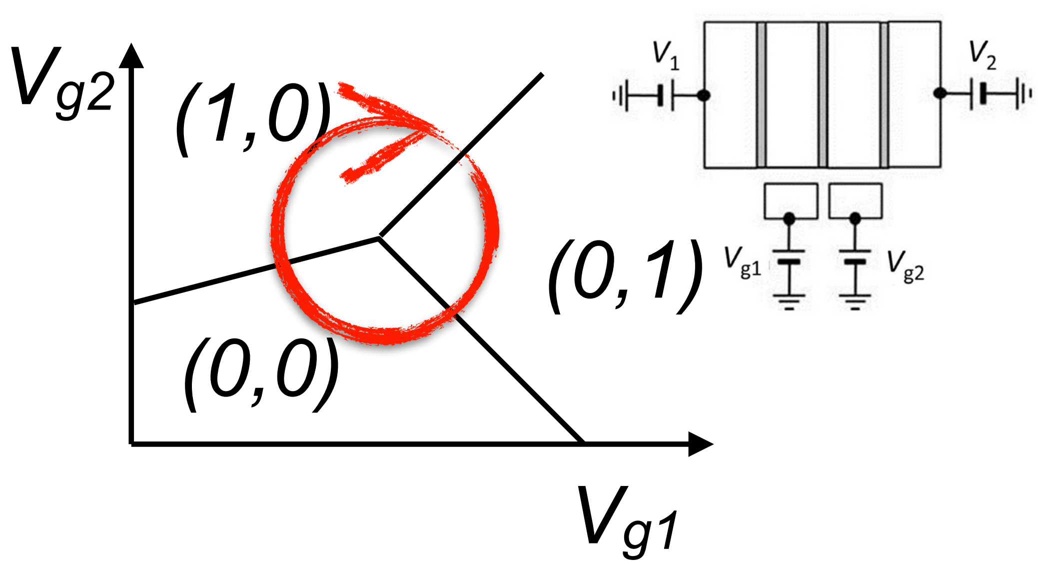
Yasuhiro Tokura

University of Tsukuba
NTT Basic Research Laboratories
Tokyo Institute of Technology

Classical pumps



http://physics.technion.ac.il/~qpump/



<u>Metallic system</u>

Single-electron pump: H. Pothier, *et al.*, EPL 17, 249 (1992).

N-I-S-I-N turnstile: J. P. Pekola, *et al.*, Nature Phys. 4, 120 (2008).

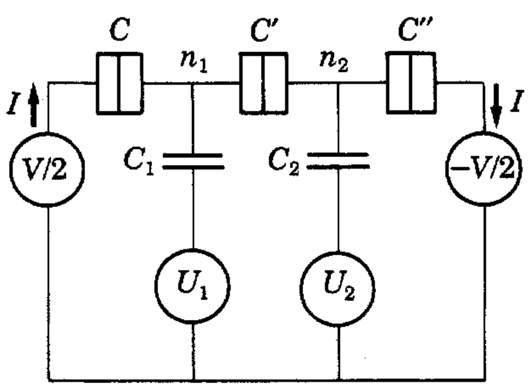
<u>Semiconductor system</u>

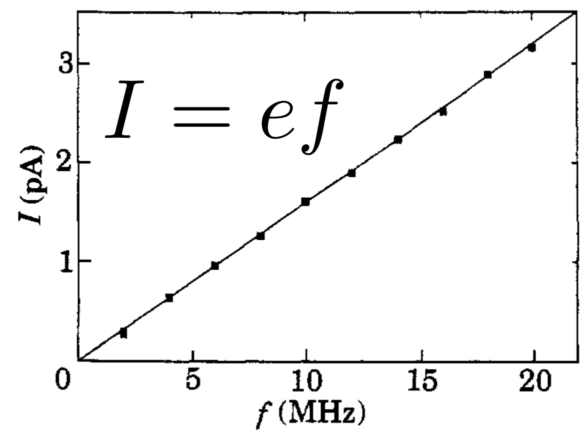
Surface acoustic wave: J. M. Shilton, *et al.*, J. Phys.: Condens. Matter 8, L531 (1996).

Tunable barrier tunneling: L. P. Kouwenhoven, *et al.*, Phys. Rev. Lett. 67, 1626 (1991).

Various pumps

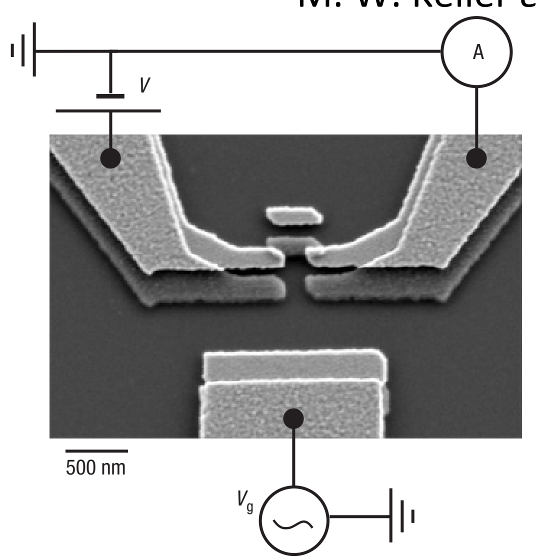
Two-parameter pump

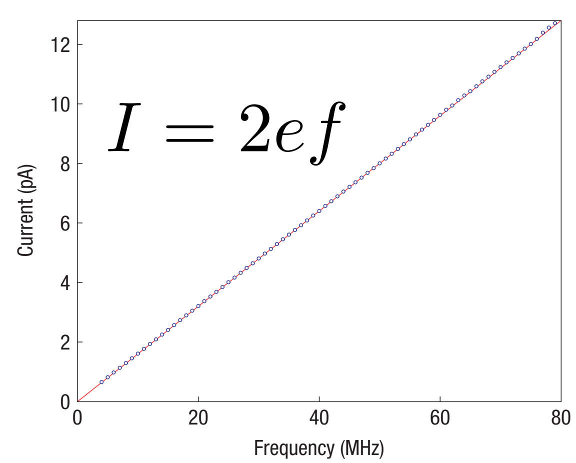




H. Pothier *et al.*, EPL **17**, (3) 249 (1992)

M. W. Keller *et al.*, APL **69**, 1804 (1996)

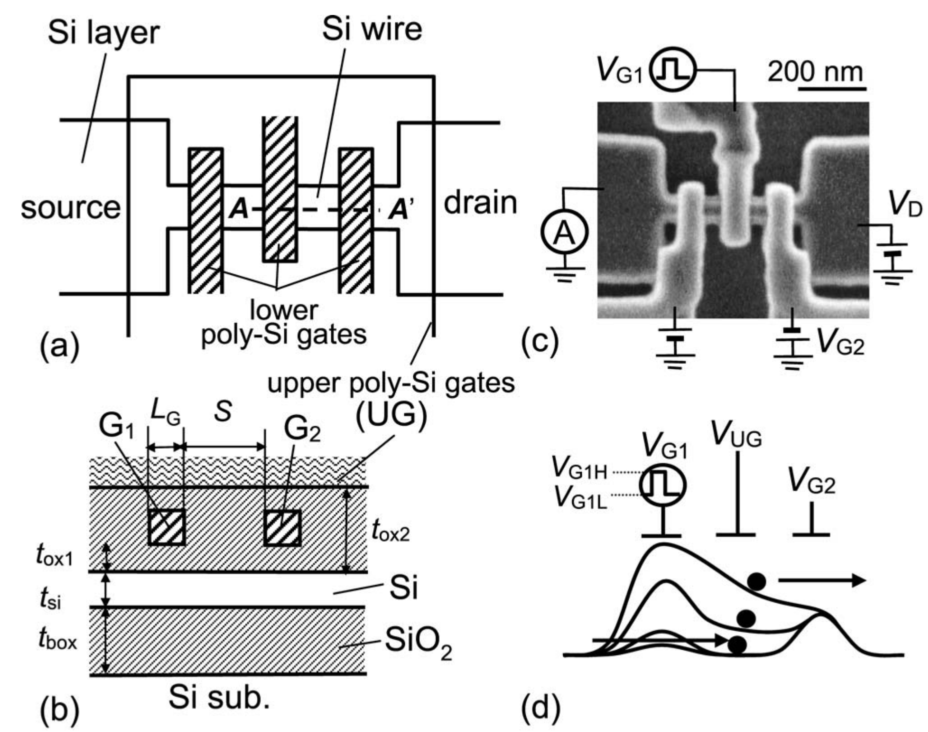




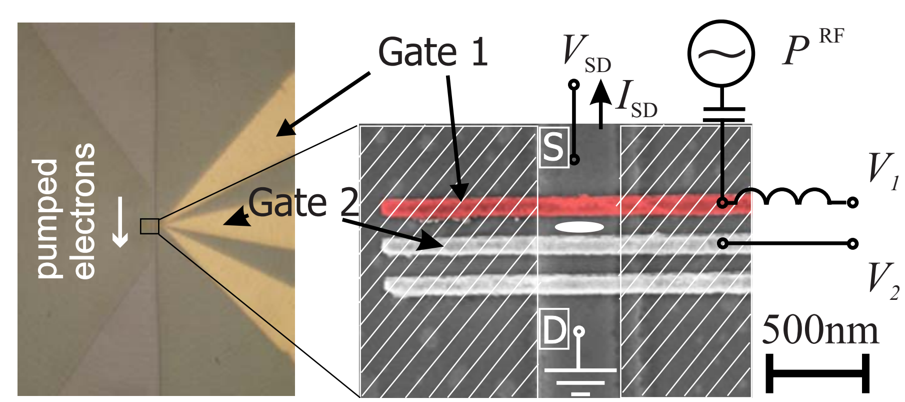
M. D. Blumenthal *et al.*, nature physics **3**, 343 (2007)

J. P. Pekola *et al.*, nature physics **4**, 120 (2008)

One-parameter pump



A. Fujiwara *et al.*, APL **92**, 042102 (2008)



B. Kaestner *et al.*, PRB **77**, 153301 (2008)

Plan of the talk

- Quantum adiabatic pumps, Brouwer's formula
- Series quantum dots, triple quantum dot ring
- Full counting statistics with quantum master equation
- Non-adiabatic pump
- Conclusions

Quantum adiabatic pump

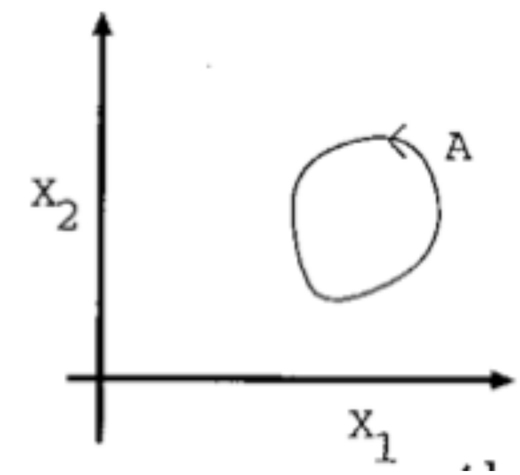
Electrons move coherently in a mesoscopic system.

Effect of quantum mechanical phase coherence in the pumping processes?

Quantum adiabatic pump is an electron transport at (net) zero bias voltage by changing various system dynamic parameters **adiabatically, periodically** in time.

Brouwer's formula

We assume a set of dynamical parameters $X_\nu(t)$ which change slowly and periodically with period $T_0 = \frac{2\pi}{\omega}$



Pumped charge per cycle (at T=0)

$$Q_\alpha = T_0 I_\alpha$$
$$= -\frac{e}{2\pi}\int_0^{T_0} dt\, \Im\{\frac{\partial S_0(E_F,t)}{\partial t} S_0^\dagger(E_F,t)\}_{\alpha\alpha}$$

Instantaneous scattering matrix

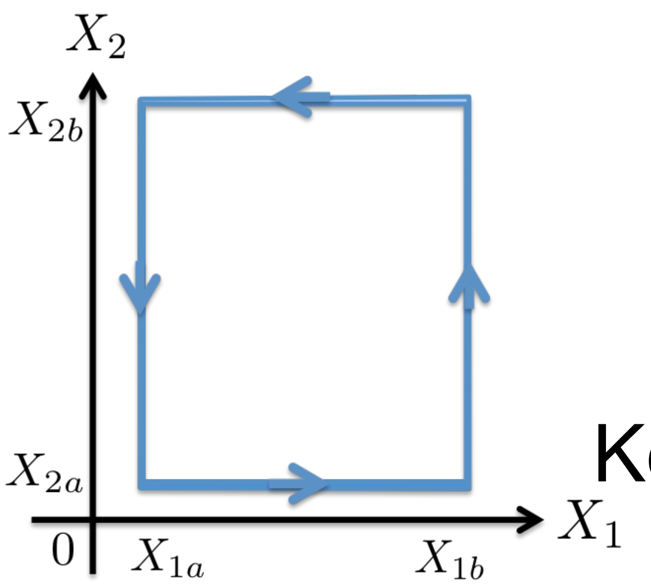
$$S_0(E,t) \equiv S_0(E,\{X_\nu(t)\})$$

For two control parameters, with Stoke's theorem, we have

$$Q_\alpha = -\frac{e}{\pi}\int_A dX_1 dX_2 \Im\{\frac{\partial S_0^\dagger(E_F,X_1,X_2)}{\partial X_1}\frac{\partial S_0(E_F,X_1,X_2)}{\partial X_2}\}_{\alpha\alpha}$$

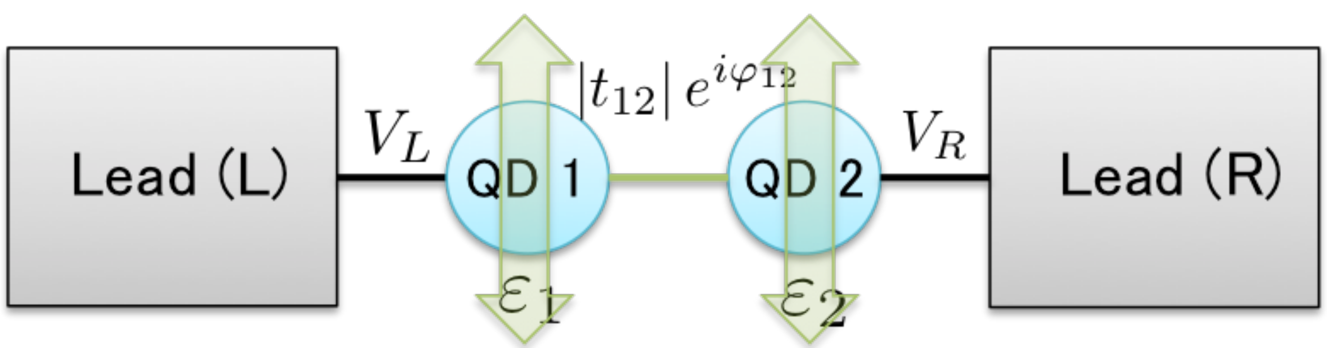
M. Buttiker *et al.*, Z.Phys. B **94**, 133 (1994).
P. W. Brouwer, Phys. Rev. B **58**, R10135 (1998).

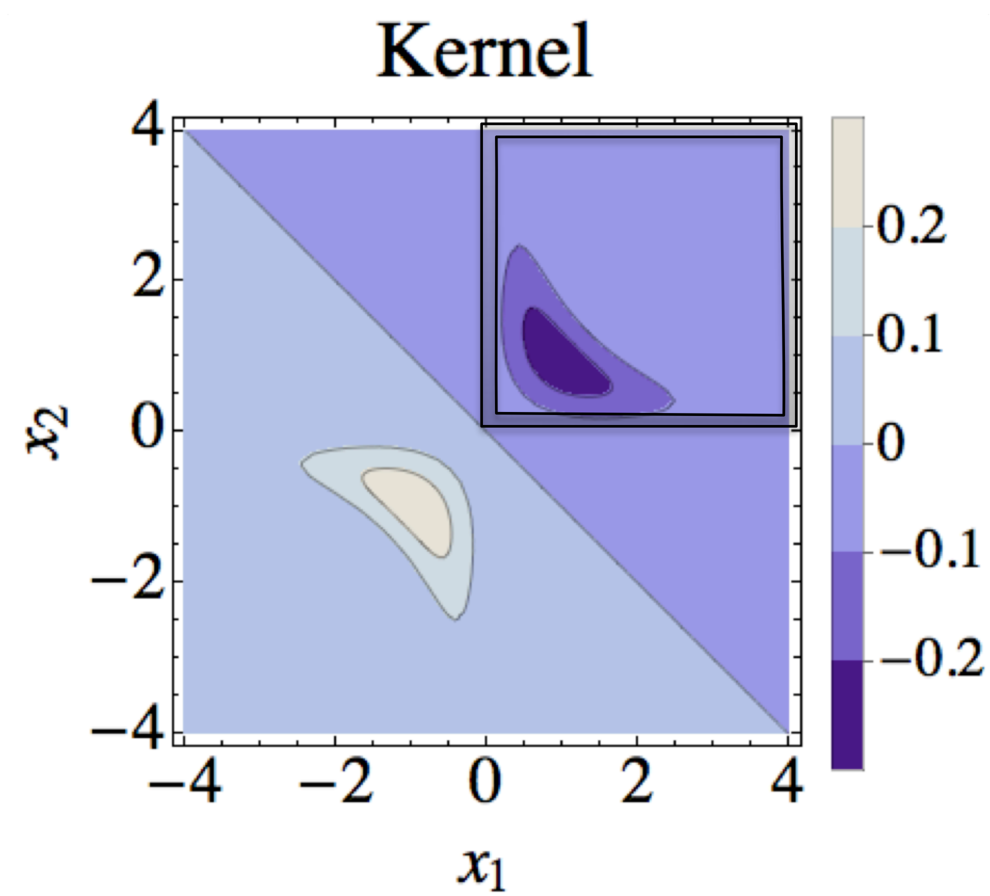
Simple example: series two QDs



$$\frac{Q}{-e} \equiv q = -\int_{X_{1a}}^{X_{1b}} \int_{X_{2a}}^{X_{2b}} dX_1 dX_2 \; \Pi\left(X_1, X_2\right)$$

Kernel: $\Pi(X_1, X_2) = \frac{1}{\pi} \mathrm{Im} \left[\frac{\partial S_{LL}^*}{\partial X_1} \frac{\partial S_{LL}}{\partial X_2} + \frac{\partial S_{LR}^*}{\partial X_1} \frac{\partial S_{LR}}{\partial X_2} \right]$





When we choose first quadrant as surface integral region,

$$0 \le x_1, x_2 \le \infty$$

$$x_\nu \equiv (E_F - \varepsilon_\nu)/\Gamma_\nu$$

$q = 1$ for $|t_{12}| \to \infty$

Scattering matrix

General form of the scattering matrix in two terminal system

$$S_0 = e^{i\gamma} \begin{pmatrix} \cos(\theta) e^{i\alpha} & i \sin(\theta) e^{-i\phi} \\ i \sin(\theta) e^{i\phi} & \cos(\theta) e^{-i\alpha} \end{pmatrix}$$

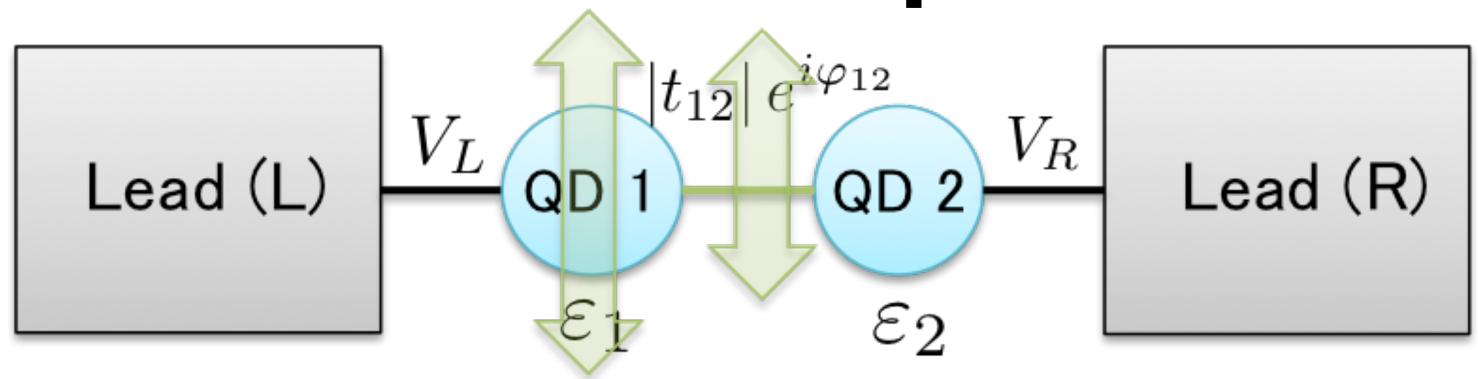
Moving the scatterer $\alpha \to \alpha + 2k_F dL$

Applying a vector potential $\phi \to \phi - \frac{e}{\hbar} \int dx A(x)$

Pump by modulating phase ϕ ?

J. E. Avron *et al.*, Phys. Rev. B 62, R10618 (2000).

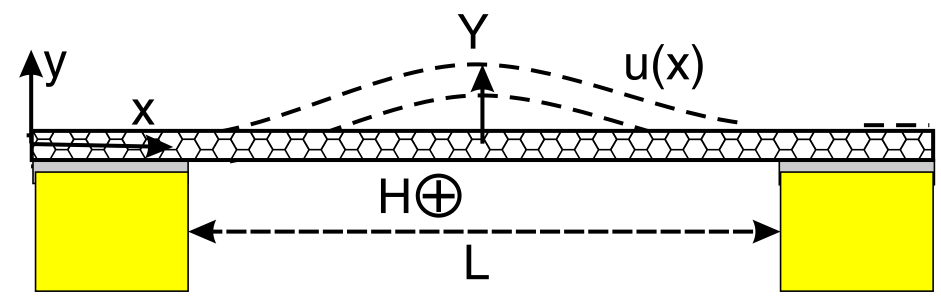
Time-dependent tunnel phase



$$X_1 = \varepsilon_1, X_2 = \varphi_{12}$$

Kernel is independent of the phase

$$\Pi(\varepsilon_1, \varphi_{12}) \equiv \Pi(\varepsilon)$$



Suspended carbon nanotube

No upper limit

$$q \propto (\varphi_{12b} - \varphi_{12a}) \equiv \delta\varphi$$

Actually, $$q = -\frac{1}{2\pi}\delta\varphi(T_b - T_a)$$

R. I. Shekhter, *et al.*,
PRL 97, 156801 (2006)

T_a

ε_1

Gauge transformation

Josephson-like relation (gauge transformation)

$$\frac{d\varphi_{12}(t)}{dt} = \frac{e}{\hbar}V$$

Define conductances by Landauer's formula

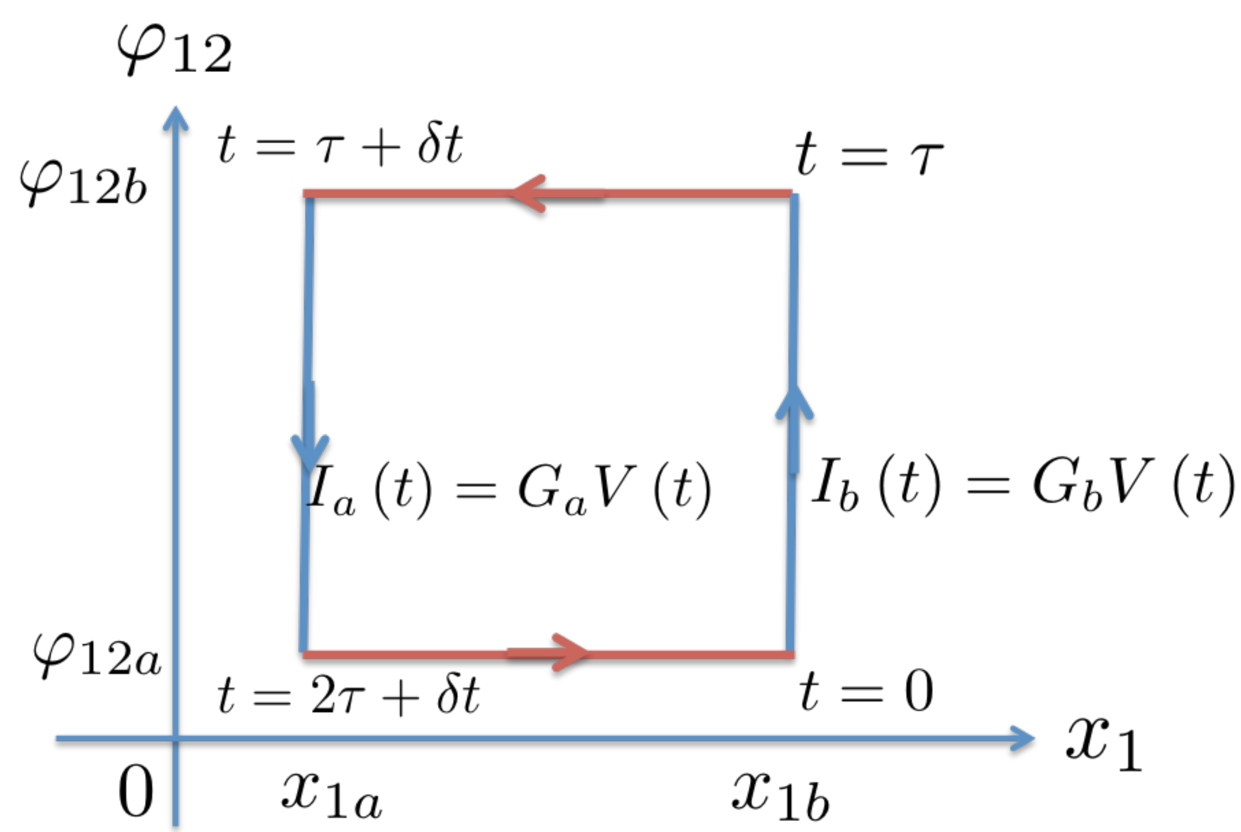
$$G_{a/b} \equiv \frac{e^2}{h}T_{a/b}$$

$$Q = \int_0^\tau dt G_b V(t) + \int_\tau^0 dt G_a V(t)$$

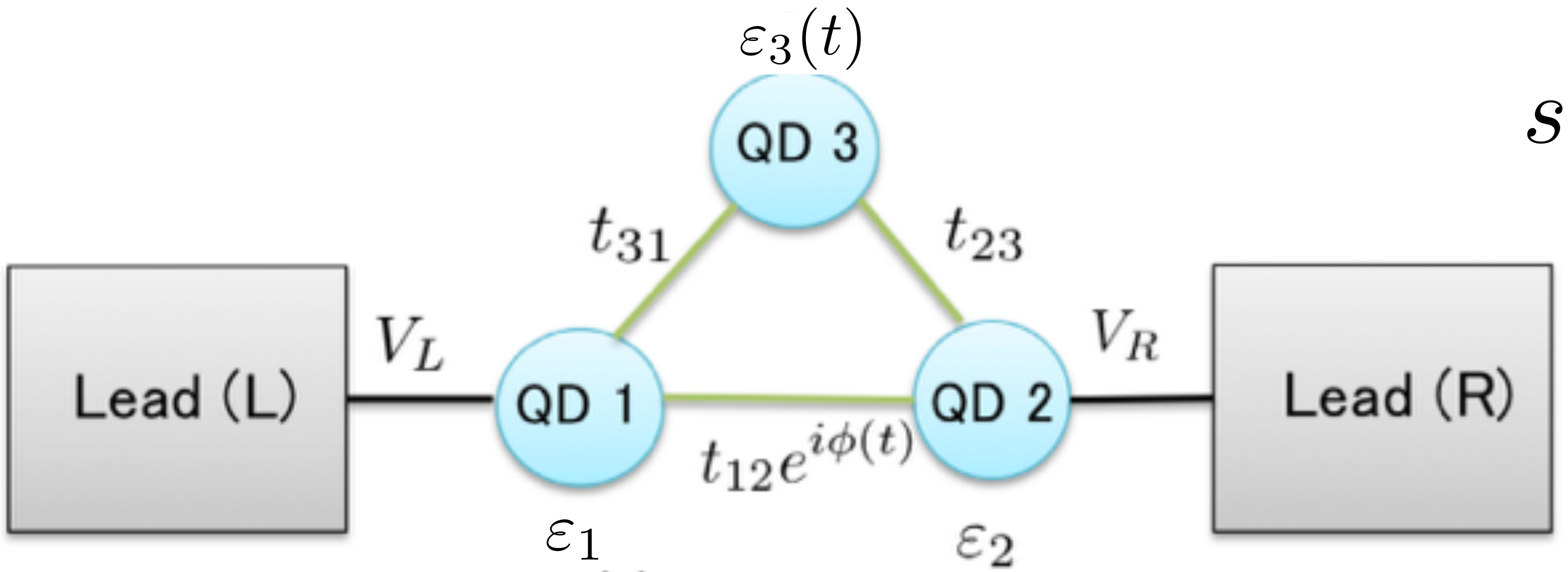
τ : phase modulation time

$$= \int_0^\tau dt\ (I_b(t) - I_a(t))$$

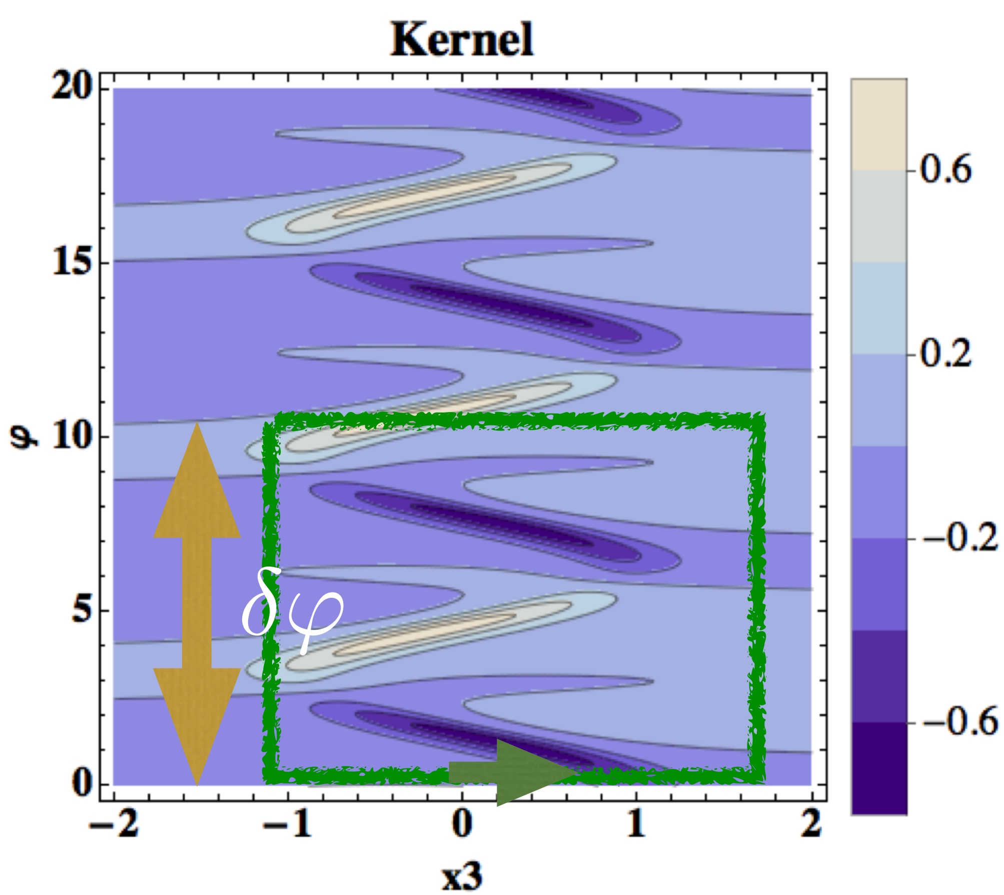
Instantaneous steady current



Three QD ring

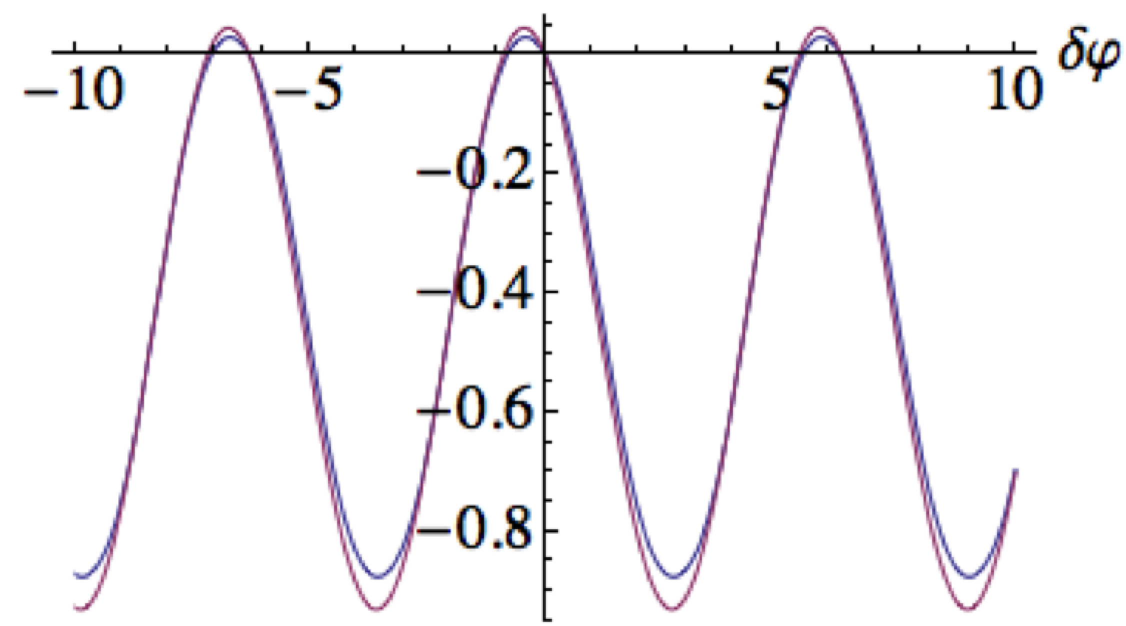


$$s_{ij} \equiv t_{ij}/\sqrt{\Gamma_i\Gamma_j}$$



$$|S_{12}| = |S_{23}| = |S_{31}| = 1 \qquad x_1 = -x_2 = 1$$

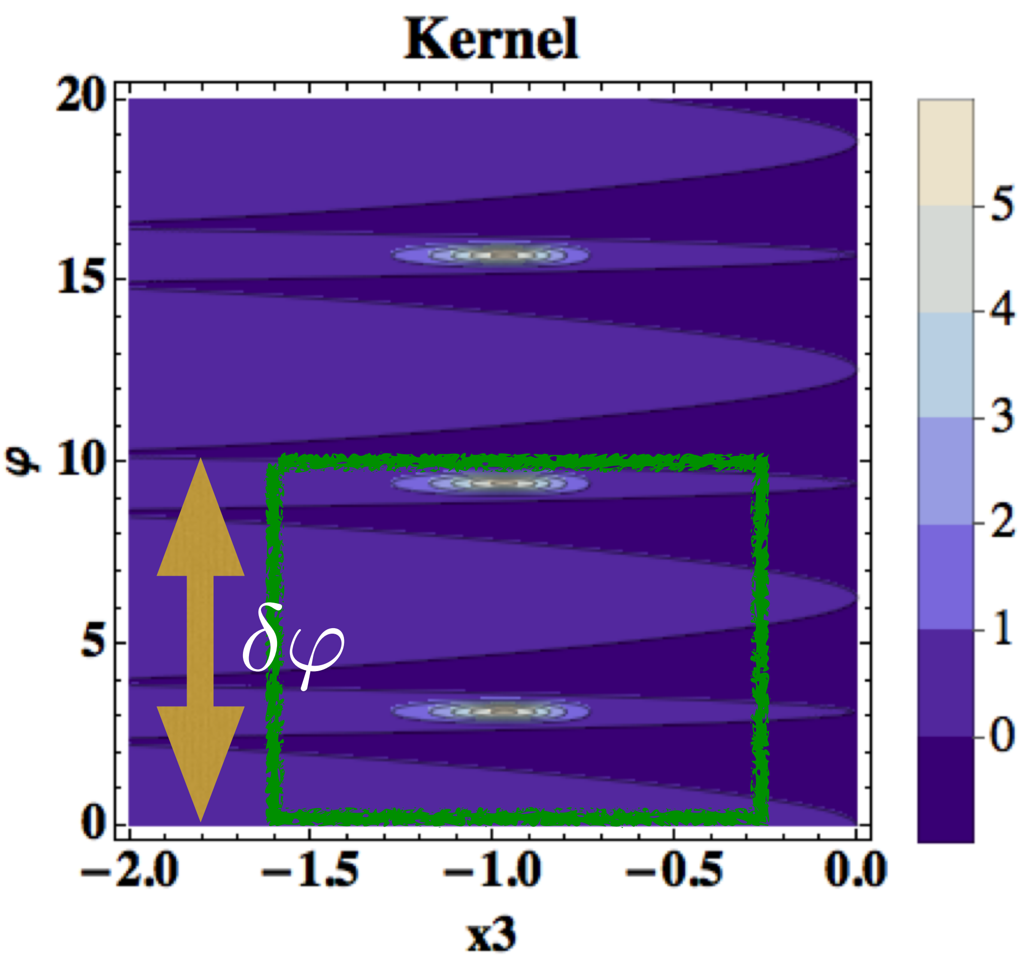
Pumped charge Q/e



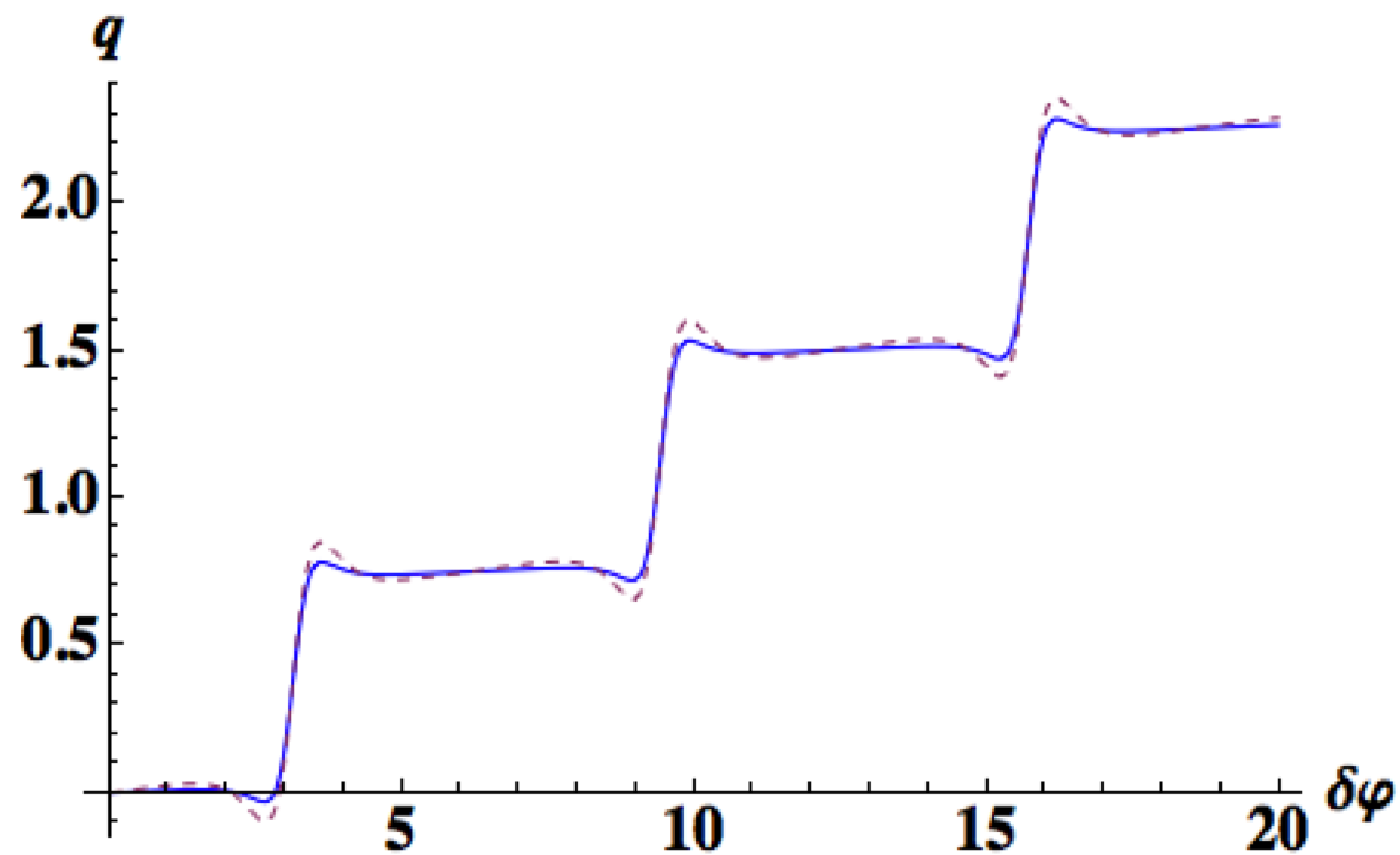
Step-like behavior

When following conditions are satisfied, the kernel shows isolated peaks:

$$|s_{12}| = |s_{23}| = |s_{31}| = x_1 = x_2 \equiv X$$



$X = -1$



Pumped charge shows quantized behavior

M. Taguchi, *et al.*, arXiv:1504.00059

The effect of interaction

Brouwer's formula is only applicable to non-interacting system.

Interaction can be treated in

- Green's function approach
  J. Splettstoesser, *et al*., Phys. Rev. Lett. **95**, 246803 (2005).

- The real-time diagrammatic approach
  H. L. Calvo, *et al.,* Phys. Rev. B **86**, 245308 (2012).

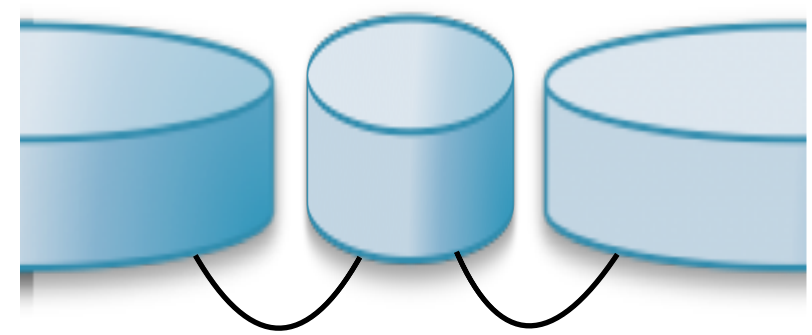
- Full-counting statistics with quantum master equation
  T. Yuge, *et al.,* Phys. Rev. B **86**, 235308 (2012).

These approaches are shown to be equivalent for the second-order treatment of the tunnel-coupling to the leads.

Full counting statistics

Hamiltonian with treating tunnel-couplings as a perturbation

$$H(t) = H_0(t) + H_1(t)$$



Total system density matrix obeys

$$\frac{d}{dt}\rho(t) = -i[H(t), \rho(t)]$$

Projection measurements of an observable *O* in leads at *t=0* and *τ*:

two outputs, o*(τ)* and o*(0)* and these difference *Δo=o(τ)-o(0)*.

Density matrix

Generating function of the probability density function $P_\tau(\Delta o)$

$$Z_\tau(\chi) \equiv \int d\Delta o P_\tau(\Delta o) e^{i\chi\Delta o} = \mathrm{Tr}_{\mathrm{tot}}[\underline{\rho(\chi, t=\tau)}]$$

χ :counting field of *O*

The density matrix modified by full-counting statistics evolves with

$$\frac{d}{dt}\rho(\chi,t) = -i[H_\chi(t)\rho(\chi,t) - \rho(\chi,t)H_{-\chi}(t)]$$

where

$$H_\chi(t) \equiv e^{i\chi O/2} H(t) e^{-i\chi O/2}$$

M. Esposito, *et al.,* Rev. Mod. Phys. 81, 1665 (2009).

Full counting statistics quantum master equation (FCS-QME)

Reduced density matrix: $\rho_S(\chi, t) \equiv \mathrm{Tr}_B \rho(\chi, t)$

Master equation by Born-Markov approximation (in Liuville space)

$$\frac{d}{dt}|\rho_S(\chi, t)\rangle\rangle = \hat{K}(\chi, \alpha_t)|\rho_S(\chi, t)\rangle\rangle$$

α_t :a set of control parameters at time *t*

$\hat{K}(\chi, \alpha_t)$:Liouvillian modified by χ

The left- and right- eigenvectors

$$\hat{K}(\chi, \alpha_t)|\rho_n^\chi(\alpha)\rangle\rangle = \lambda_n^\chi(\alpha)|\rho_n^\chi\rangle\rangle$$

$$\langle\langle \ell_n^\chi(\alpha)|\hat{K}(\chi, \alpha_t) = \lambda_n^\chi(\alpha)\langle\langle \ell_n^\chi(\alpha)|$$

Ortho-normalization $\langle\langle \ell_m^\chi(\alpha)|\rho_n^\chi(\alpha)\rangle\rangle = \delta_{mn}$

Statistical average

Formal solution of the GQME:

$$|\rho_S(\chi,\tau)\rangle\rangle = \mathrm{T}\exp[\int_0^t ds\hat{K}(\chi,\alpha_s)]|\rho_S(\chi,0)\rangle\rangle$$

Generating function is now obtained by

$$Z_\tau(\chi) = \mathrm{Tr}_S[\rho_S(\chi,t=\tau)] \equiv \langle\langle 1|\rho_S(\chi,\tau)\rangle\rangle$$

Average of a physical quantity:

$$X^\mu(\alpha) \equiv \left.\frac{\partial X(\chi,\alpha)}{\partial(i\chi_\mu)}\right|_{\chi=0}$$

$$\langle\Delta o_\mu\rangle_t = \frac{\partial}{\partial(i\chi_\mu)} \langle\langle 1|\rho_S(\chi,t)\rangle\rangle|_{\chi=0}$$

$$= \int_0^t du\langle\langle 1|\hat{K}^\mu(\alpha_u)|\rho_S(u)\rangle\rangle \equiv \int_0^t du I_\mu(u)$$

Steady state

Probability should be conserved for $\chi = 0$

$$\mathrm{Tr}\rho_S(t) \equiv \langle\langle 1|\rho_S(t)\rangle\rangle = 1$$

$$0 = \frac{d}{dt}\langle\langle 1|\rho_S(t)\rangle\rangle = \langle\langle 1|\hat{K}(0,\alpha_t)|\rho_S(t)\rangle\rangle$$

$$\Rightarrow \quad \langle\langle 1|\hat{K}(0,\alpha_t) = 0$$

$$\langle\langle \ell_0^0(\alpha)| = \langle\langle 1| \qquad \lambda_0^0(\alpha) = 0$$

Steady state condition: $\hat{K}(\alpha)|\rho_0^0(\alpha)\rangle\rangle = 0$

We define the *n=0* eigenfunction with an eigenvalue of largest real part

$$\{\langle\langle \ell_0^\chi(\alpha), |\rho_0^\chi(\alpha)\rangle\rangle\} \to \{\langle\langle \ell_0^0(\alpha), |\rho_0^0(\alpha)\rangle\rangle\}$$
$$\chi \to 0$$

Pseudo inverse

Quantum master equation is obtained when χ=0:

$$\frac{d}{dt}|\rho_S(t)\rangle\rangle = \hat{K}(\alpha)|\rho_S(t)\rangle\rangle$$

Separate adiabatic and non-adiabatic parts

$$|\rho_S(t)\rangle\rangle = |\rho_0^0(\alpha_t)\rangle\rangle + |\rho^a(t)\rangle\rangle$$

Pseudo inverse operator: $\mathcal{R}(\alpha)\hat{K}(\alpha) = 1 - |\rho_0^0\rangle\rangle\langle\langle 1|$

$$\frac{d}{dt}|\rho_S(t)\rangle\rangle = \hat{K}(\alpha)|\rho^a(t)\rangle\rangle \quad \Longleftrightarrow \quad |\rho^a(t)\rangle\rangle = \mathcal{R}(\alpha)\frac{d}{dt}|\rho_S(t)\rangle\rangle$$

$$\begin{aligned}|\rho^a(t)\rangle\rangle &= \mathcal{R}(\alpha_t)\frac{d}{dt}(|\rho_0^0(\alpha_t)\rangle\rangle + |\rho^a(t)\rangle\rangle \\ &= \sum_{n=1}^{\infty}(\mathcal{R}(\alpha_t)\frac{d}{dt})^n|\rho_0(\alpha_t)\rangle\rangle\end{aligned}$$

Hierarchy of adiabatic currents

$$I_\mu(t) \equiv \langle\langle 1|\hat{K}^\mu(\alpha_t)|\rho_S(t)\rangle\rangle$$

current operator

$$\langle\langle 1|\hat{K}^\mu(\alpha) = \lambda_0^\mu(\alpha)\langle\langle 1| - \langle\langle \ell_0^\mu(\alpha)|\hat{K}(\alpha) \equiv \langle\langle 1|W_\mu(\alpha)$$

By putting adiabatic expansion of the density matrix:

$$|\rho_S(t)\rangle\rangle = \left|\rho_0^0(\alpha_t)\right\rangle\rangle + \sum_{n=1}^{\infty}(\mathcal{R}(\alpha_t)\frac{d}{dt})^n\,|\rho_0(\alpha_t)\rangle\rangle$$

$$I(t) = \langle\langle 1|W_\mu(\alpha)|\rho_S(t)\rangle\rangle = I^{\mathrm{Steady}}(\alpha_t) + \sum_{n=1}^{\infty} I^{a(n)}(t)$$

Adiabatic limit: Instantaneous steady current:

$$I^{\mathrm{Steady}}(\alpha_t) = \lambda_0^\mu(\alpha) = \langle\langle 1|W_\mu(\alpha)|\rho_0(\alpha)\rangle\rangle$$

Berry-Sinitsyn-Nemenman vector

First non-adiabatic correction:

$$I^{a(1)}(t) = \langle\langle 1|W_\mu(\alpha_t)\mathcal{R}(\alpha_t)\frac{d}{dt}|\rho_0(\alpha_t)\rangle\rangle$$
$$= -\langle\langle \ell_0^\mu(\alpha_t)|\frac{d}{dt}|\rho_0(\alpha_t)\rangle\rangle$$

We used the identity:

$$\langle\langle 1|W_\mu(\alpha)\mathcal{R}(\alpha) = -\langle\langle \ell_0^\mu(\alpha)| + C_\mu(\alpha)\langle\langle 1|$$

Changing the integration variable to α:

$$\int_0^\tau dt I^{a(1)}(t) = -\oint_C d\alpha^n \langle\langle \ell_0^\mu(\alpha)|\frac{\partial}{\partial a^n}|\rho_0(\alpha)\rangle\rangle$$

Berry-Sinitsyn-Nemenman (BSN) vector:

$$A_n(\alpha) \equiv -\langle\langle \ell_0^\mu(\alpha)|\frac{\partial}{\partial \alpha^n}|\rho_0(\alpha)\rangle\rangle$$

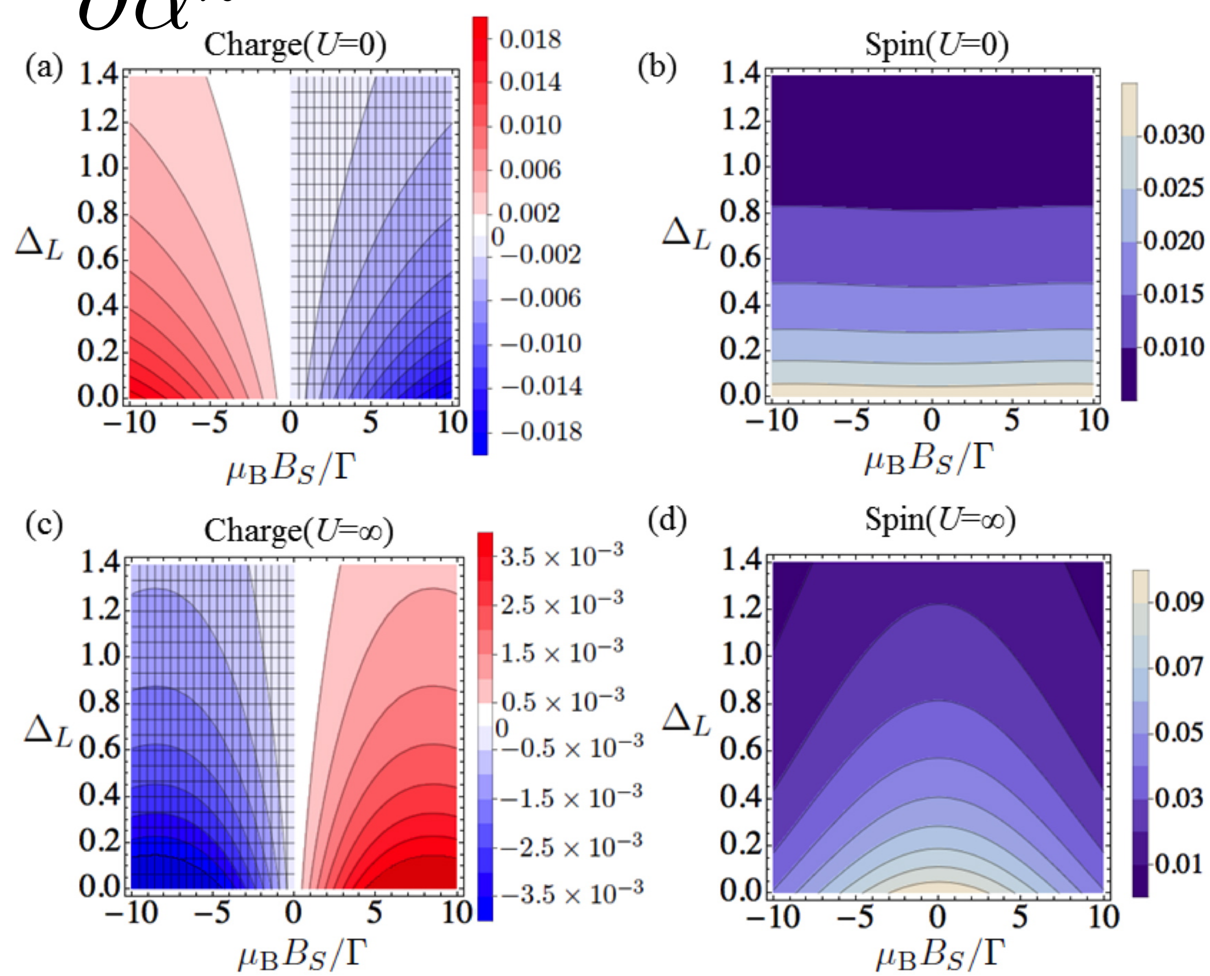
BSN curvature

Average (pumped curve) - geometric contribution

$$\langle \Delta o \rangle_{\tau}^{\mathrm{BSN}} = -\int_S d\alpha^m \wedge d\alpha^n \frac{1}{2} F_{mn}(\alpha)$$

$$F_{mn}(\alpha) \equiv \frac{\partial A_n(\alpha)}{\partial \alpha^m} - \frac{\partial A_m(\alpha)}{\partial \alpha^n}$$

By modulating magnetic field in the reservoir and tunnel coupling strength



pumped charge

$$\langle \Delta (N_{b\uparrow} + N_{b\downarrow}) \rangle$$

pumped spin

$$\langle \Delta (N_{b\uparrow} - N_{b\downarrow}) \rangle$$

Dynamic/thermodynamic

Dynamic parameters (at zero bias)

Tunnel coupling, level energy, etc

$$\int_0^\tau dt I^{\mathrm{Steady}}(\alpha_t) = 0$$

Thermodynamic parameters

Chemical potentials, temperatures of leads

In general,

$$\int_0^\tau dt I^{\mathrm{Steady}}(\alpha_t) \neq 0$$

S. Nakajima *et al.*, arXiv:1501.06181.

Conclusions

- Average physical values, resulting from a slow modulation of the system parameters, are discussed.

- In non-interacting system, Brouwer's formula is useful, which is applied to the system with a time dependent tunneling-phase.

- In quantum and interacting system, we introduced full-counting statistics with quantum master equation and applied it to the charge/spin transport through quantum dots.

Collaborators

- Satoshi Nakajima
- Masahiko Taguchi
- Toshihiro Kubo

arXiv: 1504.00059

arXiv: 1501.06181

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