2015/6/3 ISSP Workshop NPSMP2015

Kondo signature in heat transport via a local two-state system

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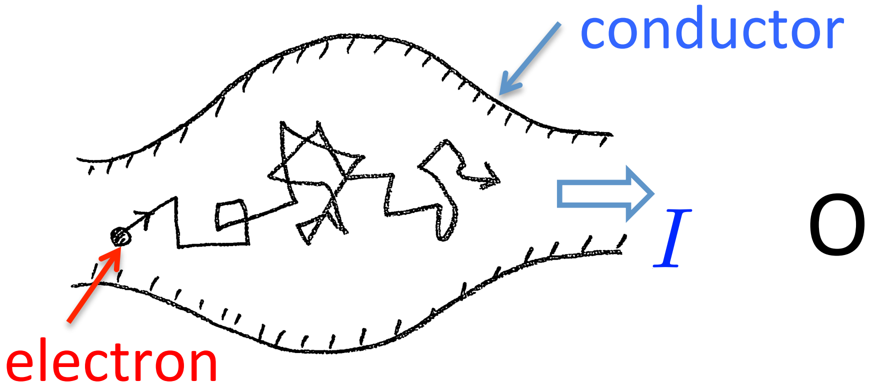
Introduction

Electronic transport

Diffusive transport $L \gg l$

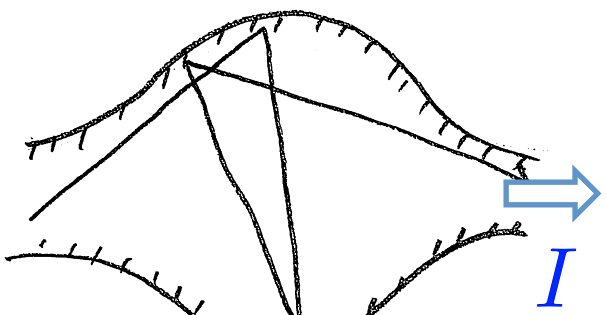
l : mean free path (impurity)

L : sample size



Ohm's law: $I = -\sigma \dfrac{dV}{dx}$

Ballistic transport $L \ll l$ mesoscopic sample

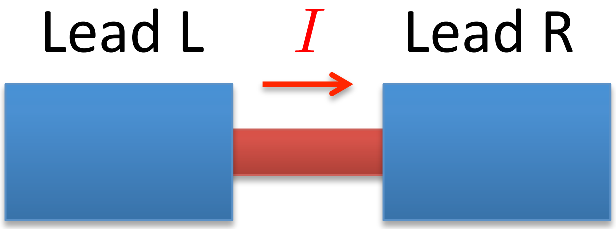


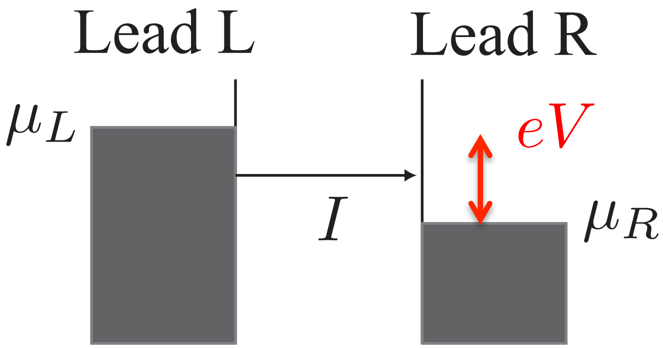
Landauer formula:

$$I = \frac{e}{2\pi} \int_{-\infty}^{\infty} d\omega \mathcal{T}(\omega)(f_L(\omega) - f_R(\omega))$$

$\mathcal{T}(\omega)$: transmission probability

More about Landauer formula



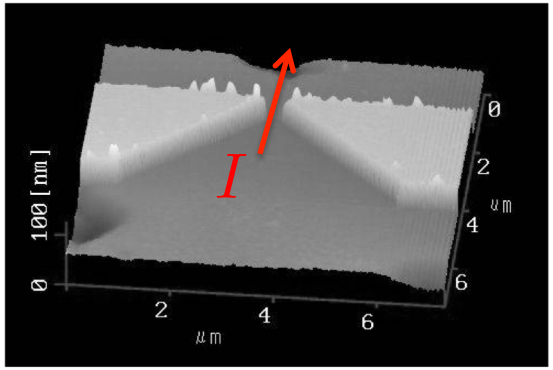


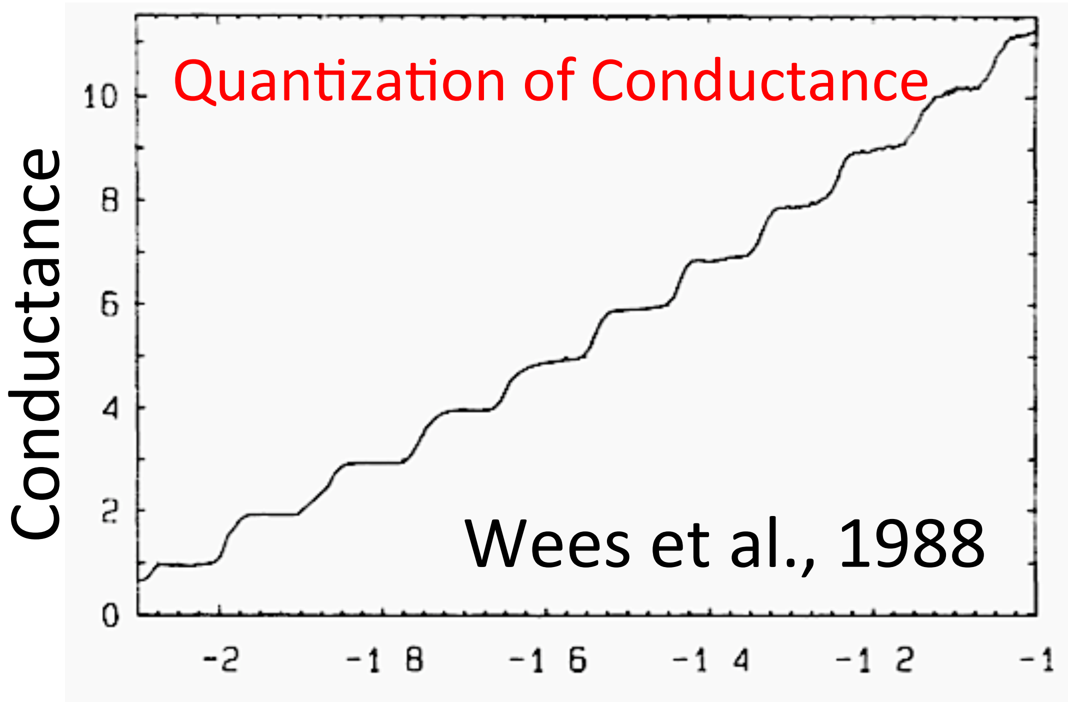
$$I = \frac{e}{2\pi}\int_{-\infty}^{\infty} d\omega \mathcal{T}(f_L - f_R)$$

energy window

$$G \equiv \frac{I}{V} \simeq \frac{2e^2}{h}\mathcal{N}$$

$\mathcal{N}$: number of channels





Electron-heat correspondence

Diffusive transport

Counterpart in heat transport

Ohm's law:

$$I = -\sigma \frac{dV}{dx}$$

→

Fourier's law:

$$I_h = -\kappa \frac{dT}{dx}$$

Ballistic transport

Landauer formula:

$$I = \frac{e}{2\pi} \int_{-\infty}^{\infty} d\omega \mathcal{T} (f_L - f_R)$$

f_L, f_R : Fermi distribution

→

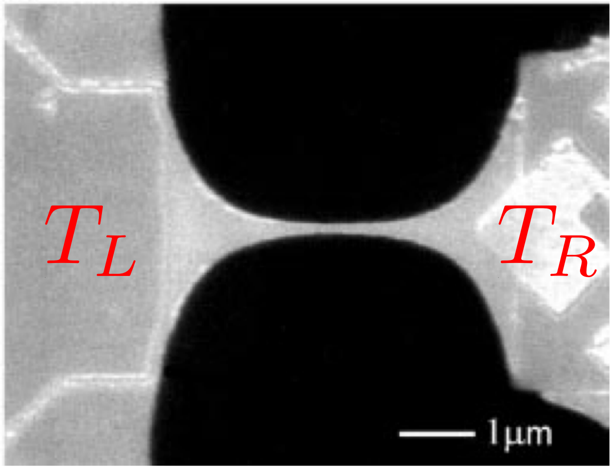
Extended Landauer formula for phonons:

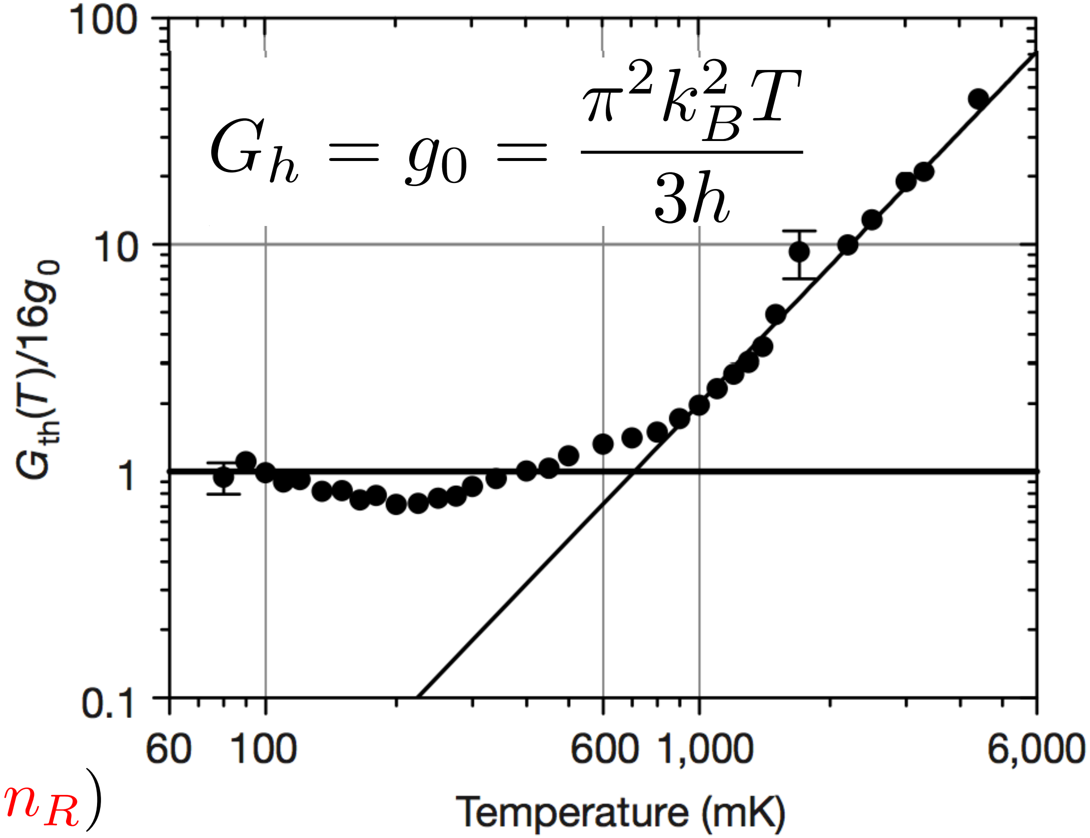
$$I_h = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \hbar\omega \mathcal{T} (n_L - n_R)$$

n_L, n_R : Bose distribution

Quantization of heat transport

Schwab et al., Nature 404, 974 (2000)





$$I_h = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \hbar\omega \mathcal{T}(n_L - n_R)$$

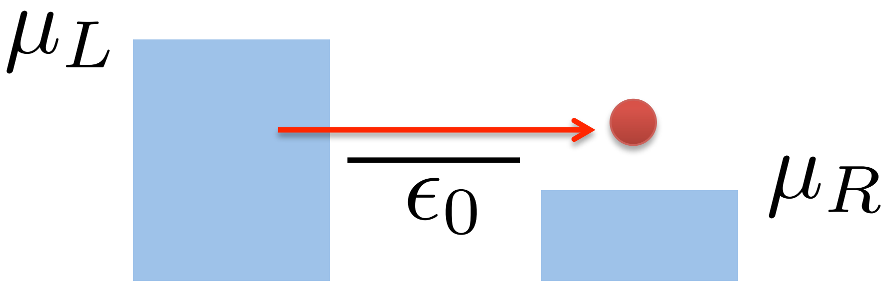
$$\longrightarrow G_h = \frac{I_h}{\Delta T} = \frac{k_B^2}{h}\sum_m \int_{x_m}^{\infty} dx \frac{x^2 e^x}{(e^2-1)^2} T_m(x k_B T/\hbar)$$

Electron conduction v.s. Heat conduction

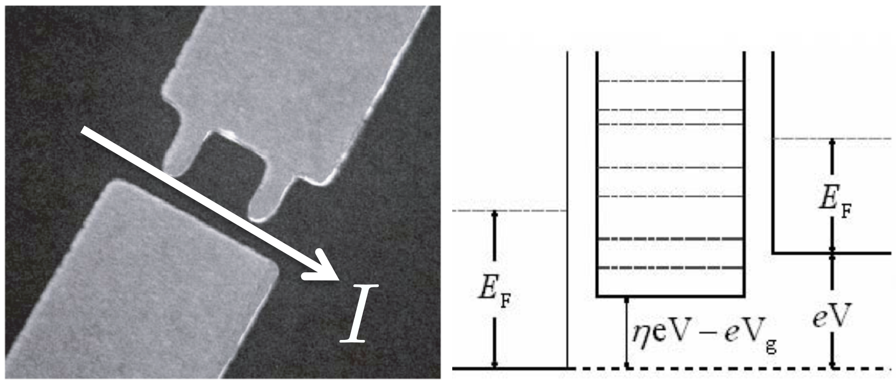
	Electroninc transport	Heat transport
Diffusive transport	Ohm's law	Fourier's law
Ballistic transport	Landauer's formula	Phonon version of Landauer's formula
Controlling device	Diode	Heat diode (Ojanen –Jauho `08)
0-dim object	Quantum dot Kondo effect	???

Zero-dimensional object

Electron transport

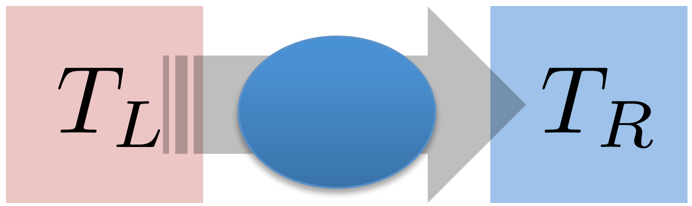


energy level



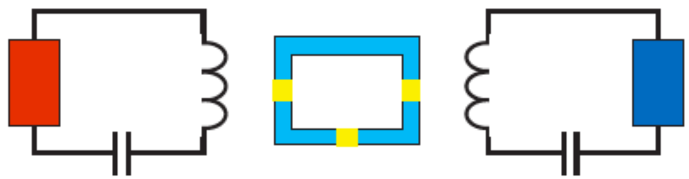
quantum dot

Heat transport



local oscillator

molecular junction

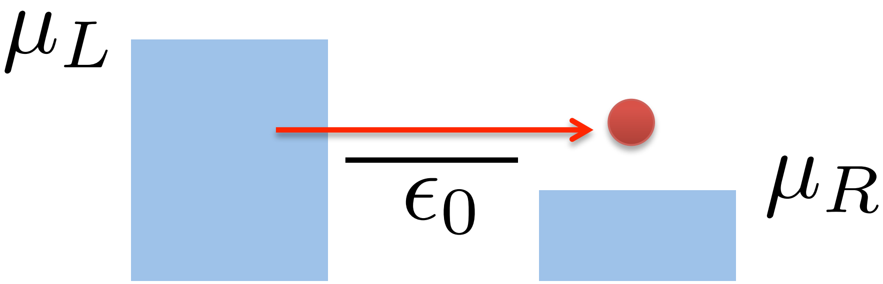


superconducting circuit

Interaction

Segal and Nitzan, PRL 2005

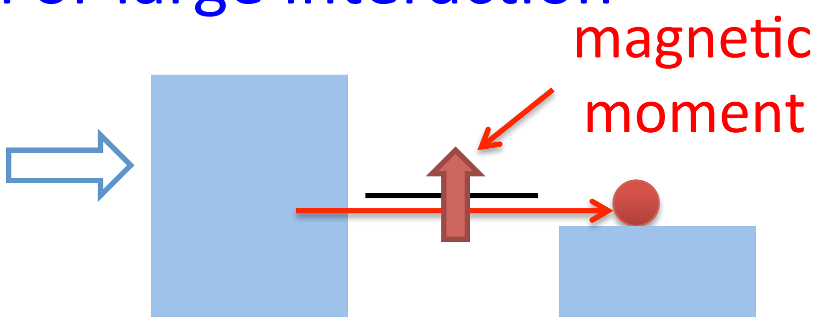
Electron transport



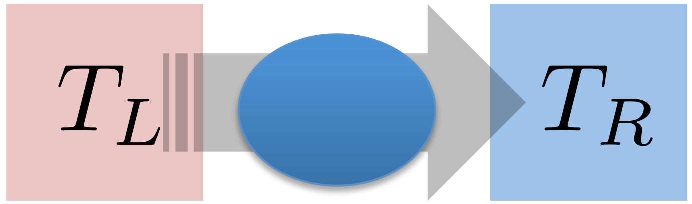
$$H_S = \sum_\sigma \varepsilon_0 d_\sigma d_\sigma + U n_\uparrow n_\downarrow$$

Coulomb interaction

For large interaction



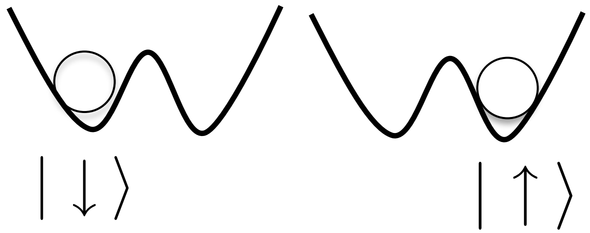
Heat transport



local oscillator

Nonlinearlity induces interaction.

→ two-state system



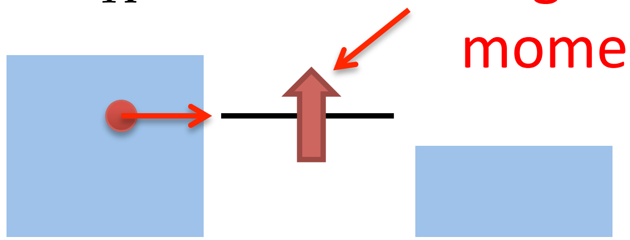
$$H_S = \Delta \sigma_x$$

Kondo effect (electronic transport)

Many-body effect well known in condensed matter physics.

$T \gg T_K$

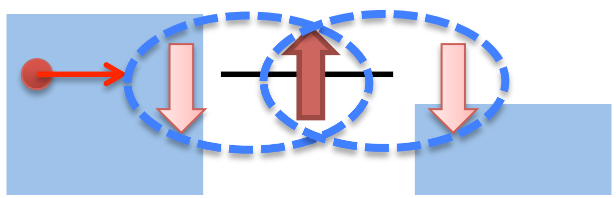
magnetic moment



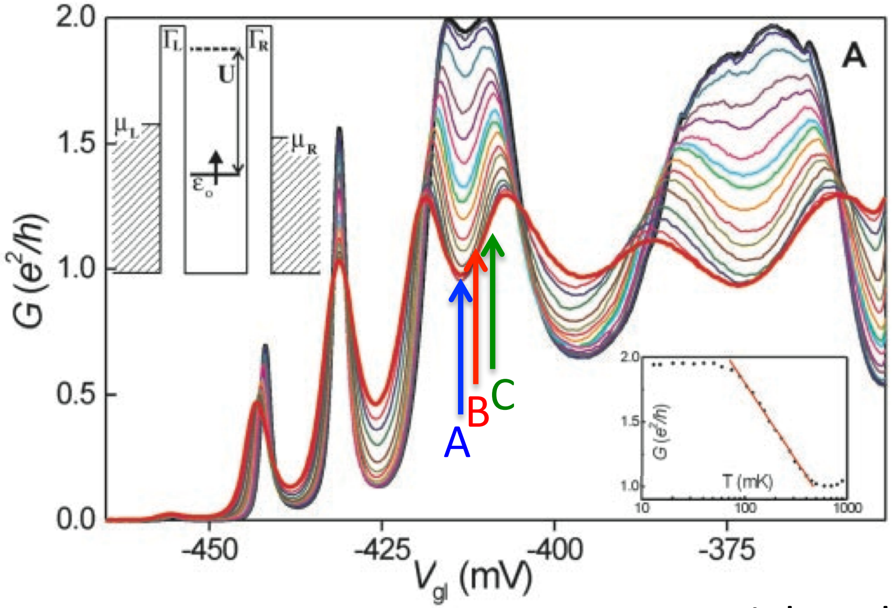
As T decreases,

$T \ll T_K$

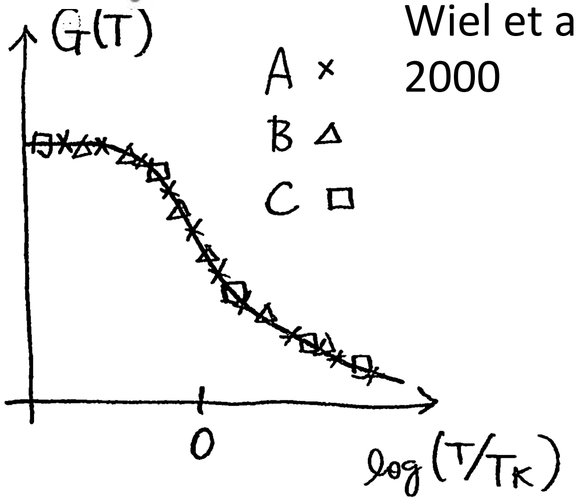
Kondo singlet



disappearance of magnetic moment



Scaling



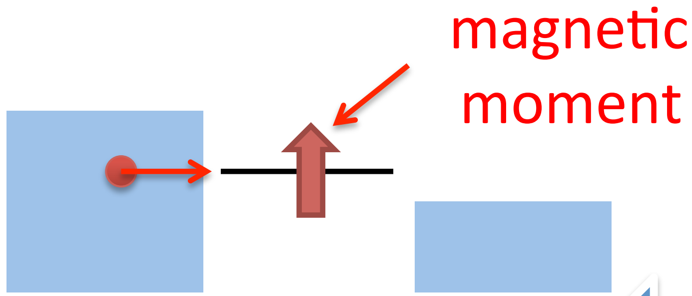
Wiel et al.
2000

Mapping

Legett et al., RMP, 1987

Bosonization technique

Electron transport

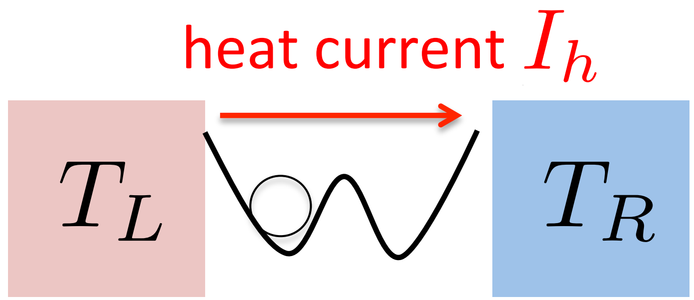


Kondo model

$$H_K = \sum_{k,\sigma} (\varepsilon_k - \mu) c^\dagger_{k\sigma} c_{k\sigma} + J_\perp \sum_{k,k'} (c^\dagger_{k\uparrow} c_{k'\downarrow} S^- + c^\dagger_{k\downarrow} c_{k'\uparrow} S^+) + \frac{J_\parallel}{2} \sum_{k,k'} (c^\dagger_{k\uparrow} c_{k'\uparrow} - c^\dagger_{k,\downarrow} c_{k'\downarrow}) S^z$$

equivalent

Heat transport



Spin-boson model

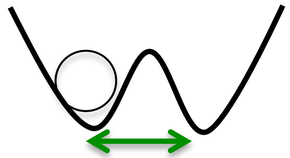
$$H = \frac{\hbar\Delta}{2}\sigma_x + \sum_k \hbar\omega_k b^\dagger_k b_k + \frac{\sigma_z}{2} \sum_k \hbar\lambda_k (b_k + b^\dagger_k)$$

Spectral function:

$$I(\omega) = \sum_k \lambda_k^2 \delta(\omega - \omega_k) = 2\alpha\omega$$

Ohmic damping

Detail of mapping



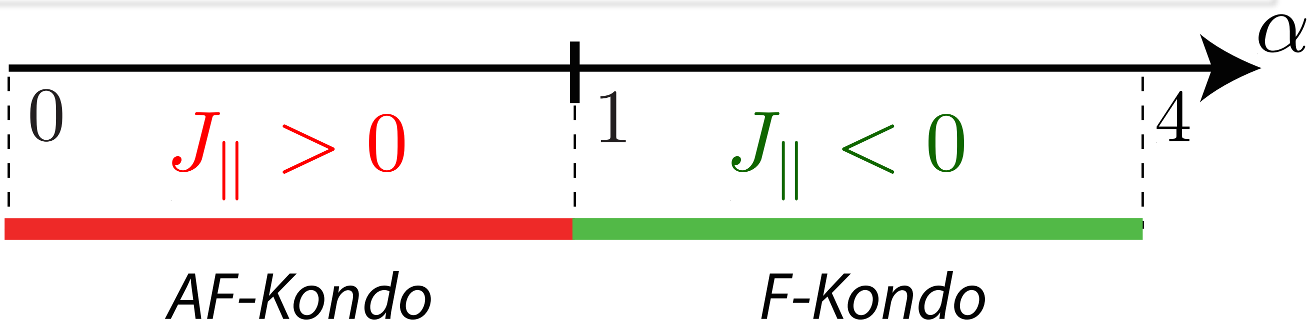
$$H_K = \sum_{k,\sigma} (\varepsilon_k - \mu) c^\dagger_{k\sigma} c_{k\sigma} + J_\perp \sum_{k,k'} (c^\dagger_{k\uparrow} c_{k'\downarrow} S^- + c^\dagger_{k\downarrow} c_{k'\uparrow} S^+) + \frac{J_\parallel}{2} \sum_{k,k'} (c^\dagger_{k\uparrow} c_{k'\uparrow} - c^\dagger_{k,\downarrow} c_{k'\downarrow}) S^z$$

$$H = \frac{\hbar\Delta}{2}\sigma_x + \sum_k \hbar\omega_k b^\dagger_k b_k + \frac{\sigma_z}{2} \sum_k \hbar\lambda_k (b_k + b^\dagger_k)$$

$$I(\omega) = \sum_k \lambda_k^2 \delta(\omega - \omega_k) = 2\alpha\omega$$

dissipation
(coupling to bath)

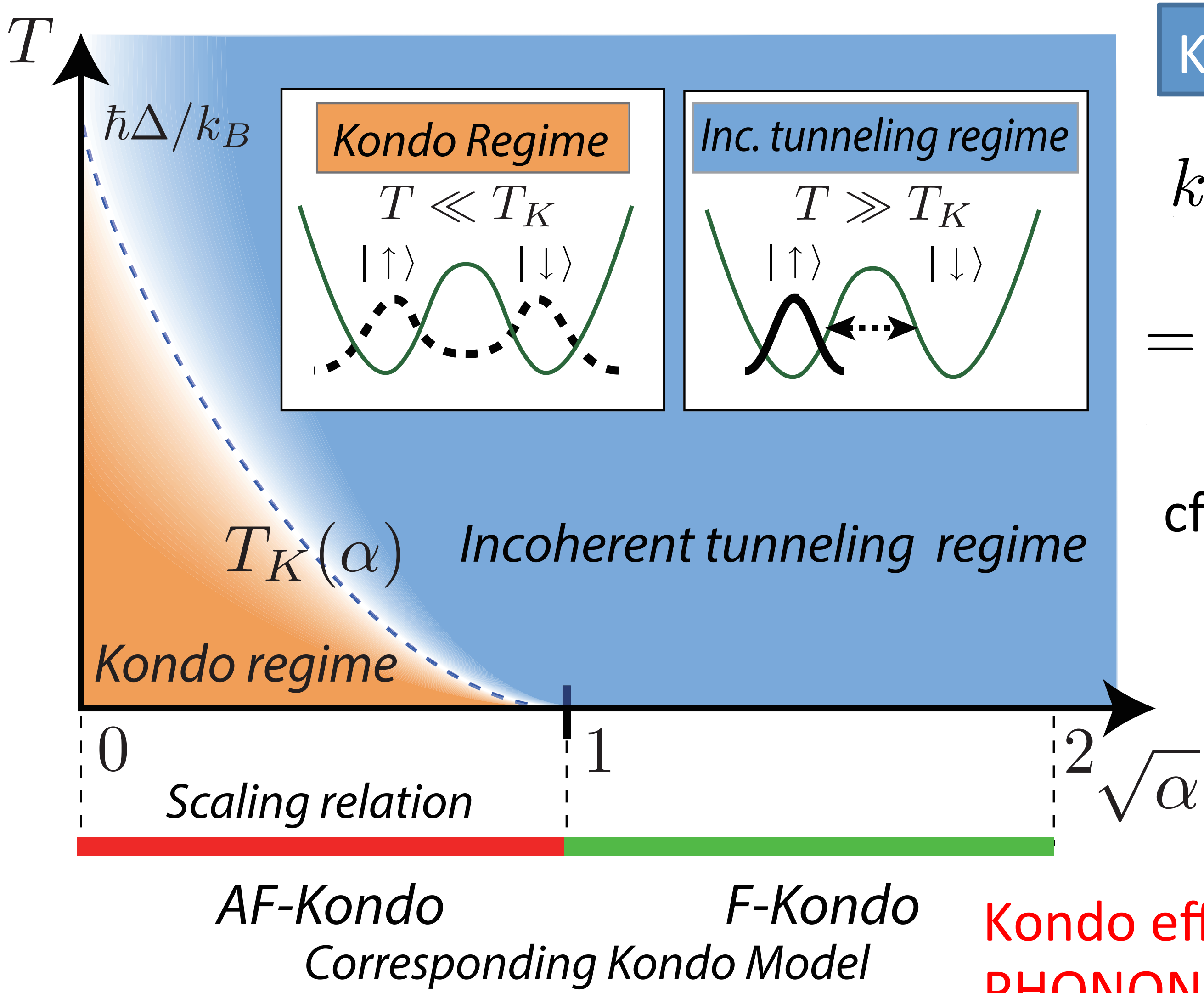
$$\Delta = \rho_0 \omega_c J_\perp \qquad \alpha = \left[1 - \frac{2}{\pi} \arctan(\pi \rho_0 J_\parallel / 4)\right]^2$$



Corresponding Kondo Model

Phase diagram

Leggett et al. RMP 1987



Kondo temerature

$$k_B T_K = \Delta \left(\frac{\Delta}{\omega_c} \right)^{\alpha/(1-\alpha)}$$

cf. poorman's scaling

Kondo effect is expected in PHONON-only systems!

Kondo effect in heat transport

Formulation

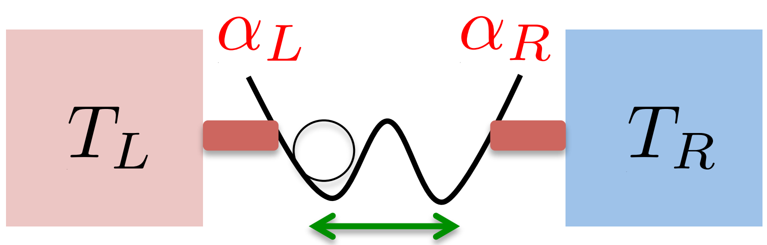
KS, EPL(2008)
Velizhanin, Yhoss, Wang, JCP (2010)
Ojanen and Jauho, PRL (2008)

Hamiltonian

$$H = \frac{\hbar\Delta}{2}\sigma_x + \sum_{\nu k} \hbar\omega_k b^\dagger_{\nu k} b_{\nu k} + \frac{\sigma_z}{2}\sum_{\nu, k} \hbar\lambda_{\nu k}(b_{\nu k} + b^\dagger_{\nu k})$$

$$I_\nu(\omega) = \sum_k \lambda^2_{\nu k}\delta(\omega - \omega_k) = 2\alpha_\nu \omega$$

Coupling strengths



tunneling amplitude Δ

Exaxt formula (Keldysh method)

$$\kappa = \frac{k_B \alpha_L \alpha_R}{2\alpha}\int_0^{\omega_c} d\omega S(\omega)\omega^2 \left[\frac{\beta\omega/2}{\sinh(\beta\omega/2)}\right]^2$$

thermal conductance

$$\kappa \equiv \lim_{\Delta T \to 0} \frac{I_h}{\Delta T}$$

$$S(\omega) = \frac{\mathrm{Im}\chi(\omega)}{\omega}$$

$$\alpha = \alpha_L + \alpha_R$$

$$\chi(t, t') = i\hbar^{-1}\theta(t - t')\langle[\sigma_z(t), \sigma_z(t')]\rangle \,.$$

Monte Carlo method(1)

Leggett et al. RMP 1987
Völker, PRB 1998

$$\chi(t,t') = \frac{i}{\hbar}\theta(t-t')\langle[\sigma_z(t),\sigma_z(t')]\rangle$$: quantity to be calculated

imaginary-time formalism

$$g(\tau) = \langle \sigma_z(t=-i\tau)\sigma_z(0)\rangle$$

$$g(i\omega_n) = \int_0^\beta d\tau g(u) e^{i\omega_n \tau}$$

$$g(i\omega_n \to \omega + i\delta) \simeq \chi(\omega)$$

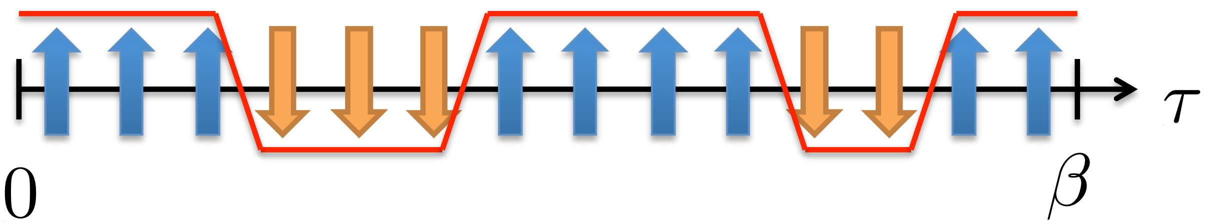
$$\langle \sigma_i \sigma_0 \rangle \simeq \langle \sigma_z(\tau)\sigma_z(0)\rangle$$

Monte Carlo method

$$\beta_I H = \sum_{\langle i,j\rangle} J_{ij}\sigma_i\sigma_j$$

path-integral expression

$\sigma_z(\tau)$



Monte Carlo method(2)

Leggett et al. RMP 1987
Völker, PRB 1998

Partition function

$$Z = \mathrm{Tr} e^{-\beta H} = Z_+ + Z_-$$

$$= \sum_{n=0}^{\infty} \mathrm{Tr}_{\mathrm{boson}} \left\{ \langle + | e^{-\beta H_z} \int_0^{\beta} d\tau_1 \cdots \int_0^{\tau_{2n-1}} d\tau_{2n} \left(\frac{\Delta}{2} \right)^{2n} \tilde{\sigma}_x(\tau_1) \cdots \tilde{\sigma}_x(\tau_{2n}) | + \rangle \right\}$$

$$= Z_0 \sum_{n=0}^{\infty} \left(\frac{\Delta \tau_c}{2} \right)^{2n} \int_0^{\beta} \frac{d\tau_1}{\tau_c} \int_0^{\tau_1 - \tau_c} \frac{d\tau_1}{\tau_c} \cdots \int_0^{\tau_{2n-1} - \tau_c} \frac{d\tau_{2n}}{\tau_c}$$

$$\exp\left\{ 2\alpha \sum (-1)^{i+j} \ln \left| \frac{\beta}{\pi \tau_c} \sin(\pi(\tau_j - \tau_i)/\beta) \right| \right\}$$

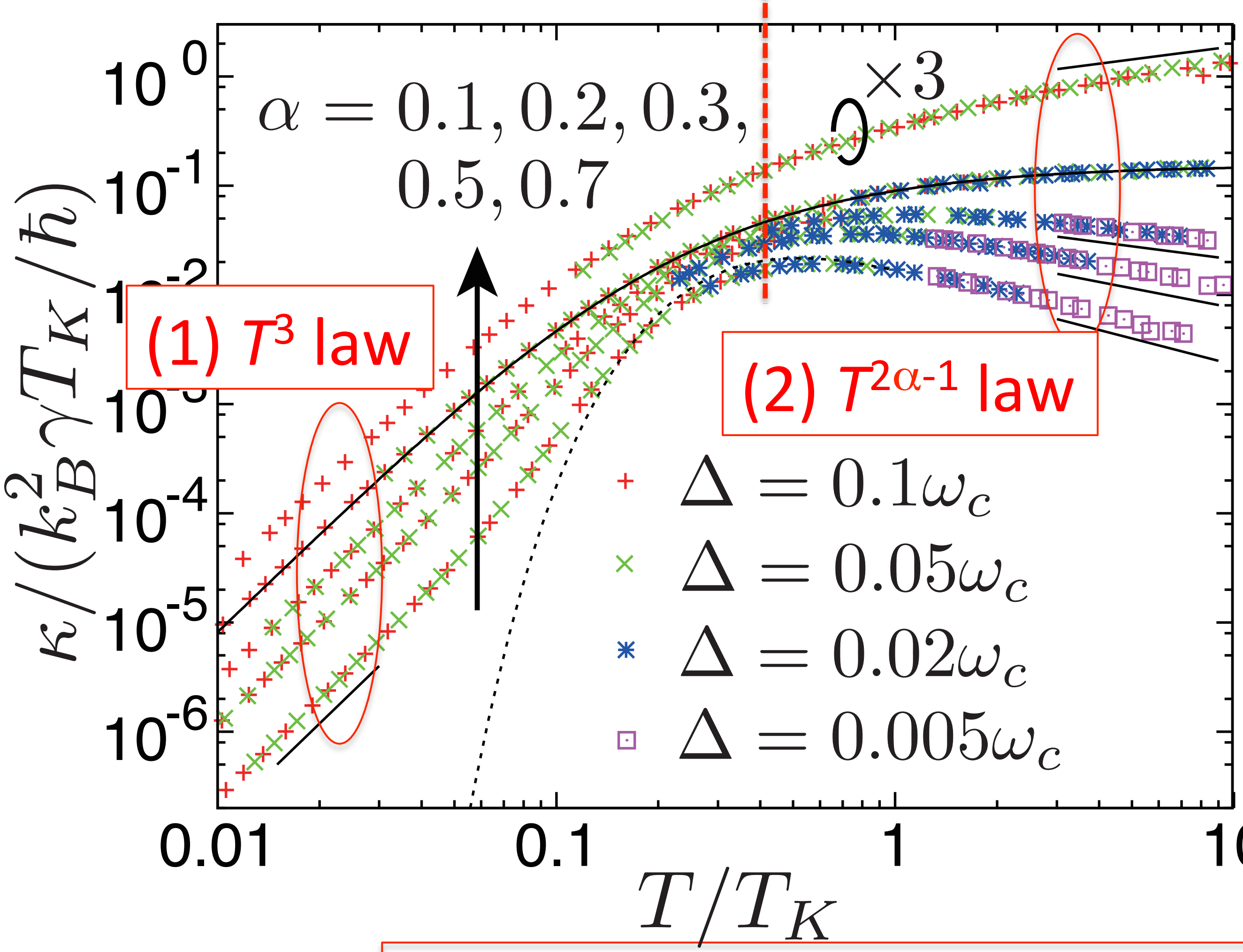
Effective Ising model

$$H_{spin-boson} = -\frac{J_{nn}}{2\beta} \sum_{i=1}^{N} \sigma_i \sigma_{i+1} - \frac{\alpha}{2\beta} \sum_{j<i} \frac{(\pi/N)^2 \sigma_i \sigma_j}{\sin^2 [\pi(j-i)/N]}$$

$$J_{nn} = -\alpha(1+\gamma) - \ln(\Delta \tau_c / 2)$$

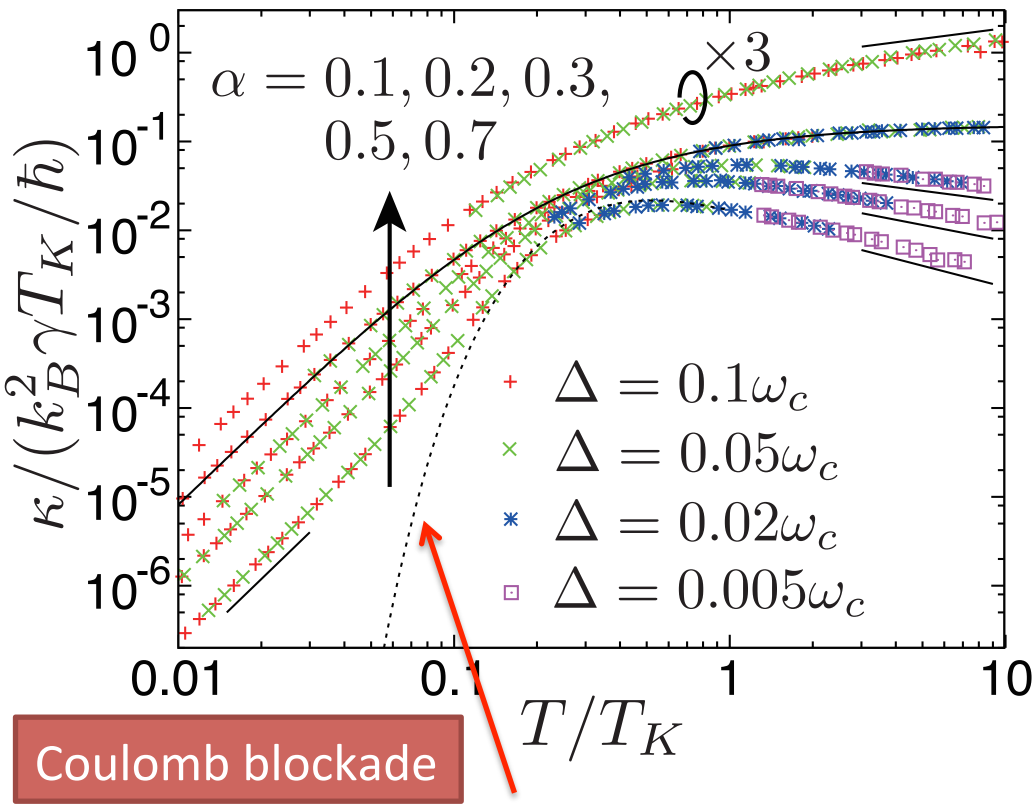
Main result

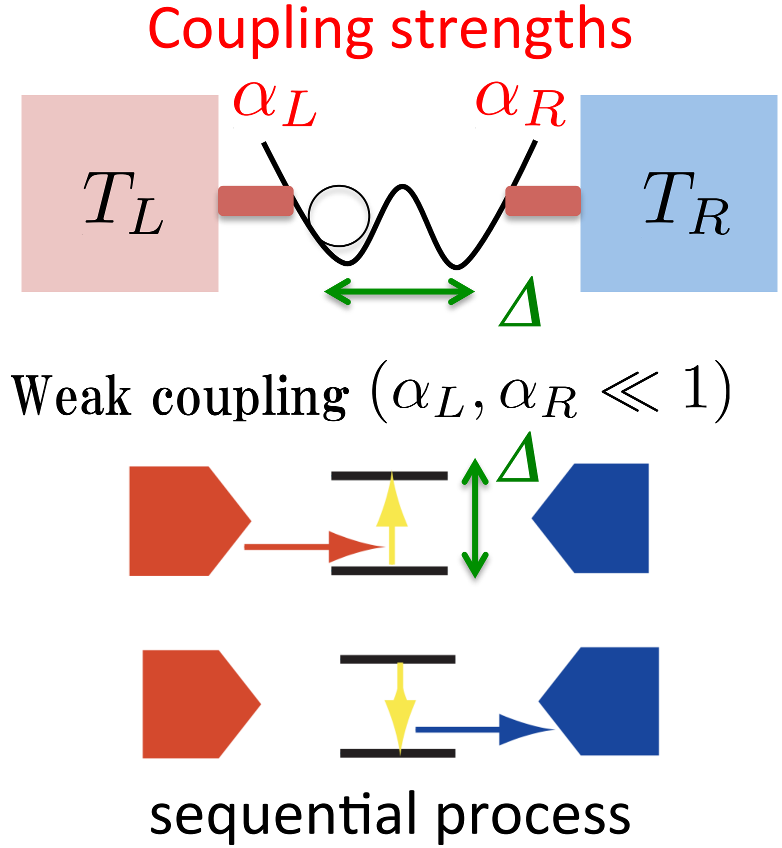
$T \sim T_K$ $\quad k_B T_K = \Delta \left(\frac{\Delta}{\omega_c} \right)^{\alpha/(1-\alpha)}$



(3) scaling form $\quad \kappa(T) \sim (k_B^2 T_K/\hbar)\, f(\alpha, T/T_K)$

Weak-coupling approach (sequential tunneling)





$$\kappa \propto \frac{\Delta}{2n(\Delta)+1}\left[\frac{\beta\hbar\Delta/2}{\sinh(\beta\hbar\Delta/2)}\right]^2 \propto \beta^2 e^{-2\beta\hbar\Delta}$$

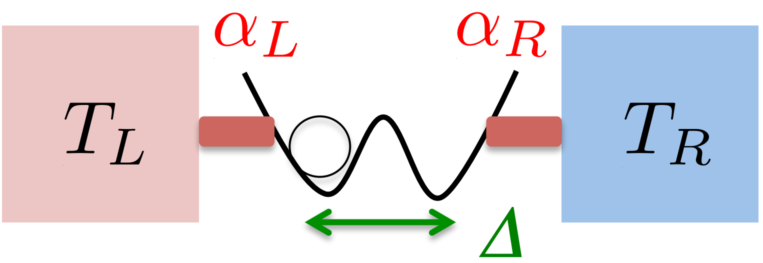
Segal & Nitzan, PRL (2005)

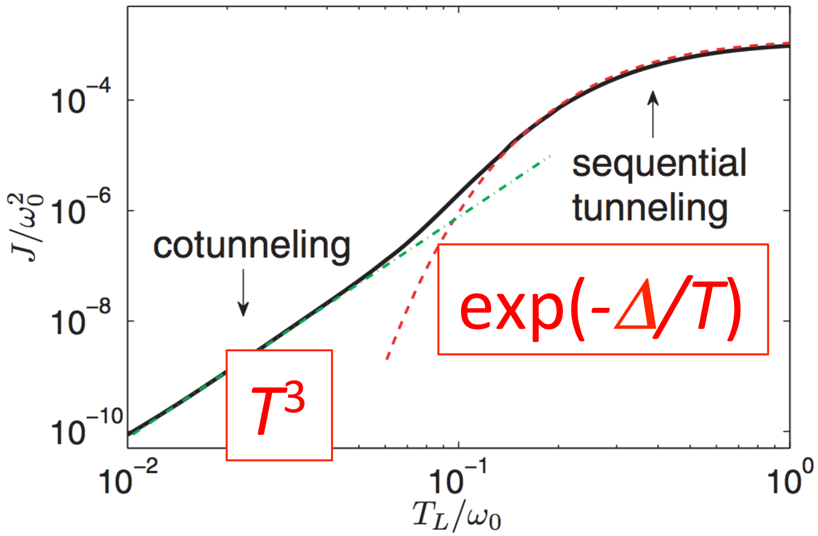
Exponential suppression

Cotunneling

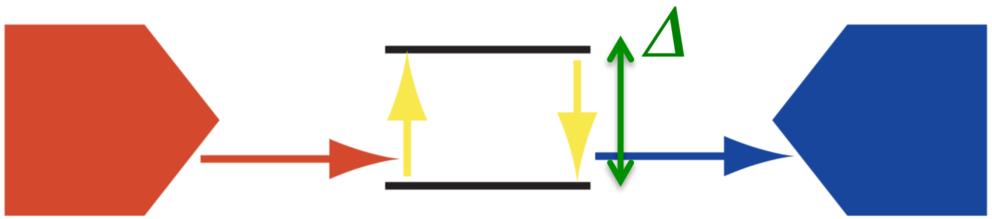
Ruokola and Ojanen, PRB (2011)

Coupling strengths





Weak coupling $(\alpha_L, \alpha_R \ll 1)$
\+ Low temperature $(T \ll \Delta)$



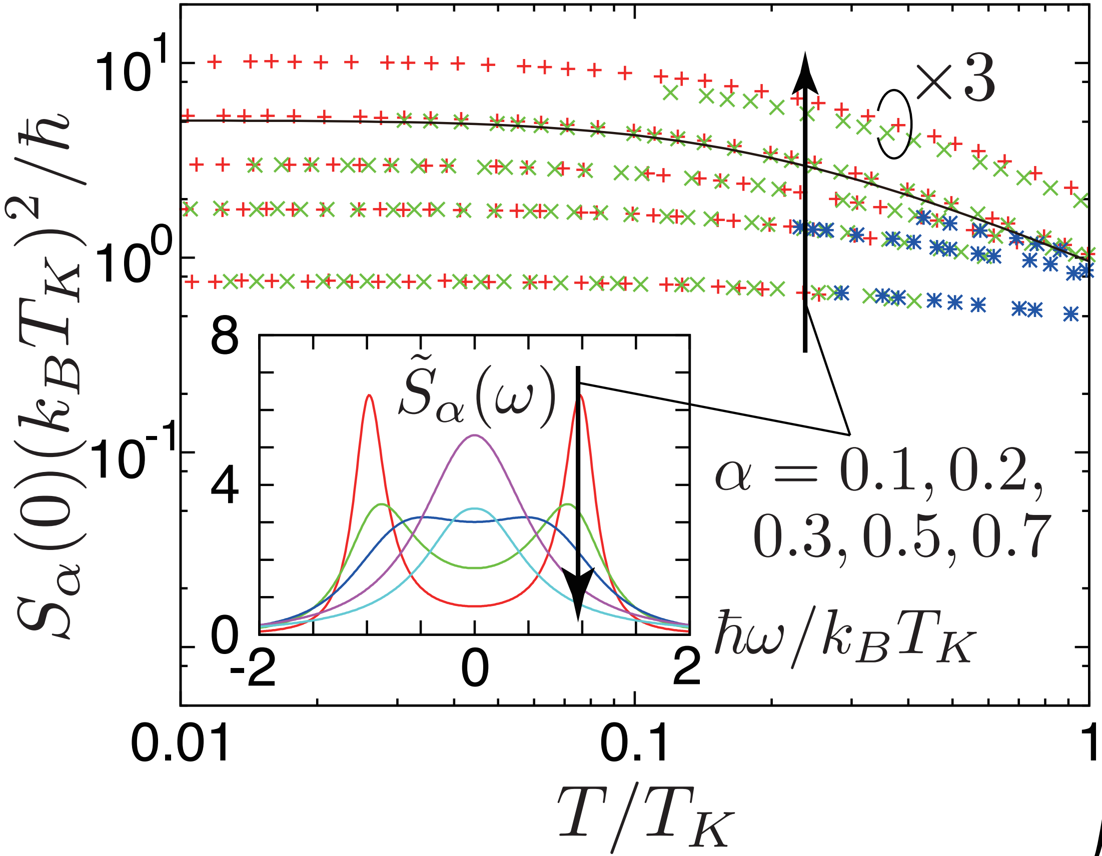
cotunneling process
(4th-order perturbation
for system-bath coupling)

$$J^{(\sigma)} = \frac{\sin^4\theta}{2\pi}\int_0^\infty d\omega\,\omega\chi_L(\omega)\chi_R(\omega)[n_L(\omega)-n_R(\omega)]$$

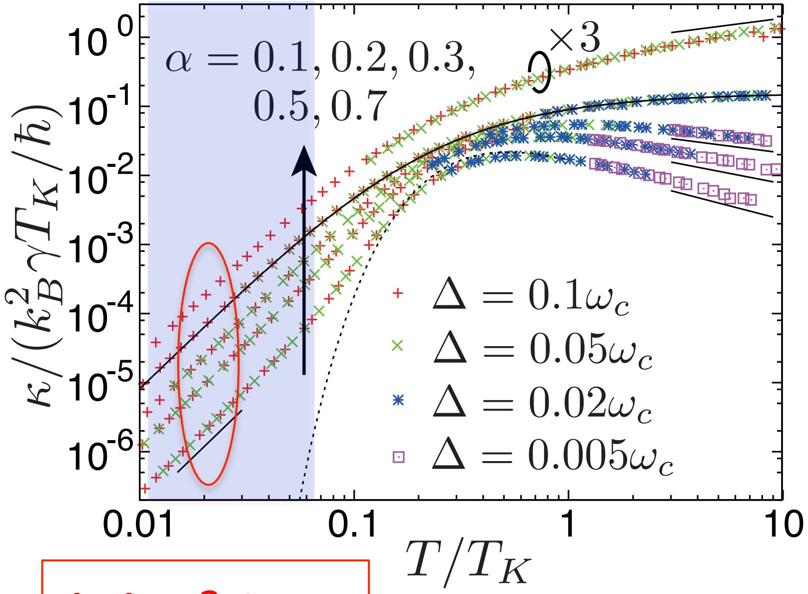
$$\times\left|\frac{1}{\omega-\omega_0\pm\frac{i}{2}\Gamma_{\bar\sigma}}-\frac{1}{\omega+\omega_0\mp\frac{i}{2}\Gamma_{\bar\sigma}}\right|^2,$$

No renormalization effect

Kondo regime (1)



$T \ll T_K$



(1) T^3 law

$$S(\omega) = \frac{\mathrm{Im}\chi(\omega)}{\omega} \qquad x = \beta\omega$$

$$\int d\omega \omega^2 \cdots \to T^3 \int dx x^2 \cdots$$

Thermal conductivity

constant for $T < T_K$

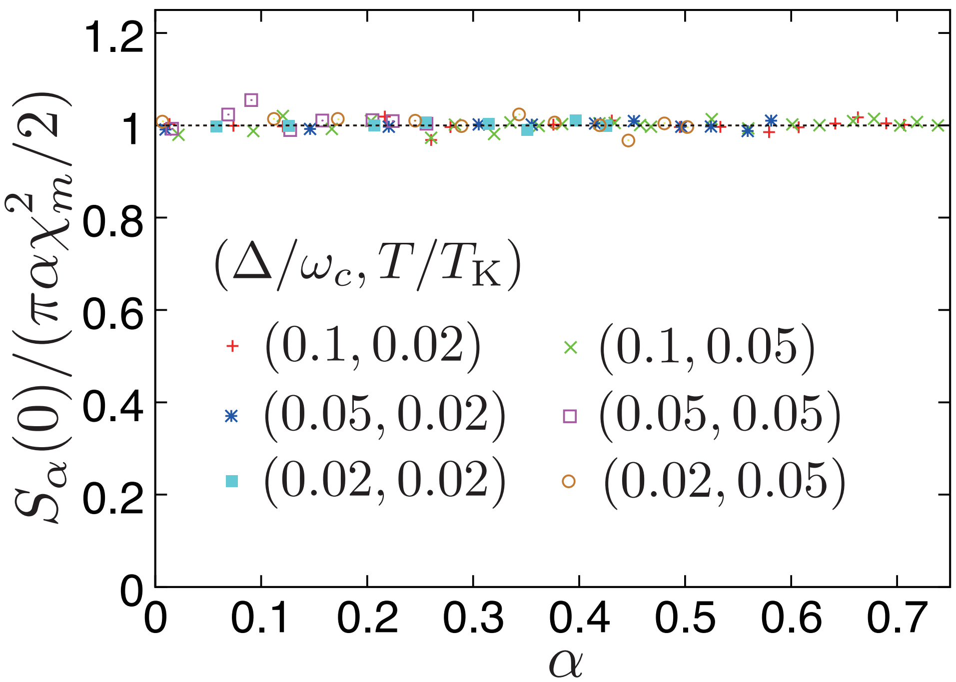
$$\kappa = \frac{k_B \alpha_L \alpha_R}{2\alpha} \int_0^{\omega_c} d\omega S(\omega) \omega^2 \left[\frac{\beta\omega/2}{\sinh(\beta\omega/2)} \right]^2$$

Kondo regime (2)

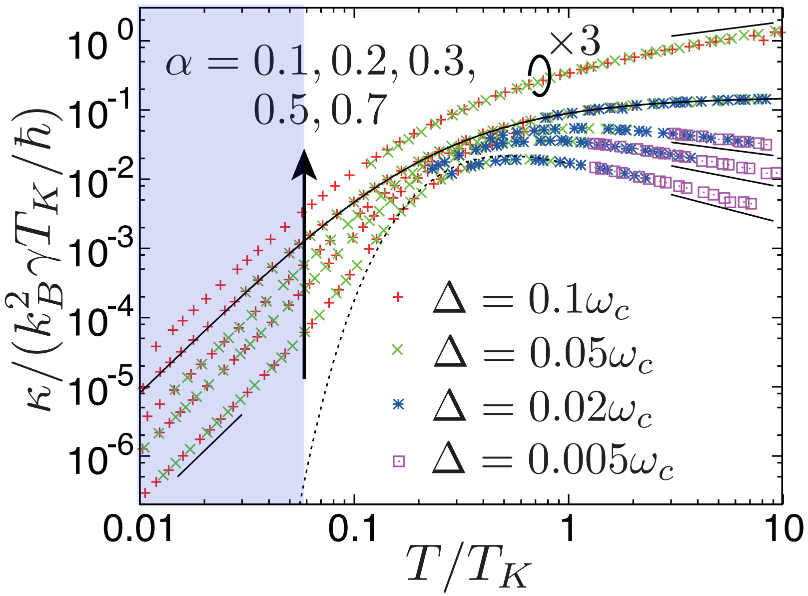
Shiba's relation

$$\chi_m = \frac{d\langle\sigma_z\rangle}{dh_z}$$

$$\lim_{\omega\to 0} S(\omega) = \frac{\alpha\pi\chi_m^2}{2}$$



$T \ll T_K$



(Local) spin susceptibility

$$\chi_m = \frac{d\langle\sigma_z\rangle}{dh_z}$$

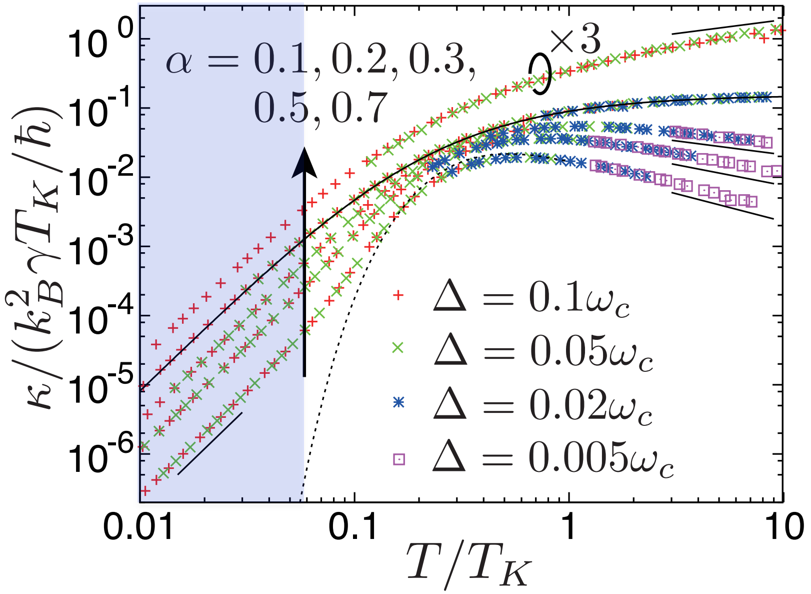
Spectral function

$$S(\omega) = \frac{\mathrm{Im}\chi(\omega)}{\omega}$$

c.f. NMR relaxation time $(\omega \to 0)$

Kondo regime (3)

$$T \ll T_K$$



Shiba's relation

$$\lim_{\omega\to 0} S_\alpha(\omega) = \frac{\alpha\pi\chi_m^2}{2}$$

Low-temperature form

$$\kappa = \frac{k_B\alpha_L\alpha_R}{2\alpha}\int_0^{\omega_c} d\omega S(\omega)\omega^2\left[\frac{\beta\omega/2}{\sinh(\beta\omega/2)}\right]^2$$

renormalization

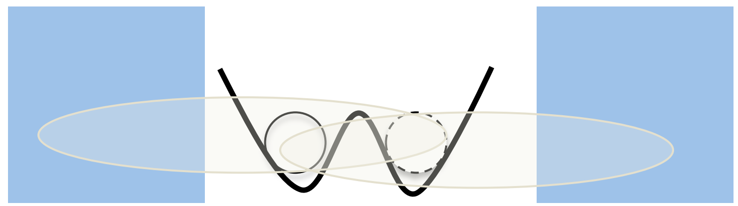
$$\kappa \sim \frac{\pi k_B \hbar^2 \chi_m^2}{8}\times\int_0^{\omega_c} d\omega I_L(\omega)I_R(\omega)\left[\frac{\beta\omega/2}{\sinh(\beta\omega/2)}\right]^2$$

cf. co-tunneling

$$\kappa \sim \frac{\pi^4\alpha k_B^2 T^3}{15 T_K^2}$$

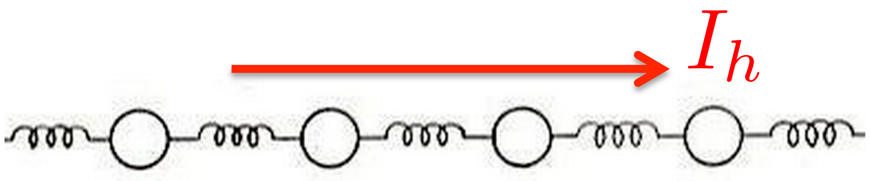
$$I_\nu(\omega) = 2\alpha_\nu\omega$$
$$\chi_m \sim 2/(k_B T_K)$$

$$T_K \simeq \Delta \quad (\alpha\to 0)$$



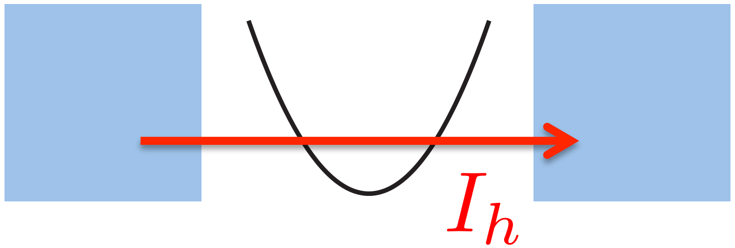
Physical meaning of T^3 law

One-dimensional harmonic oscillator chain



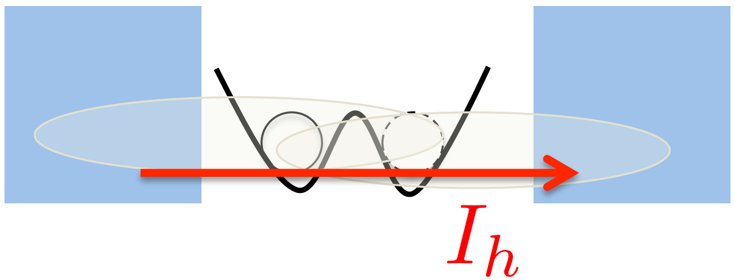
$$\kappa = \frac{I_h}{\nabla T} \propto T$$

Phonon transport via a local harmonic oscillator



$$\kappa = \frac{I_h}{\Delta T} \propto T^3$$

Phonon transport via a local two-state system for $T \ll T_K$



$$\kappa = \frac{I_h}{\Delta T} \propto T^3$$

Phonon-phonon interaction(= nonlinearity) is *hidden* in the Kondo regime.

cf. Fermi liquid theory for the standard Kondo effect

Incoherent tunneling regime

High-temperature form

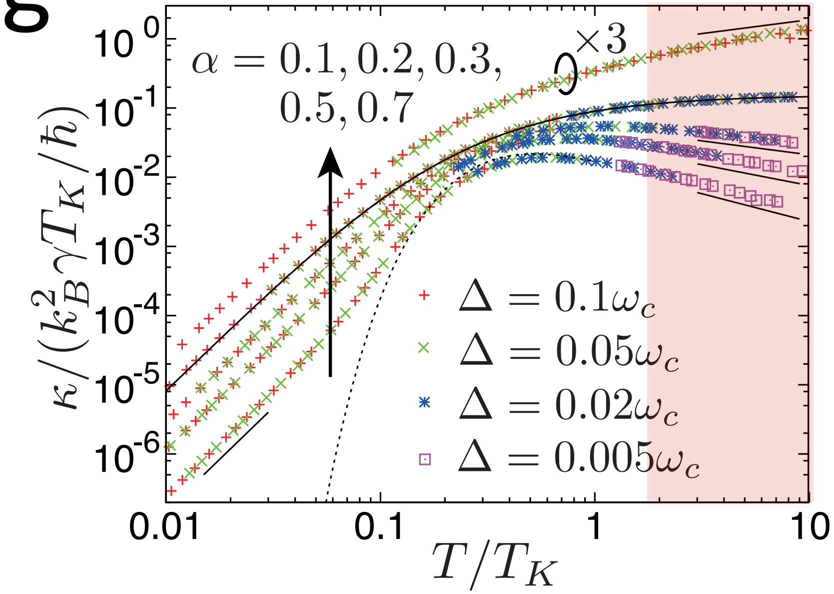
$$\Gamma \sim \frac{\Delta^2}{\omega_c}\left(\frac{\hbar\omega_c}{k_B T}\right)^{2\alpha-1}$$

Fermi's golden rule

$$\kappa \sim \frac{k_B \Delta^2}{\omega_c}\left(\frac{k_B T}{\hbar\omega_c}\right)^{2\alpha-1}$$

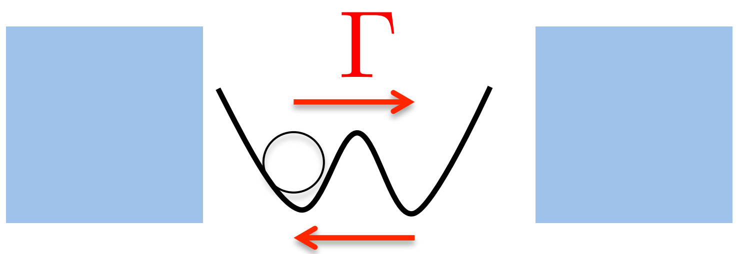
Dynamics can be described by stochastic transitions

$T \gg T_K$



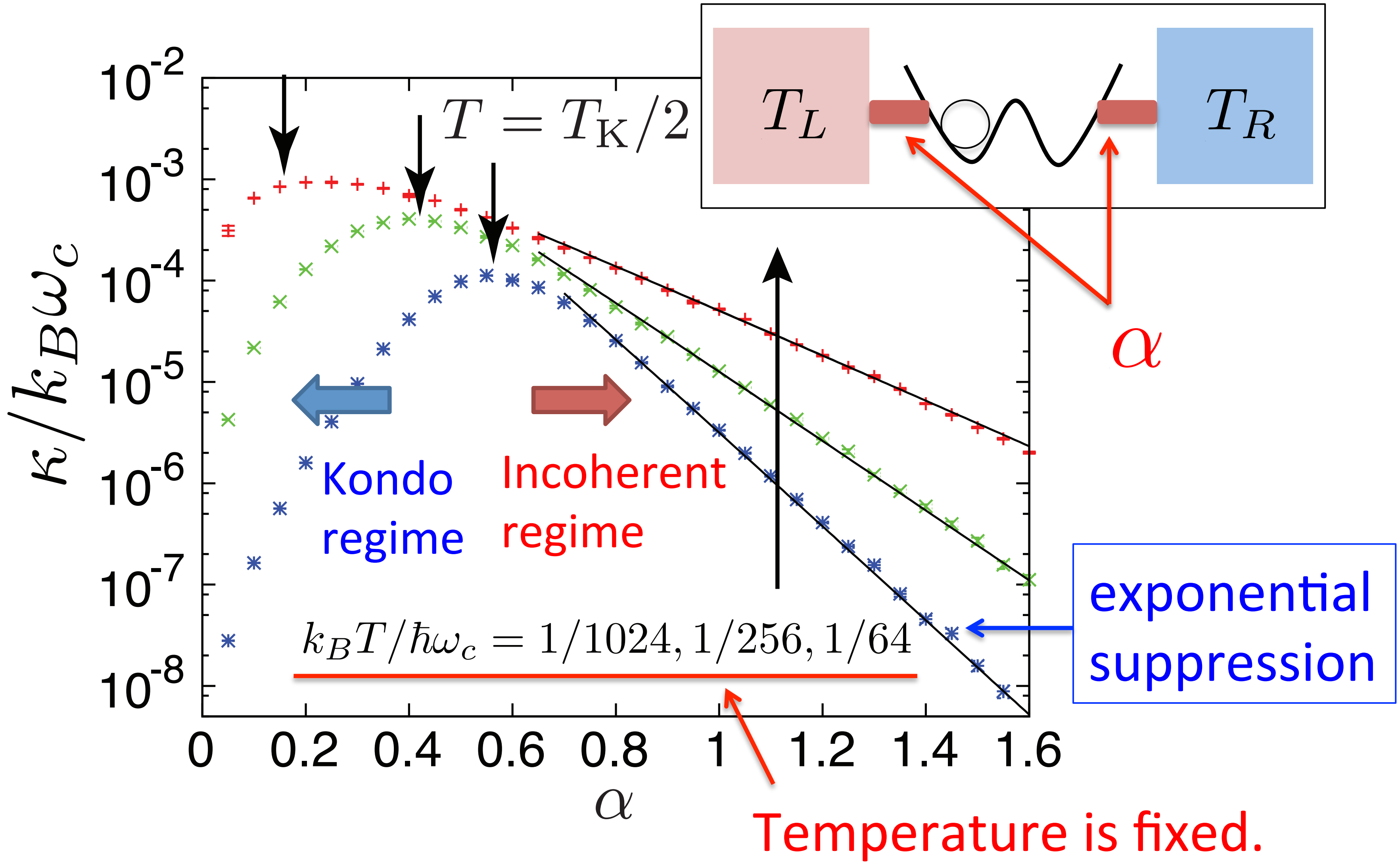
(2) $T^{2\alpha-1}$ law

transition prob.



Coupling-strength dependence

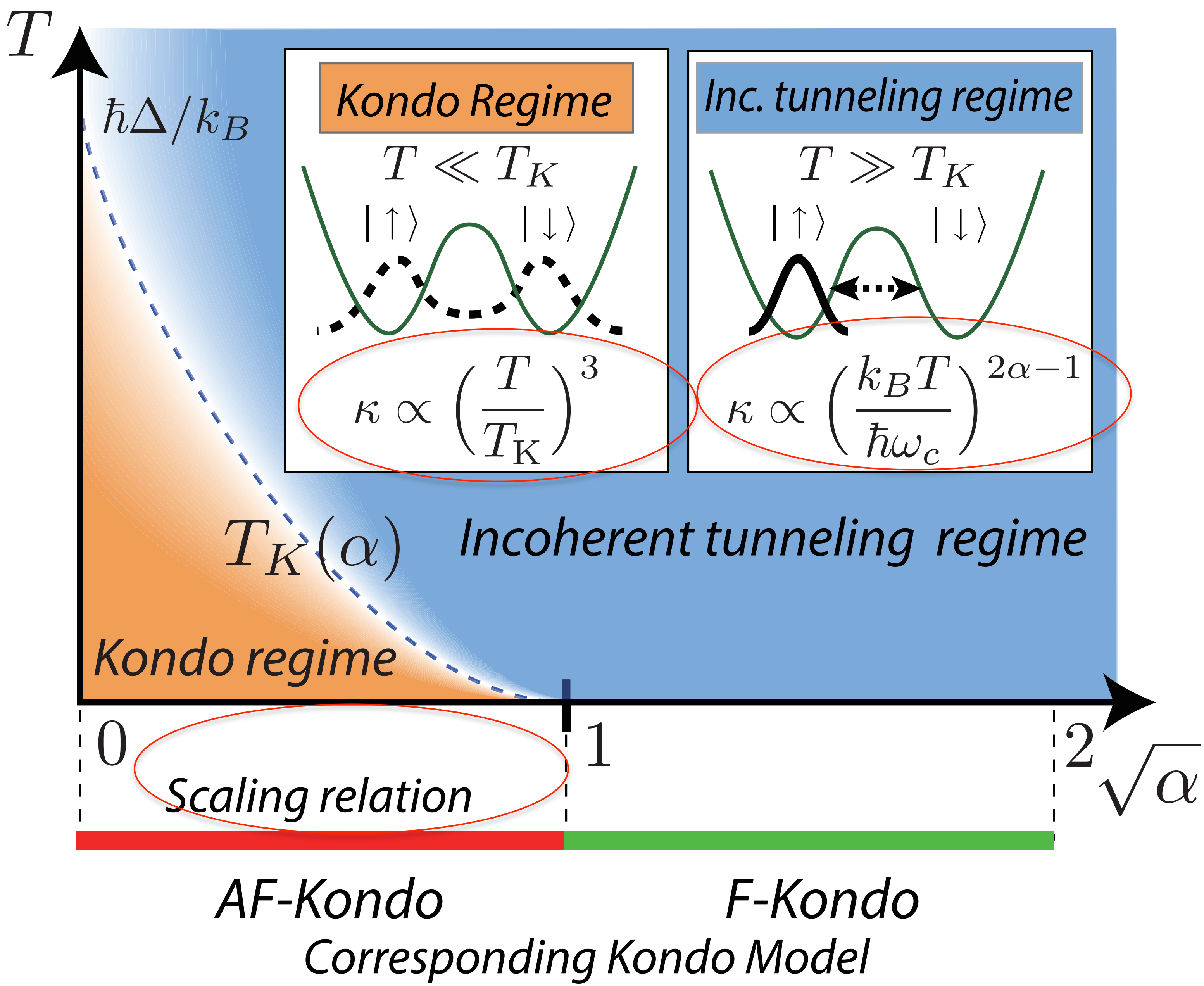
$$k_B T_K = \Delta \left(\frac{\Delta}{\omega_c} \right)^{\alpha/(1-\alpha)}$$



Summary

- Two-state system coupled to the bath

  = Correspondance of the Kondo effect
- Low-temperature: $\kappa \propto T^3 \ \ (T \ll T_K)$
- High-temperature: $\kappa \propto T^{2\alpha-1} \ (T \gg T_K)$
- Scaling relation: $\kappa(T) \sim (k_B^2 T_K/\hbar)\, f(\alpha, T/T_K)$
- Future problem
  - non-ohmic coupling
  - non-equilibrium heat transport



Thank you for your attention