Negative Coulomb drag in coupled quantum wires

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work done in collaboration with
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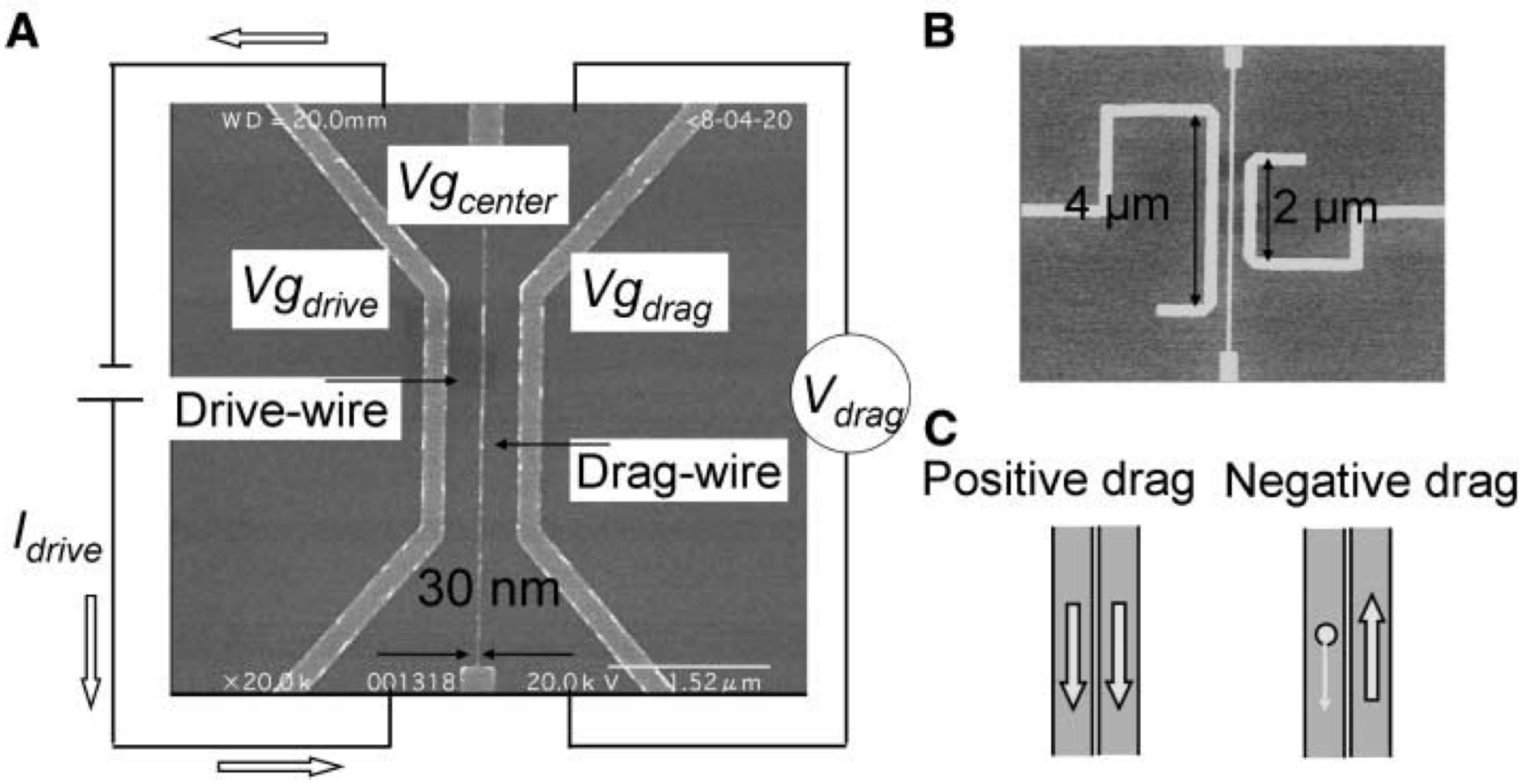
REFERENCE: S. C. Furuya, H. Matsuura and M. Ogata, arXiv: 1503.02499

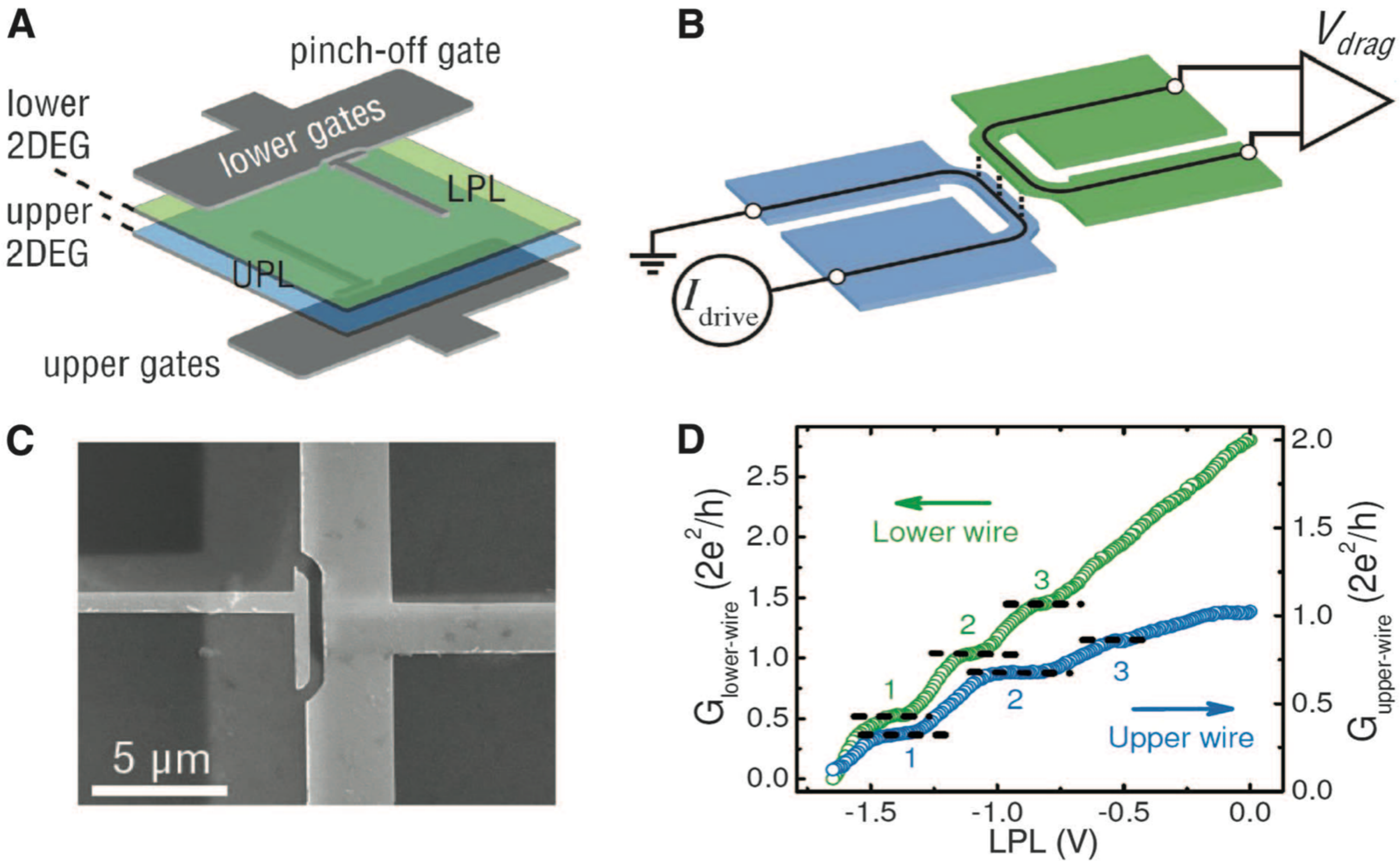
Coupled quantum wires

M. Yamamoto *et al.*, Science (2006)

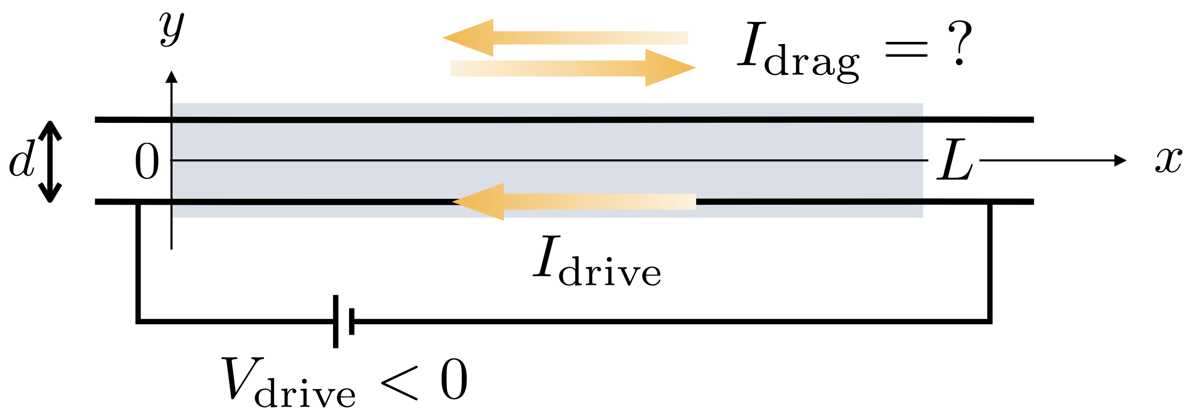


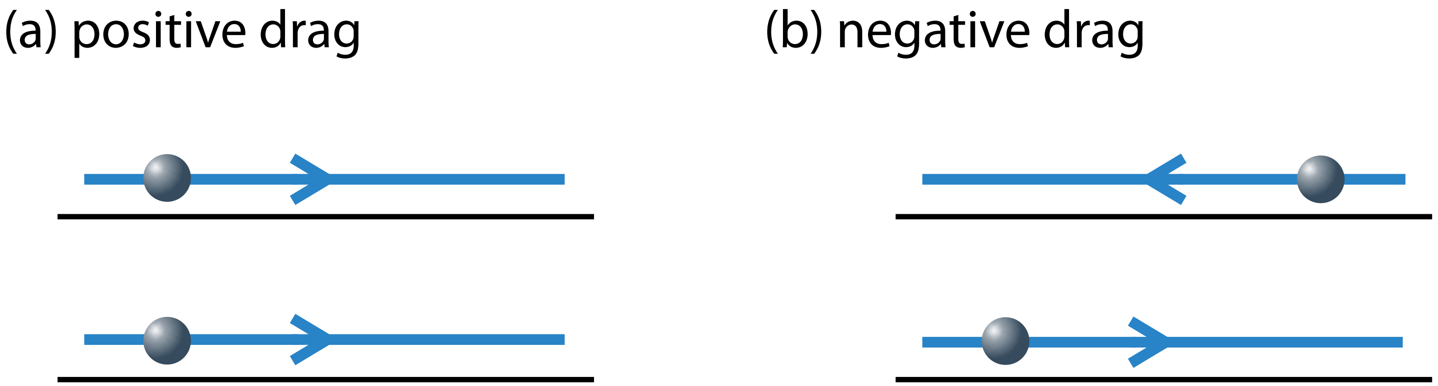
Coupled quantum wires

D. Laroche *et al.*, Science (2014)



Coulomb drag in coupled quantum wires



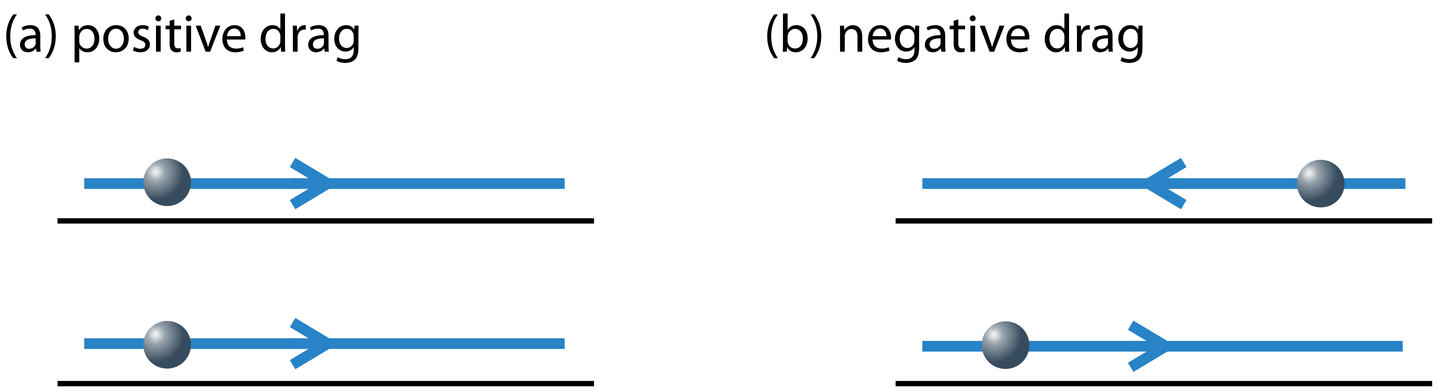


- no electron hopping between wires
- long-range nature of the Coulomb int.

Coulomb drag in coupled quantum wires

The preceding theories predict the positive drag only.

- Y. V. Nazarov and D. V. Averin, PRL **81**, 653 (1998)
- V. V. Ponomarenko and D. V. Averin, PRL **85**, 4928 (2000)
- G. A. Fiete, K. Le Hur, and L. Balents, PRB **73**, 165104 (2006)
- J. Peguiron, C. Bruder, and B. Trauzettel, PRL **99**, 086404 (2007)



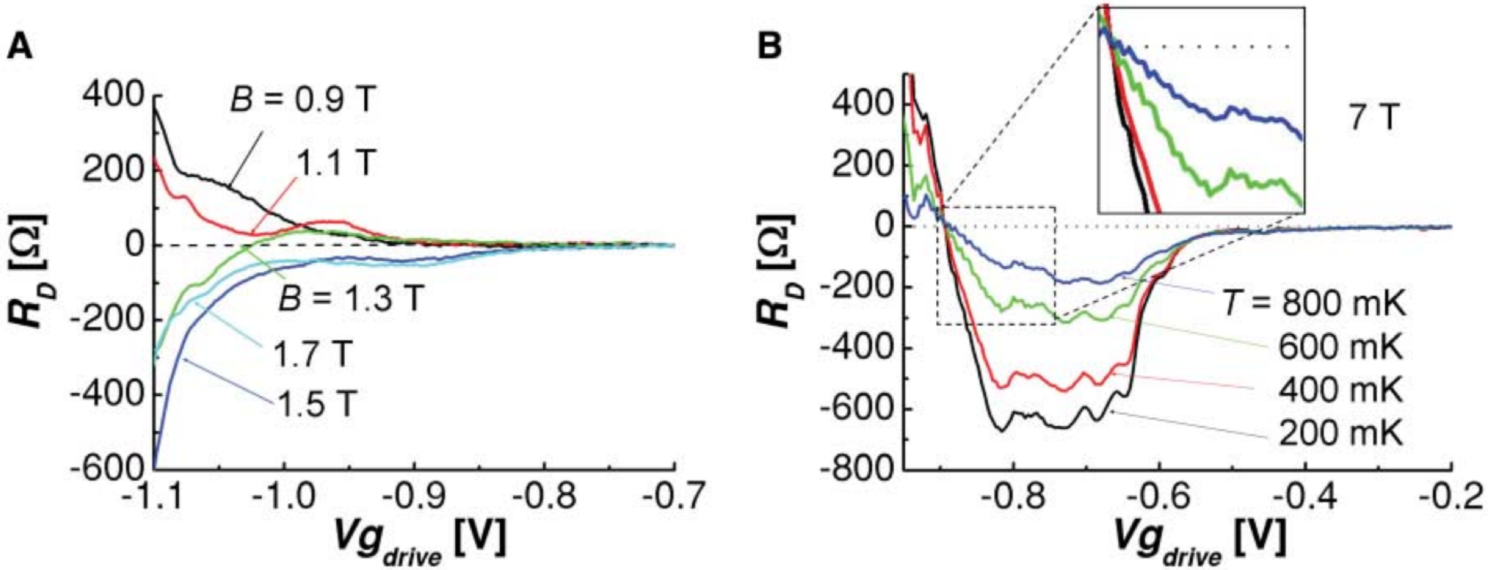
- no electron hopping between wires
- long-range nature of the Coulomb int.

Experimental evidence

$$R_D = -\frac{V_{\text{drag}}}{I_{\text{drive}}}$$

$R_D > 0$: positive drag

$R_D < 0$: negative drag

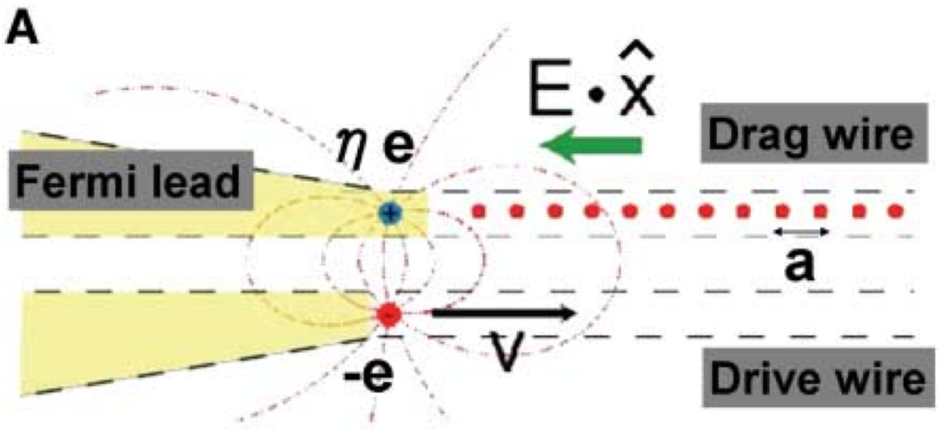


M. Yamamoto *et al.*, Science (2006)

Wigner crystal

M. Yamamoto *et al.*, Science (2006)

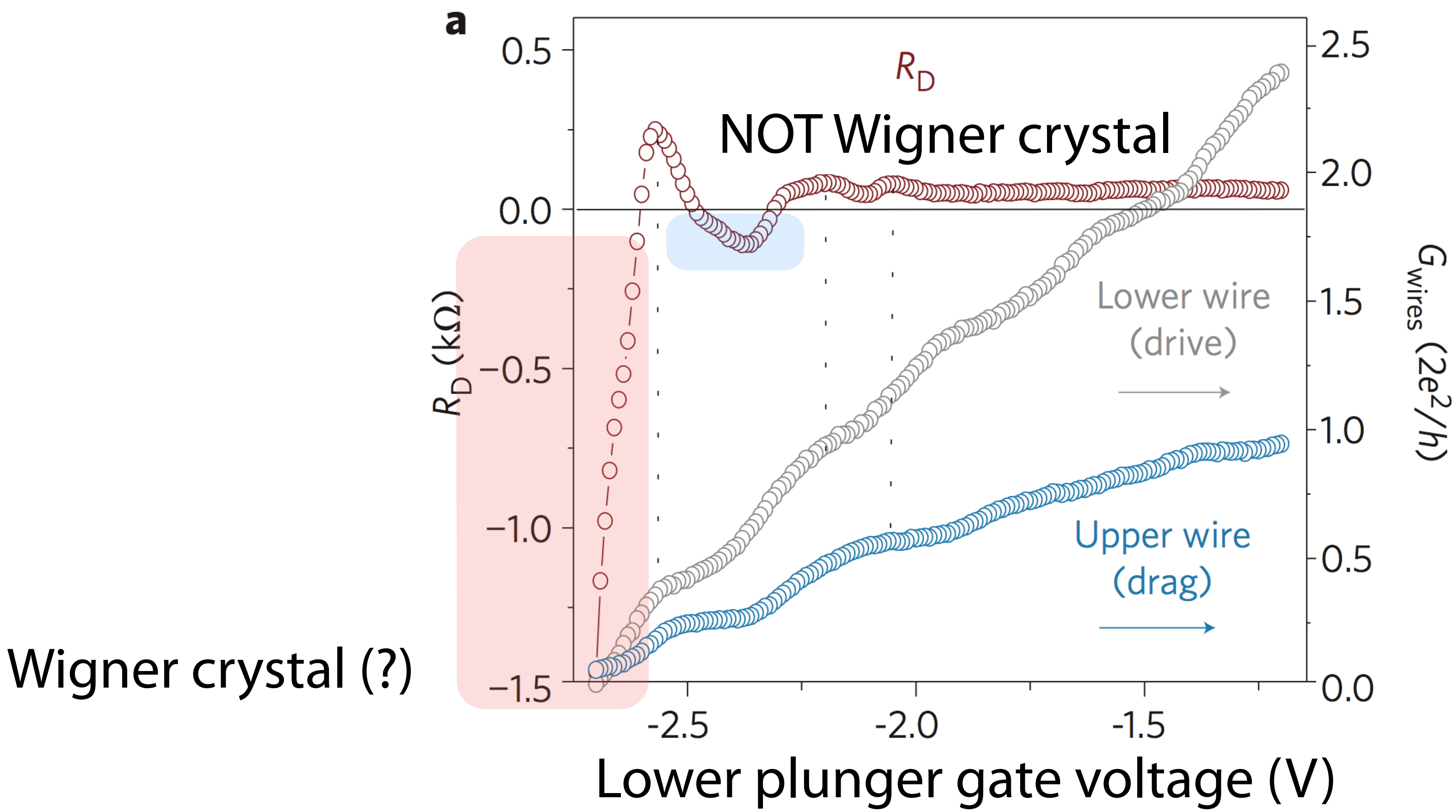
Fig. 4. (**A**) Electrons in the drive (lower) wire propagating from left to right are screened in the lead of the drag wire by a fraction η of a charge. The drive wire electron and this screening hole then induce the dipole field **E**, whose lateral component **E•x** is indicated by a green arrow. Lattice spacing *a* is 50 to 100 nm with wire width of a few 10 nm, and the distance between the wires is about 100 to 150 nm. (**B**) Effective



Wigner crystal is formed when the electron density is low.

Another experimental result

D. Laroche *et al.*, Nature nanotech. (2011)



Quantized conductance

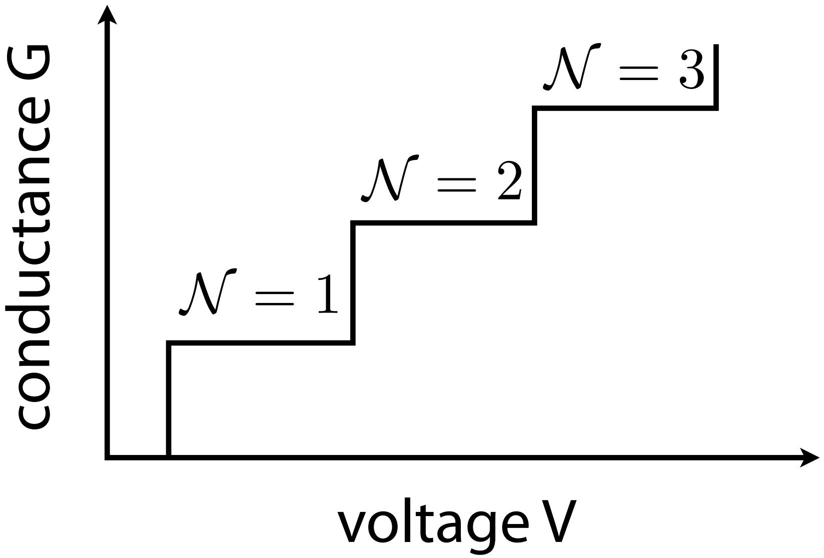
The Tomonaga-Luttinger liquid has a quantized conductance G=I/V.

$$G = \frac{e^2}{h}\mathcal{N}, \qquad (\mathcal{N} \in \mathbb{N}).$$

$\mathcal{N}$: number of TLLs

$\mathcal{N} = 1, 2, 3, \cdots,$ for spinless fermion

$\mathcal{N} = 2, 4, 6, \cdots.$ for fermion w/ spin

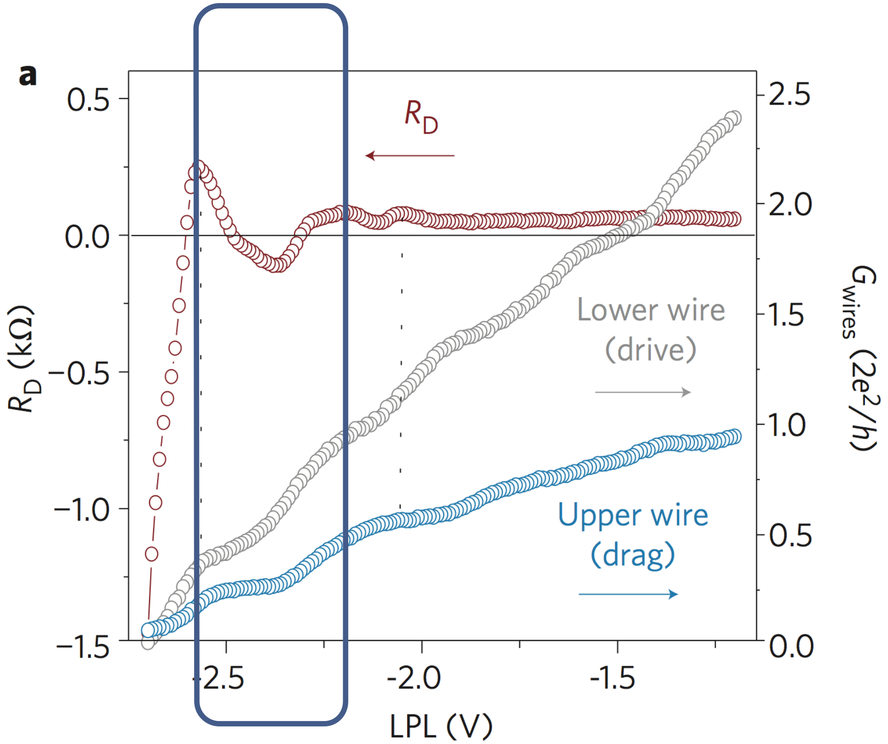


The stepwise V dependence of G.
= an evidence for the existence of TLLs.

TLL is identical to collective motions of electrons.

Negative drag of coupled TLLs

D. Laroche *et al.*, Nature nanotech. (2011)



- Tomonaga-Luttinger liquid (TLL) can cause the negative drag.
- The magnetic field is not necessary for the negative drag.

Simplest setup for the negative drag of the TLL?

Setup

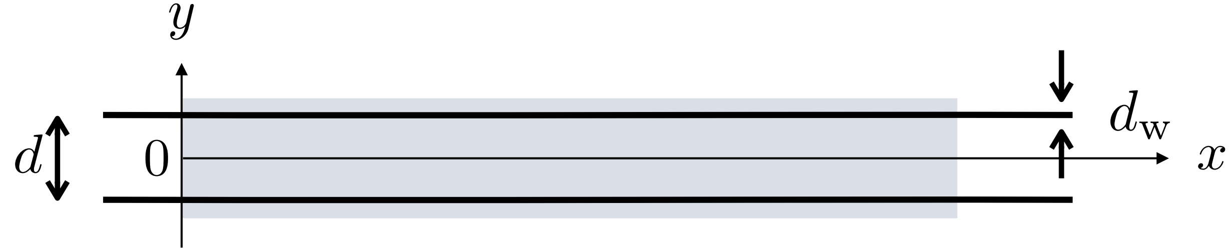
Coupled 1D spinless fermions

$$\mathcal{H} = \mathcal{H}^1_{\mathrm{fermion}} + \mathcal{H}^2_{\mathrm{fermion}} + \int dr dx\, V_\perp(r)\rho_1(x)\rho_2(x+r)$$

$$\mathcal{H}^n_{\mathrm{fermion}} = -iv_F \int dx(\psi^\dagger_{n,R}\partial_x\psi_{n,R} - \psi^\dagger_{n,L}\partial_x\psi_{n,L}) + \int dr dx V_\parallel(r)\rho_n(x)\rho_n(x+r)$$

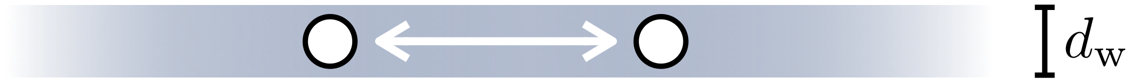
Long-range Coulomb interactions

$$V_\parallel(r) = \frac{e^2}{4\pi\epsilon}\frac{1}{\sqrt{r^2+d_{\mathrm{w}}^2}}, \qquad V_\perp(r) = \frac{e^2}{4\pi\epsilon'}\frac{1}{\sqrt{r^2+d^2}}$$



Single quantum wire w/ long-range Coulomb int.

1D spinless fermion model



$$\mathcal{H}_{\rm fermion} = -iv_F \int dx(\psi_R^\dagger \partial_x \psi_R - \psi_L^\dagger \partial_x \psi_L) + \int dx dr V_{\parallel}(r)\rho(x)\rho(x+r)$$

intrawire Coulomb interaction

Bosonization $\mathcal{H}_{\rm fermion} = \mathcal{H}_{\rm boson}$

1D boson model

$$\mathcal{H}_{\rm boson} = \int \frac{dq}{2\pi}\frac{u(q)}{2\pi}\left[K(q)q^2|\theta(q)|^2 + \frac{1}{K(q)}q^2|\phi(q)|^2\right]$$

$$K(q) = \left[\frac{e^2}{\pi^2 \epsilon v_F}\log\left(\frac{1}{|q|d_{\rm w}}\right)\right]^{-1/2}, \quad u(q) = v_F/K(q)$$

cf. TLL with short-range interaction

Hamiltonian of the TLL equivalent to the spinless fermion w/ short-range int.:

$$\mathcal{H} = \frac{u}{2\pi} \int dx \left[K(\partial_x \theta)^2 + \frac{1}{K} (\partial_x \phi)^2 \right]$$

ϕ describes the density fluctuation.

$$\rho(x) = \bar{\rho} + \frac{1}{\pi} \partial_x \phi + 2\bar{\rho} \sum_{p=1}^{\infty} \cos[p\{2\pi\bar{\rho}x + 2\phi(x)\}]$$

ϕ and θ are dual:

$$\partial_t \phi = iuK \partial_x \theta,$$
$$\partial_t \theta = -iuK \partial_x \phi.$$

cf. TLL with short-range interaction

Hamiltonian of the TLL equivalent to the spinless fermion w/ short-range int.:

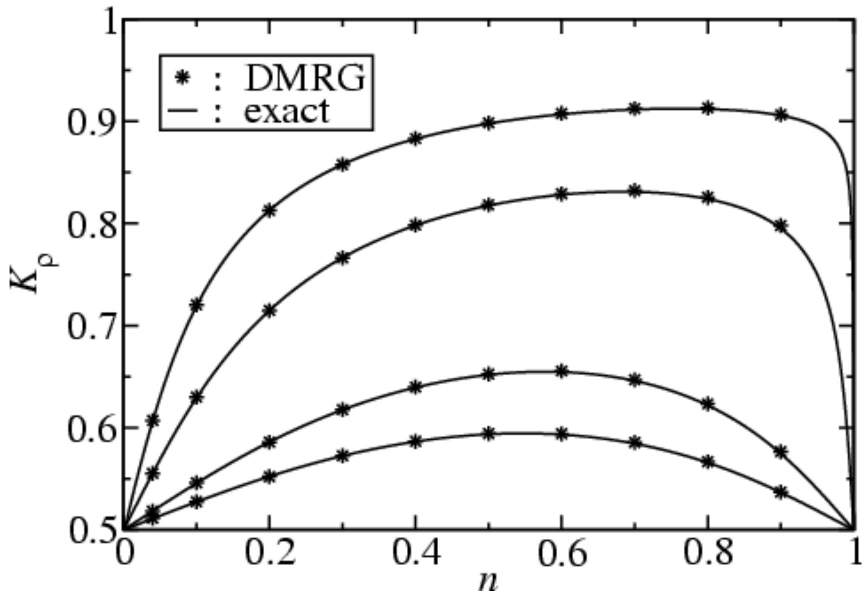
$$\mathcal{H} = \frac{u}{2\pi} \int dx \left[K(\partial_x \theta)^2 + \frac{1}{K}(\partial_x \phi)^2 \right]$$

TLL parameter "K": a measure of interaction strength

K=1: non-interacting
K<1: repulsive
K>1: attractive

e.g.: Hubbard chain

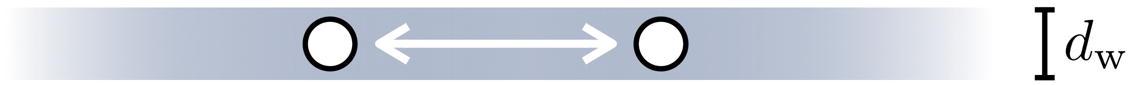
parameter range 1/2<K<1



S. Ejima, F. Gebhard, and S. Nishimoto, EPL (2005)

Single quantum wire w/ long-range Coulomb int.

1D spinless fermion model



$$\mathcal{H}_{\mathrm{fermion}} = -iv_F \int dx(\psi_R^\dagger \partial_x \psi_R - \psi_L^\dagger \partial_x \psi_L) + \int dx dr V_{\|}(r)\rho(x)\rho(x+r)$$

intrawire Coulomb interaction

Bosonization

$$\mathcal{H}_{\mathrm{fermion}} = \mathcal{H}_{\mathrm{boson}}$$

1D boson model

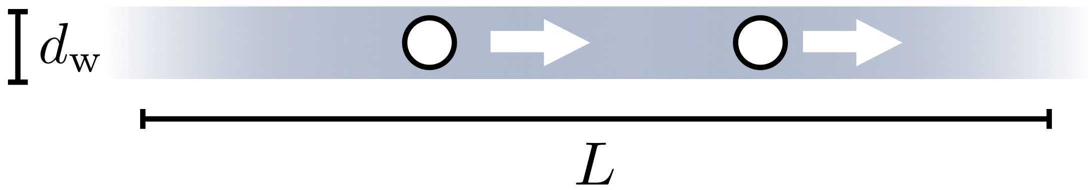
Infrared divergence

$$\mathcal{H}_{\mathrm{boson}} = \int \frac{dq}{2\pi} \frac{u(q)}{2\pi} \left[K(q) q^2 |\theta(q)|^2 + \frac{1}{K(q)} q^2 |\phi(q)|^2 \right]$$

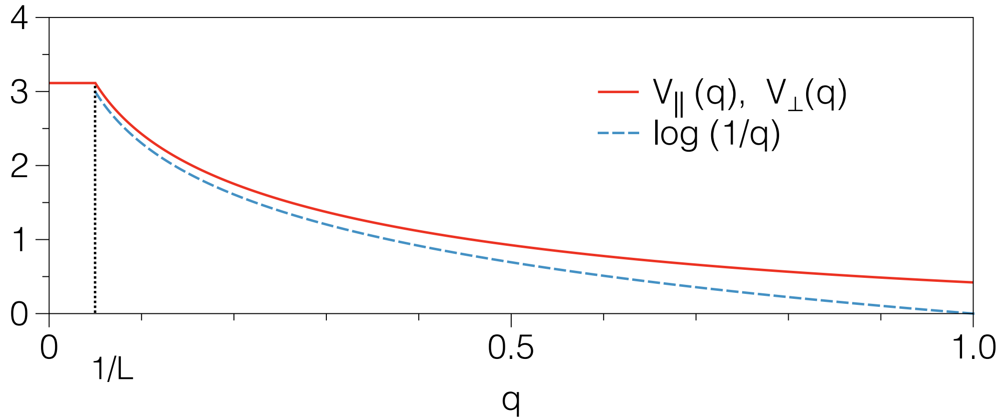
$$K(q) = \left[\frac{e^2}{\pi^2 \epsilon v_F} \log\left(\frac{1}{|q| d_{\mathrm{w}}} \right) \right]^{-1/2}, \quad u(q) = v_F / K(q)$$

Infrared divergence of Coulomb interaction

The infrared divergence is cut off in the finite-length wire.



$$V_{\|}(q \to 0) \simeq \int_{d_{\mathrm{w}}}^{L} dx\, \frac{e^2}{2\pi\epsilon} \frac{1}{x}$$
$$= \frac{e^2}{2\pi\epsilon} \log\left(\frac{L}{d_{\mathrm{w}}}\right)$$



The TLL parameter depends on geometrical parameters of the wire.

$$K = \left[\frac{e^2}{2\pi\epsilon} \log\left(\frac{L}{d_{\mathrm{w}}}\right)\right]^{-1/2}$$

In the low-q limit, the Hamiltonian is given by

$$\mathcal{H} = \frac{u}{2\pi} \int dx \left[K(\partial_x\theta)^2 + \frac{1}{K}(\partial_x\phi)^2\right].$$

Effective bosonized model of coupled quantum wires

Coupled 1D spinless fermions

$$\mathcal{H} = \mathcal{H}^1_{\text{fermion}} + \mathcal{H}^2_{\text{fermion}} + \int dr dx\, V_\perp(r)\rho_1(x)\rho_2(x+r)$$

Coupled 1D bosons

$$\mathcal{H} = \mathcal{H}^1_{\text{boson}} + \mathcal{H}^2_{\text{boson}} + \lambda \int dr dx \rho_1(x)\rho_2(x)$$

$$\lambda = V_\perp(q \to 0) = \frac{e^2}{2\pi\epsilon'} \log(L/d_{\text{w}})$$

$$\mathcal{H}^n_{\text{boson}} = \frac{u}{2\pi} \int dx \left[K(\partial_x \theta_n)^2 + \frac{1}{K}(\partial_x \phi_n)^2 \right] \quad \text{with} \quad K = \left[\frac{e^2}{2\pi\epsilon} \log\left(\frac{L}{d_{\text{w}}}\right) \right]^{-1/2}$$

The low-energy effective model does not suffer from the infrared divergence.
The long-range nature of the interaction is encoded into the TLL parameter.

Density-density interaction

The bosonization formula of the electron density.

$$\rho_n(x) = \bar{\rho}_n + \frac{1}{\pi}\partial_x \phi_n(x) + 2\bar{\rho}_n \sum_{p_n=1}^{\infty} \cos\left[p_n\{2\pi\bar{\rho}_n x + 2\phi_n(x)\}\right]$$

F. D. M. Haldane, PRL (1981)

$\bar{\rho}_n$: average electron density of the nth wire

Most terms of $\lambda\rho_1(x)\rho_2(x)$ suffer from the incommensurate spatial oscillation of $\exp(i2\pi p_1\bar{\rho}_1 x)\exp(i2\pi p_2\bar{\rho}_2 x)$.

A few terms survives the spatial integration.

e.g. ($p_1 = 0,\ p_2 = 0$):

$$\frac{1}{\pi^2}\partial_x\phi_1\partial_x\phi_2 = \frac{1}{2\pi^2}\{(\partial_x\phi_+)^2 - (\partial_x\phi_-)^2\}$$

Symmetric and anti-symmetric modes

Reconstruction of the effective boson model

$$\mathcal{H} = \mathcal{H}_+ + \mathcal{H}_-$$

The symmetric and the anti-symmetric sectors:

$$\phi_\pm = \frac{\phi_1 \pm \phi_2}{\sqrt{2}}$$

$$\mathcal{H}_\pm = \frac{u_\pm}{2\pi} \int dx \left[K_\pm (\partial_x \theta_\pm)^2 + \frac{1}{K_\pm} (\partial_x \phi_\pm)^2 \right] + V^p_\pm$$

Two cosines are kept.

$$V^p_\pm = 2\lambda \bar{\rho}_1 \bar{\rho}_2 \int dx \, \cos(4p\pi \bar{\rho}_\pm x + p\sqrt{8}\phi_\pm) \quad \text{with} \quad \rho_\pm = \frac{\bar{\rho}_1 \pm \bar{\rho}_2}{2}.$$

the $p_1 = p_2 = p$ term in $\lambda \rho_1(x) \rho_2(x)$

Commensurability

The cosine in the anti-symmetric sector

$$V_-^p = 2\lambda\bar{\rho}_1\bar{\rho}_2 \int dx\ \cos(4p\pi\bar{\rho}_- x + p\sqrt{8}\phi_-)$$

is free from the incommensurate oscillation when $\bar{\rho}_- = 0$, that is,

$$\bar{\rho}_1 - \bar{\rho}_2 = 0.$$

The cosine in the symmetric sector

$$V_+^p = 2\lambda\bar{\rho}_1\bar{\rho}_2 \int dx\ \cos(4p\pi\bar{\rho}_+ x + p\sqrt{8}\phi_+)$$

is free from the incommensurate oscillation when $\bar{\rho}_+ = 0$, that is,

$$2a_0 p(\bar{\rho}_1 + \bar{\rho}_2) = 1, 2, \cdots$$

(a_0 : lattice spacing)

Locking effect

The cosine interaction determines the fate of the currents.

$$V_\nu^p = 2\lambda\bar{\rho}_1\bar{\rho}_2 \int dx\, \cos(p\sqrt{8}\phi_\nu)$$ for a commensurate $\bar{\rho}_\nu$

The cosine locks ϕ_ν to a constant in space and time (locking effect).

Locking effect on ϕ_ν

↓

no current $J_\nu = \langle \partial_t \phi_\nu \rangle / \pi = 0$

- locking of ϕ_- leads to the positive drag.
- locking of ϕ_+ leads to the negative drag.

Drive and drag currents

Number currents of symmetric and antisymmetric sectors:

$$J_\pm = \frac{1}{\pi}\langle \partial_t \phi_\pm \rangle$$

In analogy, number currents of the nth wire:

$$J_n = \frac{1}{\pi}\langle \partial_t \phi_n \rangle$$

Drive and drag currents (i.e. the charge currents on the wires)

$$I_{\text{drive}} = -eJ_1, \quad I_{\text{drag}} = -eJ_2.$$

$$I_{\text{drive}} = -\frac{e}{\sqrt{2}}(J_+ + J_-), \quad I_{\text{drag}} = -\frac{e}{\sqrt{2}}(J_+ - J_-).$$

Positive and negative drags

Locking effect on ϕ_ν

↓

no current $J_\nu = \langle \partial_t \phi_\nu \rangle / \pi = 0$

- locking of ϕ_- leads to the positive drag.
- locking of ϕ_+ leads to the negative drag.

$$I_{\rm drive} = -\frac{e}{\sqrt{2}}(J_+ + J_-), \quad I_{\rm drag} = -\frac{e}{\sqrt{2}}(J_+ - J_-).$$

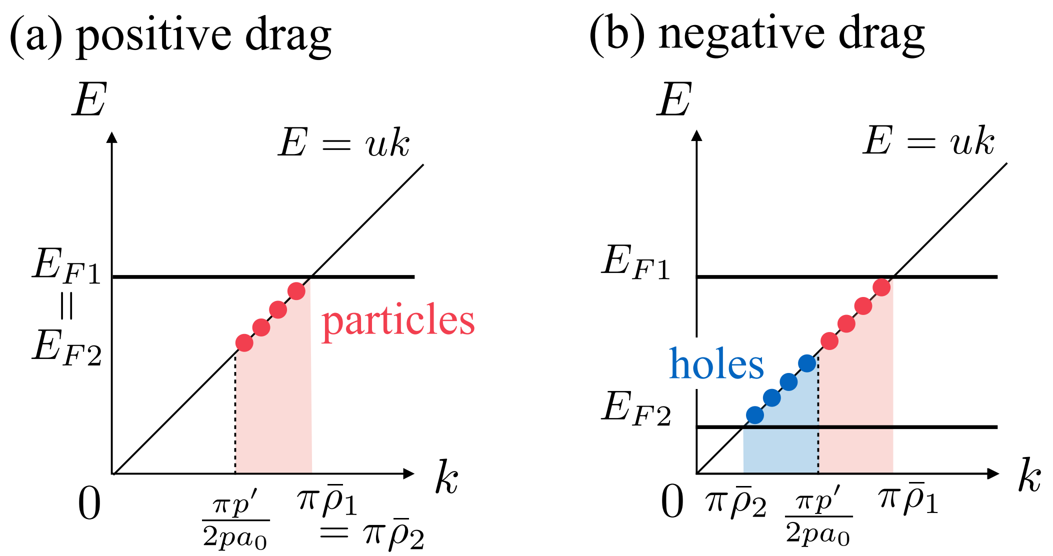
Particles and holes

- positive drag: $\bar{\rho}_1 = \bar{\rho}_2$
- negative drag: $\bar{\rho}_1 + \bar{\rho}_2 = \dfrac{p'}{2pa_0}$ (a_0 : lattice spacing)

Physical interpretations of the conditions: balance of particles and holes

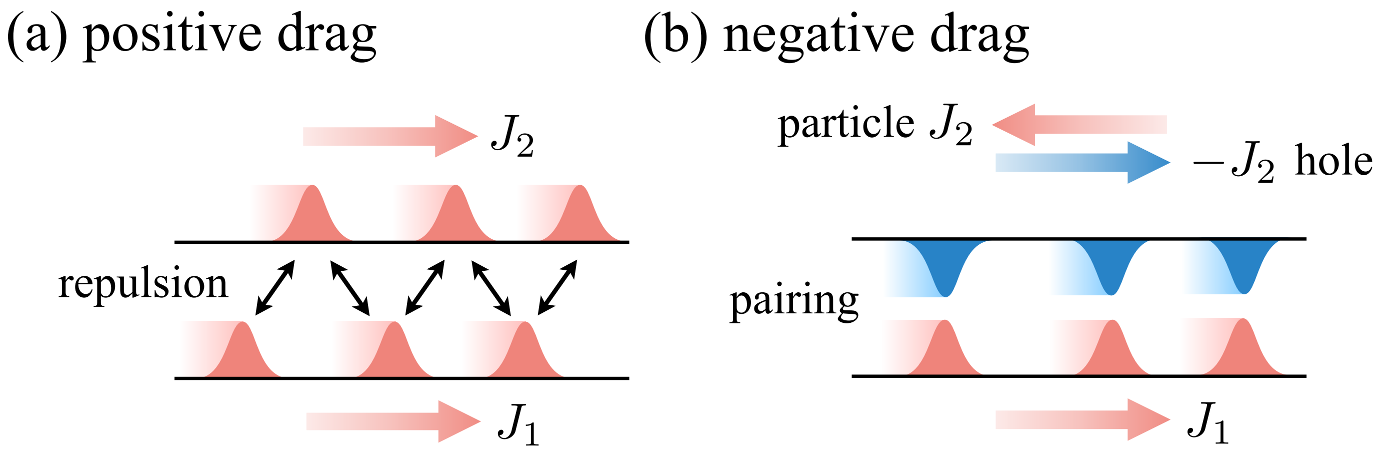
Particle and hole densities:

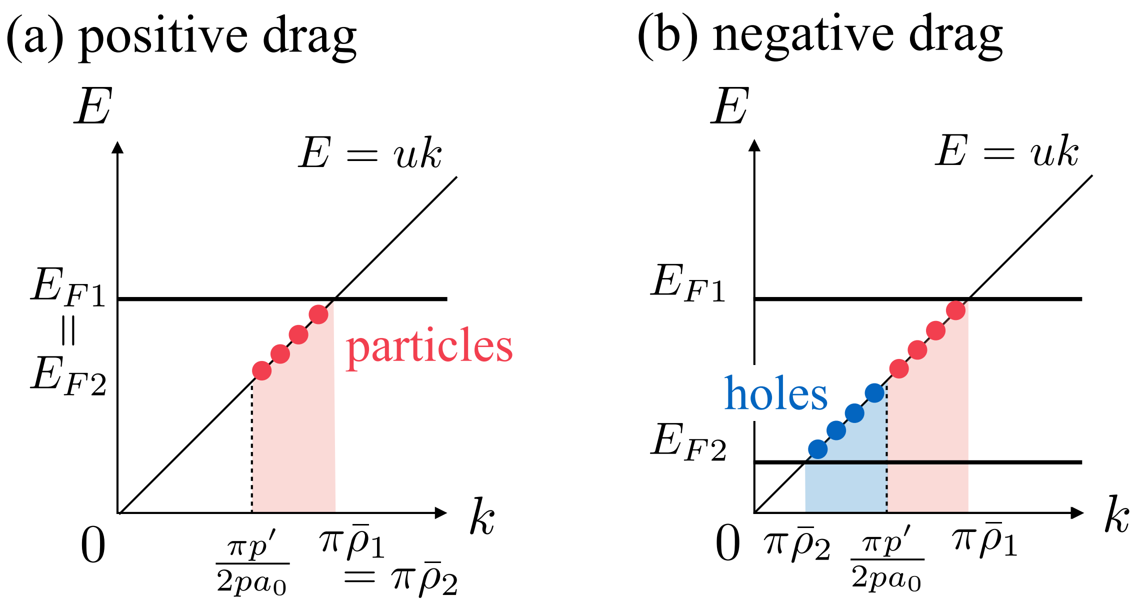
$$\rho_n^{\mathrm{p}} = \bar{\rho}_n - \frac{p'}{2pa_0},$$
$$\rho_n^{\mathrm{h}} = -\rho_n^{\mathrm{p}}.$$



- positive drag: $\rho_1^{\mathrm{p}} = \rho_2^{\mathrm{p}}$
- negative drag: $\rho_1^{\mathrm{p}} = \rho_2^{\mathrm{h}}$

Particles and holes





Relevance of the cosines

In order to induce the locking effect, the cosine interaction needs to be relevant in the sense of the renormalization group.

$$V_{\pm}^{p} = \lambda \bar{\rho}_1 \bar{\rho}_2 \int dx \, \cos(4\pi p \bar{\rho}_{\pm} x + p\sqrt{8}\phi_{\pm})$$

A condition for the relevance:

$$K_{\pm} < p^{-2}$$

- positive drag: $\bar{\rho}_1 = \bar{\rho}_2$
- negative drag: $\bar{\rho}_1 + \bar{\rho}_2 = \dfrac{p'}{2pa_0}$

Relevance of the cosines

In order to induce the locking effect, the cosine interaction needs to be relevant in the sense of the renormalization group.

$$V^p_\pm = \lambda \bar{\rho}_1 \bar{\rho}_2 \int dx \ \cos(4\pi p \bar{\rho}_\pm x + p\sqrt{8}\phi_\pm)$$

A condition for the relevance:

$$K_\pm < p^{-2}$$

- positive drag: $p = 1$
- negative drag: $p \sim$ (filling)$^{-1}$

The long-range Coulomb interaction makes K_+ small.

Typical value of filling in experiments

- electron density $= 1 \times 10^{15}\,\mathrm{m}^{-2}$
- width $d_{\mathrm{w}} = 0.5\,\mu\mathrm{m}$
- lattice spacing ($Al_xGa_{1-x}As$) $\simeq 5.7\,\mathrm{\AA}$

D. Laroche *et al.*, Nature Nanotech. (2011)
D. Laroche *et al.*, Science (2014)

$$\bar{\rho}_n = 5 \times 10^8 \mathrm{m}^{-1} \sim 0.3 a_0^{-1}$$

The condition for the negative drag:

$$K_+ < 0.1$$

cf.: $K_- \simeq 0.08 \pm 0.02$ in the experiment. D. Laroche *et al.*, Science (2014)

In addition, $K_+ < K_-$ holds in our theory. cf. Hubbard chain: 1/2 < K < 1

Summary

- We proposed the simple model that explains the negative drag.

- With the aid of the long-range Coulomb interaction, one can tune the interaction strength of the TLL as one wishes.

$$K_+ \sim [\log(L/d_{\rm w})]^{-1/2}$$

- Thus far the experimental results are consistent with our theory.

- Future problem: Inclusion of the spin degree of freedom and the spin-orbit interaction is required to understand the negative drag under the magnetic field.

Appendices

Robustness against incommensurability

The incommensurate oscillation induces the destruction of the locking and thus weakens the Coulomb drag.

For the negative drag,

$$\frac{|\bar{\rho}_+ - \bar{\rho}_+^0|}{\bar{\rho}_+} < \mathcal{A} \equiv \pi \left[\frac{2\epsilon}{\epsilon'} \frac{\log(L/d)}{\log(L/d_{\mathrm{w}})} \right]^{1/2} \quad \text{with} \quad \bar{\rho}_+^0 = \frac{p'}{2pa_0}.$$

For the positive drag,

$$\frac{|\bar{\rho}_1 - \bar{\rho}_2|}{\bar{\rho}_+} < \mathcal{A}.$$

Instantons

Instanton that induces the tunneling between neighboring minima of the cosine potential weakens the Coulomb drag.

We need to prepare low enough temperature in order not to excite instantons.

$$k_B T \ll M \simeq \frac{2e^2\bar{\rho}_+}{p\pi^2\epsilon}\left[\frac{2\epsilon}{\epsilon'}\log(L/d_{\rm w})\log(L/d)\right]^{1/2}$$