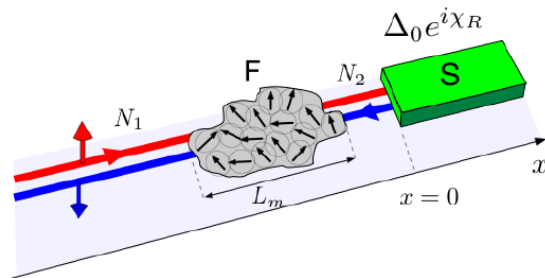




Superconducting hybrid structures based on QSH systems

New Perspectives in Spintronic and Mesoscopic Physics

June 1-19, 2015
Kashiwa, Japan



$$F^R = G_{eh}^R i\sigma_2 = \begin{pmatrix} F_{\uparrow\uparrow}^R & F_{\uparrow\downarrow}^R \\ F_{\downarrow\downarrow}^R & F_{\downarrow\uparrow}^R \end{pmatrix}$$

Björn Trauzettel

Pablo Burset (Uni Würzburg)
François Crépin (Uni Würzburg)
Fabrizio Dolcini (PolyTech Torino)
Felix Keidel (Uni Würzburg)

Elitenetzwerk
Bayern



Alexander von Humboldt
Stiftung/Foundation



DFG Deutsche
Forschungsgemeinschaft

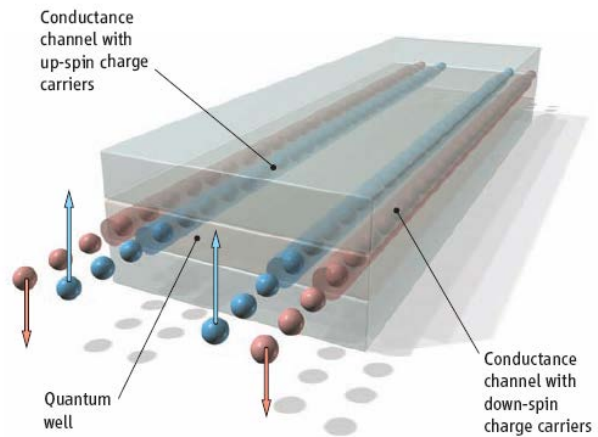


HELMHOLTZ
ASSOCIATION





QSHE (in a nutshell)



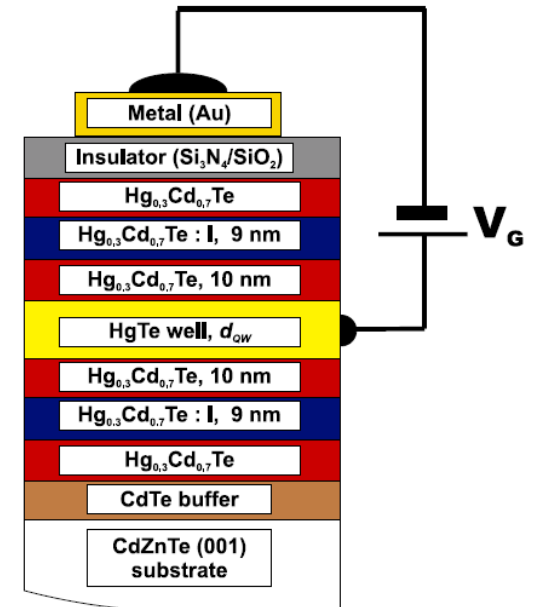
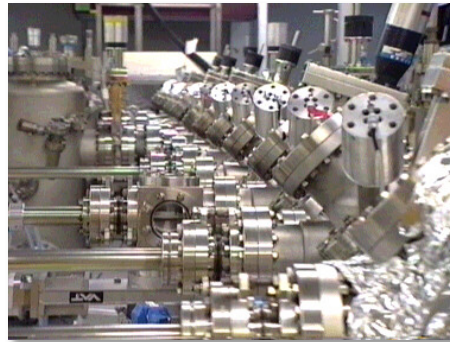
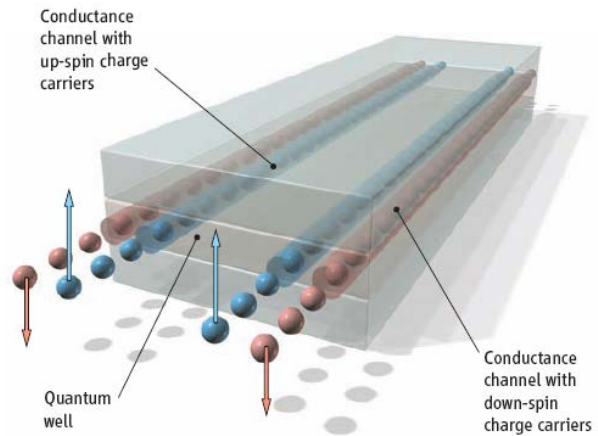
symmetry protected
topological state of matter

Kane & Mele PRL 2005

Bernevig, Hughes & Zhang Science 2006



QSHE (in a nutshell)



symmetry protected
topological state of matter

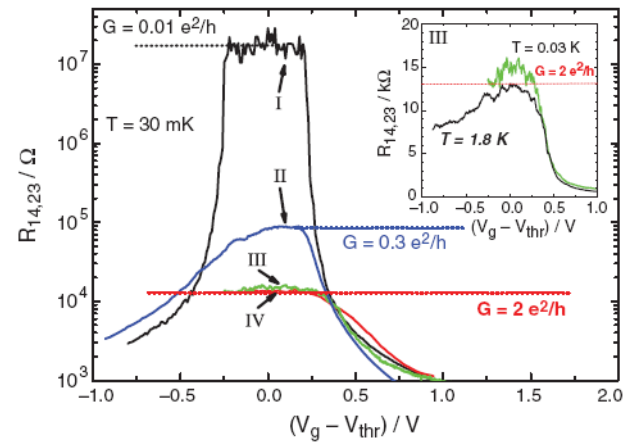
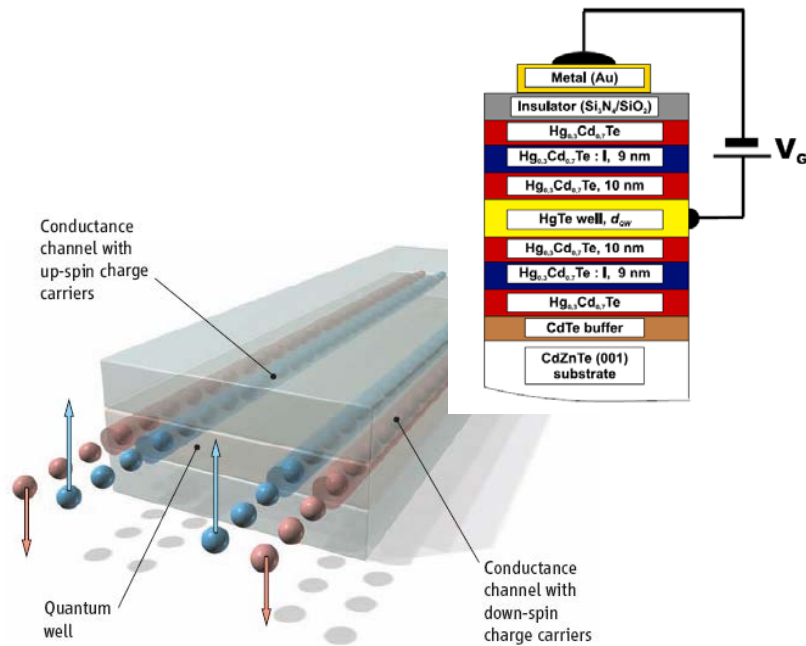
König, Molenkamp et al. *JPSJ* 2008

Kane & Mele *PRL* 2005

Bernevig, Hughes & Zhang *Science* 2006



QSHE (in a nutshell)



König, Molenkamp et al. *Science* 2007

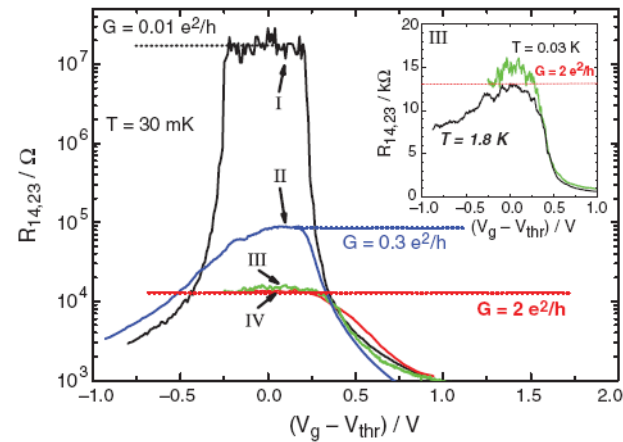
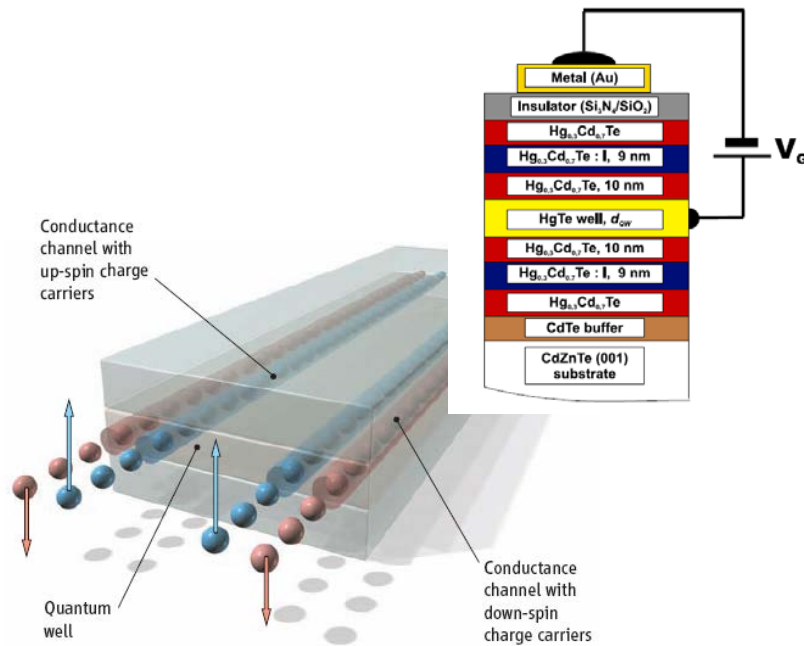
symmetry protected
topological state of matter

Kane & Mele *PRL* 2005

Bernevig, Hughes & Zhang *Science* 2006



QSHE (in a nutshell)

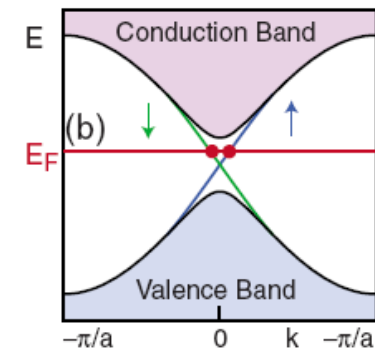
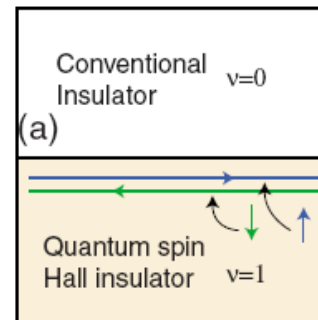


König, Molenkamp et al. *Science* 2007

symmetry protected
topological state of matter

Kane & Mele *PRL* 2005

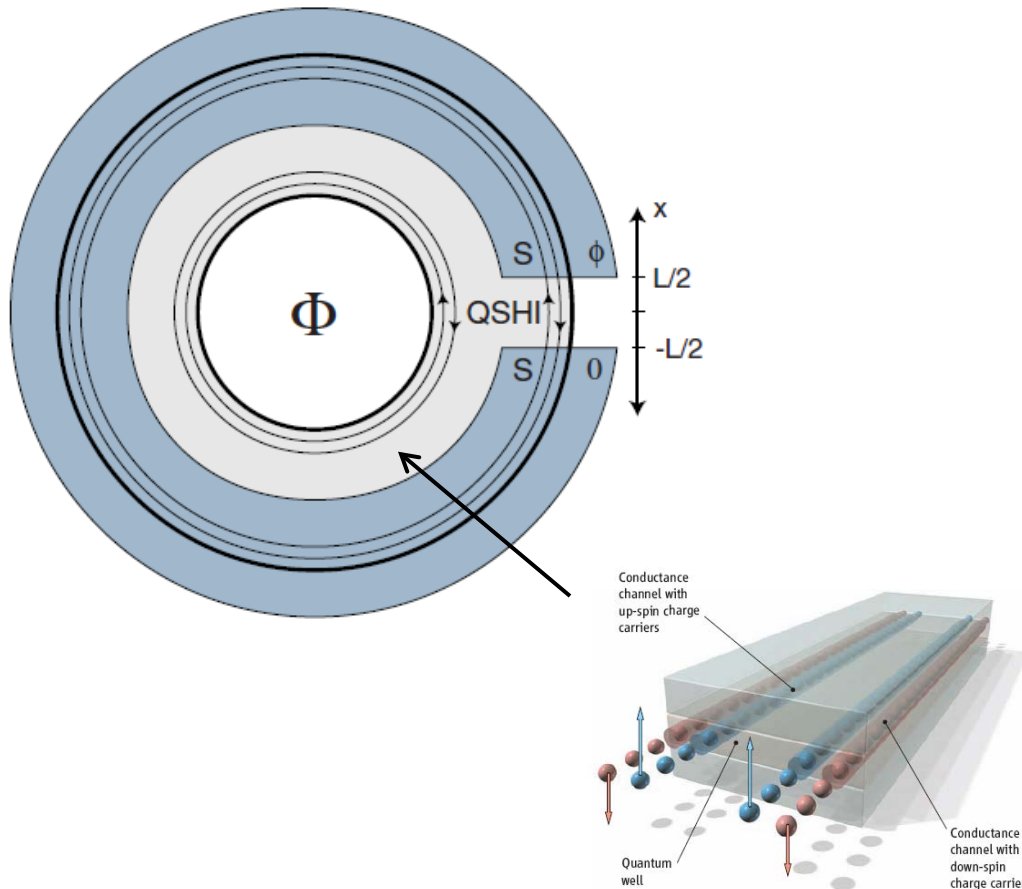
Bernevig, Hughes & Zhang *Science* 2006



Hasan & Kane *RMP* 2010

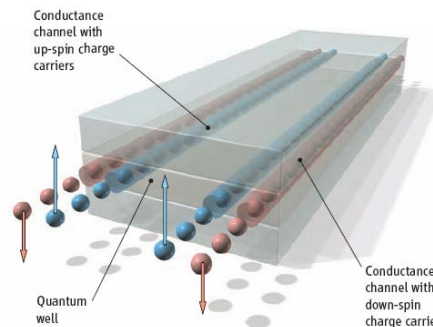
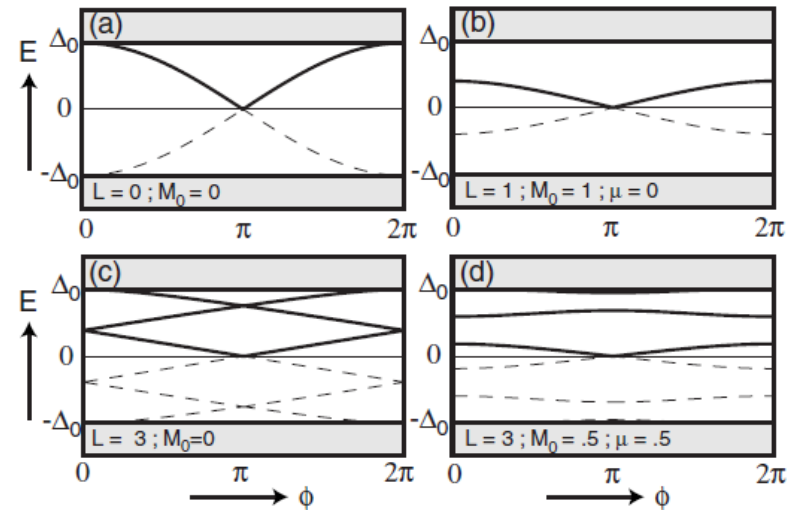
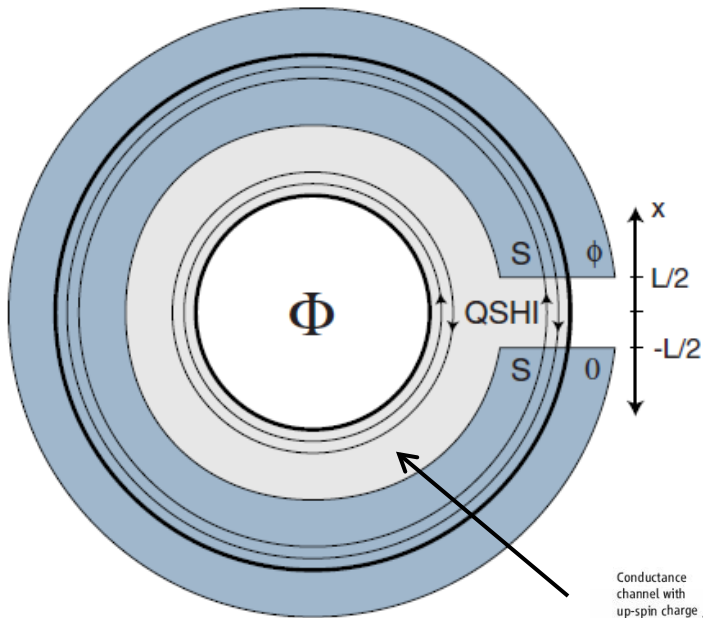


Pioneering prediction





Pioneering prediction

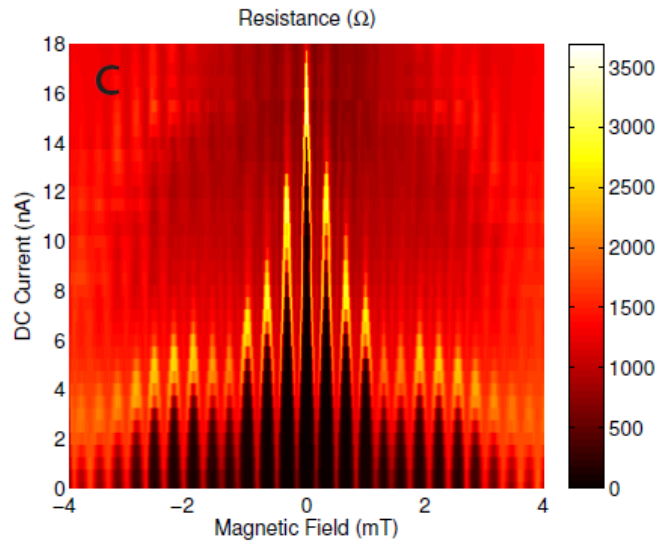


signatures of p-wave
superconductivity?



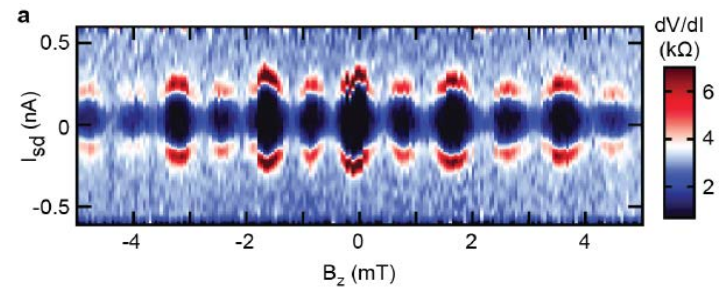
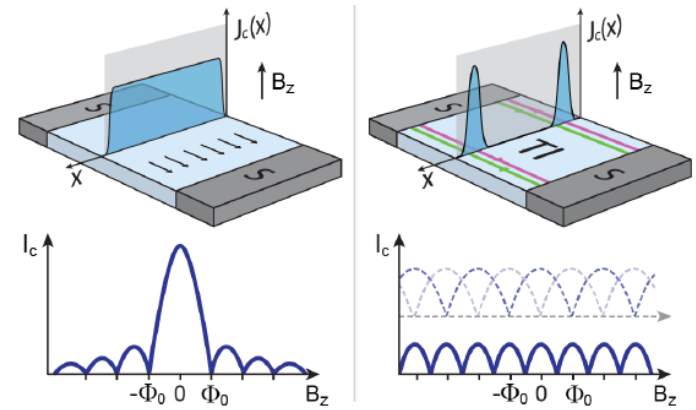
Inspiring experiments

Hg(Cd)Te QWs



Hart, Molenkamp, Yacoby et al. *Nature Phys* 2014

InAs/GaSb QWs



Pribyl, Kouwenhoven et al. *Nature Nano* 2015

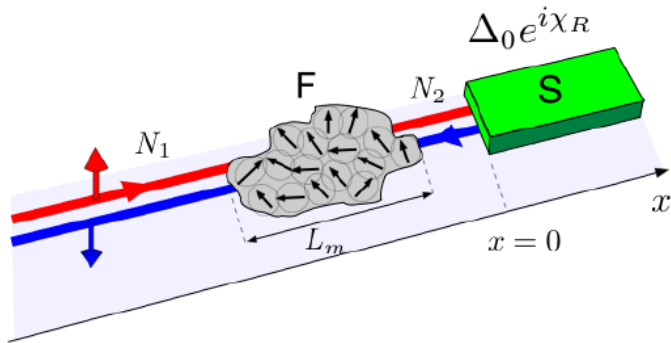


Outline

- **Transport signatures** of NS junctions -> Majorana bound states
- **Crossed Andreev reflection** in NSN setups -> odd-frequency triplet superconductivity



Setup & Hamiltonian

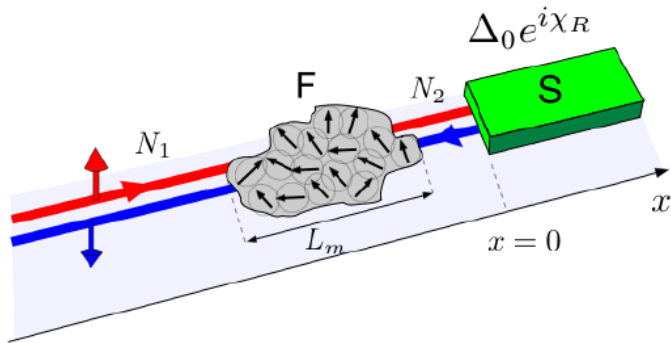


$$H = H_0 + H_{FM} + H_{SC}$$

$$H_0 = \int dx \left(\psi_{R\uparrow}^\dagger, \psi_{L\downarrow}^\dagger \right) \left[v_F p_x \sigma_z - \mu \right] \begin{pmatrix} \psi_{R\uparrow} \\ \psi_{L\downarrow} \end{pmatrix}$$



Setup & Hamiltonian



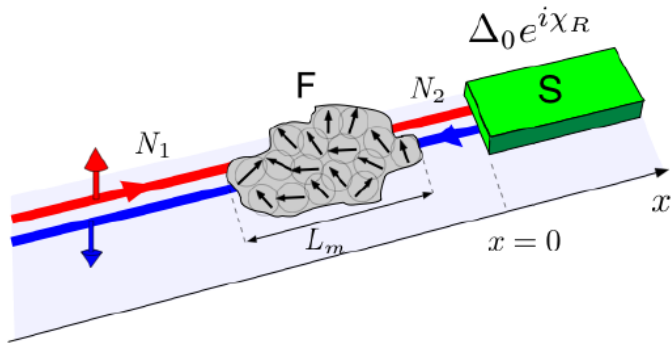
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$$H_{FM} = \int dx \left(\psi_{R\uparrow}^\dagger, \psi_{L\downarrow}^\dagger \right) \vec{m}(x) \cdot \vec{\sigma} \begin{pmatrix} \psi_{R\uparrow} \\ \psi_{L\downarrow} \end{pmatrix}$$



Setup & Hamiltonian



$$H = H_0 + H_{FM} + H_{SC}$$

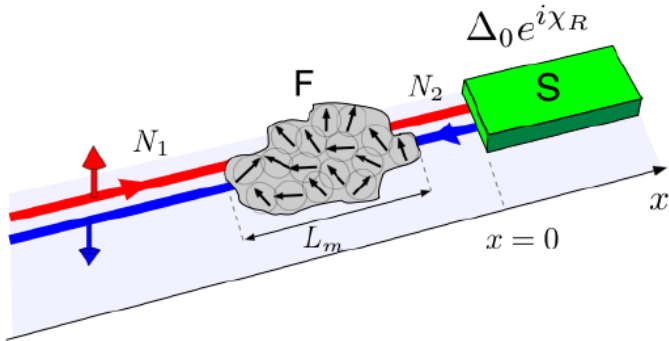
$$H_0 = \int dx \left(\psi_{R\uparrow}^\dagger, \psi_{L\downarrow}^\dagger \right) \left[v_F p_x \sigma_z - \mu \right] \begin{pmatrix} \psi_{R\uparrow} \\ \psi_{L\downarrow} \end{pmatrix}$$

$$H_{FM} = \int dx \left(\psi_{R\uparrow}^\dagger, \psi_{L\downarrow}^\dagger \right) \vec{m}(x) \cdot \vec{\sigma} \begin{pmatrix} \psi_{R\uparrow} \\ \psi_{L\downarrow} \end{pmatrix}$$

$$H_{SC} = \int dx \left[\Delta(x) \psi_{R\uparrow}^\dagger \psi_{L\downarrow}^\dagger + \Delta^*(x) \psi_{L\downarrow} \psi_{R\uparrow} \right]$$



BdG Hamiltonian



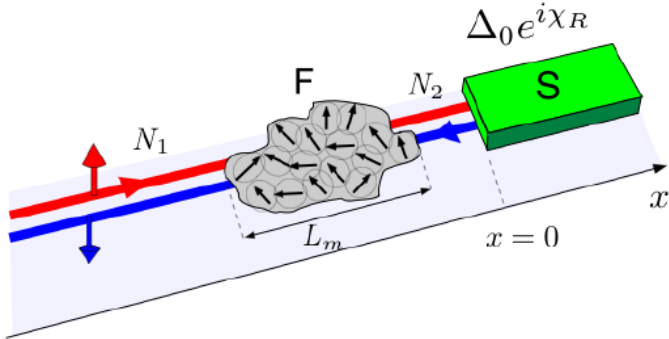
$$H = \frac{1}{2} \int dx \Psi^\dagger H_{BdG} \Psi$$

$$\Psi^\dagger = (\psi_{R\uparrow}^\dagger, \psi_{L\downarrow}^\dagger, \psi_{L\downarrow}, -\psi_{R\uparrow})$$

$$H_{BdG} = \begin{pmatrix} H_{0+FM}^e & \Delta(x) \sigma_0 \\ \Delta^*(x) \sigma_0 & H_{0+FM}^h \end{pmatrix}$$



BdG Hamiltonian



$$H = \frac{1}{2} \int dx \Psi^\dagger H_{BdG} \Psi$$

$$\Psi^\dagger = (\psi_{R\uparrow}^\dagger, \psi_{L\downarrow}^\dagger, \psi_{L\downarrow}, -\psi_{R\uparrow})$$

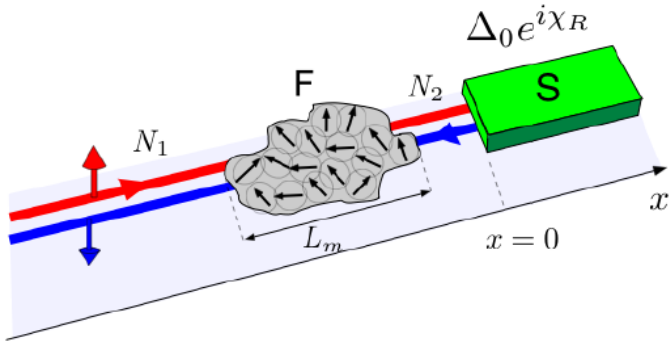
$$H_{0+FM}^e = v_F \sigma_z p_x - \mu \sigma_0 + \vec{m}(x) \cdot \vec{\sigma}$$

$$H_{BdG} = \begin{pmatrix} H_{0+FM}^e & \Delta(x) \sigma_0 \\ \Delta^*(x) \sigma_0 & H_{0+FM}^h \end{pmatrix}$$

$$H_{0+FM}^h = -T H_{0+FM}^e T^{-1}$$



BdG Hamiltonian



$$H = \frac{1}{2} \int dx \Psi^\dagger H_{BdG} \Psi$$

$$\Psi^\dagger = (\psi_{R\uparrow}^\dagger, \psi_{L\downarrow}^\dagger, \psi_{L\downarrow}, -\psi_{R\uparrow})$$

$$H_{BdG} = \begin{pmatrix} H_{0+FM}^e & \Delta(x) \sigma_0 \\ \Delta^*(x) \sigma_0 & H_{0+FM}^h \end{pmatrix}$$

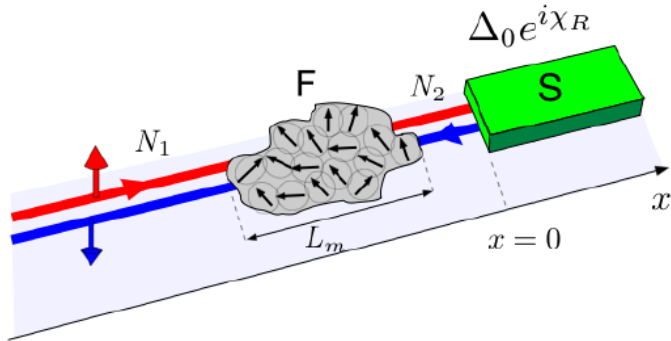
$$H = \sum_{\varepsilon_n \geq 0, j} \varepsilon_n \gamma_{\varepsilon_n, j}^\dagger \gamma_{\varepsilon_n, j}$$

$$\Psi(x) = \sum_{\varepsilon_n \geq 0, j} \left\{ \varphi_{\varepsilon_n, j}(x) \gamma_{\varepsilon_n, j} + [C\varphi_{\varepsilon_n, j}](x) \gamma_{\varepsilon_n, j}^\dagger \right\}$$

charge conjugated wave function



Symmetries & Majoranas



Particle-hole symmetry

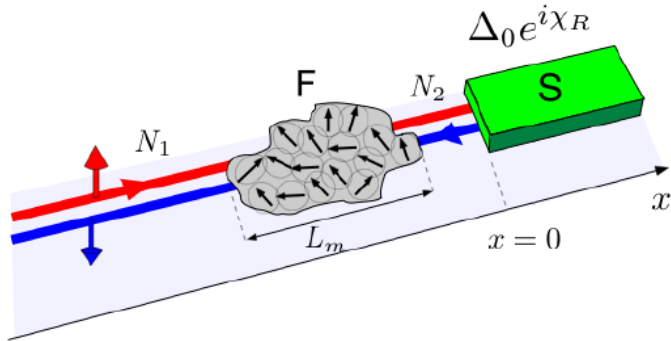
$$C H_{BdG} C^{-1} = -H_{BdG}$$

$$C = K \tau_y \otimes \sigma_y = K U_C$$

$$\gamma_{\varepsilon_n, j}^\dagger = \gamma_{-\varepsilon_n, j}$$



Symmetries & Majoranas



Particle-hole symmetry

$$C H_{BdG} C^{-1} = -H_{BdG}$$

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Majorana fermion



Majorana fermions vs. anyons



Majorana fermions

$$\Psi(x) = \sum_{\varepsilon_n \geq 0, j} \left\{ \varphi_{\varepsilon_n, j}(x) \gamma_{\varepsilon_n, j} + [C\varphi_{\varepsilon_n, j}](x) \gamma_{\varepsilon_n, j}^\dagger \right\}$$

fermions

$$\Psi^\dagger(x) = U_C \Psi(x)$$

$$\gamma_{\varepsilon_n, j}^\dagger = \gamma_{-\varepsilon_n, j}$$



Majorana fermions vs. anyons



Majorana fermions

$$\Psi(x) = \sum_{\varepsilon_n \geq 0, j} \left\{ \varphi_{\varepsilon_n, j}(x) \gamma_{\varepsilon_n, j} + [C \varphi_{\varepsilon_n, j}](x) \gamma_{\varepsilon_n, j}^\dagger \right\}$$

fermions

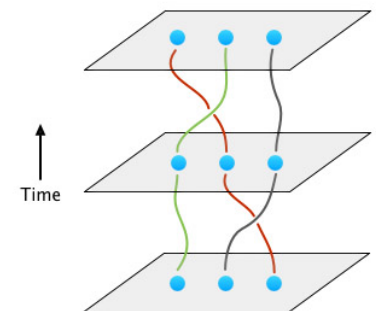
$$\Psi^\dagger(x) = U_C \Psi(x)$$

$$\gamma_{\varepsilon_n, j}^\dagger = \gamma_{-\varepsilon_n, j}$$

Majorana bound states

$$\gamma_{0, j}^\dagger = \gamma_{0, j}$$

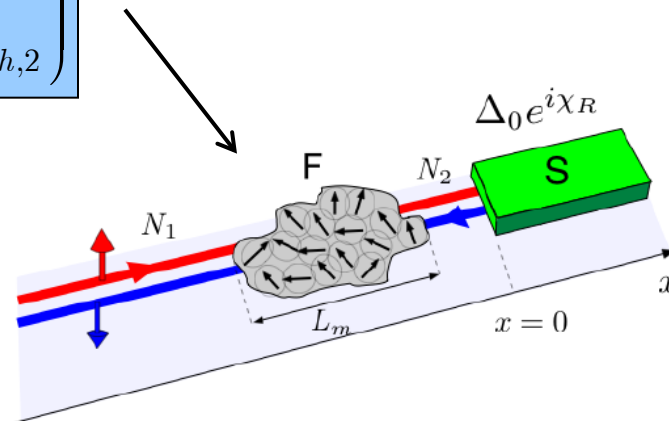
anyons





S-matrix construction

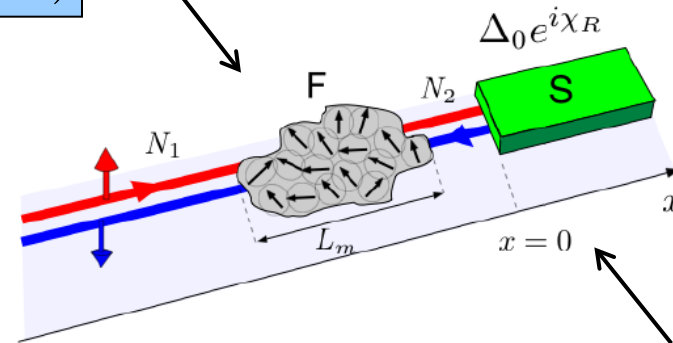
$$\begin{pmatrix} b_{e,1} \\ b_{e,2} \\ b_{h,1} \\ b_{h,2} \end{pmatrix} = \begin{pmatrix} S_0^e(\varepsilon) & 0 \\ 0 & S_0^h(\varepsilon) \end{pmatrix} \begin{pmatrix} a_{e,1} \\ a_{e,2} \\ a_{h,1} \\ a_{h,2} \end{pmatrix}$$





S-matrix construction

$$\begin{pmatrix} b_{e,1} \\ b_{e,2} \\ b_{h,1} \\ b_{h,2} \end{pmatrix} = \begin{pmatrix} S_0^e(\varepsilon) & 0 \\ 0 & S_0^h(\varepsilon) \end{pmatrix} \begin{pmatrix} a_{e,1} \\ a_{e,2} \\ a_{h,1} \\ a_{h,2} \end{pmatrix}$$



perfect AR

$$\begin{pmatrix} a_{e,2} \\ a_{h,2} \end{pmatrix} = \exp\left(-i \arccos\left(\frac{\varepsilon}{\Delta_0}\right)\right) \begin{pmatrix} 0 & e^{i\chi_R} \\ e^{-i\chi_R} & 0 \end{pmatrix} \begin{pmatrix} b_{e,2} \\ b_{h,2} \end{pmatrix}$$



S-matrix of FM domain

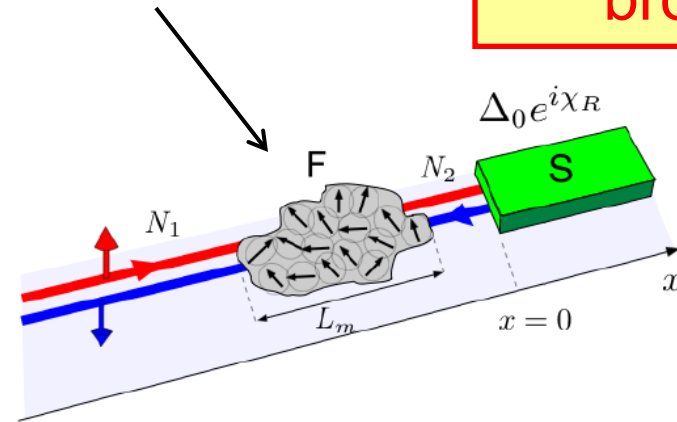
$$\Gamma_m(\varepsilon) \sim k_F L_m$$

$$\Phi_m(\varepsilon) \sim k_F x_0$$

$$S_0^e(\varepsilon) = e^{i\Gamma_m(\varepsilon)} \begin{pmatrix} -ie^{i\Phi_m(\varepsilon)} \sqrt{1-T_\varepsilon} & e^{i\chi_m(\varepsilon)} \sqrt{T_\varepsilon} \\ e^{-i\chi_m(\varepsilon)} \sqrt{T_\varepsilon} & -ie^{-i\Phi_m(\varepsilon)} \sqrt{1-T_\varepsilon} \end{pmatrix}$$

$$\chi_m(\varepsilon) \sim m_z L_m$$

all symmetries
broken

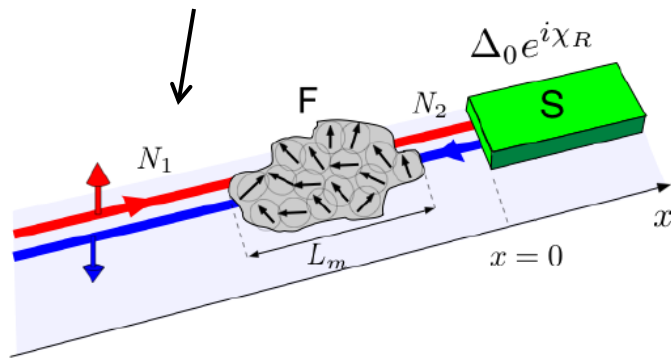




Andreev reflection

$$\begin{pmatrix} b_{e,1} \\ b_{h,1} \end{pmatrix} = \begin{pmatrix} r_{ee} & r_{eh} \\ r_{he} & r_{hh} \end{pmatrix} \begin{pmatrix} a_{e,1} \\ a_{h,1} \end{pmatrix}$$

$$R_A = |r_{eh}|^2 = |r_{he}|^2$$

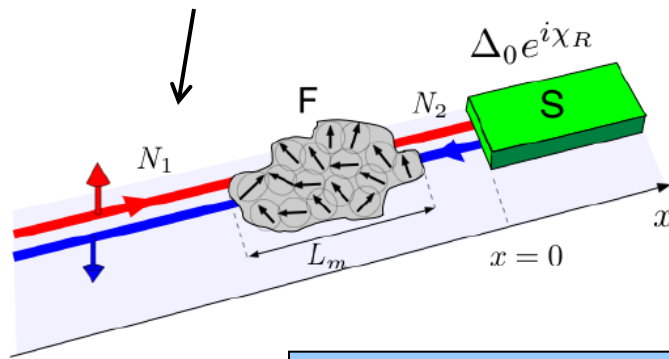




Andreev reflection

$$\begin{pmatrix} b_{e,1} \\ b_{h,1} \end{pmatrix} = \begin{pmatrix} r_{ee} & r_{eh} \\ r_{he} & r_{hh} \end{pmatrix} \begin{pmatrix} a_{e,1} \\ a_{h,1} \end{pmatrix}$$

$$R_A = |r_{eh}|^2 = |r_{he}|^2$$



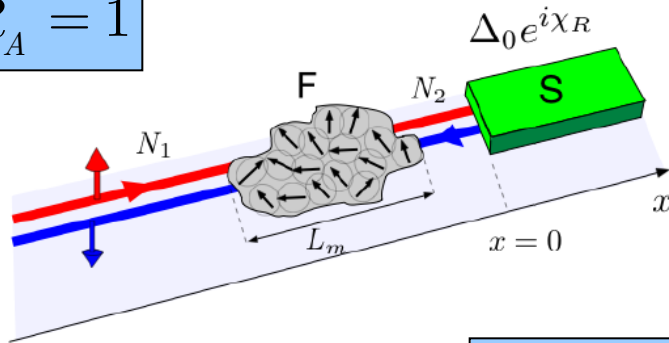
$$\Phi_m^A(\varepsilon) = \frac{1}{2}(\Phi_m(\varepsilon) - \Phi_m(-\varepsilon))$$

$$R_A = \frac{T_\varepsilon T_{-\varepsilon}}{\left(1 - \sqrt{R_\varepsilon R_{-\varepsilon}}\right)^2 + 4 \cos^2 \left[\arccos \left(\frac{\varepsilon}{\Delta_0} \right) + \Phi_m^A(\varepsilon) \right] \sqrt{R_\varepsilon R_{-\varepsilon}}}$$



Resonance condition

$$R_A = 1$$

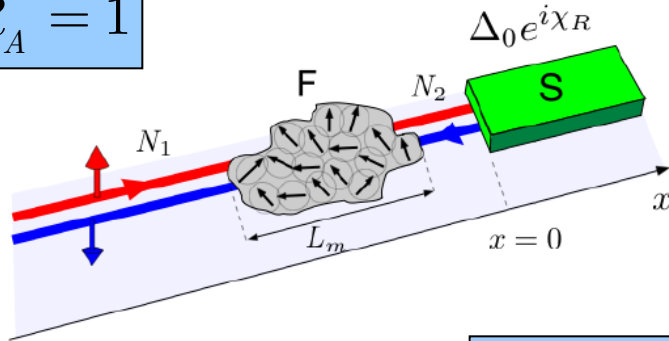


$$\left(\sqrt{R_\varepsilon} - \sqrt{R_{-\varepsilon}}\right)^2 + 4 \cos^2 \left[\arccos \left(\frac{\varepsilon}{\Delta_0} \right) + \Phi_m^A(\varepsilon) \right] \sqrt{R_\varepsilon R_{-\varepsilon}} = 0$$



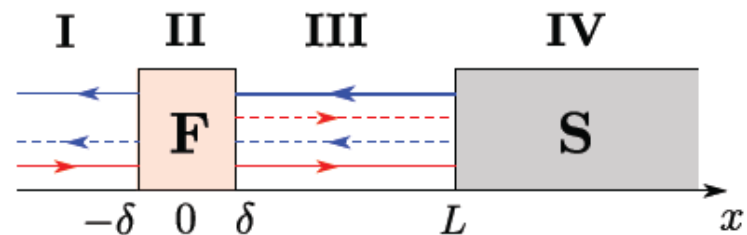
Resonance condition

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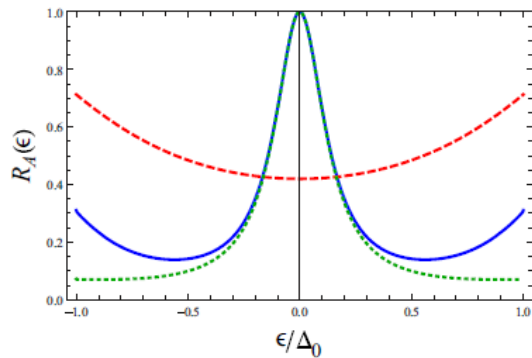
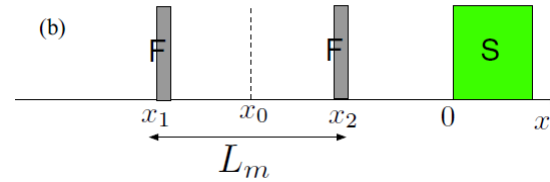
$$\left(\sqrt{R_\varepsilon} - \sqrt{R_{-\varepsilon}}\right)^2 + 4 \cos^2 \left[\arccos \left(\frac{\varepsilon}{\Delta_0} \right) + \Phi_m^A(\varepsilon) \right] \sqrt{R_\varepsilon R_{-\varepsilon}} = 0$$

**Fabry-Perot resonator
for electrons/holes**

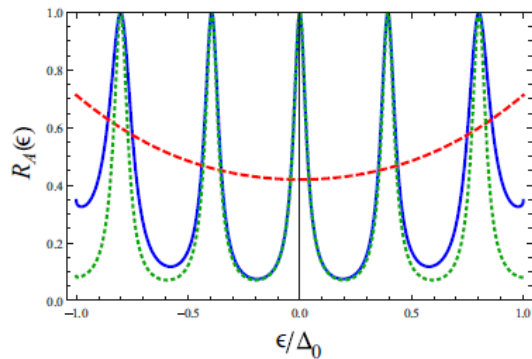




Detection of MBS



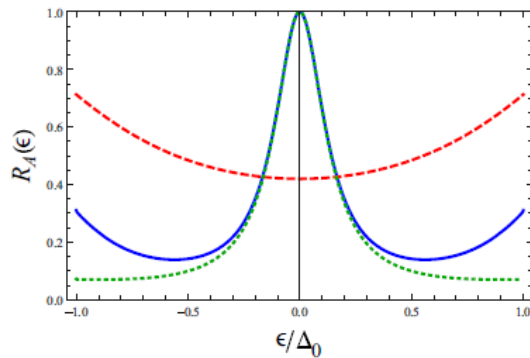
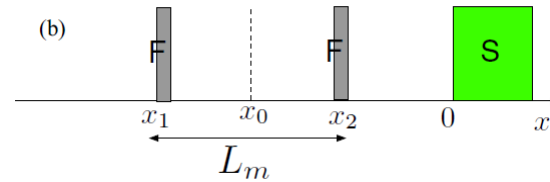
(a) $\mu_0 = 0.5, \mu = 0, L_m = 1, x_0 = 1.5, \Delta\phi = 0$



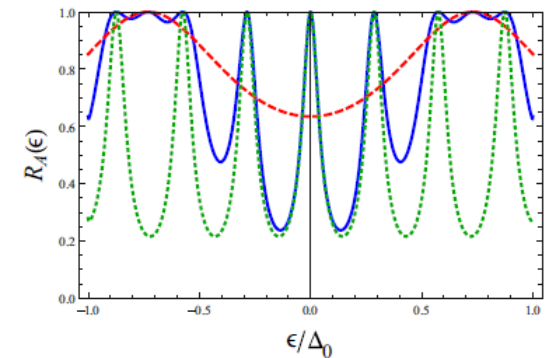
(b) $\mu_0 = 0.5, \mu = 0, L_m = 1, x_0 = 4.5, \Delta\phi = 0$



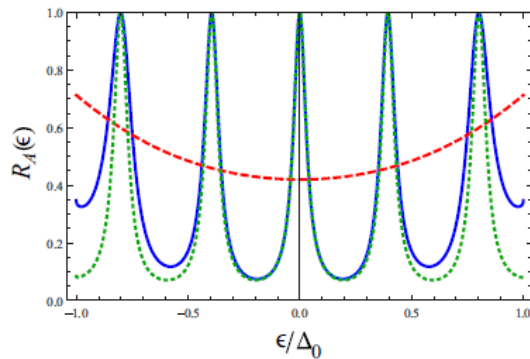
Detection of MBS



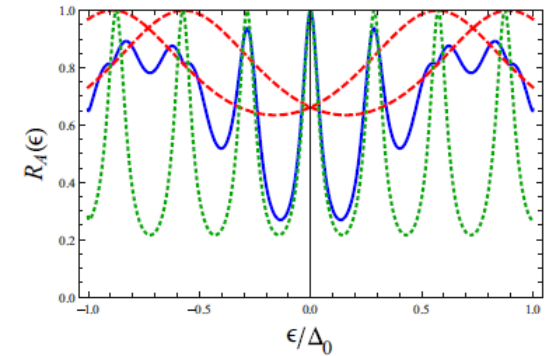
(a) $\mu_0 = 0.5, \mu = 0, L_m = 1, x_0 = 1.5, \Delta\phi = 0$



(a) $\mu_0 = 0.35, \mu = 0, L_m = 2.15, x_0 = 6$



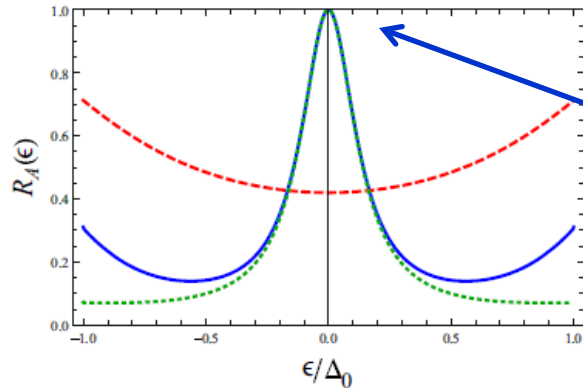
(b) $\mu_0 = 0.5, \mu = 0, L_m = 1, x_0 = 4.5, \Delta\phi = 0$



(b) $\mu_0 = 0.35, \mu = 0.16, L_m = 2.15, x_0 = 6$

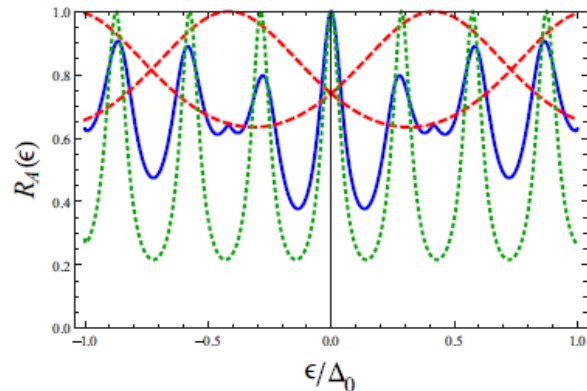
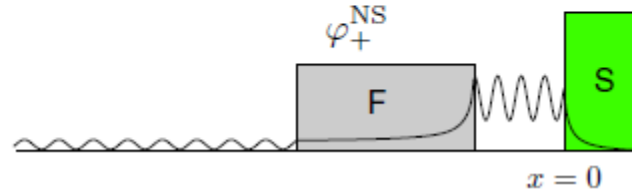
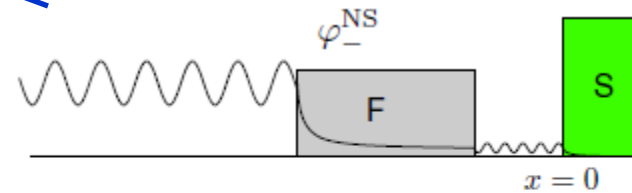


Robust MBS signature



(a) $\mu_0 = 0.5, \mu = 0, L_m = 1, x_0 = 1.5, \Delta\phi = 0$

$$C\varphi_{\pm}^{NS} = \varphi_{\pm}^{NS}$$



(c) $\mu_0 = 0.35, \mu = 0.3, L_m = 2.15, x_0 = 6$

Majorana bound states

$$\epsilon = 0$$



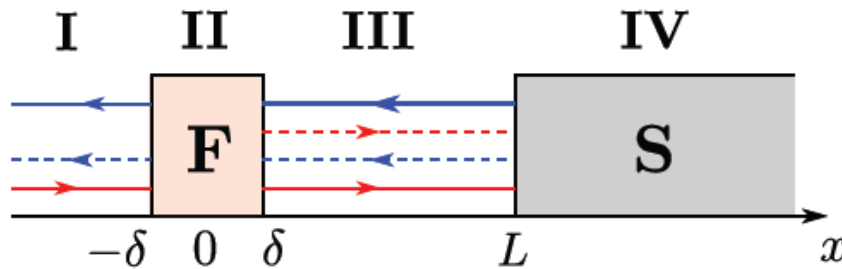
Outline

- Transport signatures of NS junctions -> Majorana bound states
- **Crossed Andreev reflection** in NSN setups -> odd-frequency triplet superconductivity



Model and setup

BdG Hamiltonian



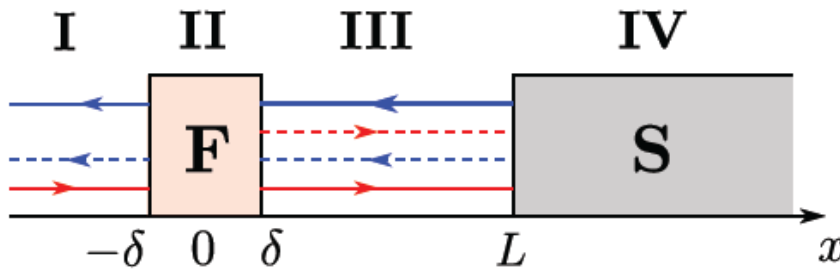
$$H = \frac{1}{2} \int dx \Psi^\dagger H_{BdG} \Psi$$

$$\Psi^\dagger = (\psi_{R\uparrow}^\dagger, \psi_{L\downarrow}^\dagger, \psi_{L\downarrow}, -\psi_{R\uparrow})$$



Model and setup

BdG Hamiltonian



$$H = \frac{1}{2} \int dx \Psi^\dagger H_{BdG} \Psi$$

$$\Psi^\dagger = (\psi_{R\uparrow}^\dagger, \psi_{L\downarrow}^\dagger, \psi_{L\downarrow}, -\psi_{R\uparrow})$$

spin space

$$H_{BdG} = \left(-iv_F \partial_x \sigma_z - \mu \right) \tau_z + \vec{m}(x) \cdot \vec{\sigma} + \Delta_1(x) \tau_x + \Delta_2(x) \tau_y$$

particle-hole space



Green's functions

$$r = (x, t)$$

$$G^R(r, r') = -i\theta(t - t') \langle \{ \Psi(r), \Psi^\dagger(r') \} \rangle$$

$$G^A(r, r') = i\theta(t - t') \langle \{ \Psi(r), \Psi^\dagger(r') \} \rangle$$

$$G^M(x, \tau, x', \tau') = - \langle T_\tau \Psi(x, \tau) \Psi^\dagger(x', \tau') \rangle$$



Green's functions

$$r = (x, t)$$

$$G^R(r, r') = -i\theta(t - t') \langle \{ \Psi(r), \Psi^\dagger(r') \} \rangle$$

$$G^A(r, r') = i\theta(t - t') \langle \{ \Psi(r), \Psi^\dagger(r') \} \rangle$$

$$G^M(x, \tau, x', \tau') = - \langle T_\tau \Psi(x, \tau) \Psi^\dagger(x', \tau') \rangle$$

$$G^X = \begin{pmatrix} G_{ee}^X & G_{eh}^X \\ G_{he}^X & G_{hh}^X \end{pmatrix}$$

4x4 matrix

$$G_{eh}^X = \begin{pmatrix} \left[G_{eh}^X \right]_{\uparrow\downarrow} & \left[G_{eh}^X \right]_{\uparrow\uparrow} \\ \left[G_{eh}^X \right]_{\downarrow\downarrow} & \left[G_{eh}^X \right]_{\downarrow\uparrow} \end{pmatrix}$$



Scattering states: N side

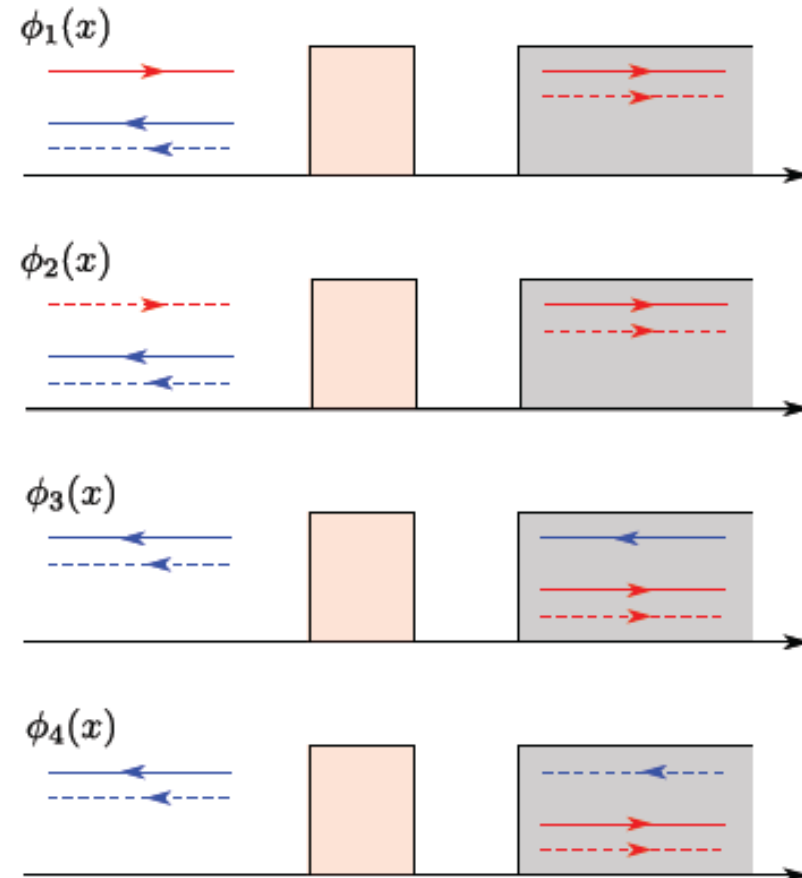
$$\phi_1(x) = \phi_e^{(+)} e^{ik_e x} + a_1 \phi_h^{(-)} e^{ik_h x} + b_1 \phi_e^{(-)} e^{-ik_e x}$$

$$\phi_2(x) = \phi_h^{(+)} e^{-ik_e x} + b_2 \phi_h^{(-)} e^{ik_h x} + a_2 \phi_e^{(-)} e^{-ik_e x}$$

$$\phi_3(x) = c_3 \phi_e^{(-)} e^{-ik_e x} + d_3 \phi_h^{(-)} e^{ik_h x}$$

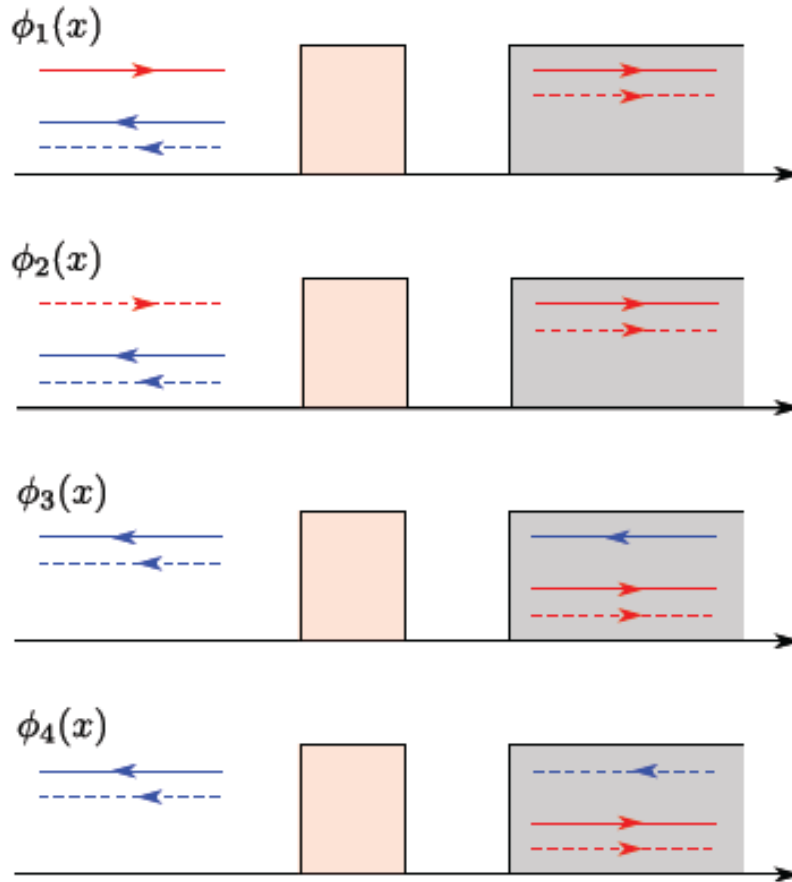
$$\phi_4(x) = d_4 \phi_e^{(-)} e^{-ik_e x} + c_4 \phi_h^{(-)} e^{ik_h x}$$

$$k_e = \mu + \varepsilon \quad k_h = \mu - \varepsilon$$





Scattering states: S side



$$\phi_1(x) = c_1 \chi_e^{(+)} e^{ik_e^S x} + d_1 \chi_h^{(+)} e^{-ik_h^S x}$$

$$\phi_2(x) = d_2 \chi_e^{(+)} e^{ik_e^S x} + c_2 \chi_h^{(+)} e^{-ik_h^S x}$$

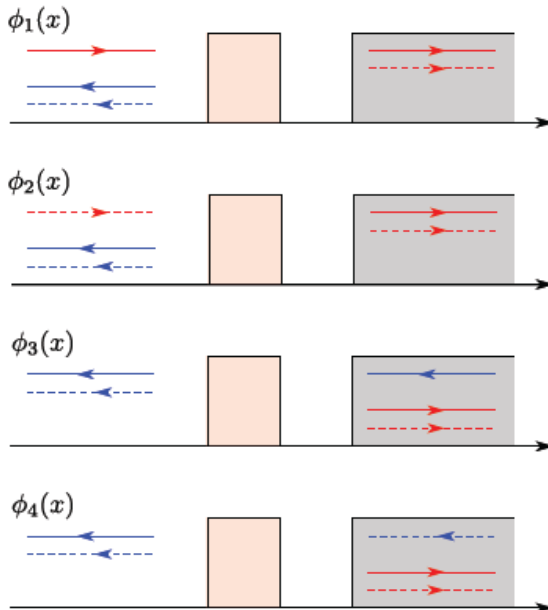
$$\phi_3(x) = \chi_e^{(-)} e^{-ik_e^S x} + a_3 \chi_h^{(+)} e^{-ik_e^S x} + b_3 \chi_h^{(+)} e^{ik_e^S x}$$

$$\phi_4(x) = \chi_h^{(-)} e^{ik_h^S x} + b_4 \chi_h^{(+)} e^{-ik_h^S x} + a_4 \chi_e^{(+)} e^{ik_e^S x}$$

$$k_e^S = \mu + \sqrt{\varepsilon^2 - \Delta^2} \quad k_h^S = \mu - \sqrt{\varepsilon^2 - \Delta^2}$$



Green's functions from scattering states



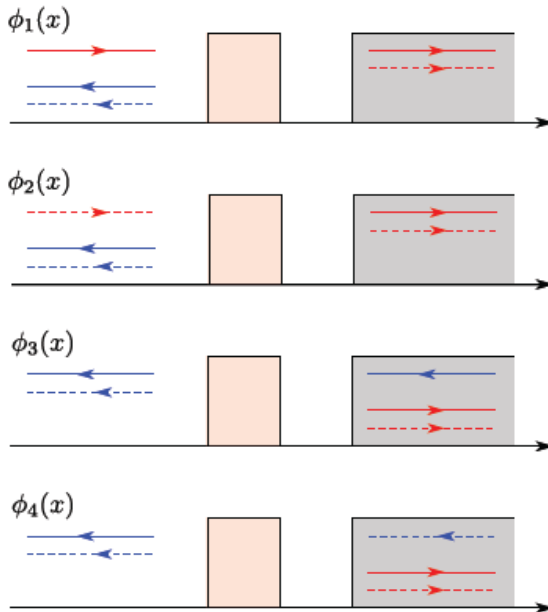
$$G_{\omega}^R(x, x') = \int dt e^{i(\omega + i\eta)(t-t')} G^R(x, t, x', t')$$

$$\left[\omega - H_{BdG}(x) \right] G_{\omega}^R(x, x') = \delta(x - x')$$

$$\lim_{\varepsilon \rightarrow 0} \left[G_{\omega}^R(x + \varepsilon, x) - G_{\omega}^R(x - \varepsilon, x) \right] = \frac{1}{i v_F} \sigma_z \tau_z$$



Green's functions from scattering states



$$G_{\omega}^R(x, x') = \int dt e^{i(\omega + i\eta)(t-t')} G^R(x, t, x', t')$$

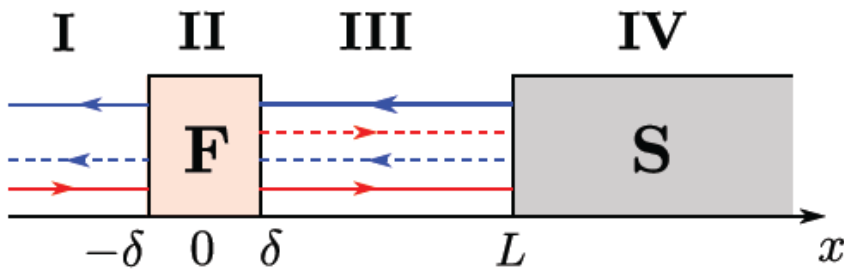
$$\left[\omega - H_{BdG}(x) \right] G_{\omega}^R(x, x') = \delta(x - x')$$

$$\lim_{\varepsilon \rightarrow 0} \left[G_{\omega}^R(x + \varepsilon, x) - G_{\omega}^R(x - \varepsilon, x) \right] = \frac{1}{i v_F} \sigma_z \tau_z$$

$$G_{\omega}^R(x, x') = \begin{cases} \phi_3(x) A_3(x')^T + \phi_4(x) A_4(x')^T & \text{if } x < x' \\ \phi_1(x) A_1(x')^T + \phi_2(x) A_2(x')^T & \text{if } x > x' \end{cases}$$



Pairing amplitude



Green's function

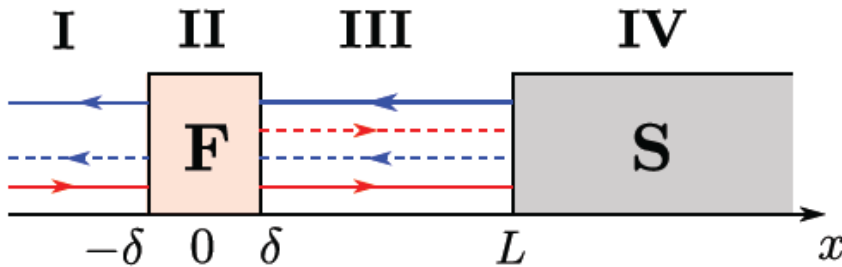
$$G^R = \begin{pmatrix} G_{ee}^R & G_{eh}^R \\ G_{he}^R & G_{hh}^R \end{pmatrix}$$

Pairing amplitude

$$F^R = G_{eh}^R i\sigma_2 = \begin{pmatrix} F_{\uparrow\uparrow}^R & F_{\uparrow\downarrow}^R \\ F_{\downarrow\uparrow}^R & F_{\downarrow\downarrow}^R \end{pmatrix}$$



Pairing amplitude



Green's function

$$G^R = \begin{pmatrix} G_{ee}^R & G_{eh}^R \\ G_{he}^R & G_{hh}^R \end{pmatrix}$$

Pairing amplitude

$$F^R = G_{eh}^R i\sigma_2 = \begin{pmatrix} F_{\uparrow\uparrow}^R & F_{\uparrow\downarrow}^R \\ F_{\downarrow\uparrow}^R & F_{\downarrow\downarrow}^R \end{pmatrix}$$

singlet

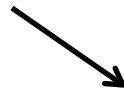
triplet

$$F^R(x, x', \omega) = \left[f_0^R(x, x', \omega) \sigma_0 + f_i^R(x, x', \omega) \sigma_i \right] i\sigma_2$$



Antisymmetry of pairing amplitude

singlet



triplet



$$F^R(x, x', \omega) = \left[f_0^R(x, x', \omega) \sigma_0 + f_i^R(x, x', \omega) \sigma_i \right] i\sigma_2$$

$$\begin{aligned} f_0^R(x, x', \omega) &= f_0^A(x', x, -\omega) \\ f_i^R(x, x', \omega) &= -f_i^A(x', x, -\omega) \end{aligned}$$



Classification of pairing amplitude

$$F^R(x, x', \omega) = \left[f_0^R(x, x', \omega) \sigma_0 + f_i^R(x, x', \omega) \sigma_i \right] i\sigma_2$$

orbital

$$\begin{aligned} f_0^R(x, x', \omega) &= \pm f_0^R(x', x, \omega) \\ f_i^R(x, x', \omega) &= \pm f_i^R(x', x, \omega) \end{aligned}$$



Classification of pairing amplitude

$$F^R(x, x', \omega) = \left[f_0^R(x, x', \omega) \sigma_0 + f_i^R(x, x', \omega) \sigma_i \right] i\sigma_2$$

orbital

$$\begin{aligned} f_0^R(x, x', \omega) &= \pm f_0^R(x', x, \omega) \\ f_i^R(x, x', \omega) &= \pm f_i^R(x', x, \omega) \end{aligned}$$

frequency

$$\begin{aligned} f_0^R(x, x', \omega) &= \pm f_0^A(x, x', -\omega) \\ f_i^R(x, x', \omega) &= \pm f_i^A(x, x', -\omega) \end{aligned}$$



Classification of pairing amplitude

$$F^R(x, x', \omega) = \left[f_0^R(x, x', \omega) \sigma_0 + f_i^R(x, x', \omega) \sigma_i \right] i\sigma_2$$

frequency, **spin**, orbital

even, **singlet**, even -> **ESE**

odd, **singlet**, odd -> **OSO**

even, **triplet**, odd -> **ETO**

odd, **triplet**, even -> **OTE**

orbital

$$f_0^R(x, x', \omega) = \pm f_0^R(x', x, \omega)$$

$$f_i^R(x, x', \omega) = \pm f_i^R(x', x, \omega)$$

frequency

$$f_0^R(x, x', \omega) = \pm f_0^A(x, x', -\omega)$$

$$f_i^R(x, x', \omega) = \pm f_i^A(x, x', -\omega)$$



Classification of pairing amplitude

$$F^R(x, x', \omega) = \left[f_0^R(x, x', \omega) \sigma_0 + f_i^R(x, x', \omega) \sigma_i \right] i\sigma_2$$

frequency, **spin**, orbital

even, **singlet**, even -> **ESE**

odd, **singlet**, odd -> **OSO**

even, **triplet**, odd -> **ETO**

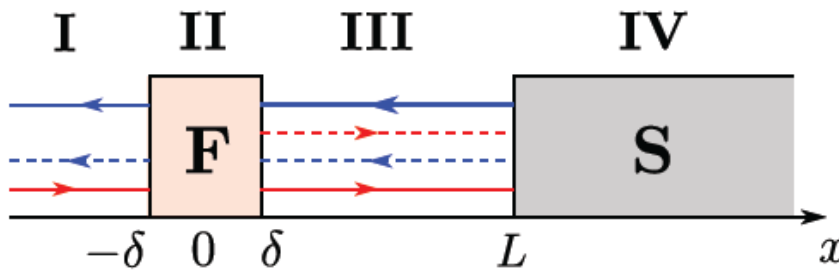
odd, **triplet**, even -> **OTE**

S and **T** mix if **spin rotational symmetry** is broken

(orbital) E and O mix if **inversion** is broken
-> NS junctions



Classification of pairing amplitude: results



frequency, **spin**, orbital

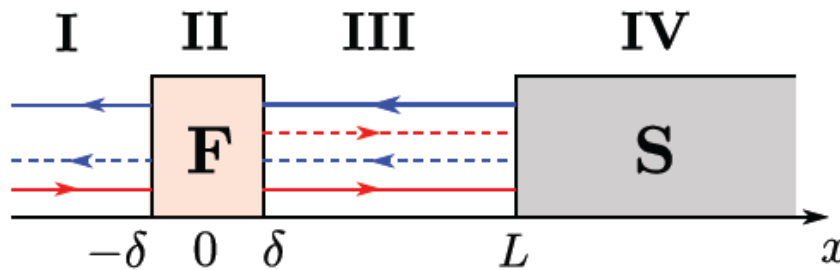
-> **ESE**, **OSO**, **ETO**, **OTE**

$$f_{\alpha}^R(x, x', \omega) = f_{\alpha, \text{bulk}}^R(x, x', \omega) + f_{\alpha, \text{edge}}^R(x, x', \omega)$$

	Pairing	Interface	Bulk
f_0	$\uparrow\downarrow - \downarrow\uparrow$	ESE+OSO	ESE
f_3	$\uparrow\downarrow + \downarrow\uparrow$	ETO+OTE	ETO
f_{\pm}	$\uparrow\uparrow, \downarrow\downarrow$	OTE	X



Classification of pairing amplitude: results



frequency, **spin**, orbital

-> **ESE**, **OSO**, **ETO**, **OTE**

$$f_{\alpha}^R(x, x', \omega) = f_{\alpha, \text{bulk}}^R(x, x', \omega) + f_{\alpha, \text{edge}}^R(x, x', \omega)$$

	Pairing	Interface	Bulk
f_0	$\uparrow\downarrow - \downarrow\uparrow$	ESE+OSO	ESE
f_3	$\uparrow\downarrow + \downarrow\uparrow$	ETO+OTE	ETO
f_{\pm}	$\uparrow\uparrow, \downarrow\downarrow$	OTE	X

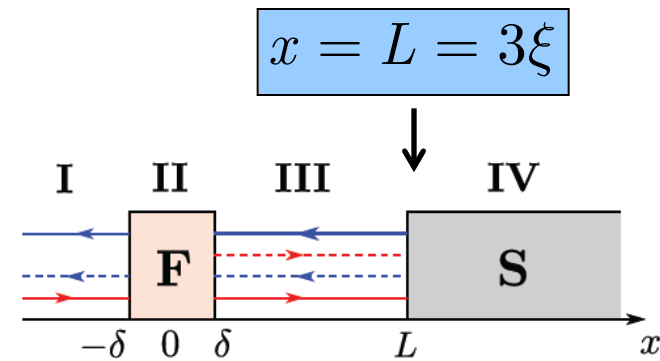
$$f_{\alpha, \text{bulk}}^R(x, x', \omega) \propto e^{-\kappa(\omega)|x-x'|}$$

$$f_{\alpha, \text{edge}}^R(x, x', \omega) \propto e^{-\kappa(\omega)(x+x')}$$

(in region IV)



Classification of pairing amplitude: results

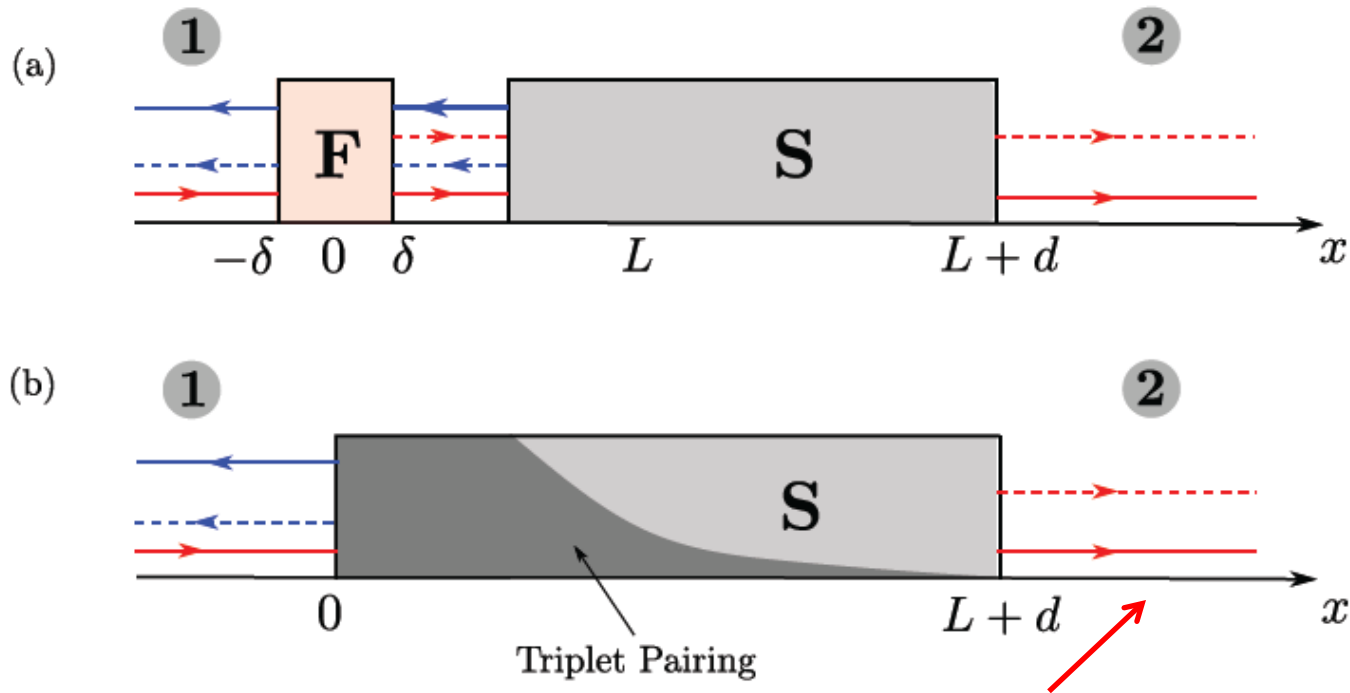


$$m_0 = 0.5\Delta$$

$$f_\alpha^R(x, x, \omega) = f_{\alpha, \text{bulk}}^R(x, x, \omega) + f_{\alpha, \text{edge}}^R(x, x, \omega)$$



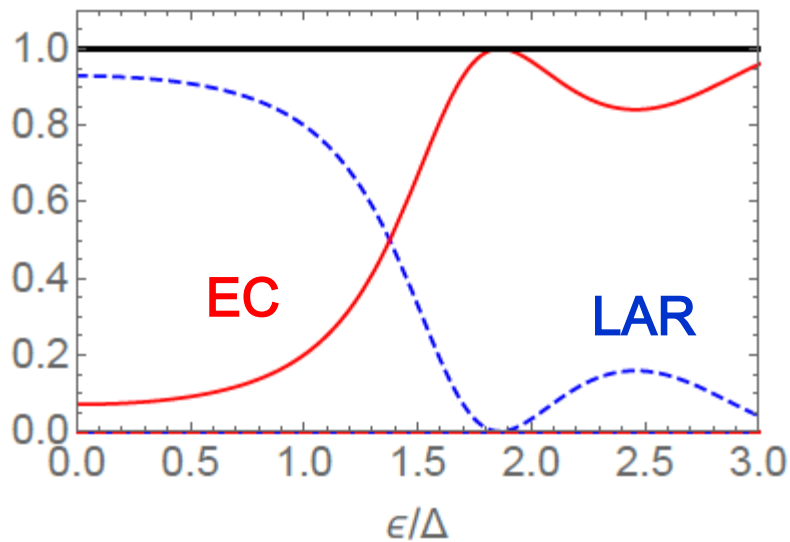
Detection of OTE: idea



use crossed Andreev reflection



Detection of OTE: results

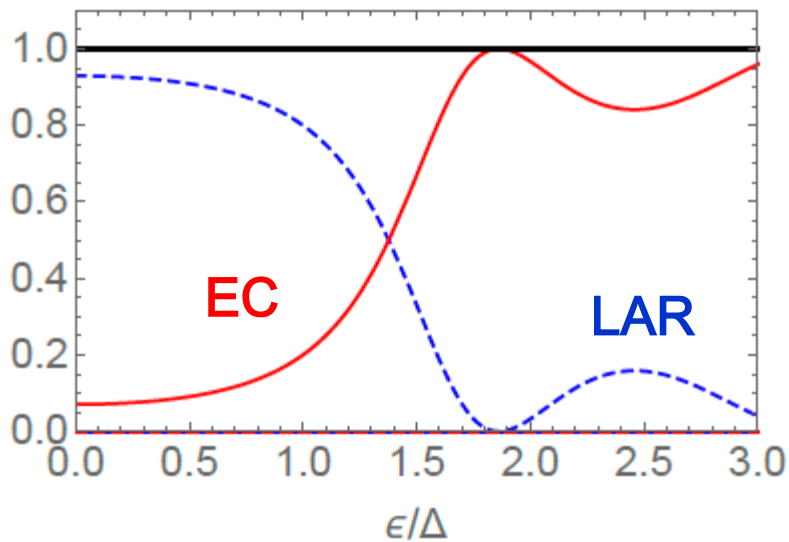


(a) $d = 2\xi$, $m_0 = 0$

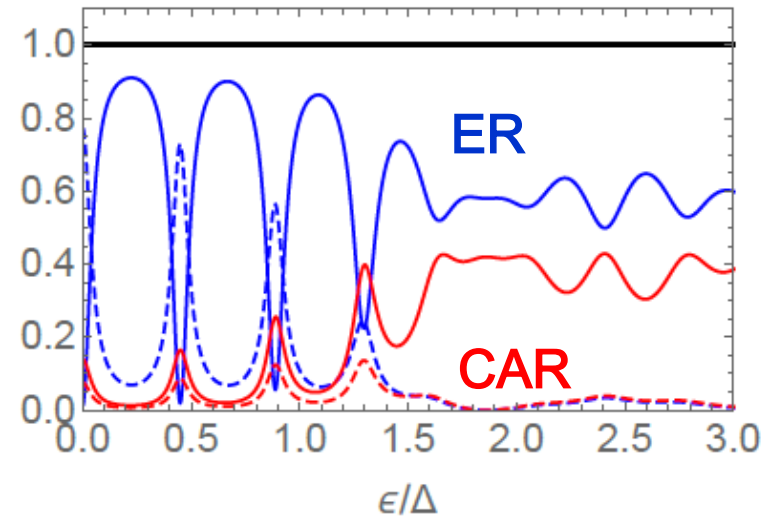
Adroguer et al. PRB 2010



Detection of OTE: results



(a) $d = 2\xi, m_0 = 0$

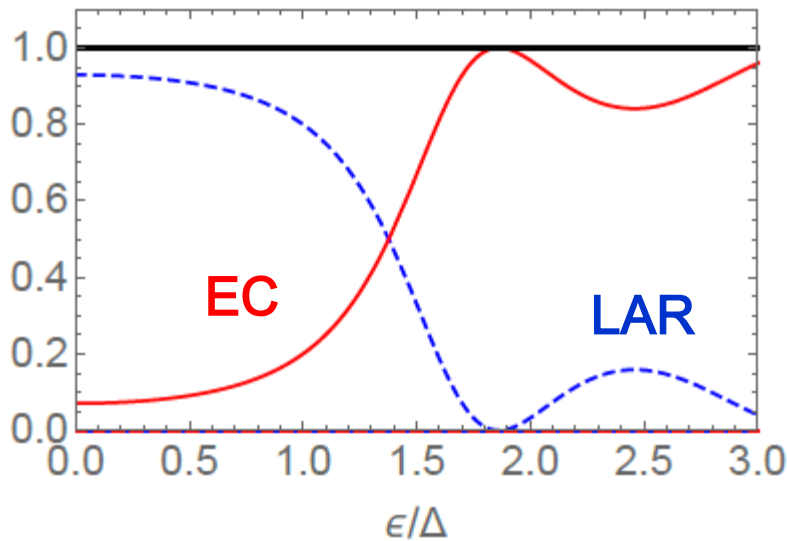


(c) $d = 2\xi, m_0 = 0.5, L = 3\xi$

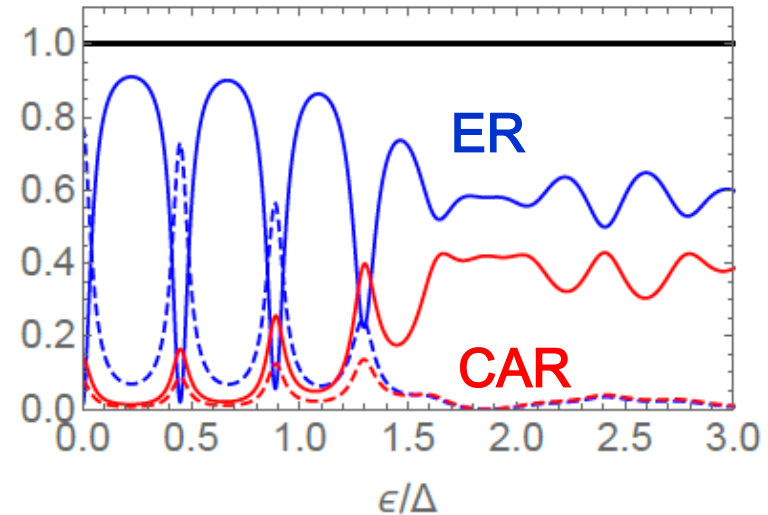
Adroguer et al. PRB 2010



Detection of OTE: results



(a) $d = 2\xi, m_0 = 0$



(c) $d = 2\xi, m_0 = 0.5, L = 3\xi$

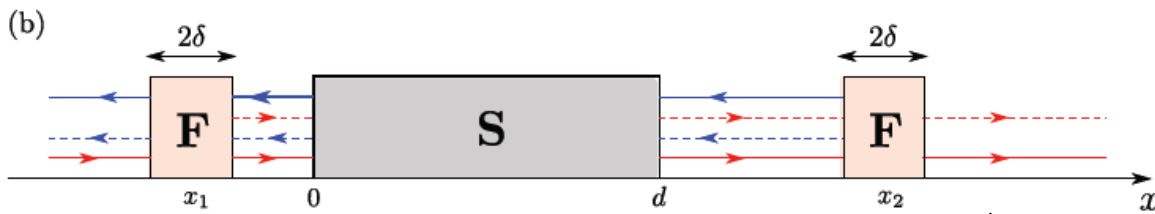
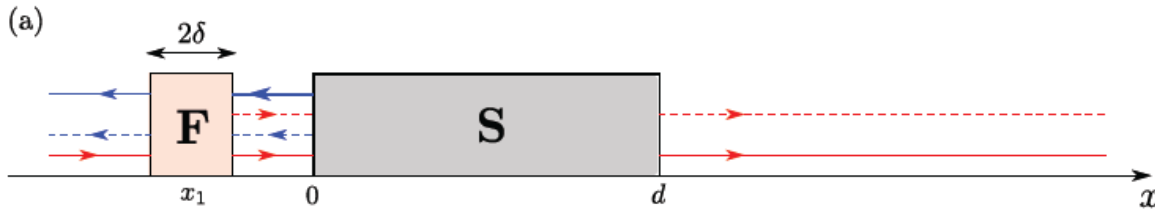
Adroguer et al. PRB 2010

-> dilemma

$$\frac{T_{CAR}}{T_{EC}} = \tanh^2(2m_0) \tanh^2\left(\frac{d}{\xi}\right) \leq 1$$

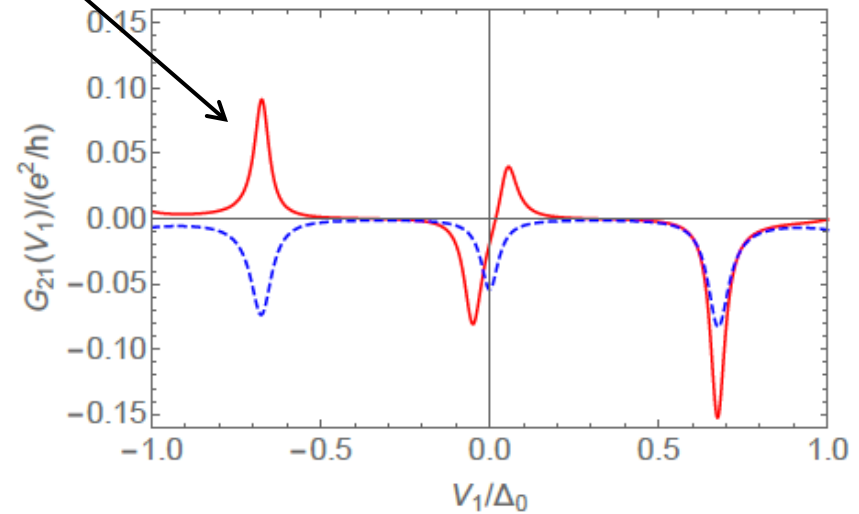


Way out ...



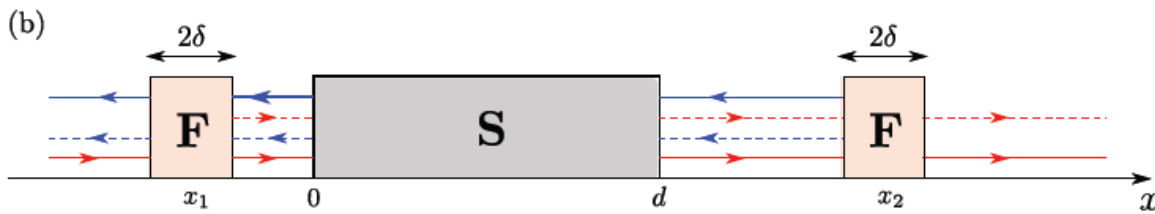
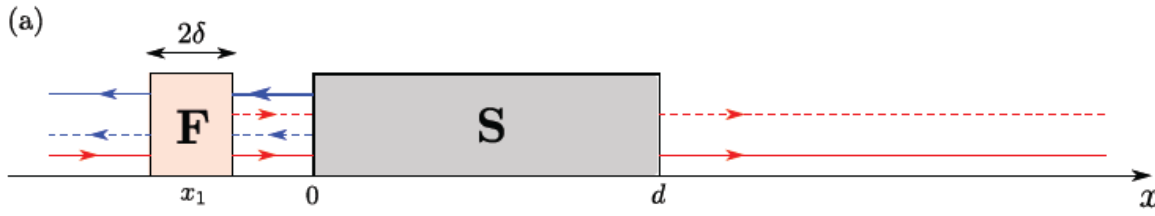
... add complexity

$$G_{21} = - \frac{\partial I_2}{\partial V_1} = T_{CAR} - T_{EC}$$





Way out ...

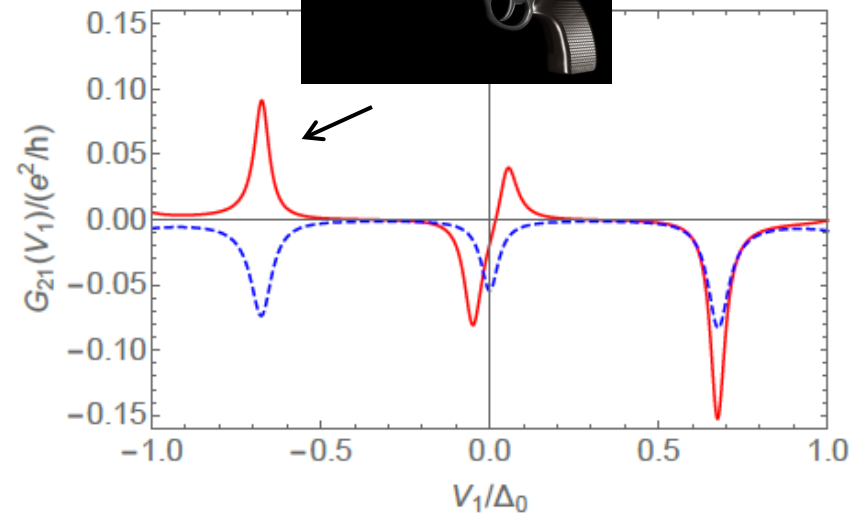


... add complexity



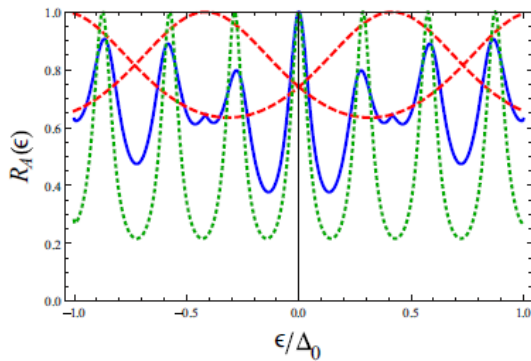
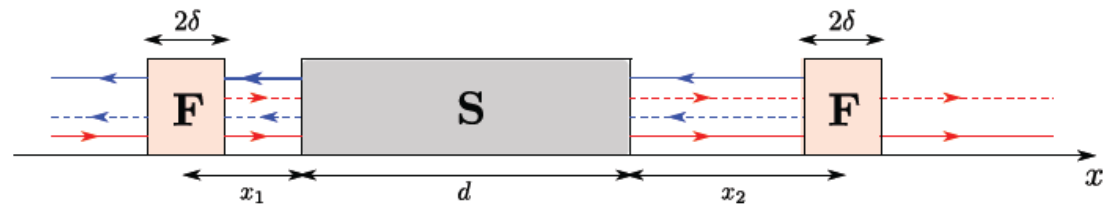
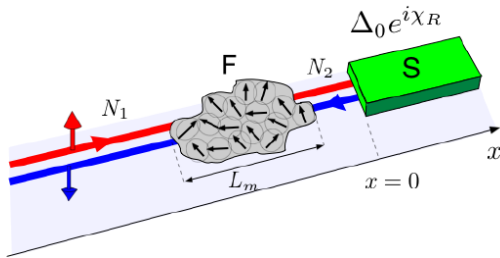
OTE

$$G_{21} = - \frac{\partial I_2}{\partial V_1} = T_{CAR} - T_{EC}$$





Summary



(c) $\mu_0 = 0.35, \mu = 0.3, L_m = 2.15, x_0 = 6$

