Interacting cold atoms on quasiperiodic lattices: dynamics and topological phases

Thursday, 3 July 2014

NHSCP2014 at ISSP, Univ. of Tokyo

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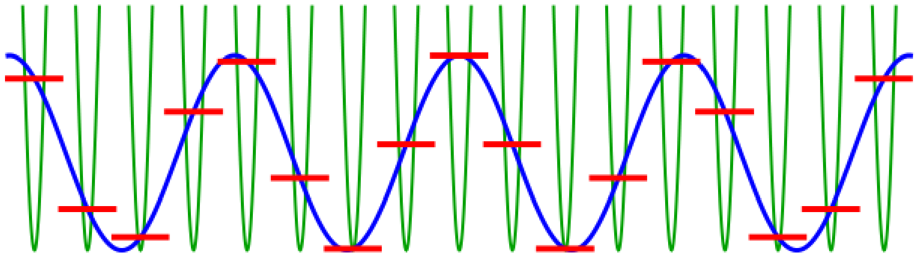
Quasiperiodic lattice



Many questions:

Free fermions ⇔ Interacting fermions

Hard-core bosons ⇔ Soft-core bosons

Cosine modulation ⇔ Two-value case

Trapped system ➔ Release dynamics?

Insulator ⇔ Superfluid

Topological features?

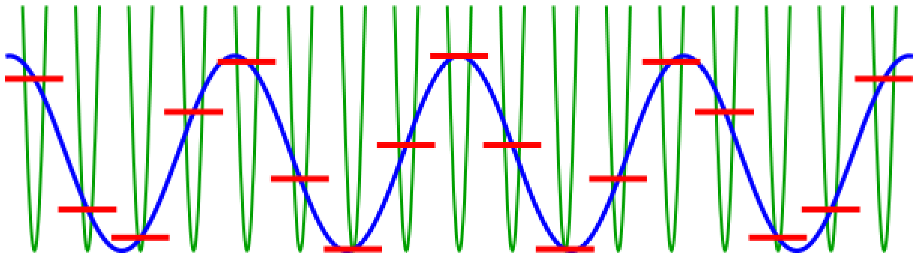
➔ Can be studied in highly tunable cold-atom systems

General motivation

- Well-defined, configurable inhomogeneity + interaction
  - New quantum phases?
  - Physics on transition line: universal exponents or not? ➔ dynamics

- Correspondence to higher dimensions: topological classification
  - 1D quasiperiodic ⬄ 2D regular lattice with magnetic field
  - *d* (>1)-dimensional quasicrystal ⬄ 2*d* (>3)-dimensional system?
    - Periodic table of topologically nontrivial phases: realization of e.g. *d*=4 system

Plan of the talk



Introduction: Interacting cold atoms on quasiperiodic lattices

1. Attractively interacting spin-1/2 fermions
   - Pairing enhanced by lattice deformation
   - Anomalous exponent after release from trap
   - ➢ MT and A. M. Garcia-Garcia: PRA **82**, 043613 (2010), PRA **85**, 031602R (2012)
2. Repulsively interacting spinless bosons
   - Topologically non-trivial incommensurate CDW phase
   - Equivalence between Harper-type and Fibonacci-type lattices
   - ➢ Fuyuki Matsuda, MT, and Norio Kawakami: arXiv:1404.6315 (JPSJ to appear)
3. Spin-1/2 fermions with proximity pairing
   - Spin-orbit coupling introduces a peculiar self-similar band structure
   - Reentrant topological transitions

     Correlation is not the main topic here, so if time allows…
   - ➢ MT and Norio Kawakami: PRB **85**, 140508R(2012), PRB **88**, 155428 (2013)

Introduction: quasiperiodic lattices

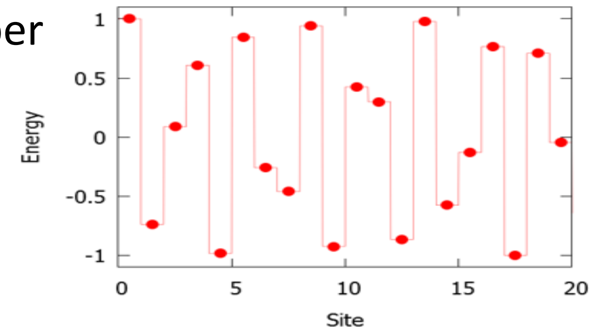
- Modulation of a tight-binding lattice with an incommensurate wavenumber
  - Here we focus on site level (diagonal) modulation
- 1D: Different modulating functions have been studied
  - Harper (or Aubry-Andre): $\cos(2\pi g j + \varphi)$
  - Fibonacci: + - + + - + + - + - + + - + + - + - + + - …
  - more complicated modulations allowing mobility edge [see e.g. Ribeiro *et al.*: PRA **87**, 043635 (2013); Wang *et al.*: 1312.0844], etc.
- Historically many theoretical papers
  - Mostly non-interacting case until recently
- Recent cold atom experiments
  - ➢ Interacting bosons and fermions

Diagonal modulation

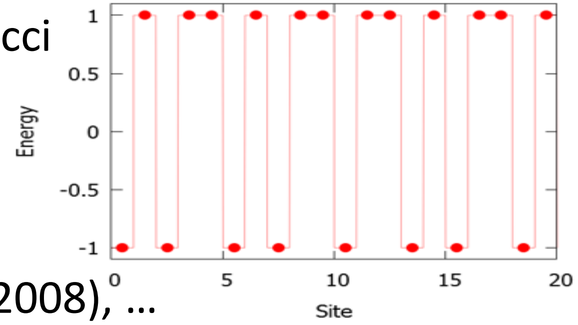
$$\hat{H} = -\sum_{\langle i,j \rangle} J_{ij} \left(\hat{c}_i^{\dagger} \hat{c}_j + \mathrm{H.c.} \right) + \sum_j \epsilon_j \hat{c}_j^{\dagger} \hat{c}_j$$

Off-diagonal modulation

Harper



Fibonacci

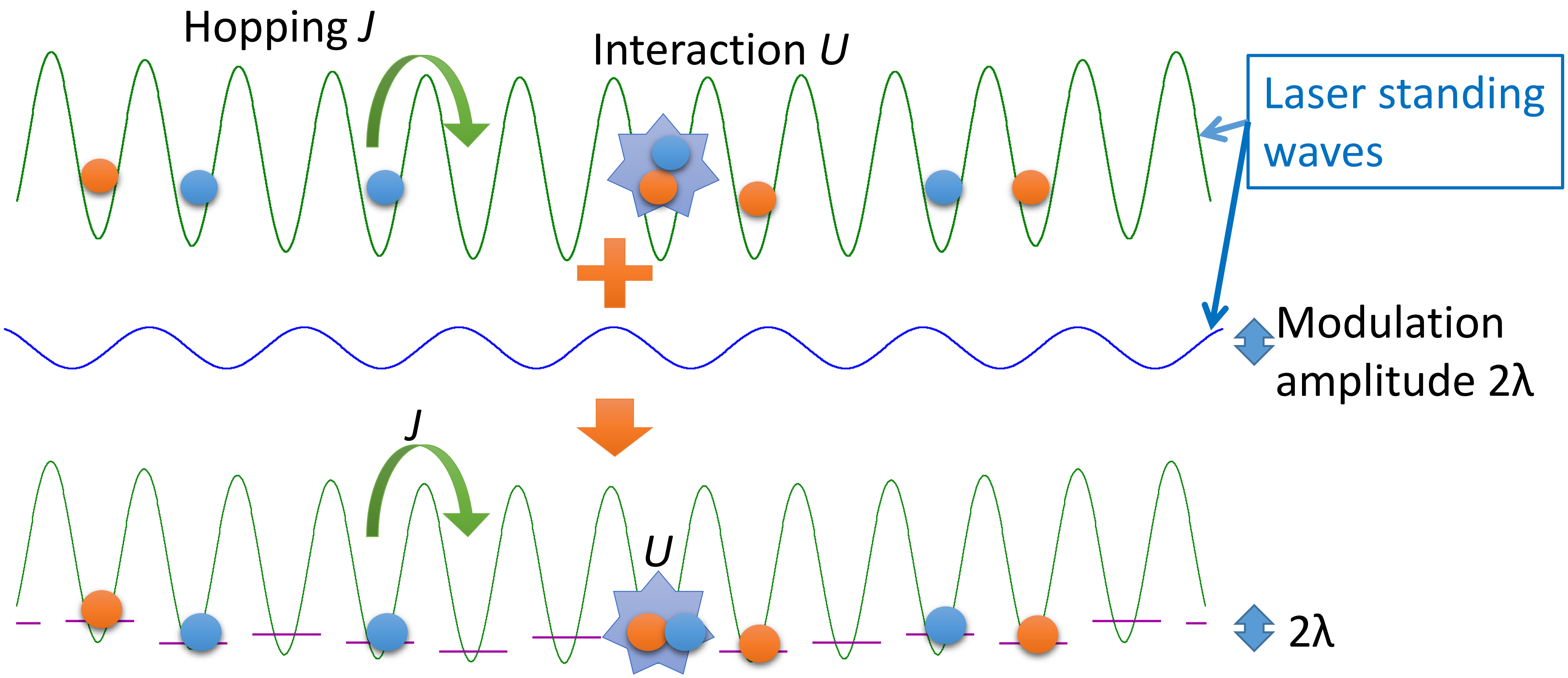


Roati *et al.*(Florence): Nature **453**, 895 (2008), …

cf. <u>temporal</u> quasiperiodicity [Gommers *et al.*: PRL **96**, 240604 (2006)]

Realization of quasiperiodic optical lattice for cold atoms

Bichromatic optical lattice



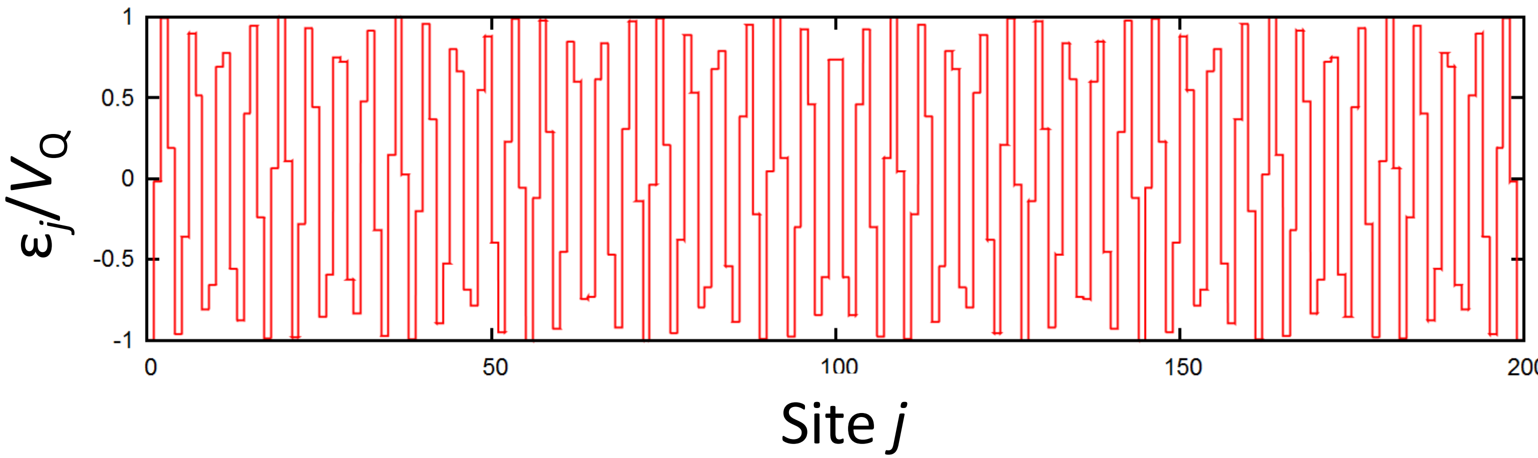
➔ Modelled by Hubbard model with Harper-type quasiperiodic site energy modulation

Theory (Bosons): X. Deng *et al.*: PRA **78**, 013625 (2008); G. Roux *et al.*: PRA **78**, 023628 (2008); …

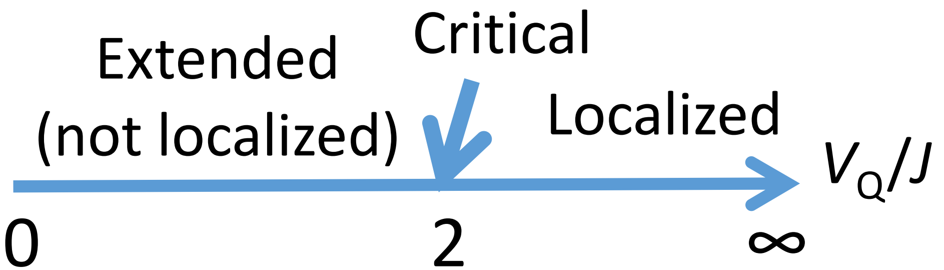
Harper model

Diagonal case:

$\varepsilon_j = V_Q \cos(2\pi g j)$ g: some irrational number (e.g. inverse golden ratio (√5-1)/2)



All single-particle levels:



Tight-binding model with hopping J [P. G. Harper: Proc. Phys. Soc. Sec. A **68**, 874 (1955)]

(Also known as the Andre – Aubry model) [Andre & Aubry: Ann. Isr. Phys. Soc. **3**, 133 (1980)]

All single-particle levels known to localize at self-dual point $V_Q = 2J$ [Kohmoto: PRL **51**, 1198 (1983)]

β→0

Smooth connection between Harper and Fibonacci types known

β→∞

$$V_j(\phi,\beta) = \frac{\tanh\beta[\cos(2\pi b j + \phi) - \cos\pi b]}{\tanh\beta}$$

cf. Fibonacci model (A→ABB, B→A) $\varepsilon_j = V_Q\, V(gj)$

$$V(z) = \begin{cases} -1 & (m - g < z \le m) \\ +1 & (m < x \le m + 1 - g) \end{cases}$$

All single-particle levels are critical regardless of V_Q

[Kohmoto, Kadanoff, and Tang: PRL **50**, 1870 (1983); Ostlund *et al.*: PRL **50**, 1873 (1983)]

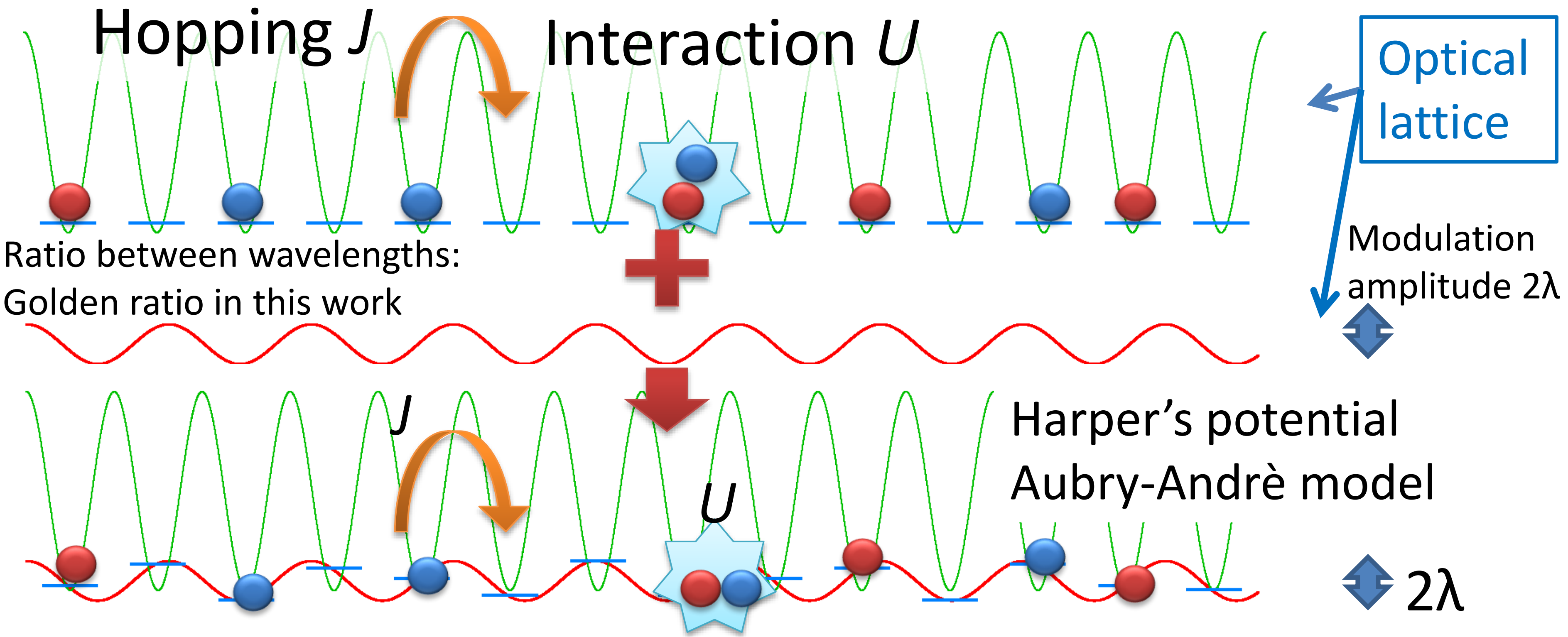
MT and A. M. Garcia-Garcia: PRA **82**, 043613 (2010), PRA **85**, 031602R (2012)

1. Attractively interacting spin-1/2 fermions

Motivation:

Pairing (Superfluid) $U(<0)$ → Dynamics at the transition point? ← Localization (insulator) Modulation λ

cf. Superconductor with disorder
e.g. Boron-doped diamond



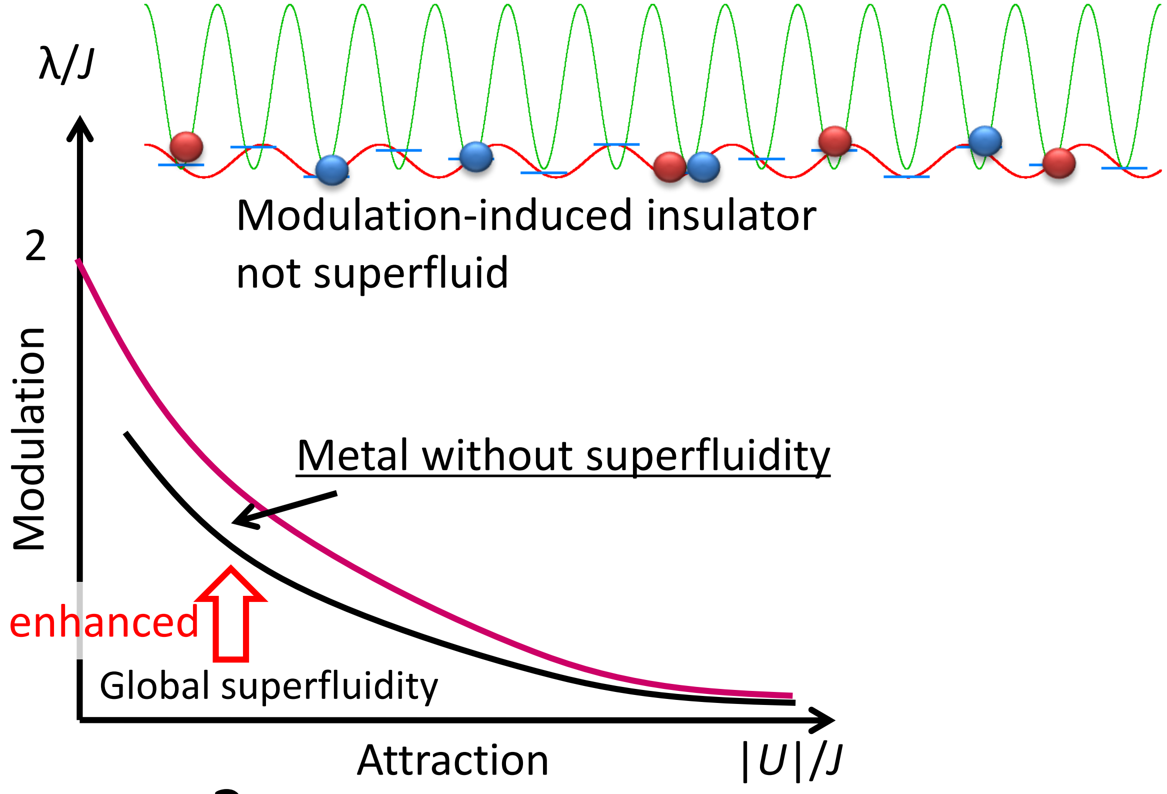
Schematic phase diagram

DMRG calculation of

- Inverse participation ratio (how much the fermions are delocalized)
- Pair structure factor (how slowly the pair correlation decay)

for different system sizes at the constant filling factor

- For strong interaction ($|U| \gg J$), pairing decreases as modulation amplitude λ is increased, and localizes at ~ insulating transition λ_c
- For weaker interaction ($|U|$~J), pairing has a peak as a function of λ, but localizes before λ_c



➔ Trap-release dynamics: diffusion process?

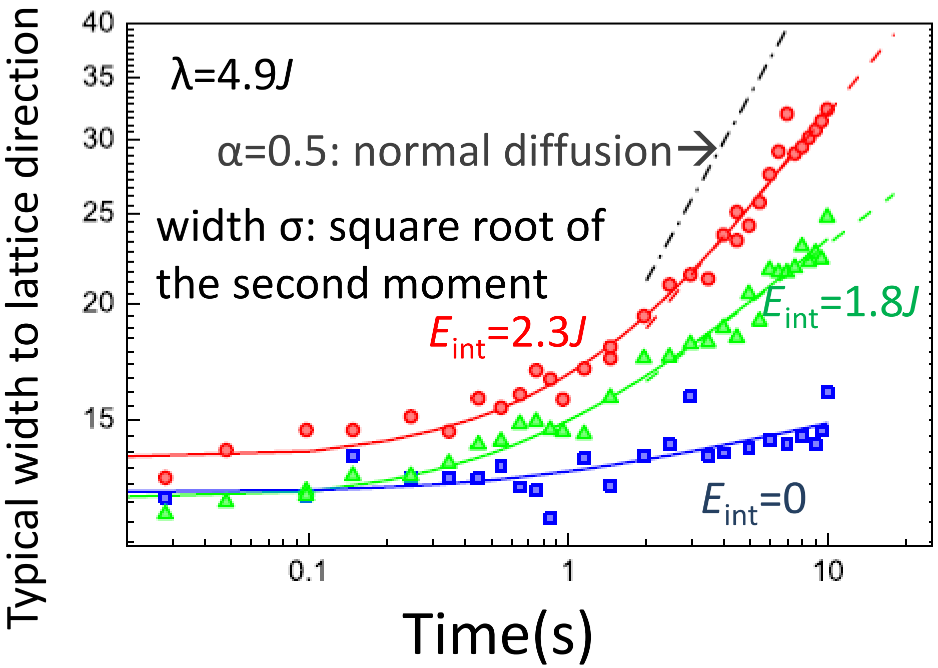
Dynamics: experiments with bosons

Trap-release experiments: dynamics of the atomic clouds observed

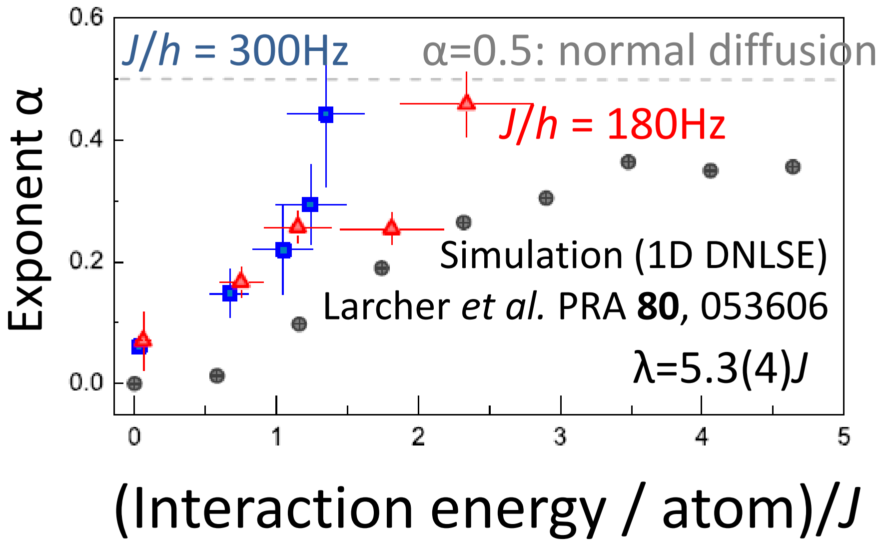
Bosons: E. Lucioni *et al.* (LENS, Florence): PRL **106**, 230403 (2011)

Subdiffusion (slower than random walk) observed in bichromatic lattice (3D)

$V(x) = V_1\cos^2(k_1x) + V_2\cos^2(k_2x)$, $k_1=2\pi/(1064.4\text{nm})$, $k_2=2\pi/(859.6\text{nm})$
50 thousand ^{39}K atoms, almost spherical trap switched off at $t=0$



$$\sigma(t) = \sigma_0(1 + t/t_0)^\alpha$$



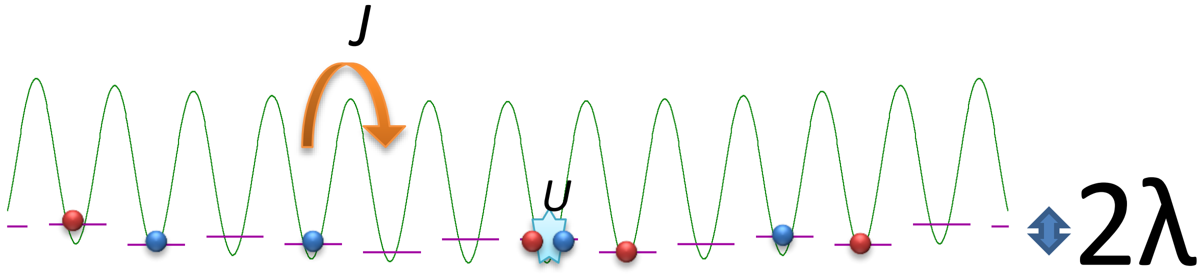
➔ What happens for interacting 1D fermions in a bichromatic potential?

Dependence on strength of attractive interation

Very weakly attractive ($|U| \ll W=4t$):
Modulation governs the conductance

Effect of modulation: relatively strong ($|U| << \lambda$)

Hopping not significantly renormalized



$\lambda_c < 2J$ but not much smaller

At transition point:

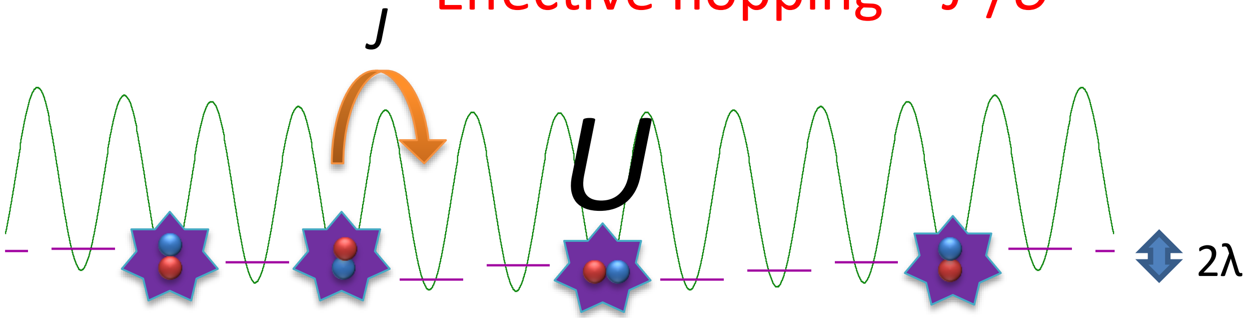
Haussdorf dim. = 1/2
for non-interacting case

Excitation spectrum still fractal;
random walk-like motion ($\langle x^2 \rangle \sim t$)
expected

Strongly attractive ($|U| \gg W$):
Tightly bound hard-core bosons formed

Effect of modulation: relatively weak ($\lambda << |U|$)

Effective hopping ~ J^2/U



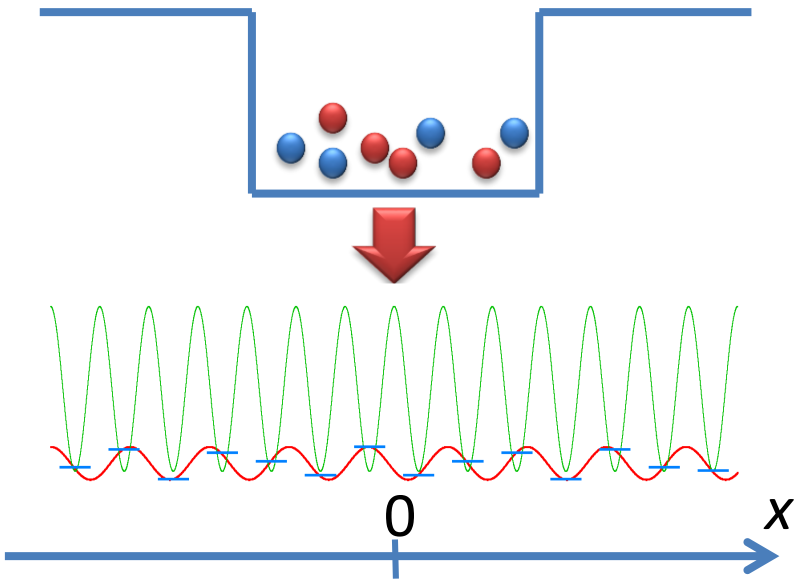
$\lambda_c \sim 2J^2/U << 2J$

At transition point:
Spectrum should be almost normal
Is the cloud expansion almost ballistic?
($\langle x^2 \rangle \sim t^2$?)

Simulation setup

- Optical lattice + Harper-type incommensurate potential

This work: 12+12 fermions on 64 sites



- On-site attractive interaction
- Initially trapped in a box potential <u>without</u> q.p. potential (initial condition does not depend on λ)

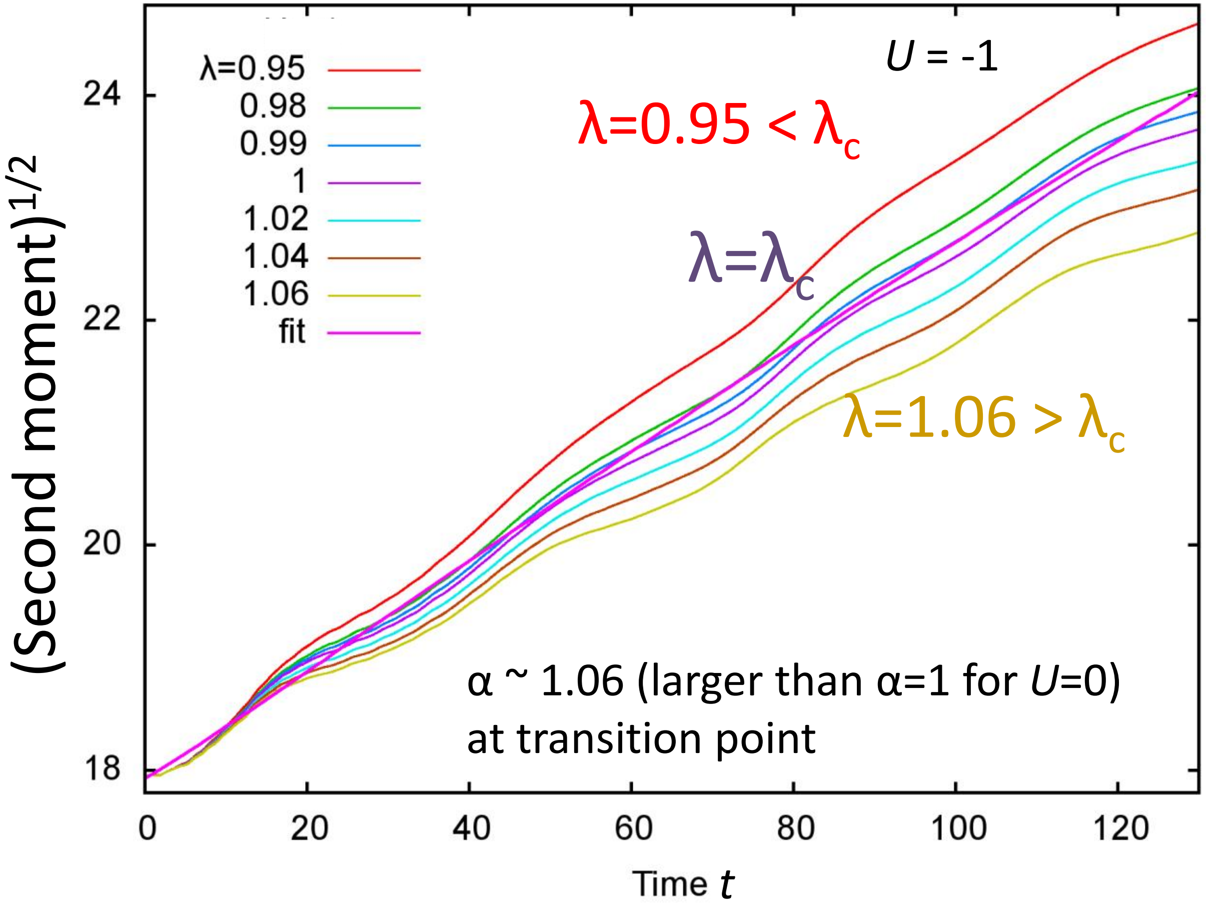
- Remove the box potential and switch the incommensurate potential on: diffusion exponent?

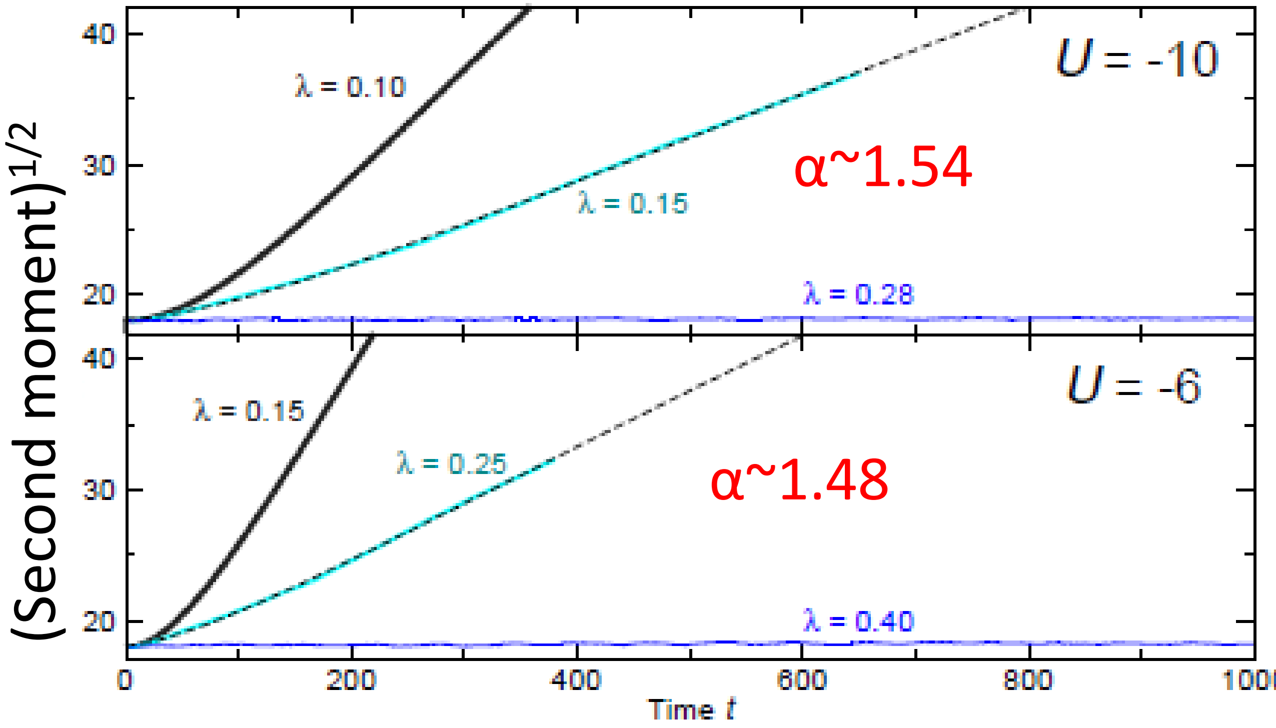
➔ Simulation by time-dependent DMRG

Tezuka and García-García: PRA **85**, 031602 (R) (2012)

Expansion exponent from second moment

$\sqrt{\langle x^2(t) \rangle}$ fit by $x_0\sqrt{1+(t/t_0)^{\alpha}}$





Value of α at transition increasing as |U| increases:

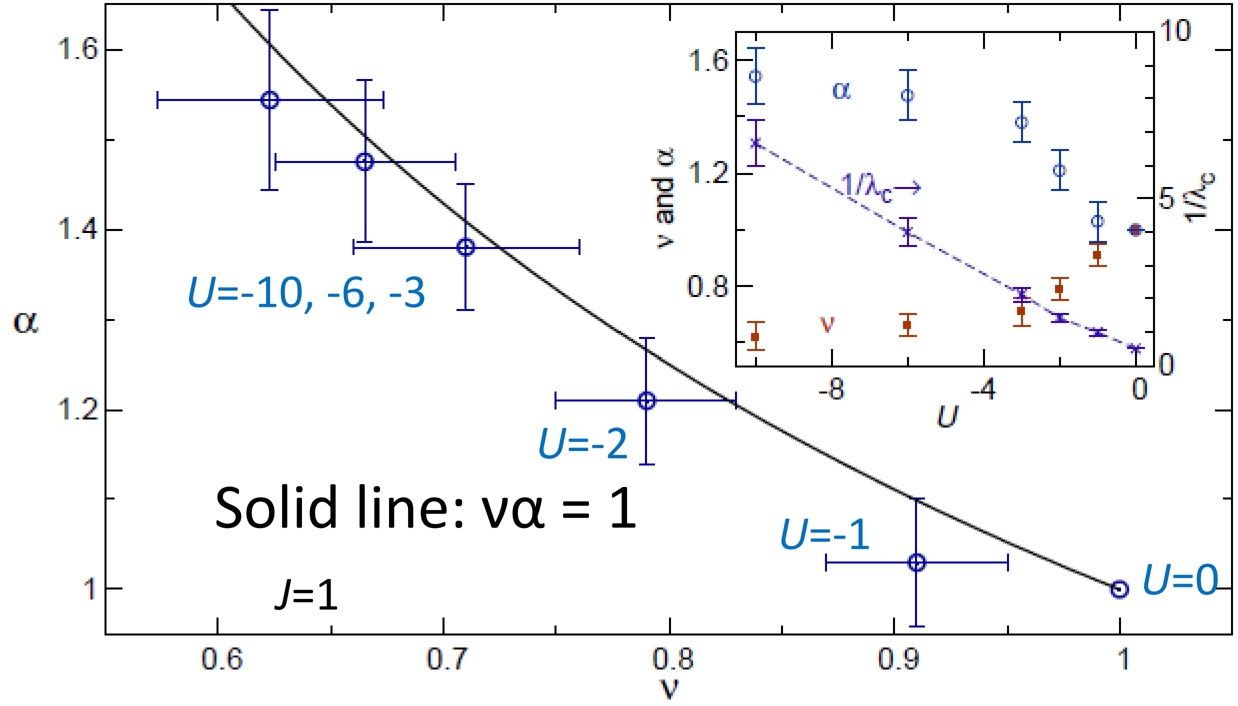
anomalous exponent! (between random walk and ballistic)

Tezuka and García-García: PRA **85**, 031602(R) (2012)

Exponents from dynamics and static property

12+12 fermions, 64→160 or 256 sites

Exponent of diffusion (dynamics)



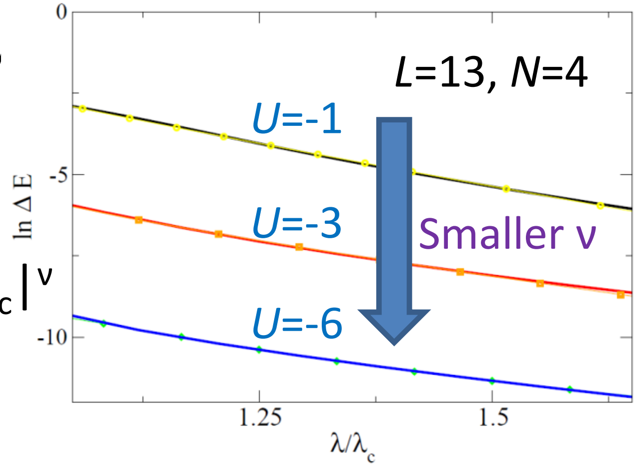
Exponent of localization length (static property)

Localization length ξ should diverge as $|\lambda-\lambda_c|^{-\nu}$
as MIT is approached from insulator side
(ν=1 at U=0; ν=1/2 in mean field limit)

Sensitivity of the ground state energy to boundary cond.
$E_{P(A)}$: ground state energy for periodic (antiperiodic) b.c.

$$\Delta E = |E_P - E_A| \propto e^{-L/}$$

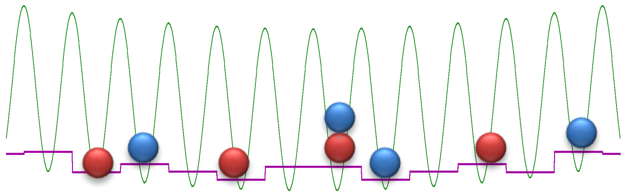
Extract ξ by fitting:
$$\ln \Delta E \sim -L/\xi \propto |\lambda-\lambda_c|^{\nu}$$



We conjecture from analogy to the non-interacting cases: να=1
(cf. Hiramoto JPSJ 1990; Hashimoto *et al.* J. Phys. A 1992; Kopidakis *et al.* PRL 2008)

➔α indeed increases while ν decreases; να = 1 ?

Tezuka and García-García: PRA **85**, 031602(R) (2012)



Quick summary (1)

(Bichromatic lattice)

Modulated 1D system, $U < 0$, at "metal"-insulator transition

	$\lvert U\rvert \to 0$	intermediate $\lvert U\rvert$	$\lvert U\rvert \to \infty$
Diffusion $\langle x^2(t)\rangle \propto t^\alpha$	$\alpha=1$ brownian motion	**increases** as $\lvert U\rvert$ increases	α~2? ballistic motion?
Hausdorff dimension of the spectrum d_H	$d_H=0.5$ see *e.g.* Artuso *et al.*: PRL **68**, 3826 (1992)	One parameter scaling $\alpha = 2d_H$ at MIT	d_H~1? Not fractal?
Localization length close to transition $\xi \propto \lvert\lambda - \lambda_c\rvert^{-\nu}$	$\nu=1$	**decreases** as $\lvert U\rvert$ increases	ν~1/2?

λ_c strongly suppressed! (~$t^2/|U|$)
(MT & AGG: PRA 82, 043613 (2010))

Our conjecture: $\nu\alpha=1$

➔**Anomalous diffusion** in modulated, interacting 1D Fermi gas observed;
Interesting relation between the **dynamic** and **static** behavior conjectured

2. Repulsively interacting spinless bosons

Fuyuki Matsuda, MT, and Norio Kawakami: arXiv:1404.6315 (to appear in JPSJ)

1D quasiperiodic ⟷ 2D topological phase

correspondence

1D quasiperiodic system (chiral: Z)

$$\mathcal{H}(\phi) = \sum_n \left[\underbrace{-t(c_{n+1}^\dagger c_n + c_{n-1}^\dagger c_n)}_{\text{hopping}} + \underbrace{\lambda \cos(2\pi b n + \phi) c_n^\dagger c_n}_{\text{quasiperiodic potential}} \right]$$

Fourier transform

$$\hat{c}_n(\phi) = \sum_m e^{-i\phi m} \hat{c}_{m,n}$$

2D integer quantum Hall system (class A: Z)

$$\mathcal{H} = \sum_{m,n} \left[-t(c_{m,n+1}^\dagger c_{m,n} + c_{m,n-1}^\dagger c_{m,n}) + \frac{\lambda}{2}(e^{i2\pi b n} c_{m+1,n}^\dagger c_{m,n} + e^{-i2\pi b n} c_{m-1,n}^\dagger c_{m,n}) \right]$$

Magnetic field

"Topological states and adiabatic pumping in quasicrystals"
Y. E. Kraus *et al.*, PRL **109**, 106402 (2012)

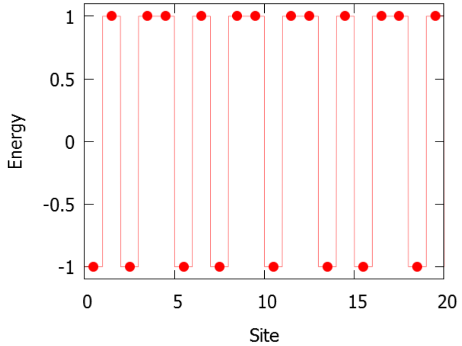
- Localization of light in a 1D array of optical waveguides
- Single-particle problem

➔ The case of interacting cold atoms?

Topological equivalence

$$\mathcal{H}(\phi,\beta) = \sum_j \left[(t + \lambda_{od} V_n(\phi + \pi b))\, \hat{c}^\dagger_{j+1} \hat{c}_j + H.c. + \lambda_d V_n(\phi) \hat{c}^\dagger_j \hat{c}_j \right.$$

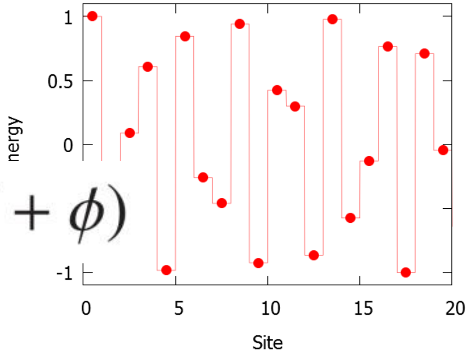
$$V_j(\phi,\beta) = \frac{\tanh\beta[\cos(2\pi b j + \phi) - \cos\pi b]}{\tanh\beta}$$ smoothly connects V_j^{Harper} and $V_j^{\mathrm{Fibonacci}}$



Harper-type

$$V_j^{Harper}(\phi) = \cos(2\pi b j + \phi)$$

β→0



Fibonacci-type

$$V_j^{Fibonacci}(\phi) = 2\left(\left\lfloor \left(j + 2 + \frac{\phi}{2\pi b}\right)/b \right\rfloor - \left\lfloor \left(j + 1 + \frac{\phi}{2\pi b}\right)/b \right\rfloor\right) - 1$$

β→∞

Equivalence for $0<\beta<\infty$, $0<\lambda_{od}<\lambda_d<\infty$ for non-interacting case [Kraus and Zilberberg: PRL 2012]

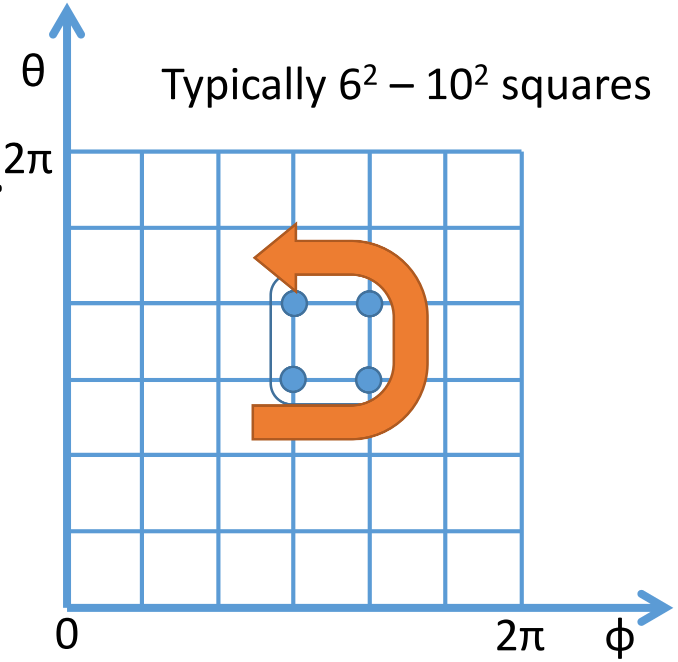
➔ <u>What happens for interacting bosons?</u> (t=1, λ_{od}=0, λ_d=λ in the following)

Calculating the Chern number for interacting case

Chern number for **many-body** ground state |Ψ>

$$C = \frac{1}{2\pi i}\int d\theta d\phi \left(\left\langle \frac{\partial \Psi}{\partial \phi} \middle| \frac{\partial \Psi}{\partial \theta} \right\rangle - \left\langle \frac{\partial \Psi}{\partial \theta} \middle| \frac{\partial \Psi}{\partial \phi} \right\rangle \right)$$

ϕ: Phase of the quasiperiodic potential
θ: Twisted boundary condition



- Approximate the quasiperiodic system by periodic systems
  2/5, 3/8, 5/11, 8/21, 13/34, … → (3-√5)/2 = 1 – g = 0.381966…
- DMRG + Fukui-Hatsugai-Suzuki method [JPSJ **74**, 1674 (2005)]
  to obtain the Chern number from a finite set of (θ, ϕ)

Obtain |Ψ> for four parameter sets at a time to estimate the U(1) link variables

$$U_\mu(k_l) \equiv \langle n(k_l)|n(k_l+\hat{\mu})\rangle / |\langle n(k_l)|n(k_l+\hat{\mu})\rangle| \,;\; k_l = (\phi, \theta)$$

Lattice field strength associated with Berry connection:

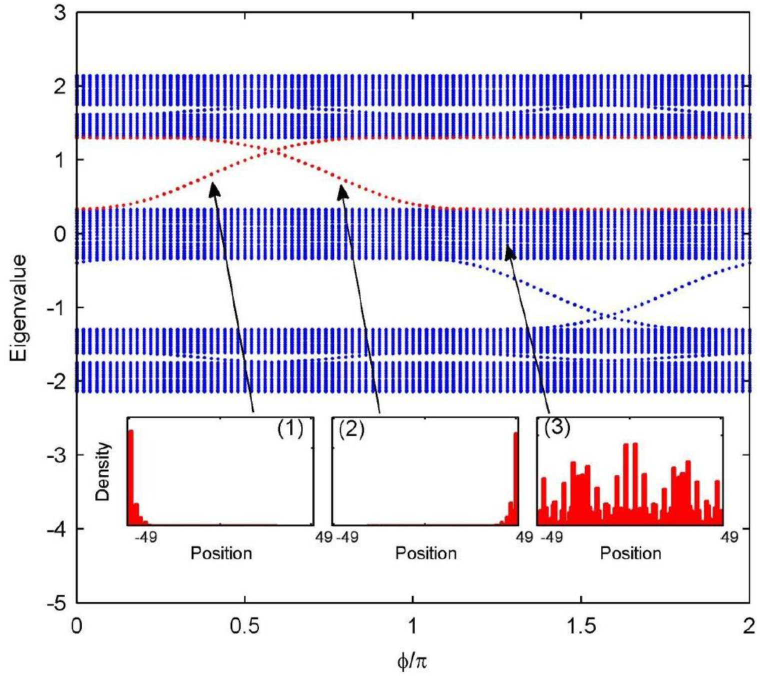
$$\widetilde{F_{\phi\theta}}(k_l) \equiv \ln\left(U_\phi(k_l) U_\theta(k_l+\hat{\phi}) U_\phi(k_l+\hat{\theta})^{-1} U_\theta(k_l)^{-1}\right); -\pi < i^{-1}\widetilde{F_{\phi\theta}}(k_l) \le \pi$$

Then the integer Chern number is obtained as

$$C = (2\pi i)^{-1} \sum_l \widetilde{F_{\phi\theta}}(k_l)$$

Phase diagram for interacting bosons: Harper type case

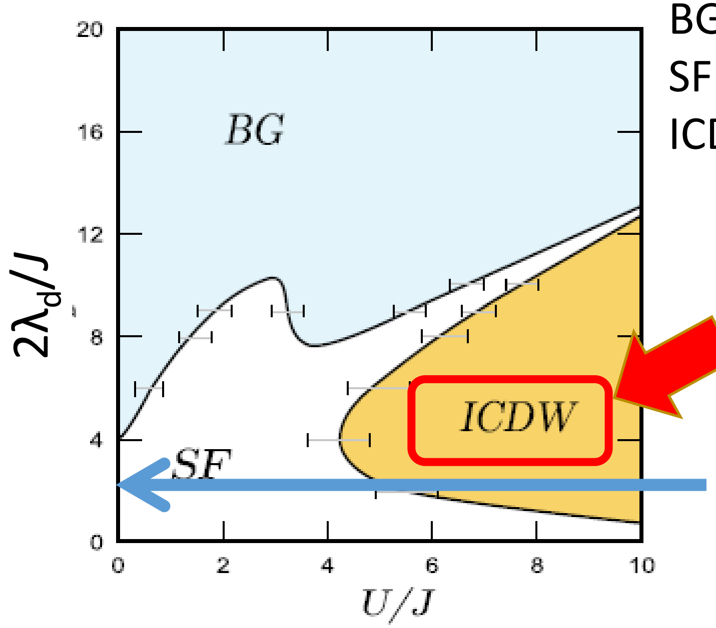
Non-interacting fermions:
bulk gaps appear at fillings $n = N/L = b$, $1-b$, …



Y. E. Kraus *et al.*: PRL **109**, 106402 (2012)

Interacting bosons: reduces to non-interacting fermions as $U \rightarrow \infty$

$n = b$



BG: Bose glass
SF: Superfluid
ICDW: Incommensurate charge-density wave

Insulating

This work

G. Roux *et al.*: PRA **78**, 023628 (2008)

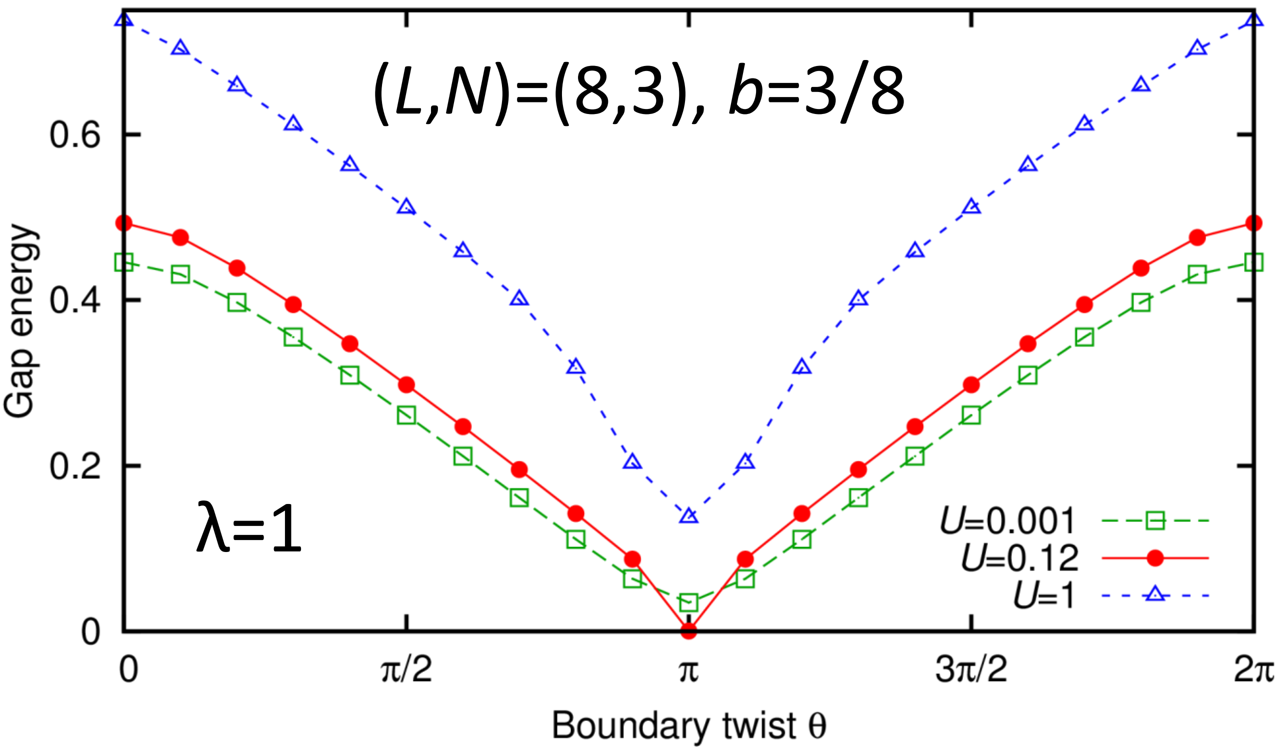
➔Topological characterization of the ICDW phase?

➔Fibonacci-type case?

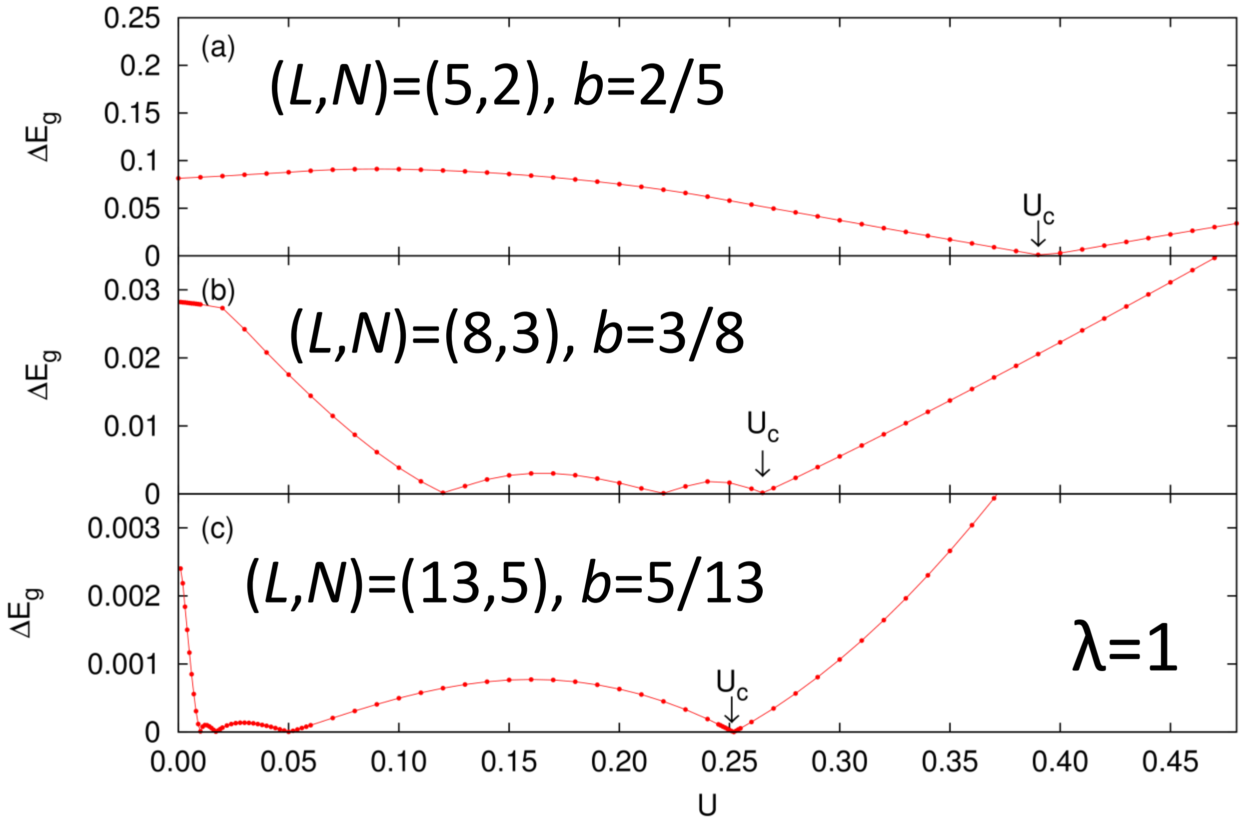
Fuyuki Matsuda, MT, and Norio Kawakami: arXiv:1404.6315 (to appear in JPSJ)

Energy gap (Chern number can change only if closed)

Boundary condition: $t_{0,L\text{-}1} = t\,\exp(i\theta)$



Minimum energy gap for a fixed θ



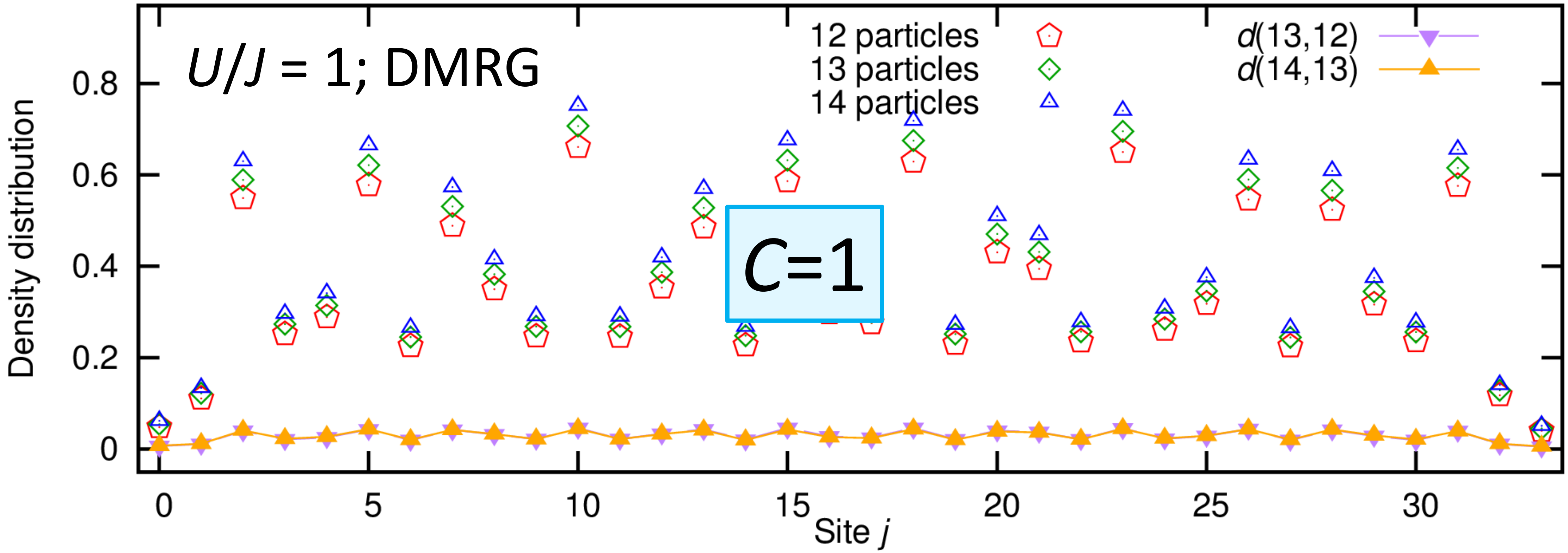
Minimum energy gap for all (θ, ϕ)

➔ Energy gap closes only for $U \ll J$; topological equivalence for larger U expected

Fuyuki Matsuda, MT, and Norio Kawakami: arXiv:1404.6315 (to appear in JPSJ)

Bulk-edge correspondence? : case of small U

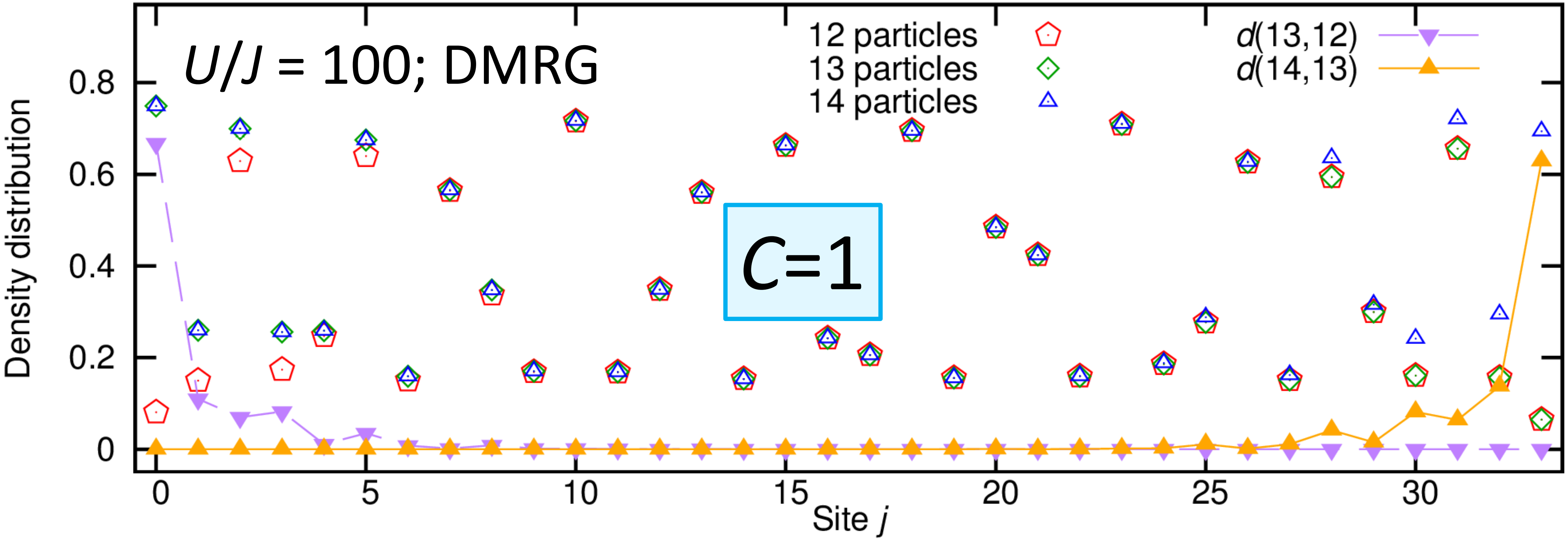
Change of particle distribution at ground state as number is changed by one



Density change not localized: particles still almost condensed

Bulk-edge correspondence? : case of larger U

Change of particle distribution at ground state as number is changed by one



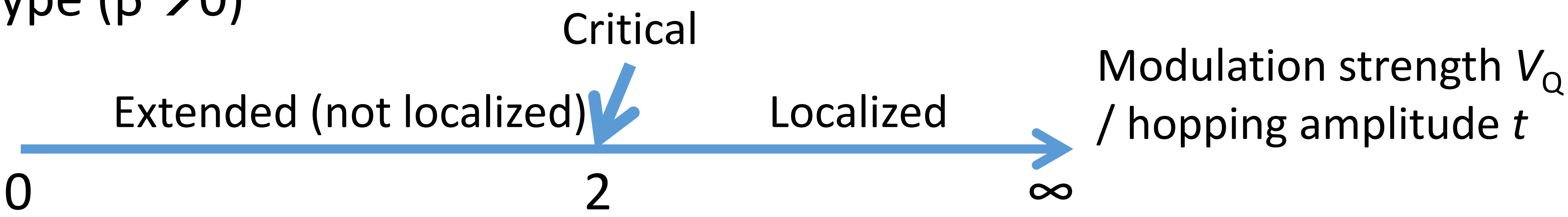
Localized structure at the end

Correspondence between non-trivial Chern # and edge modes

Harper type and Fibonacci type

<u>1D, diagonal (site level) modulation</u>

- Harper type (β→0)

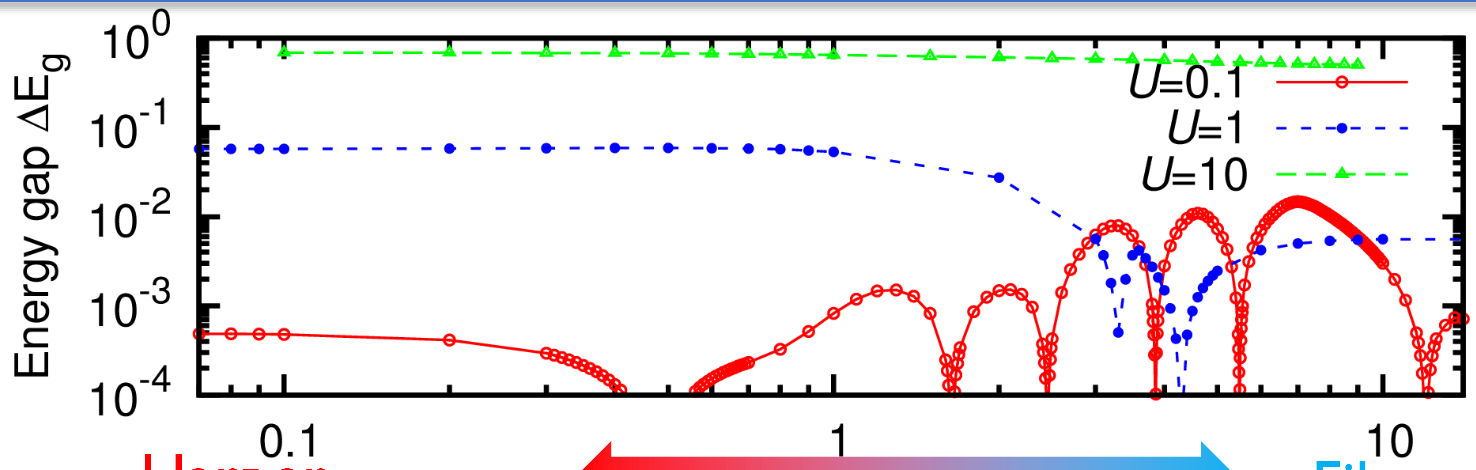


- Fibonacci type (β→∞)
  - All single-particle states critical regardless of V/W
  - (Fractal wavefunctions)
- Smooth connection between Harper and Fibonacci types known

$$V_j(\phi, \beta) = \frac{\tanh \beta [\cos(2\pi b j + \phi) - \cos \pi b]}{\tanh \beta}$$

Fuyuki Matsuda, MT, and Norio Kawakami: arXiv:1404.6315 (to appear in JPSJ)

Result: Harper-type and Fibonacci-type

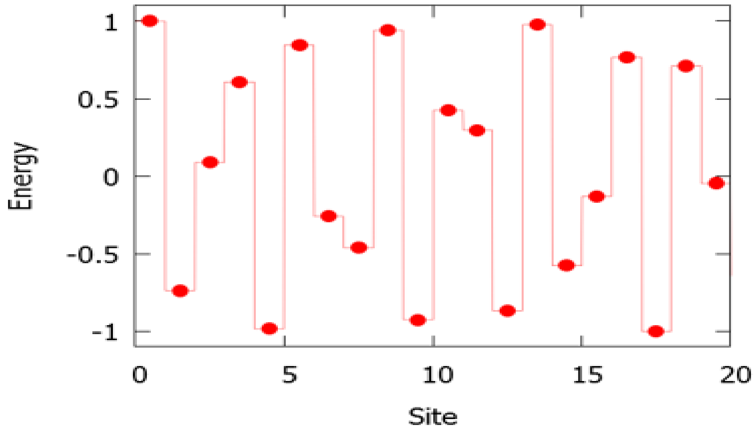


b=5/13, (L, N)=(13,5), λ=1, U=0.1, 1, 10

Harper

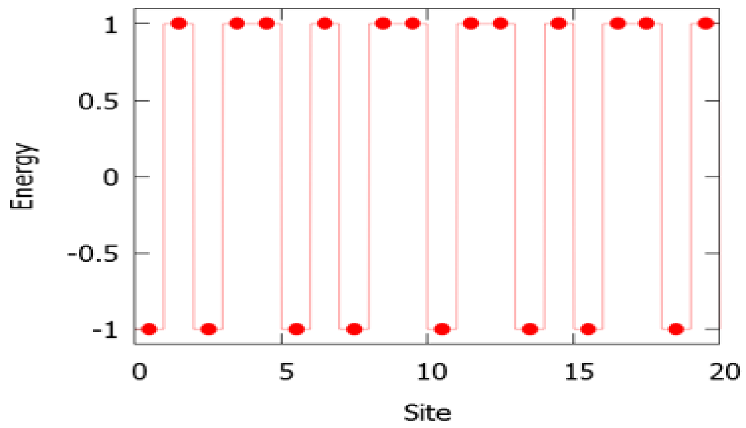
β

Fibonacci



$$V_j(\phi,\beta) = \frac{\tanh\beta[\cos(2\pi b j + \phi) - \cos\pi b]}{\tanh\beta}$$

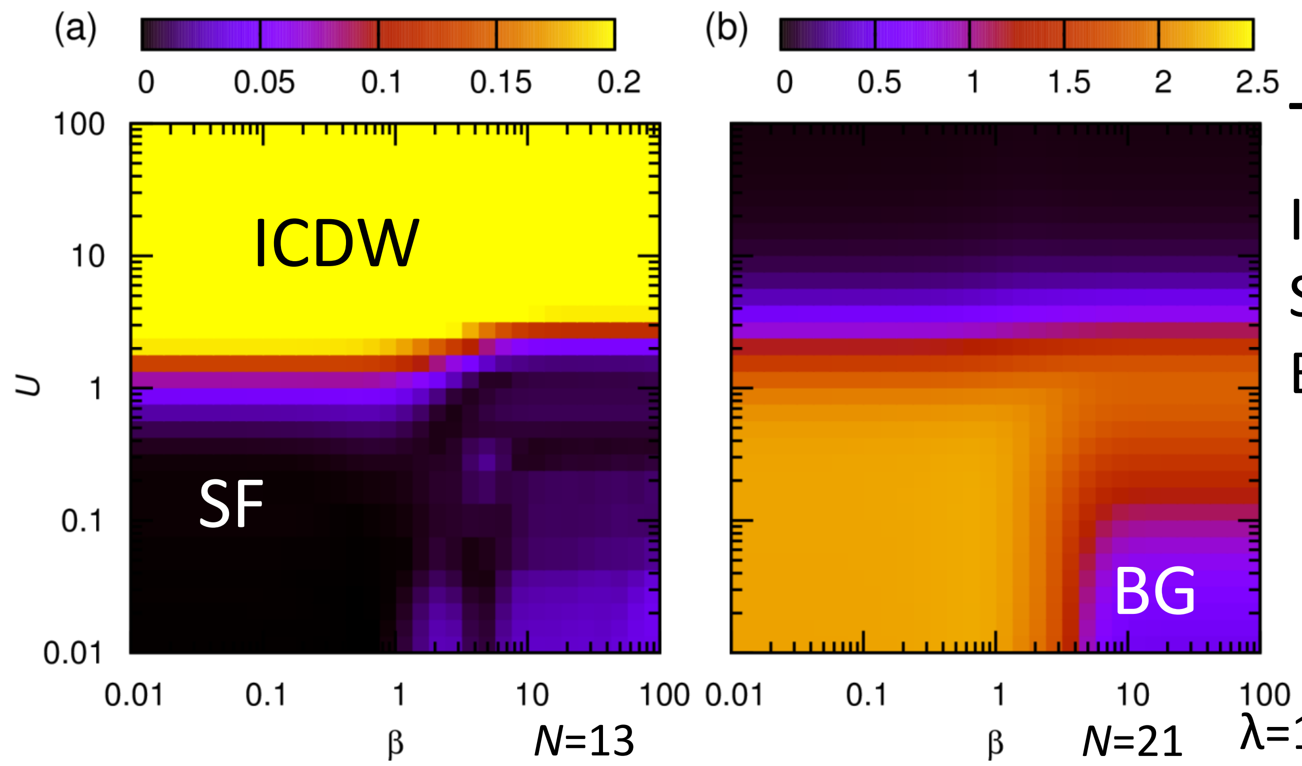
Kraus *et al.*: PRL **109**, 116404 (2012)



Gap does not close if $U \gtrsim 4J$: no change in Chern number

Fuyuki Matsuda, MT, and Norio Kawakami: arXiv:1404.6315 (to appear in JPSJ)

Phase diagram and topological equivalence



λ=1, b~(3-√5)/2

Energy gap minimum

Superfluid density

$$\rho_s = \frac{2L}{\pi}\left(E^{\mathrm{apbc}} - E^{\mathrm{pbc}}\right)$$

Three phases observed

ICDW: Incommensurate Charge Density Wave

SF: Superfluid (quasi-condensate)

BG: Bose Glass

k	ΔE_g	SF density
ICDW	✔	✘
SF	✘	✔
BG	✘	✘

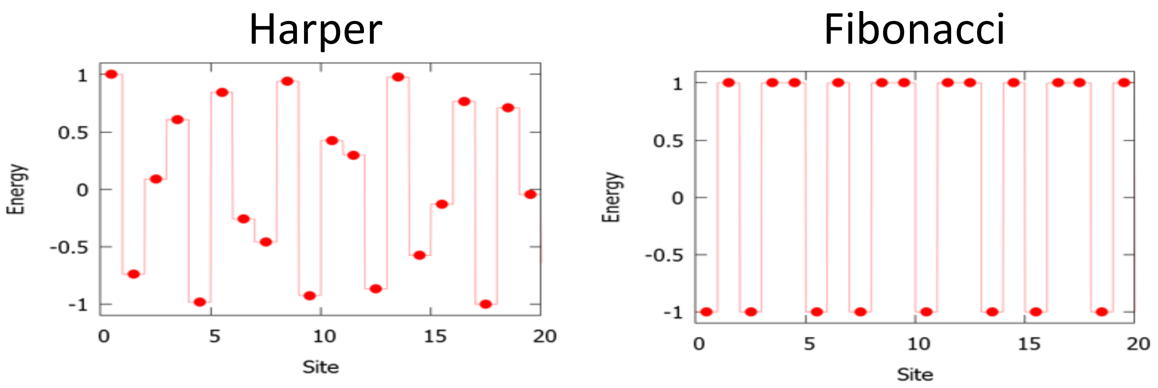
Topological equivalence: inside the ICDW phase (no gap closing)

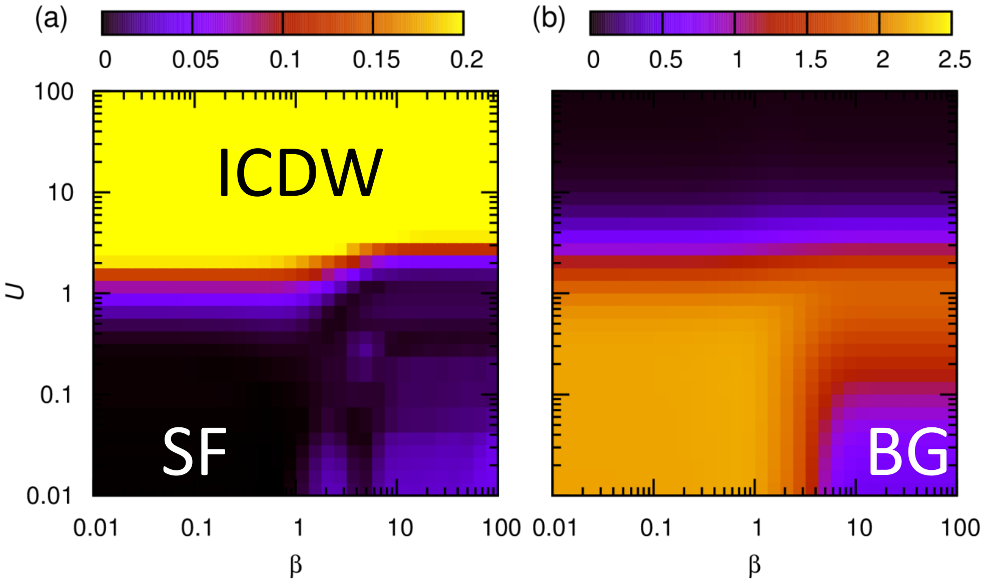
➔ Continuously connected to the non-interacting fermion case

Quick summary (2)

- Topological classification of 1D interacting boson systems with quasiperiodic modulation

- Excitation gap closes at small values of *U*
- Bulk-edge correspondence for larger *U*
- Phase diagram with respect to
  - Interaction *U*
  - Harper-Fibonacci transformation parameter β
- Incommensurate “charge” density wave phase: topologically nontrivial and equivalent

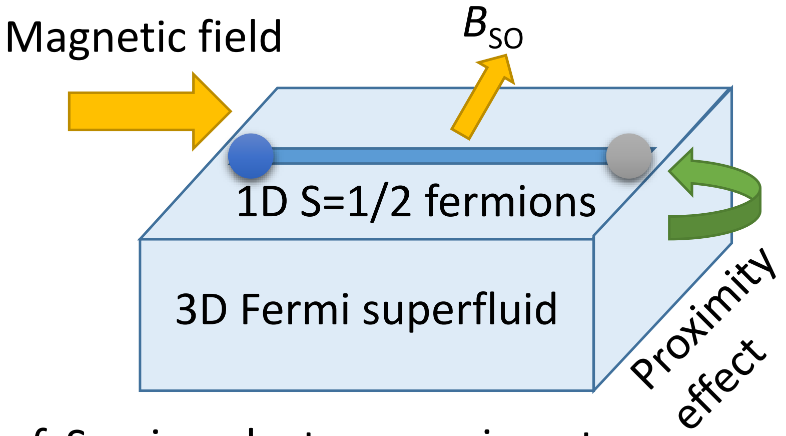




MT and Norio Kawakami: PRB **85**, 140508R(2012), PRB **88**, 155428 (2013)

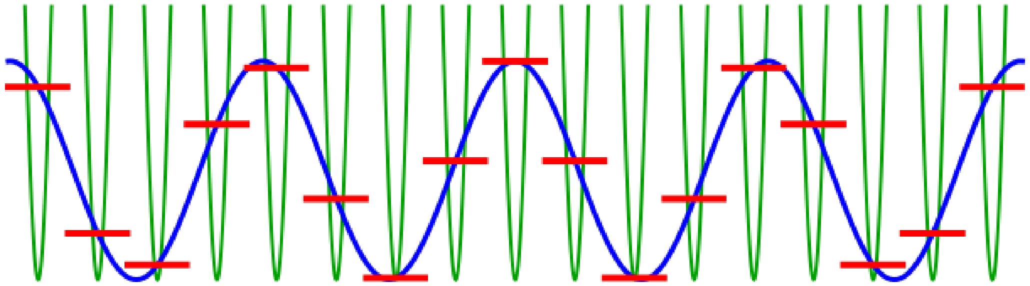
3. Spin-1/2 fermions with proximity pairing

Majorana fermions (MF) expected at the ends of 1D topological superfluid (TS)



cf. Semiconductor experiment
Mourik *et al.* : Science **336**, 1003 (2012), …

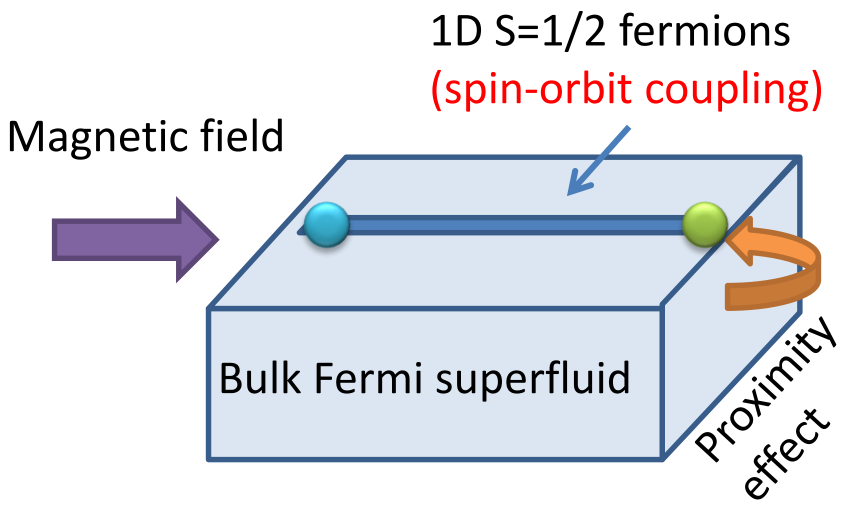
<u>Q. Effect of lattice modulation?</u>

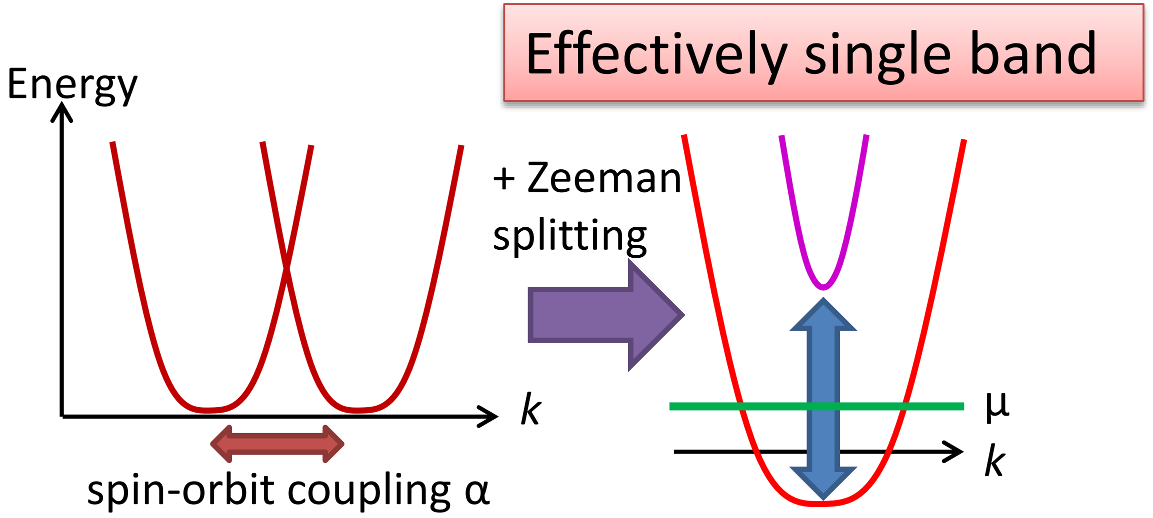


End Majorana fermions of a 1D topological superconductor with spin

Kitaev: Physics-Uspenski **44**, 131 (2001)

1D spinless superconductor: can have end Majorana fermions





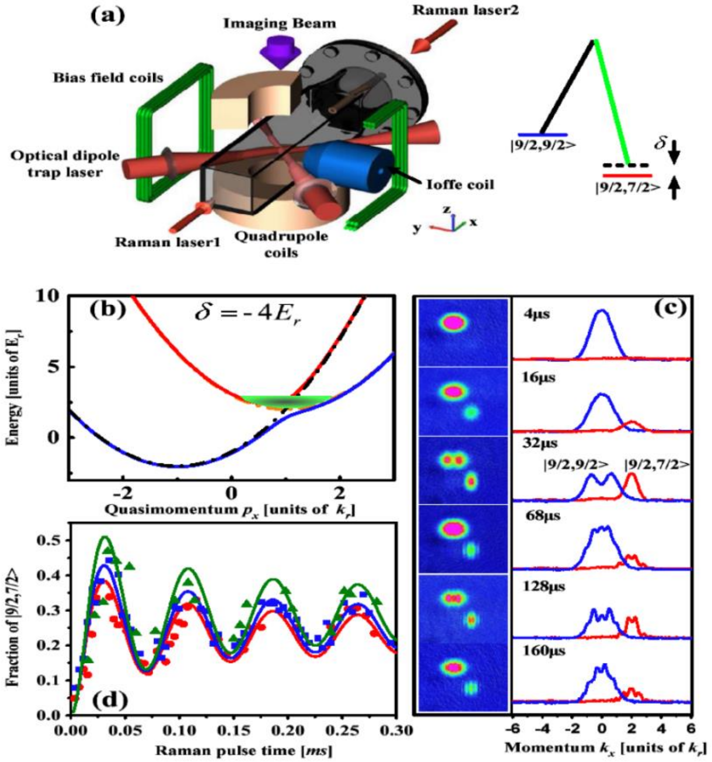
Review (including 2D, 3D, QHE, …): Alicea: Rep. Prog. Phys. **75**, 076501 (2012)

Theory (1D): Lutchyn *et al.*: PRL **105**, 077001 (2010); Oreg *et al.*: PRL **105**, 177002 (2010), …

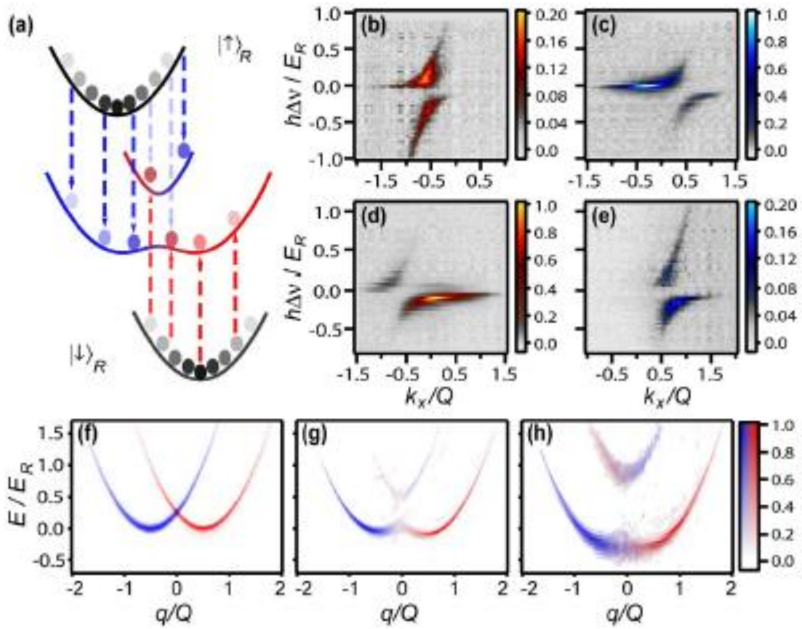
cf. 2D Tewari *et al.* (2007); Sato *et al.* (2008); Fu and Kane (2008); Tanaka *et al.* (2009); etc.

Experimental realization of spin-orbit coupling in degenerate Fermi gases

^{40}K: Pengjun Wang *et al.* (Shanxi): PRL **109**, 095301 (2012)



^{6}Li: Lawrence W. Chunk *et al.* (MIT): PRL **109**, 095302 (2012)

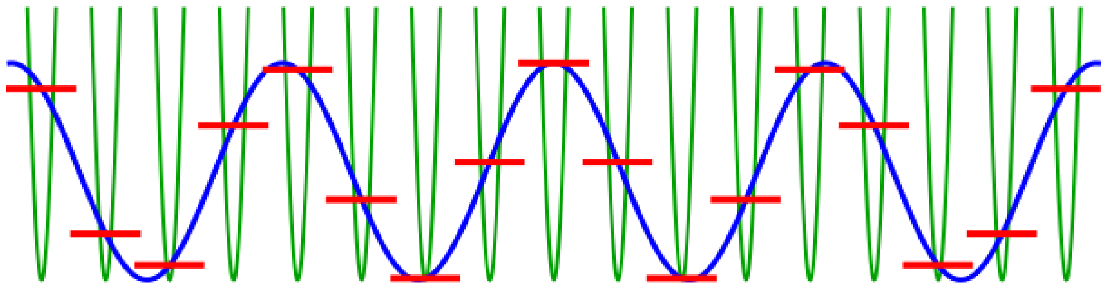


➔ Topological states (as in electron systems in solid state physics), *e.g.* topological superfluid with Majorana edge fermions? Their reaction to quasiperiodic modulation?

Quasiperiodic modulation

V_Q: quasiperiodic potential amplitude
V_z: Zeeman energy
$2t = 2$: band width (hopping=$t/2$)
α: spin-orbit coupling
Δ: proximity pairing

$$\varepsilon_{\sigma,l} = V_Q \cos(\kappa x + \varphi_0);\ x = l - (L-1)/2$$



Hamiltonian

(Tight binding model)

hopping (t = 1)

Site l (=0, 1, …, L-1)

Zeeman

$$\begin{aligned}\mathcal{H} = &-\frac{t}{2}\sum_{l=0}^{L-2}\sum_{\sigma=\uparrow,\downarrow}(\hat{c}^{\dagger}_{\sigma,l}\hat{c}_{\sigma,l+1} + \text{h.c.})\\ &+\sum_{l=0}^{L-1}\left[V_z(\hat{n}_{\uparrow,l} - \hat{n}_{\downarrow,l}) + \sum_{\sigma=\uparrow,\downarrow}(t - \mu + \epsilon_{\sigma,l})\hat{n}_{\sigma,l}\right]\\ &+\frac{\alpha}{2}\sum_{l=0}^{L-2}[(\hat{c}^{\dagger}_{\downarrow,l}\hat{c}_{\uparrow,l+1} - \hat{c}^{\dagger}_{\uparrow,l}\hat{c}_{\downarrow,l+1}) + \text{h.c.}].\\ &+\Delta\sum_{l=0}^{L-1}(\hat{c}_{\uparrow,l}\hat{c}_{\downarrow,l} + \text{h.c.})\\ &+U\sum_{l=0}^{L-1}\hat{n}_{\uparrow,l}\hat{n}_{\downarrow,l}\end{aligned}$$

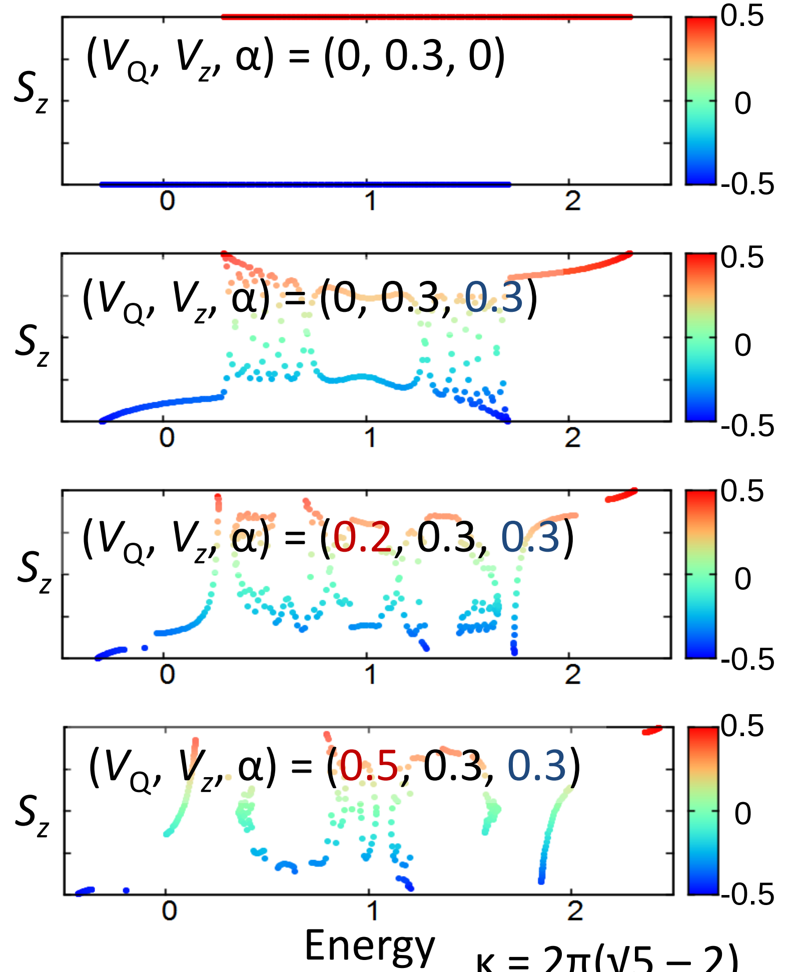
Quasiperiodic potential

Spin-orbit coupling (Rashba type)

proximity pairing

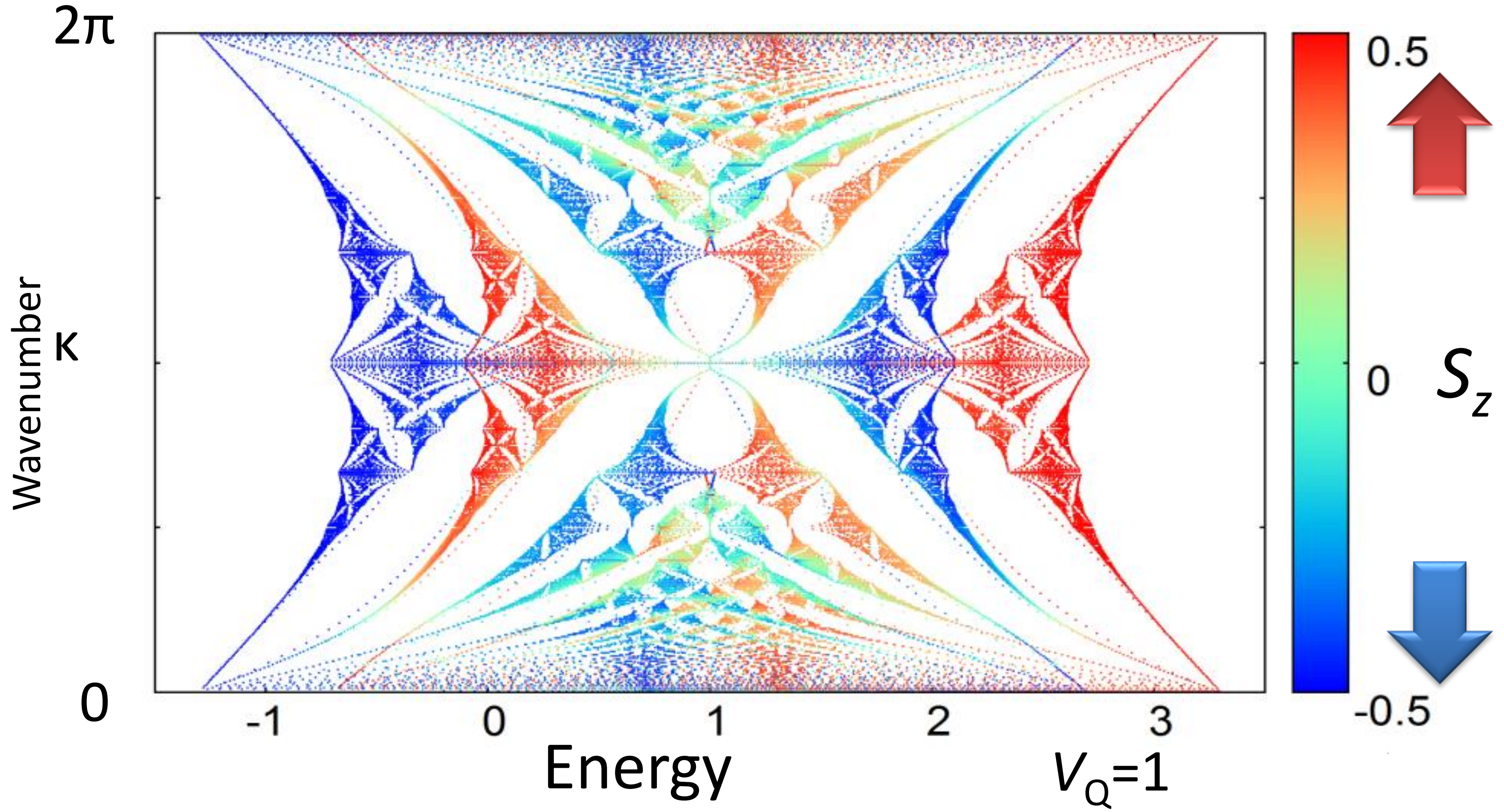
On-site interaction

Single particle states: energy and $\langle S_z \rangle$



$\kappa = 2\pi(\sqrt{5} - 2)$

Single-particle states: "Double Hofstadter butterfly"



Dots: single particle states for each value of κ

Tezuka and Kawakami: PRB **88**, 155428 (2013)

Method (1): BdG equation

U=0 case:

Hamiltonian: bilinear in (c, $c^\dagger$)

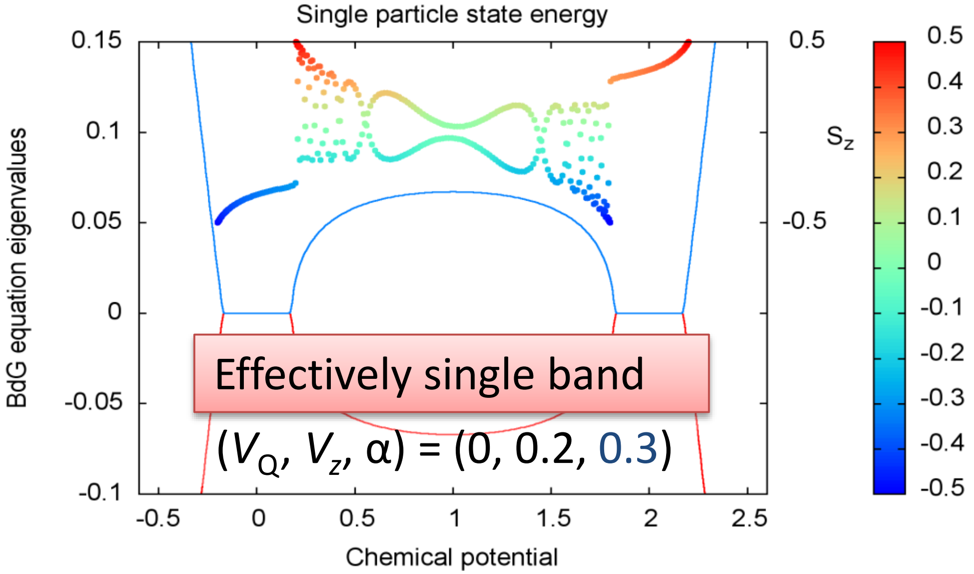
$$
\begin{aligned}
\mathcal{H} = & -\frac{t}{2}\sum_{l=0}^{L-2}\sum_{\sigma=\uparrow,\downarrow}(\hat{c}^\dagger_{\sigma,l}\hat{c}_{\sigma,l+1} + \text{h.c.}) \\
& + \sum_{l=0}^{L-1}\left[V_z(\hat{n}_{\uparrow,l} - \hat{n}_{\downarrow,l}) + \sum_{\sigma=\uparrow,\downarrow}(t - \mu + \epsilon_{\sigma,l})\hat{n}_{\sigma,l}\right] \\
& + \frac{\alpha}{2}\sum_{l=0}^{L-2}[(\hat{c}^\dagger_{\downarrow,l}\hat{c}_{\uparrow,l+1} - \hat{c}^\dagger_{\uparrow,l}\hat{c}_{\downarrow,l+1}) + \text{h.c.}]. \\
& + \Delta\sum_{l=0}^{L-1}(\hat{c}_{\uparrow,l}\hat{c}_{\downarrow,l} + \text{h.c.})
\end{aligned}
$$

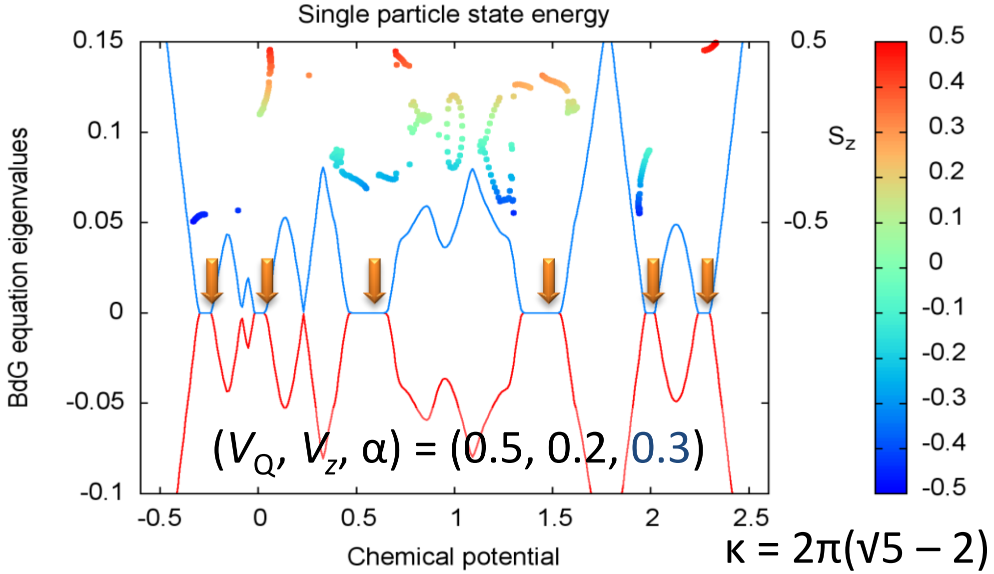
Bogoliubov-de Gennes equation (with fixed, real Δ)

$$
\begin{pmatrix}
H_\uparrow & \alpha & \Delta & \\
\alpha & H_\downarrow & & \Delta \\
\Delta & & -H_\uparrow & -\alpha \\
& \Delta & -\alpha & -H_\downarrow
\end{pmatrix}
\begin{pmatrix} u_\uparrow \\ u_\downarrow \\ v_\uparrow \\ v_\downarrow \end{pmatrix}
= E \begin{pmatrix} u_\uparrow \\ u_\downarrow \\ v_\uparrow \\ v_\downarrow \end{pmatrix}
$$

L sites: 2*L* pairs of eigenvalues ($+E_j$, $-E_j$); Majorana mode candidates : $E \sim 0$

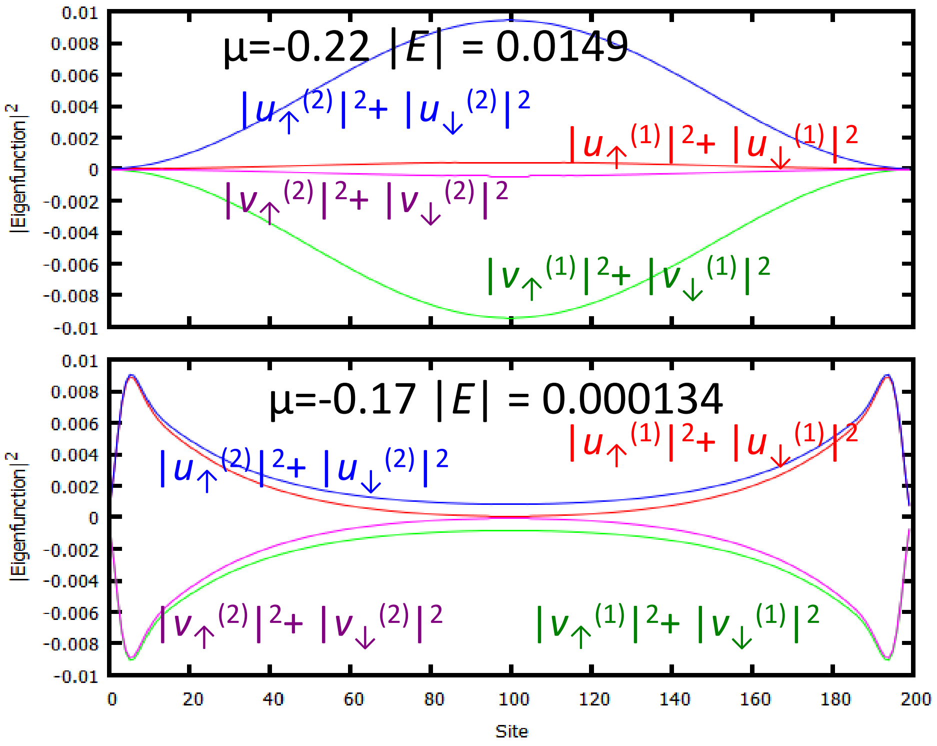
Δ=0.1





$\kappa = 2\pi(\sqrt{5} - 2)$

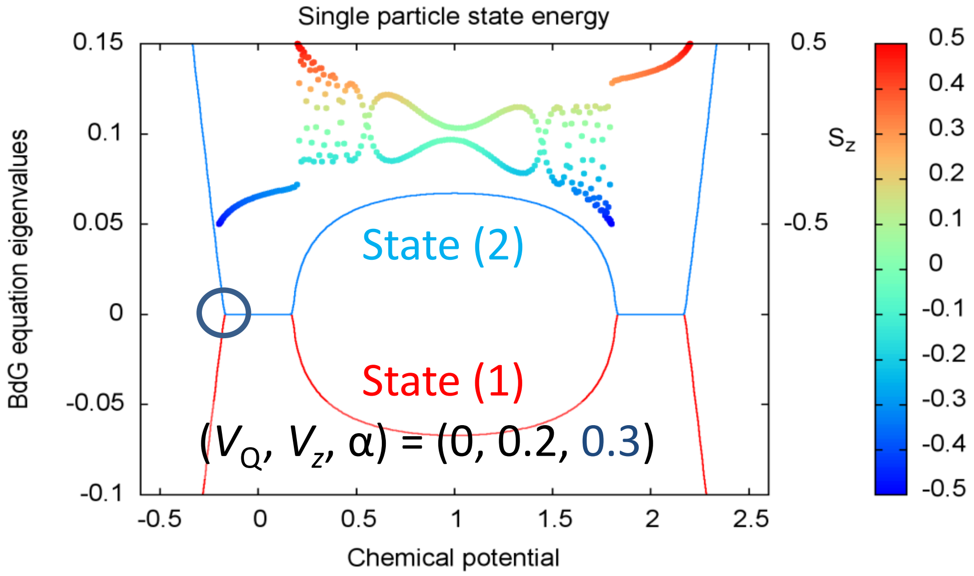
E = 0 states: without q. p. modulation

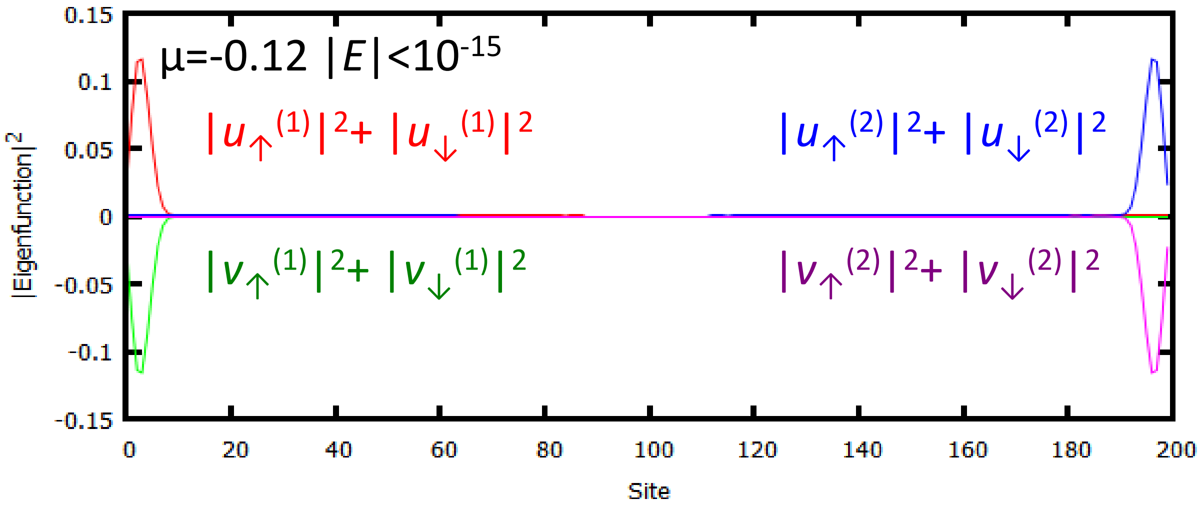


State (1): largest $E < 0$

State (2): smallest $E > 0$

Δ=0.1





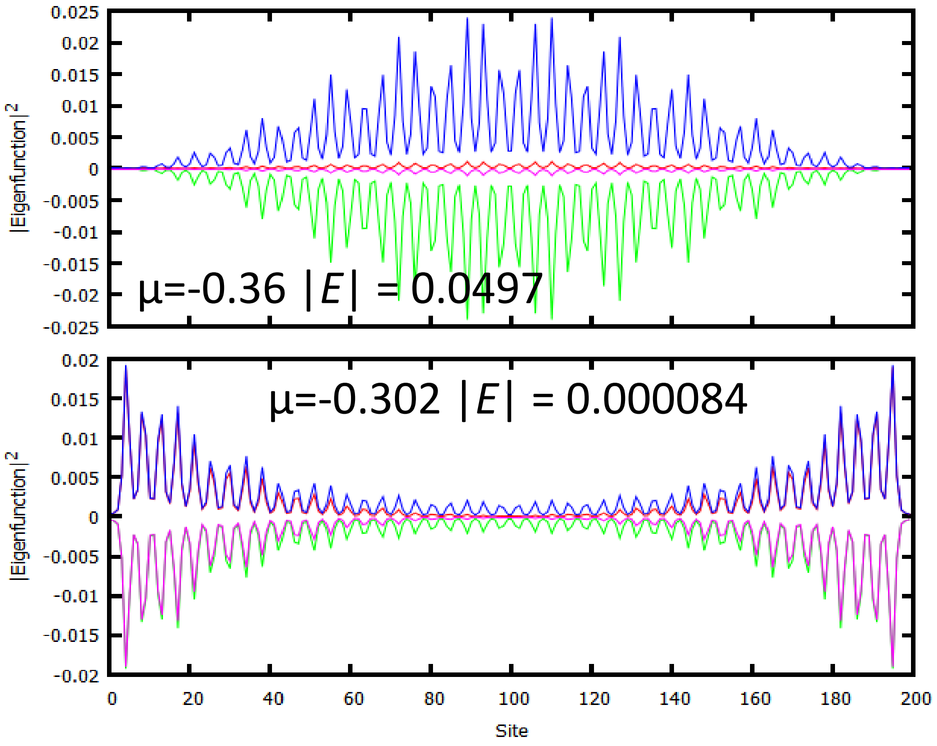
☑ Antiparticle = particle

☑ Localized at edges

➔ Majorana modes!

Tezuka and Kawakami: PRB **88**, 155428 (2013)

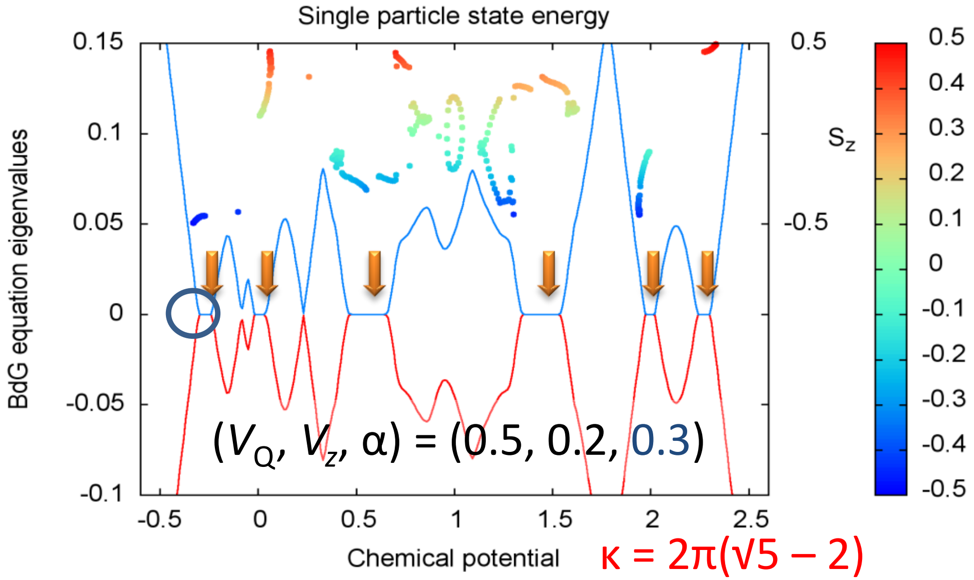
E=0 states: with q. p. modulation

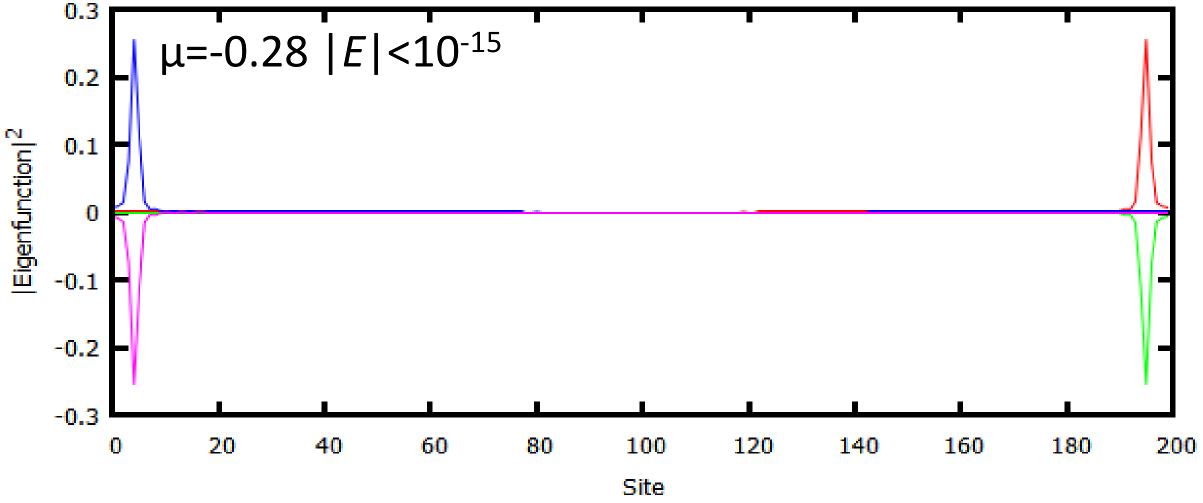


State (1): largest $E < 0$

State (2): smallest $E > 0$

Δ=0.1





☑ Antiparticle = particle

☑ Localized at edges

Majorana modes appear even under quasiperiodic modulation!

➔Dependence on κ?

➔Effect of correlation?

Method(2): DMRG

Tezuka and Kawakami: Phys. Rev. B **85**, 140508(R) (2012)

(Density-matrix renormalization group)

Pairing and **on-site interaction** introduced → Many body states

$$\Delta \sum_{l=0}^{L-1} (\hat{c}_{\uparrow,l}\hat{c}_{\downarrow,l} + \mathrm{h.c.})$$

The number of fermions is not preserved;
the parity of the number is.

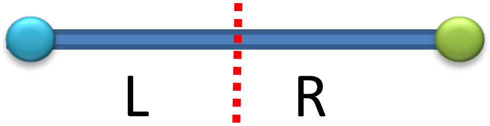
Ground states: Energy

$$|\Psi_{\mathrm{e}}\rangle = \sum_j f_j^{(N=0)}|\phi_j^{(N=0)}\rangle + \sum_j f_j^{(N=2)}|\phi_j^{(N=2)}\rangle + \sum_j f_j^{(N=4)}|\phi_j^{(N=4)}\rangle + \cdots \quad E=E_{\mathrm{even}}$$

$$|\Psi_{\mathrm{o}}\rangle = \sum_j f_j^{(N=1)}|\phi_j^{(N=1)}\rangle + \sum_j f_j^{(N=3)}|\phi_j^{(N=3)}\rangle + \sum_j f_j^{(N=5)}|\phi_j^{(N=5)}\rangle + \cdots \quad E=E_{\mathrm{odd}}$$

Majorana fermion operators $\hat{\gamma}_1, \hat{\gamma}_2$ $\quad \hat{\gamma}_1 = \hat{\gamma}_1^\dagger, \hat{\gamma}_2 = \hat{\gamma}_2^\dagger$ localized at the ends

$$\hat{\gamma}_1|\Psi_{\mathrm{e}}\rangle \propto |\Psi_{\mathrm{o}}\rangle \propto \hat{\gamma}_2|\Psi_{\mathrm{e}}\rangle \qquad \hat{\gamma}_1|\Psi_{\mathrm{o}}\rangle \propto |\Psi_{\mathrm{e}}\rangle \propto \hat{\gamma}_2|\Psi_{\mathrm{o}}\rangle$$



For the case with lattice site level inhomogeneity,

(1) $\Delta E = E_{\mathrm{even}}\text{-}E_{\mathrm{odd}}$: $|\Delta E| \ll 1$ (Ground state degeneracy) ← corresponds to E~0 in BdG

(2) Reduced density matrices $\rho_{\mathrm{e}} \equiv \mathrm{Tr_R}|\Psi_{\mathrm{e}}\rangle\langle\Psi_{\mathrm{e}}|, \rho_{\mathrm{o}} \equiv \mathrm{Tr_R}|\Psi_{\mathrm{o}}\rangle\langle\Psi_{\mathrm{o}}|$: degenerate eigenstates

(3) Majorana mode distribution: localized

$$\hat{\gamma}_1 = \sum_{\sigma,l}(a_{\sigma,l}^{(1)}\hat{c}_{\sigma,l} + \mathrm{c.c.}); a_{\sigma,l}^{(1)} = \langle\Psi_{\mathrm{o}}|\hat{c}_{\sigma,l}^\dagger|\Psi_{\mathrm{e}}\rangle + \langle\Psi_{\mathrm{e}}|\hat{c}_{\sigma,l}^\dagger|\Psi_{\mathrm{o}}\rangle$$

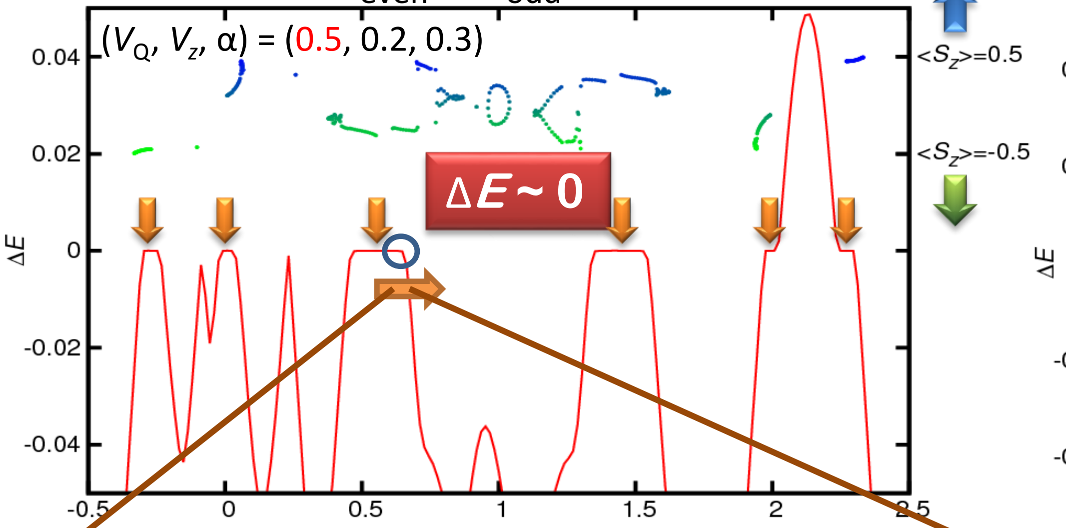
$$\hat{\gamma}_2 = \sum_{\sigma,l}(a_{\sigma,l}^{(2)}\hat{c}_{\sigma,l} + \mathrm{c.c.}); a_{\sigma,l}^{(2)} = i\left(\langle\Psi_{\mathrm{o}}|\hat{c}_{\sigma,l}^\dagger|\Psi_{\mathrm{e}}\rangle - \langle\Psi_{\mathrm{e}}|\hat{c}_{\sigma,l}^\dagger|\Psi_{\mathrm{o}}\rangle\right)$$

as have been done in [Stoudenmire *et al.*: PRB **84**, 014503 (2011)] (no inhomogeneity).

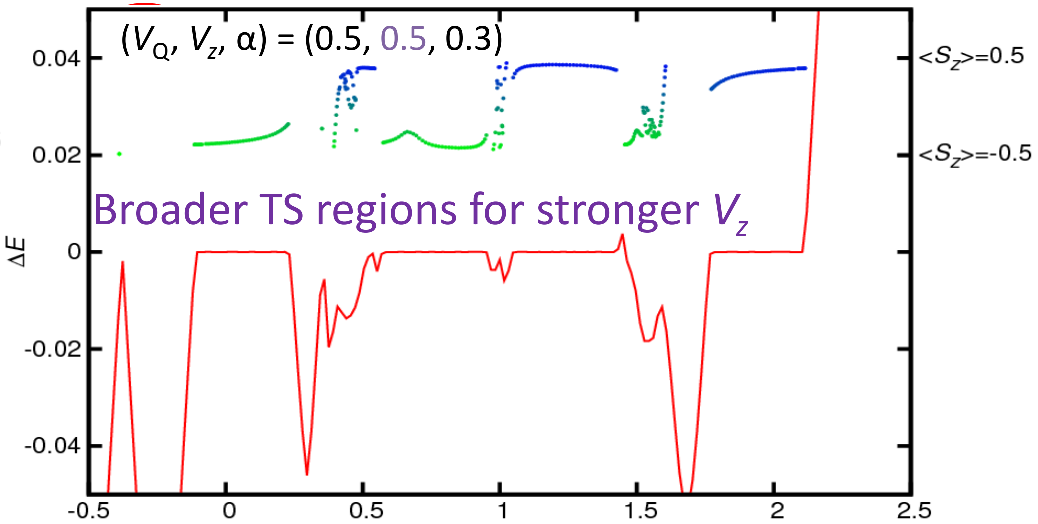
☑ U=0 case: agrees with BdG results for all parameter ranges studied

Multiple regions with degeneracy (ΔE=0)
➔ End Majorana fermions

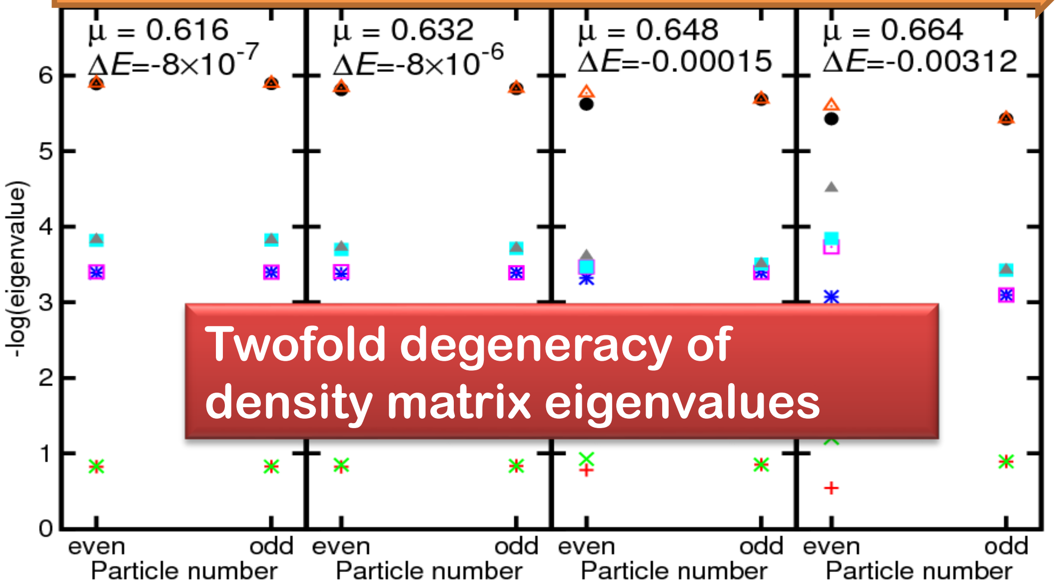
Δ = 0.1 $\qquad \Delta E = E_{\text{even}} - E_{\text{odd}} \qquad \Delta$=0.1 $\qquad \kappa = \sqrt{5} - 2$



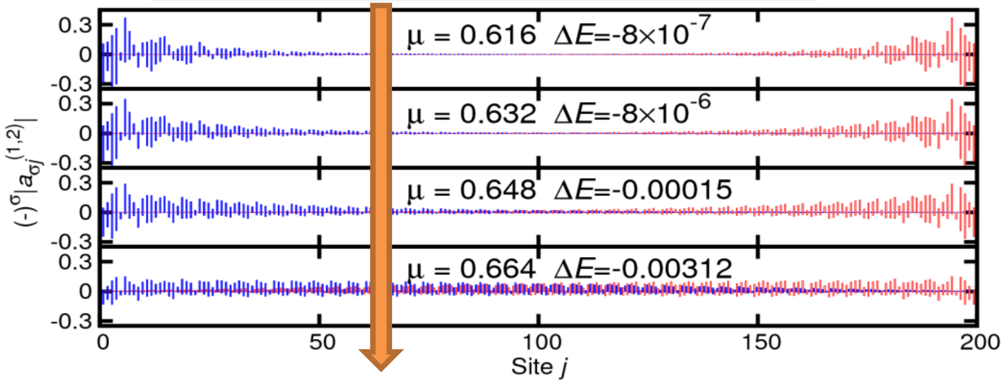
Chemical potential, Single particle energy



Chemical potential, Single particle energy



Localized Majorana modes



$\langle \Psi_o | \hat{c}^{\dagger}_{\sigma,l} | \Psi_e \rangle + \langle \Psi_e | \hat{c}^{\dagger}_{\sigma,l} | \Psi_o \rangle \qquad i \left(\langle \Psi_o | \hat{c}^{\dagger}_{\sigma,l} | \Psi_e \rangle - \langle \Psi_e | \hat{c}^{\dagger}_{\sigma,l} | \Psi_o \rangle \right)$

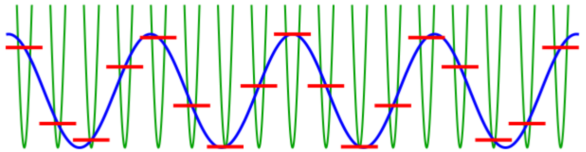
On-site interaction

V_Q: quasiperiodic potential amplitude
V_z: Zeeman energy
$2t = 2$: band width (hopping=$t/2$)
α: spin-orbit coupling
Δ: proximity pairing

$\kappa = \sqrt{5} - 2$

Hamiltonian
(Tight binding model)

$\varepsilon_{\sigma,l} = V_Q \cos(\kappa x + \varphi_0)$; $x = l - (L-1)/2$



Site l (=0, 1, …, L-1)

hopping (t = 1)

$$\mathcal{H} = -\frac{t}{2}\sum_{l=0}^{L-2}\sum_{\sigma=\uparrow,\downarrow}(\hat{c}^\dagger_{\sigma,l}\hat{c}_{\sigma,l+1} + \text{h.c.})$$

Zeeman

$$+\sum_{l=0}^{L-1}\left[V_z(\hat{n}_{\uparrow,l} - \hat{n}_{\downarrow,l}) + \sum_{\sigma=\uparrow,\downarrow}(t - \mu + \epsilon_{\sigma,l})\hat{n}_{\sigma,l}\right]$$

Quasiperiodic potential

$$+\frac{\alpha}{2}\sum_{l=0}^{L-2}[(\hat{c}^\dagger_{\downarrow,l}\hat{c}_{\uparrow,l+1} - \hat{c}^\dagger_{\uparrow,l}\hat{c}_{\downarrow,l+1}) + \text{h.c.}].$$

Spin-orbit coupling (Rashba type)

$$+\Delta\sum_{l=0}^{L-1}(\hat{c}_{\uparrow,l}\hat{c}_{\downarrow,l} + \text{h.c.})$$

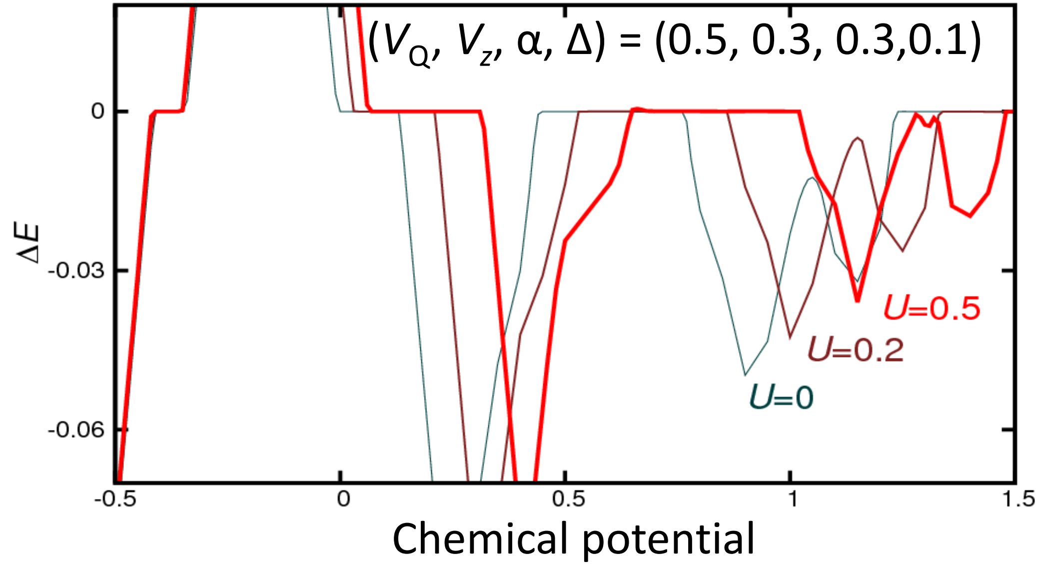
proximity pairing

$$+U\sum_{l=0}^{L-1}\hat{n}_{\uparrow,l}\hat{n}_{\downarrow,l}$$

On-site repulsion

"Enhances magnetism ➔ wider TS region"
[Stoudenmire *et al.*: PRB 84, 014503 (2011)]
(no inhomogeneity)

Our result for quasiperiodic modulation

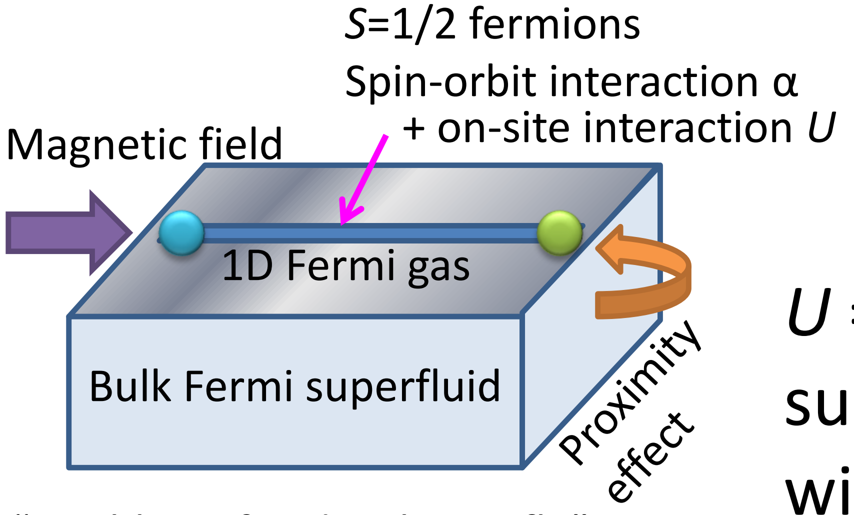


➔ Topological superconductor phase:
also widened by $U > 0$; end MFs observed

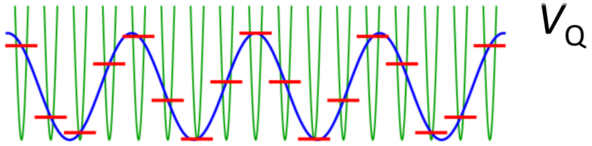
Quick summary 3: 1D topological superfluid with Majorana end fermions

Effect of (quasi)periodic site level modulation

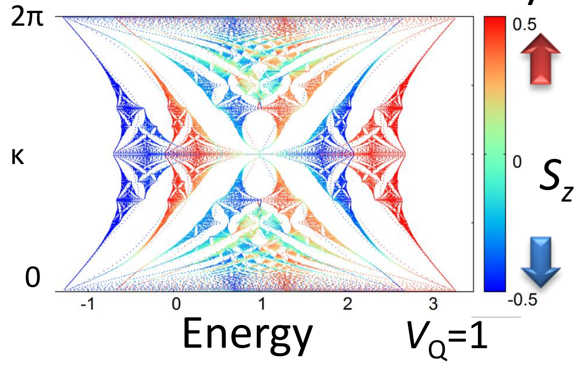
M. Tezuka and N. Kawakami: PRB **85**, 140508(R) (2012); PRB **88**, 155428 (2013)



$\varepsilon_{\sigma,l} = V_Q \cos(\kappa x + \varphi_0)$; $x = l - (L-1)/2$



"Double Hofstadter butterfly"



U = 0 (BdG OK): Multiple topological superconductor phases with end Majorana fermions

(stable against phase jumps)

U > 0 (DMRG needed): TS phases are broadened

Summary

Interacting cold atoms on quasiperiodic lattices exhibit various phases:

1. Attractively interacting spin-1/2 fermions
   - Pairing enhanced by lattice deformation
   - Anomalous exponent after release from trap
   - ➢ MT and A. M. Garcia-Garcia: PRA **82**, 043613 (2010), PRA **85**, 031602R (2012)
2. Repulsively interacting spinless bosons
   - Topologically non-trivial incommensurate CDW phase
   - Equivalence between Harper-type and Fibonacci-type lattices
   - ➢ Fuyuki Matsuda, MT, and Norio Kawakami: arXiv:1404.6315 (to appear in JPSJ)
3. Spin-1/2 fermions with proximity pairing
   - Spin-orbit coupling introduces a peculiar self-similar band structure
   - Reentrant topological transitions
   - ➢ MT and Norio Kawakami: PRB **85**, 140508R(2012), PRB **88**, 155428 (2013)