Anderson pseudospin resonance with Higgs mode in superconductors

Matsunaga, Tsuji, et al., to be published in Science.
Tsuji, Aoki, arXiv:1404.2711

03 July 2014 @ ISSP workshop
"New Horizon of Strongly Correlated Physics"

Naoto Tsuji (Univ. of Tokyo)

Plan of the talk

- What is "Higgs" in superconductors? — Brief introduction.

- Very recent THz laser Experiment:
  Observation of "Anderson pseudospin resonance" with Higgs mode in a superconductor. [Matsunaga, Tsuji, et al., Science (2014)]

- Theory:
  Analytically solve BCS equation of motion. [Tsuji, Aoki, arXiv:1404.2711]
  Discuss effects of electron-electron scattering (nonequilibrium DMFT) and impurity scattering (Abrikosov-Gor'kov theory).

Superconductivity

- Ginzburg-Landau functional:

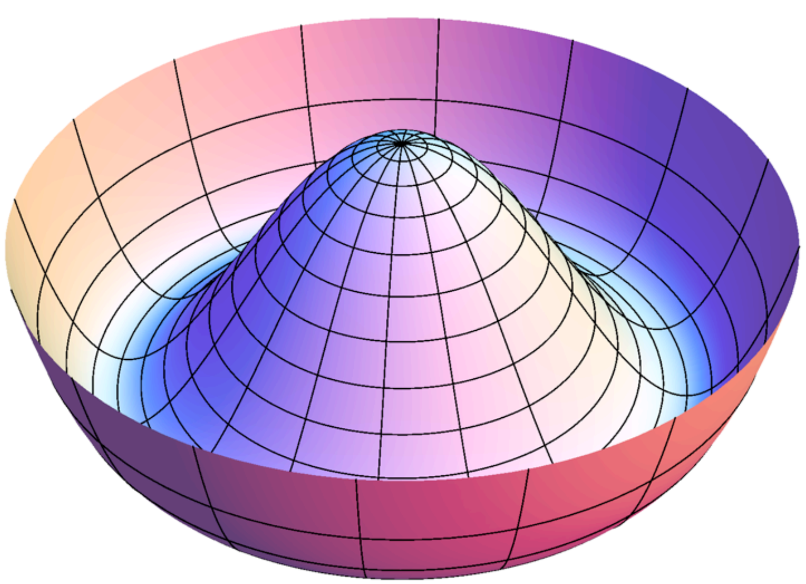
$$\mathcal{F}_{GL} = \int d\boldsymbol{r} \left[\frac{1}{2m}(\nabla - ie^* \boldsymbol{A})\psi \cdot (\nabla + ie^* \boldsymbol{A})\psi^\dagger + V(\psi) + \frac{1}{2\mu}(\nabla \times \boldsymbol{A})^2 \right]$$

- Gauge invariant:

$$\psi \longrightarrow \psi e^{ie^*\chi} \qquad \boldsymbol{A} \longrightarrow \boldsymbol{A} + \nabla\chi$$

- Mexican hat potential → Spontaneous symmetry breaking

$$V(\psi) = a_0(T - T_c)|\psi|^2 + b|\psi|^4$$



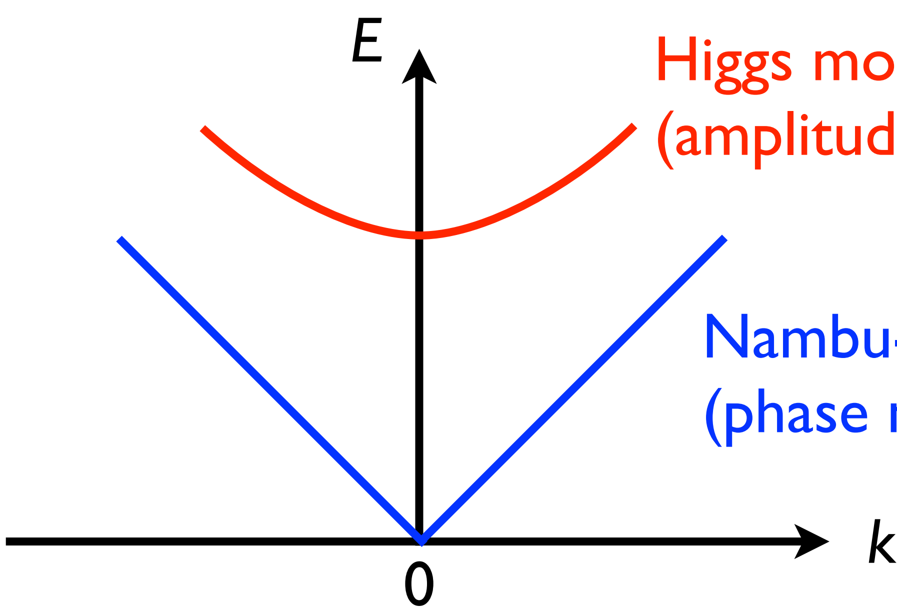
Higgs & Nambu-Goldstone

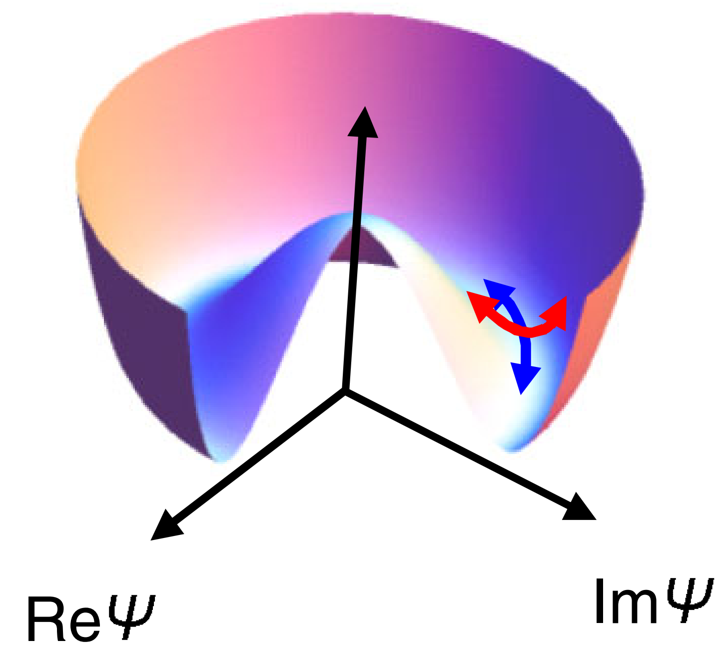
- Without coupling to gauge fields:

$$\mathcal{F} = \int d\boldsymbol{r} \left[\frac{1}{2m} \nabla\psi \cdot \nabla\psi^\dagger + V(\psi) \right] \qquad V(\psi) = a|\psi|^2 + b|\psi|^4 \quad (a < 0)$$

- Elementary excitation:

$$\psi = \psi_0 + \delta\psi_1 + i\delta\psi_2$$

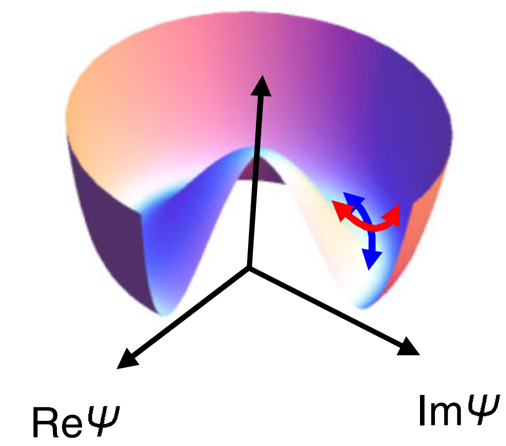




2 massive modes

$$\longrightarrow \begin{cases} 1 \text{ massive mode} \\ + 1 \text{ massless mode} \end{cases}$$

Higgs mechanism



- Elementary excitation:

$$\psi = \psi_0 + \delta\psi_1 + i\delta\psi_2$$

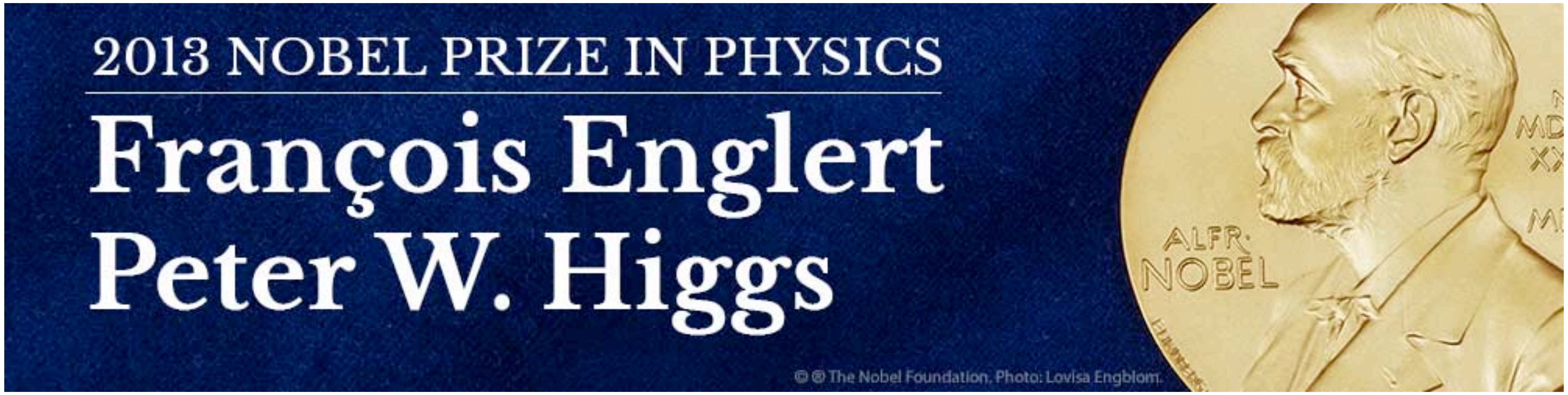
- Effective free energy:

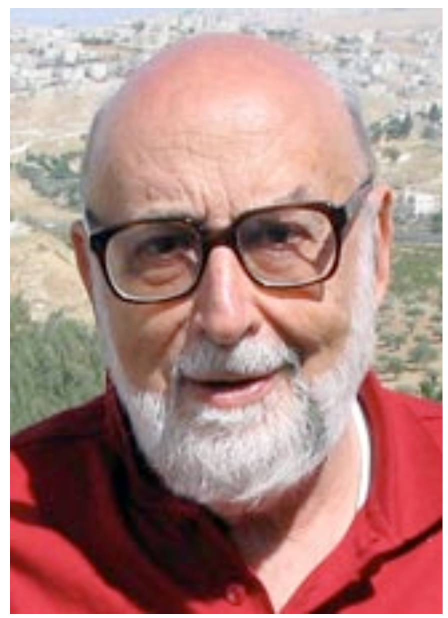
$$\mathcal{F}_{GL} = \int d\boldsymbol{r}\left[\frac{1}{2m}(\nabla\delta\psi_1)^2 - 2a(\delta\psi_1)^2 + \frac{1}{2\mu}(\nabla \times \boldsymbol{A})^2 + \frac{e^{*2}\psi_0^2}{2m}\boldsymbol{A}^2 + \cdots\right]$$

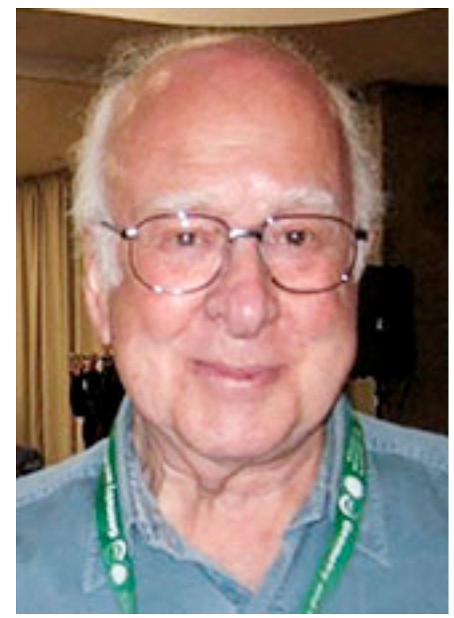
- Gauge transf. to eliminate the unphysical $\delta\psi_2$

$$\left\{\begin{array}{l}\text{2 massive modes}\\ \text{+ 1 photon (transverse)}\end{array}\right. \longrightarrow \left\{\begin{array}{l}\text{1 massive mode (Higgs)}\\ \text{+ 1 massive photon (transverse \& longitudinal)}\end{array}\right.$$

2 + 2 $\qquad\qquad$ 1 + 3







Here, at last!

The Nobel Prize in Physics 2013 was awarded jointly to François Englert and Peter W. Higgs *"for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider"*

Higgs mode

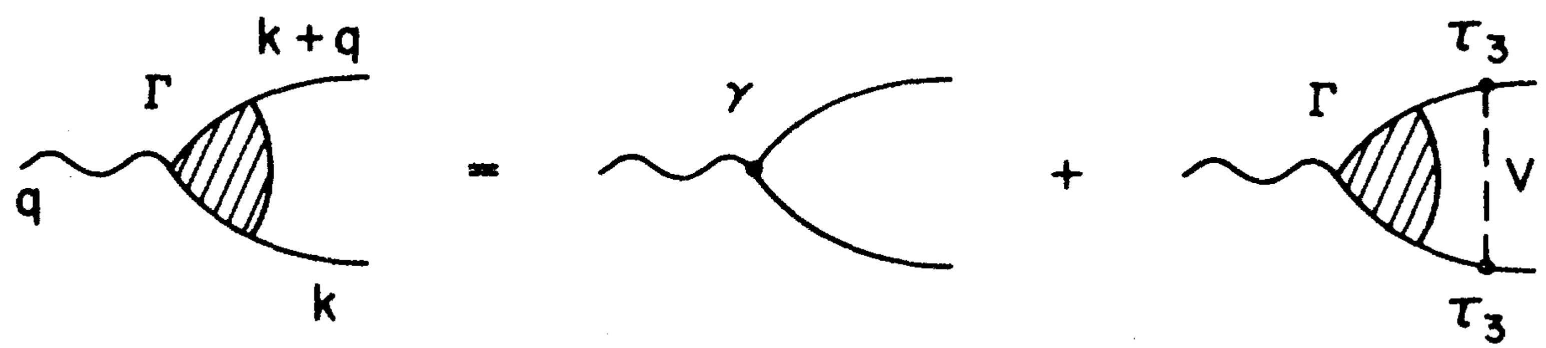
Littlewood, Varma (1981, 1982).

- The system is invariant under a gauge transf.:

$$\Psi(\boldsymbol{r},t) \to \exp(\alpha(\boldsymbol{r},t)\tau_1)\Psi(\boldsymbol{r},t)$$

- This leads to a continuity equation, which is satisfied when one requires

$$\phi(q)\tau_1 = -V\int \frac{d^4k}{(2\pi)^4}\tau_3 G(k+q)\phi(q)\tau_1 G(k)\tau_3$$



$$1 + \frac{1}{2}V\int \frac{d^3k}{(2\pi)^3}\frac{\epsilon^2}{E(\nu^2/4 - E^2)} = 0$$

- When $\nu = 2\Delta$, it reduces to the gap equation.
  → The existence of the Higgs mode at $\nu = 2\Delta$ in all (s-wave BCS) superconductors.

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Raman Scattering by Superconducting-Gap Excitations and Their Coupling to Charge-Density Waves

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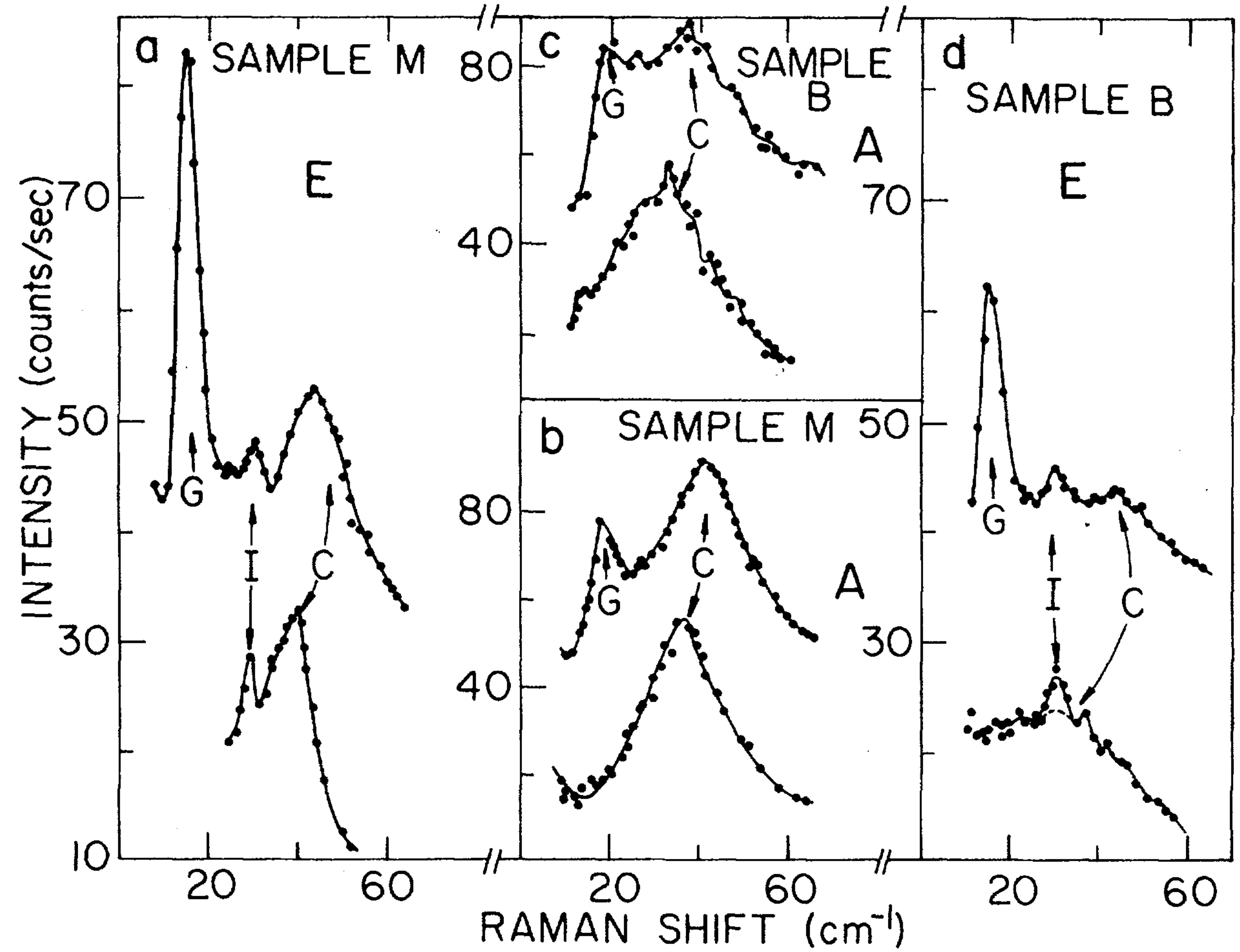
(Received 24 March 1980)



2H-NbSe2
(SC+CDW)

See also Measson et al. PRB (2014).

Higgs Amplitude Mode in the BCS Superconductors $Nb_{1-x}Ti_xN$ Induced by Terahertz Pulse Excitation

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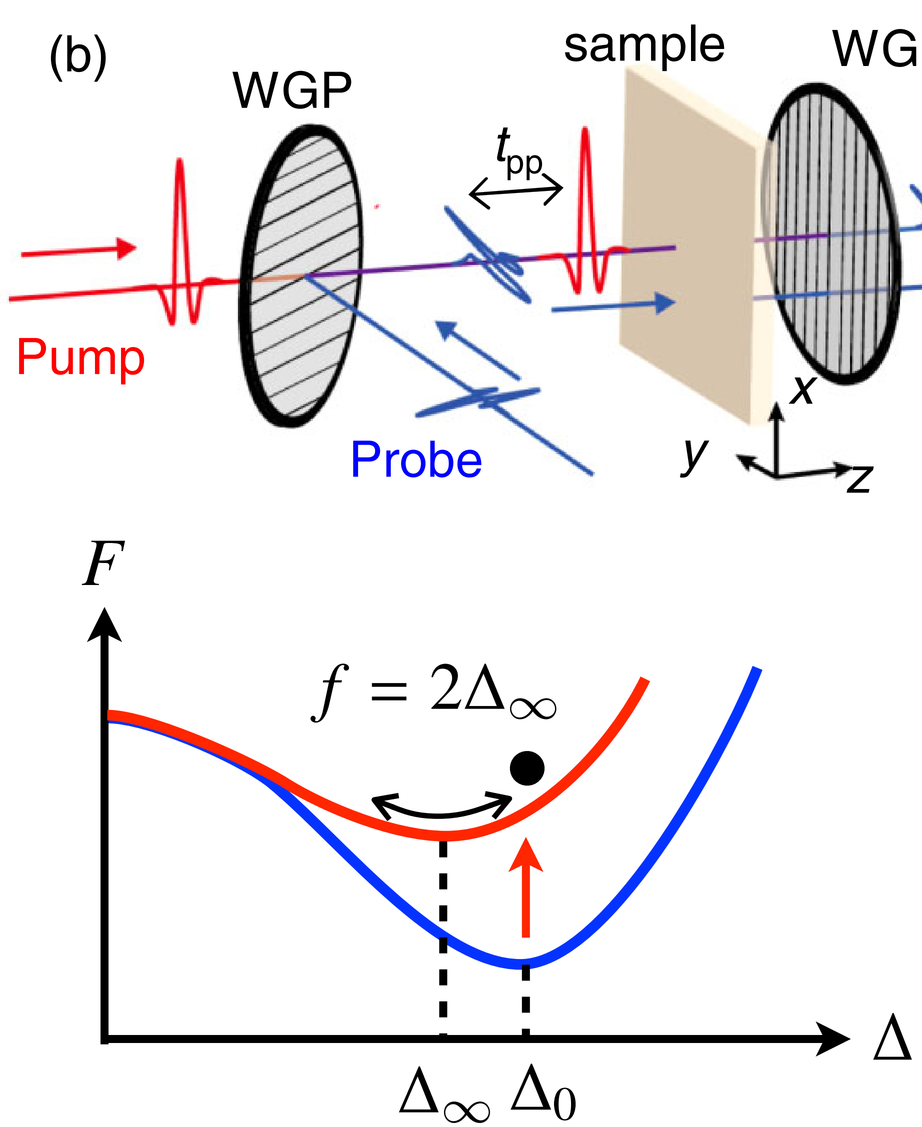

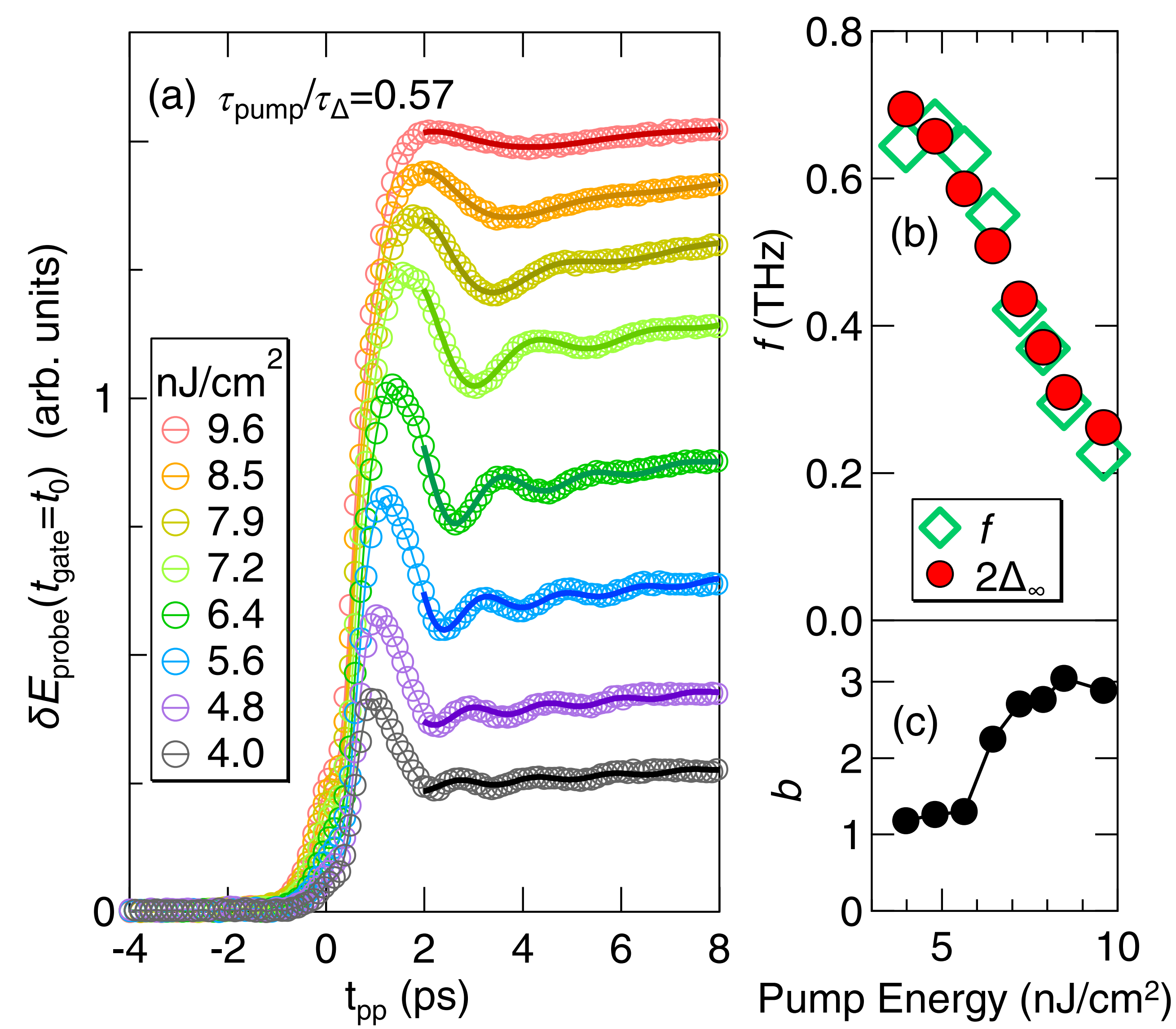

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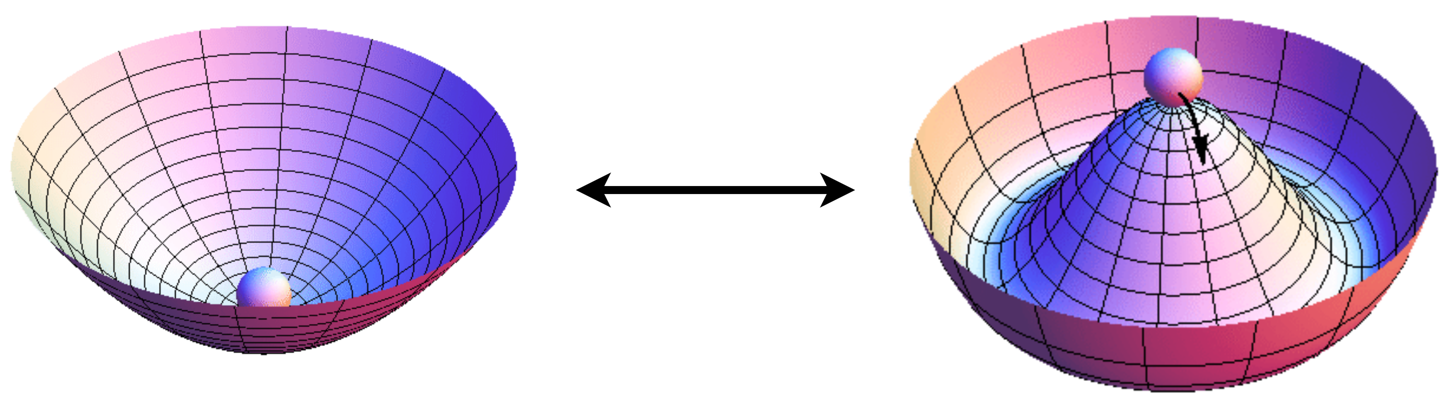
Dynamics of superconductors

- Time-dependent Ginzburg-Landau equation

$$-\Gamma\frac{\partial\Delta}{\partial t}=\frac{\delta\mathcal{F}_{GL}}{\delta\Delta^*}=a\Delta+b|\Delta|^2\Delta-\frac{c}{2m}\nabla^2\Delta$$



- Microscopically justified
  - near the critical point (Ginzburg condition: $|T\text{-}T_c|<T_G$)
  - when (time scale of order parameter) >> (quasiparticle relaxation time)

Dynamics of superconductors

- Bogoliubov-de Gennes equation coupled to an electric field

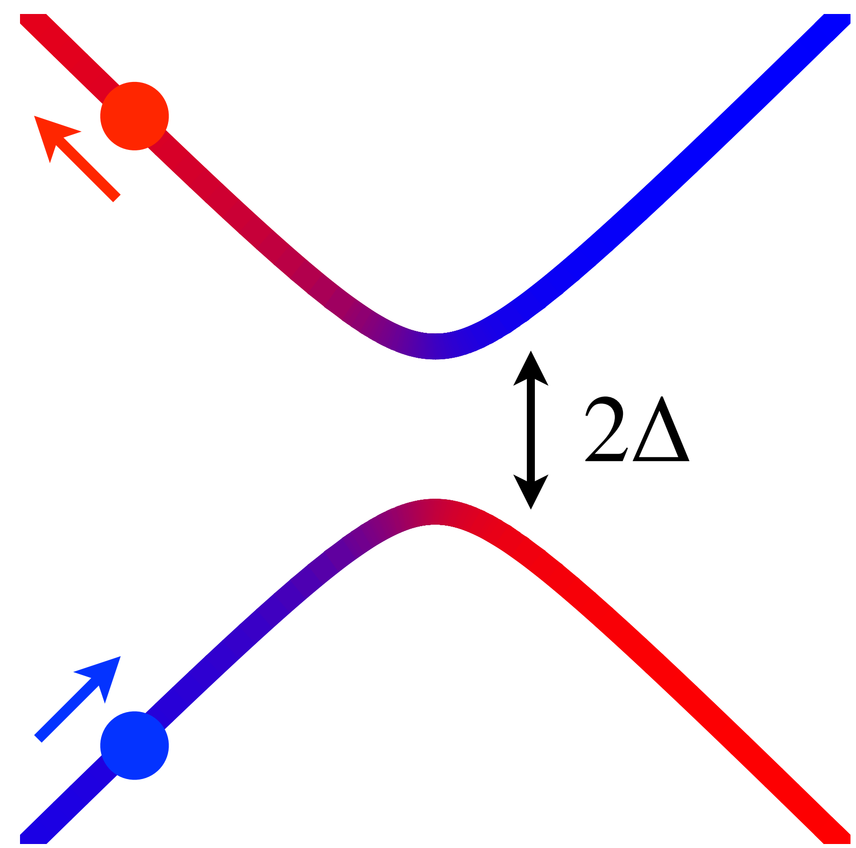
electron

$$i\partial_t \Psi_{\boldsymbol{k}} = \begin{pmatrix} \epsilon_{\boldsymbol{k}-eA(t)} & -\Delta^* \\ -\Delta & -\epsilon_{\boldsymbol{k}+eA(t)} \end{pmatrix} \Psi_{\boldsymbol{k}}$$

hole

$$\Psi_{\boldsymbol{k}} = \begin{pmatrix} c_{\boldsymbol{k}\uparrow} \\ c^\dagger_{-\boldsymbol{k}\downarrow} \end{pmatrix}$$: Nambu spinor

$$A(t) = A \sin \Omega t$$: ac electric field



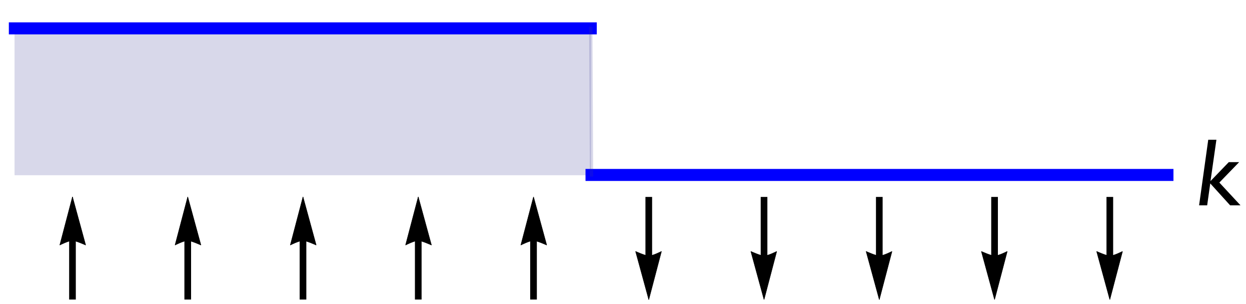
Anderson pseudospin

$$\boldsymbol{\sigma}_{\boldsymbol{k}} = \frac{1}{2}\Psi_{\boldsymbol{k}}^{\dagger} \cdot \boldsymbol{\tau} \cdot \Psi_{\boldsymbol{k}}$$

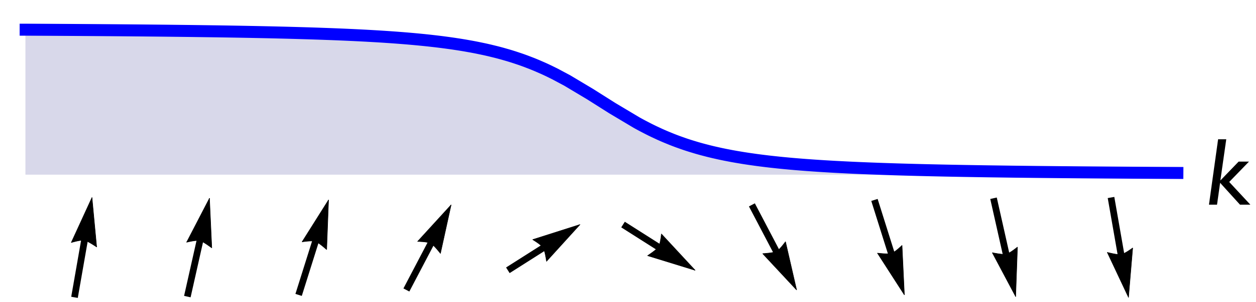
Anderson, Phys. Rev. 112, 1900 (1958)

$$\boxed{\partial_t \boldsymbol{\sigma}_{\boldsymbol{k}} = 2\boldsymbol{b}_{\boldsymbol{k}} \times \boldsymbol{\sigma}_{\boldsymbol{k}} \qquad \boldsymbol{b}_{\boldsymbol{k}} = \left(-\Delta', -\Delta'', \frac{\epsilon_{\boldsymbol{k}-e\boldsymbol{A}(t)} + \epsilon_{\boldsymbol{k}+e\boldsymbol{A}(t)}}{2}\right)}$$

Tsuji, Aoki, arXiv:1404.2711



normal state



BCS state

- Particle-hole symmetric by construction.
- Linear response vanishes.

Light-pseudospin coupling

$$\boxed{\partial_t \boldsymbol{\sigma}_{\boldsymbol{k}} = 2\boldsymbol{b}_{\boldsymbol{k}} \times \boldsymbol{\sigma}_{\boldsymbol{k}} \qquad \boldsymbol{b}_{\boldsymbol{k}} = \left(-\Delta', -\Delta'', \frac{\epsilon_{\boldsymbol{k}-e\boldsymbol{A}(t)} + \epsilon_{\boldsymbol{k}+e\boldsymbol{A}(t)}}{2}\right)}$$

$$b^z_{\boldsymbol{k}} = \epsilon_{\boldsymbol{k}} + \frac{1}{2}\sum_{ij} \frac{\partial^2 \epsilon_{\boldsymbol{k}}}{\partial k_i \partial k_j} e^2 A_i(t) A_j(t) + O(A^4)$$

- Let *x* be the polarization direction of the electric field.

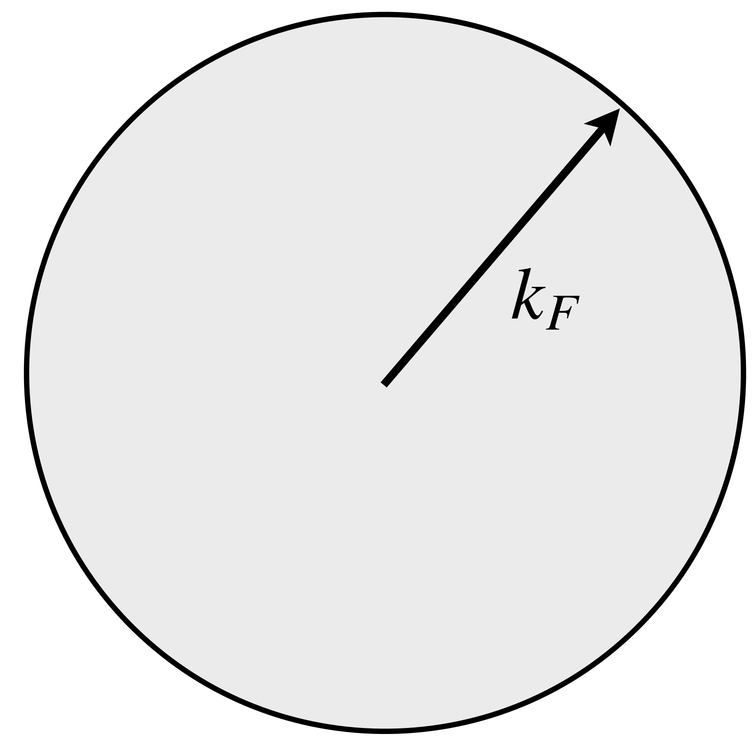
- When all the directions are equivalent, one can symmetrize

$$\frac{\partial^2 \epsilon_{\boldsymbol{k}}}{\partial k_x^2} \to \frac{1}{d}\nabla^2_{\boldsymbol{k}} \epsilon_{\boldsymbol{k}}$$

$$\boxed{\partial_t \boldsymbol{\sigma}_{\boldsymbol{k}} = 2\boldsymbol{b}_{\boldsymbol{k}} \times \boldsymbol{\sigma}_{\boldsymbol{k}} \quad \boldsymbol{b}_{\boldsymbol{k}} = \left(-\Delta', -\Delta'', \epsilon_{\boldsymbol{k}} + \frac{1}{2}e^2 A(t)^2 \cdot \frac{1}{d}\nabla^2_{\boldsymbol{k}}\epsilon_{\boldsymbol{k}}\right)}$$

- Consider an isotropic system [$\epsilon_{\boldsymbol{k}} = \epsilon(|\boldsymbol{k}|)$]
- Expand $\epsilon_{\boldsymbol{k}}$ near the Fermi surface:

$$\epsilon_{\boldsymbol{k}} = \sum_{n=1}^{\infty} c_n(|\boldsymbol{k}| - k_F)^n$$



- Let us define an expansion

$$\frac{1}{d}\nabla^2_{\boldsymbol{k}}\epsilon_{\boldsymbol{k}} = \alpha_0 + \alpha_1\epsilon_{\boldsymbol{k}} + \alpha_2\epsilon^2_{\boldsymbol{k}} + \cdots$$

with $\alpha_0 = 2c_2 d^{-1} + c_1(1 - d^{-1})k_F^{-1}$

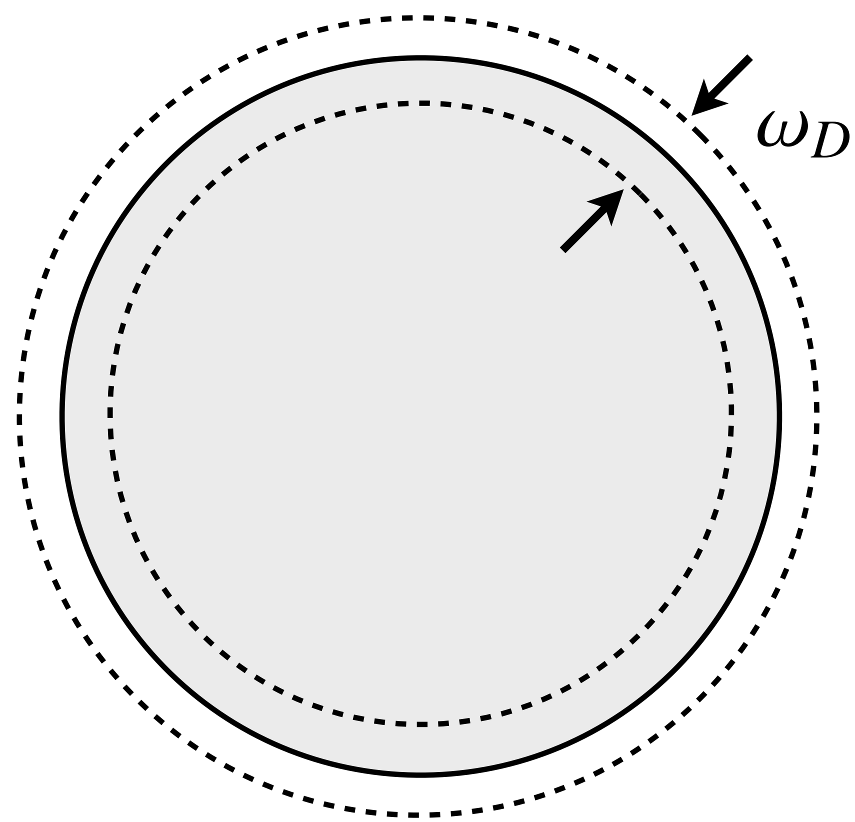
$\alpha_1 = c_1^{-1}[6c_3 d^{-1} + (1 - d^{-1})(2c_2 k_F^{-1} - c_1 k_F^{-2})]$, etc.

$$\boxed{\partial_t \boldsymbol{\sigma}_{\boldsymbol{k}} = 2\boldsymbol{b}_{\boldsymbol{k}} \times \boldsymbol{\sigma}_{\boldsymbol{k}} \quad \boldsymbol{b}_{\boldsymbol{k}} = \left(-\Delta', -\Delta'', \epsilon_{\boldsymbol{k}} + \frac{1}{2}e^2 A(t)^2 \cdot \frac{1}{d}\nabla_{\boldsymbol{k}}^2 \epsilon_{\boldsymbol{k}}\right)}$$

$$\frac{1}{d}\nabla_{\boldsymbol{k}}^2 \epsilon_{\boldsymbol{k}} = \alpha_0 + \alpha_1 \epsilon_{\boldsymbol{k}} + \alpha_2 \epsilon_{\boldsymbol{k}}^2 + \cdots$$

Remarks:

- The α_n term has a contribution of order $(\omega_D/\epsilon_F)^n$.
- The first term (potential shift) can be gauged out.
- The α_1 term is the leading.
- For the ideal parabolic band $\epsilon_{\boldsymbol{k}} = \boldsymbol{k}^2/2m$, $\alpha_1 = 0$.



Anderson pseudospin precession

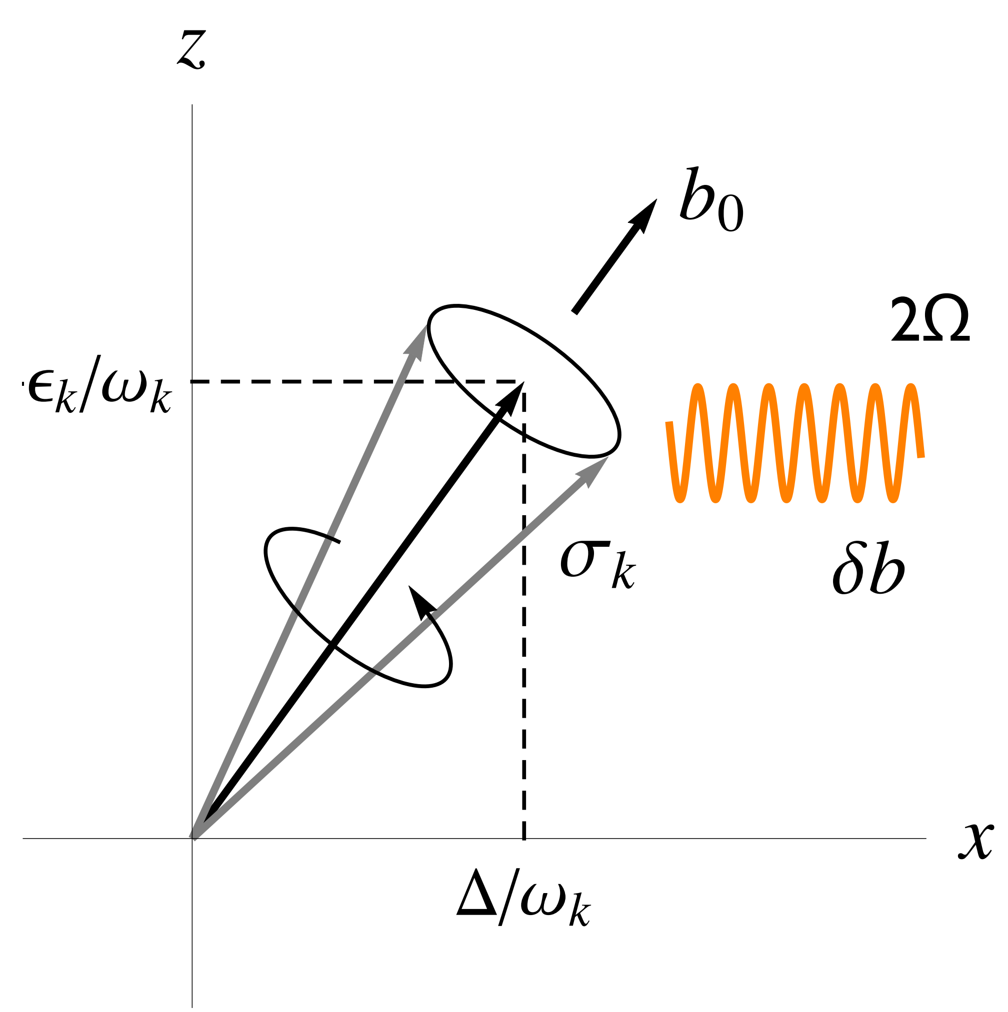
$$\boxed{\partial_t \boldsymbol{\sigma}_{\boldsymbol{k}} = 2\boldsymbol{b}_{\boldsymbol{k}} \times \boldsymbol{\sigma}_{\boldsymbol{k}} \quad \boldsymbol{b}_{\boldsymbol{k}} = \left(-\Delta', -\Delta'', \epsilon_{\boldsymbol{k}} + \frac{1}{2}\alpha_1 \epsilon_{\boldsymbol{k}}\, e^2 A(t)^2\right)}$$

- Let us first take Δ to be time independent (no self-consistency for Δ).

→ Usual spin resonance problem.

$$\delta\sigma_{\boldsymbol{k}}(t) \sim \frac{1}{(2\Omega)^2 - \omega_{\boldsymbol{k}}^2}\delta b_{\boldsymbol{k}}(t)$$

$$\omega_{\boldsymbol{k}} = 2\sqrt{\epsilon_{\boldsymbol{k}}^2 + \Delta^2}$$



$$\partial_t \boldsymbol{\sigma}_{\boldsymbol{k}} = 2\boldsymbol{b}_{\boldsymbol{k}} \times \boldsymbol{\sigma}_{\boldsymbol{k}} \qquad \boldsymbol{b}_{\boldsymbol{k}} = \left(-\Delta', -\Delta'', \epsilon_{\boldsymbol{k}} + \frac{1}{2}\alpha_1 \epsilon_{\boldsymbol{k}} e^2 A(t)^2\right)$$

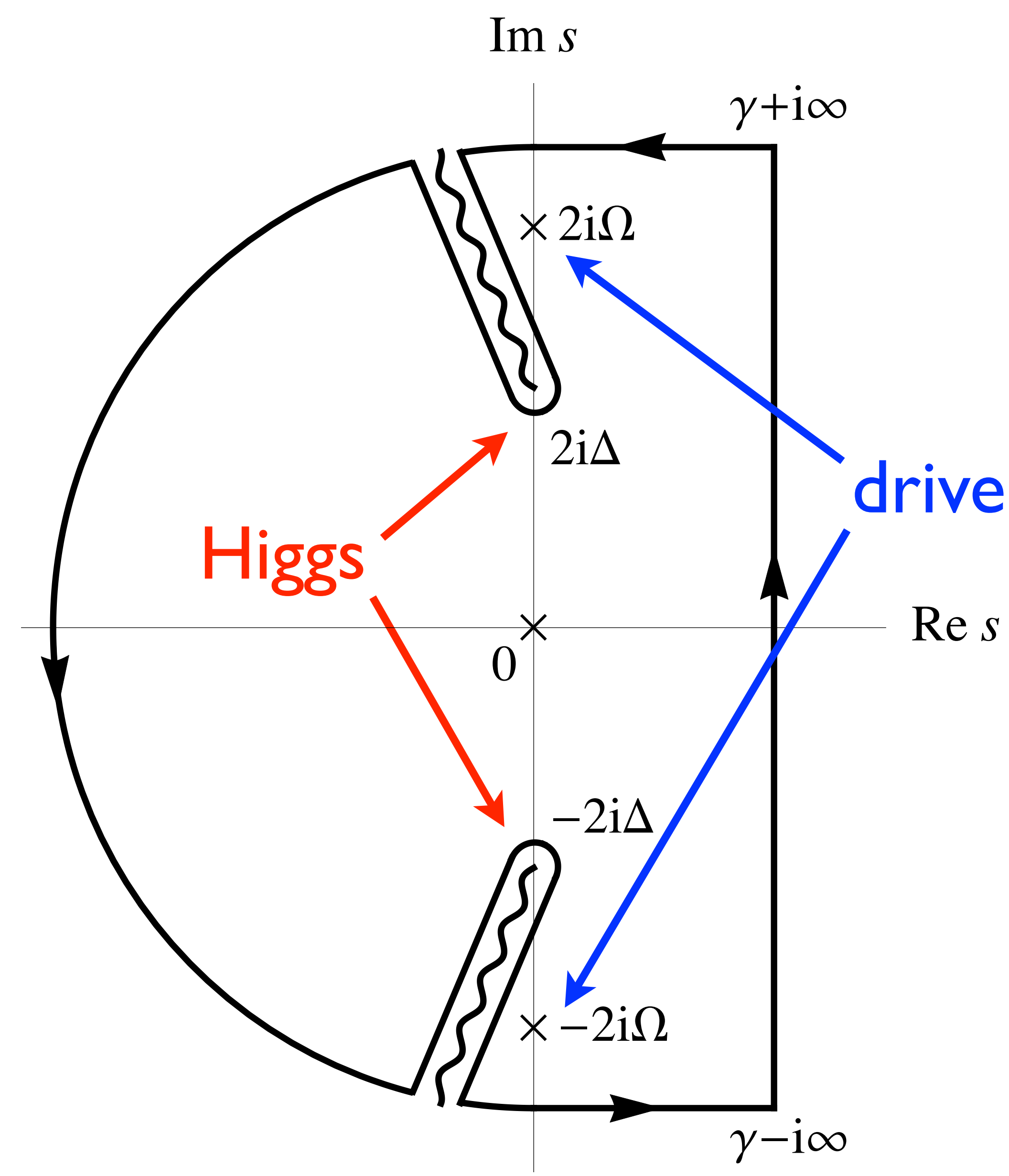
$$\Delta = U \sum_{\boldsymbol{k}} (\sigma^x_{\boldsymbol{k}} + i\sigma^y_{\boldsymbol{k}})$$ (self-consistency condition)

- Solve the equation of motion up to O(A^2).
- Laplace transformation:

$$\frac{\delta\Delta(s)}{\alpha_1 e^2 A^2 \Delta} = \frac{\Omega^2}{s(s^2+4\Omega^2)}\left[1 - \frac{1}{\lambda(s^2+4\Delta^2)F(s)}\right]$$

$$F(s) = \frac{1}{s\sqrt{s^2+4\Delta^2}} \sinh^{-1}\left(\frac{s}{2\Delta}\right)$$

cf. Volkov, Kogan (1973)



$$\partial_t \boldsymbol{\sigma}_{\boldsymbol{k}} = 2\boldsymbol{b}_{\boldsymbol{k}} \times \boldsymbol{\sigma}_{\boldsymbol{k}} \qquad \boldsymbol{b}_{\boldsymbol{k}} = \left(-\Delta', -\Delta'', \epsilon_{\boldsymbol{k}} + \frac{1}{2}\alpha_1 \epsilon_{\boldsymbol{k}}\, e^2 A(t)^2\right)$$

$$\Delta = U \sum_{\boldsymbol{k}} (\sigma^x_{\boldsymbol{k}} + i\sigma^y_{\boldsymbol{k}})$$ (self-consistency condition)

Analytical solution:

For U quench, see Yuzbashyan, Dzero (2006); Barankov, Levitov (2006).

$$\frac{\delta\Delta(t)}{\alpha_1 e^2 A^2 \Delta} \sim \frac{1}{4\lambda}\left[\frac{2}{\pi^{3/2}}\frac{\Omega^2}{\Omega^2-\Delta^2}\frac{1}{\sqrt{\Delta t}}\cos\left(2\Delta t + \frac{\pi}{4}\right) - 1\right]$$

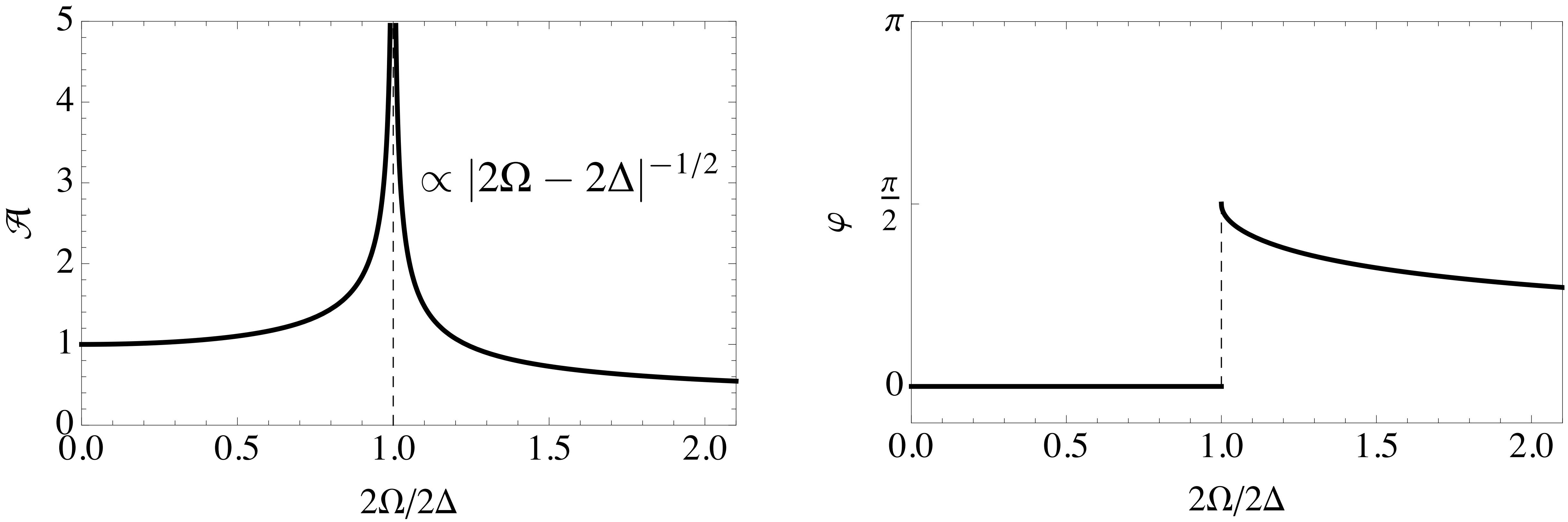
$$+\frac{1-\cos 2\Omega t}{4} + \boxed{\frac{1}{4\lambda} \times \begin{cases} \dfrac{\Omega}{\sqrt{\Delta^2-\Omega^2}}\dfrac{\cos 2\Omega t}{\sin^{-1}\left(\frac{\Omega}{\Delta}\right)} & \Omega < \Delta \\ \dfrac{\Omega}{\sqrt{\Omega^2-\Delta^2}}\dfrac{\cos(2\Omega t-\varphi)}{\sqrt{\left[\cosh^{-1}\left(\frac{\Omega}{\Delta}\right)\right]^2+\left(\frac{\pi}{2}\right)^2}} & \Omega > \Delta \end{cases}}$$

$$\varphi = \tan^{-1}\left(\frac{\pi/2}{\cosh^{-1}\left(\frac{\Omega}{\Delta}\right)}\right)$$

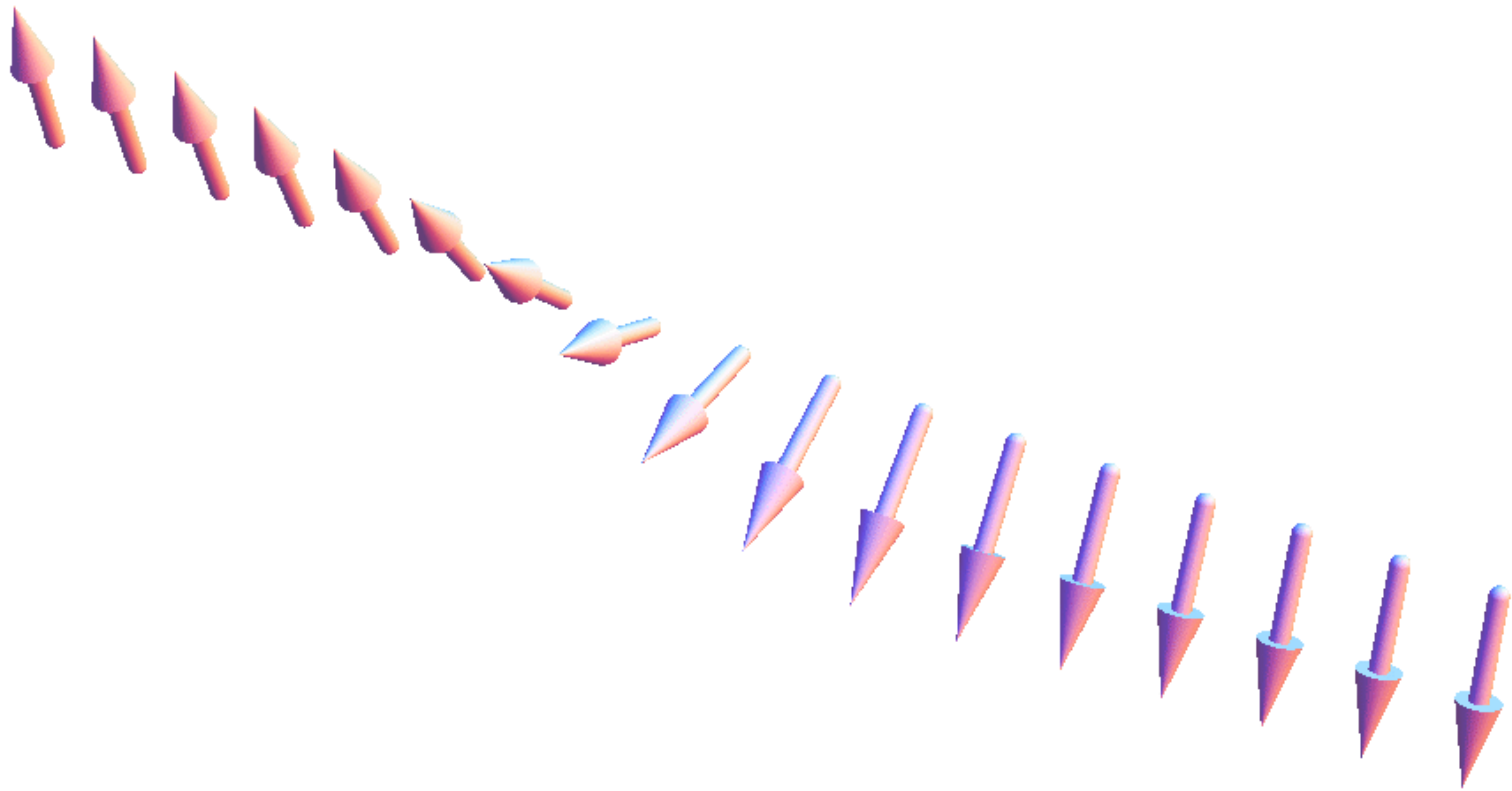
→ resonance with Higgs mode at 2Δ=2Ω

Anderson pseudospin resonance

- 2Ω oscillation of the order parameter: $\delta\Delta(t) \propto \mathscr{A} \cos(2\Omega t - \varphi)$
- $\mathscr{A}$ and φ are the universal function of $2\Omega/2\Delta$.

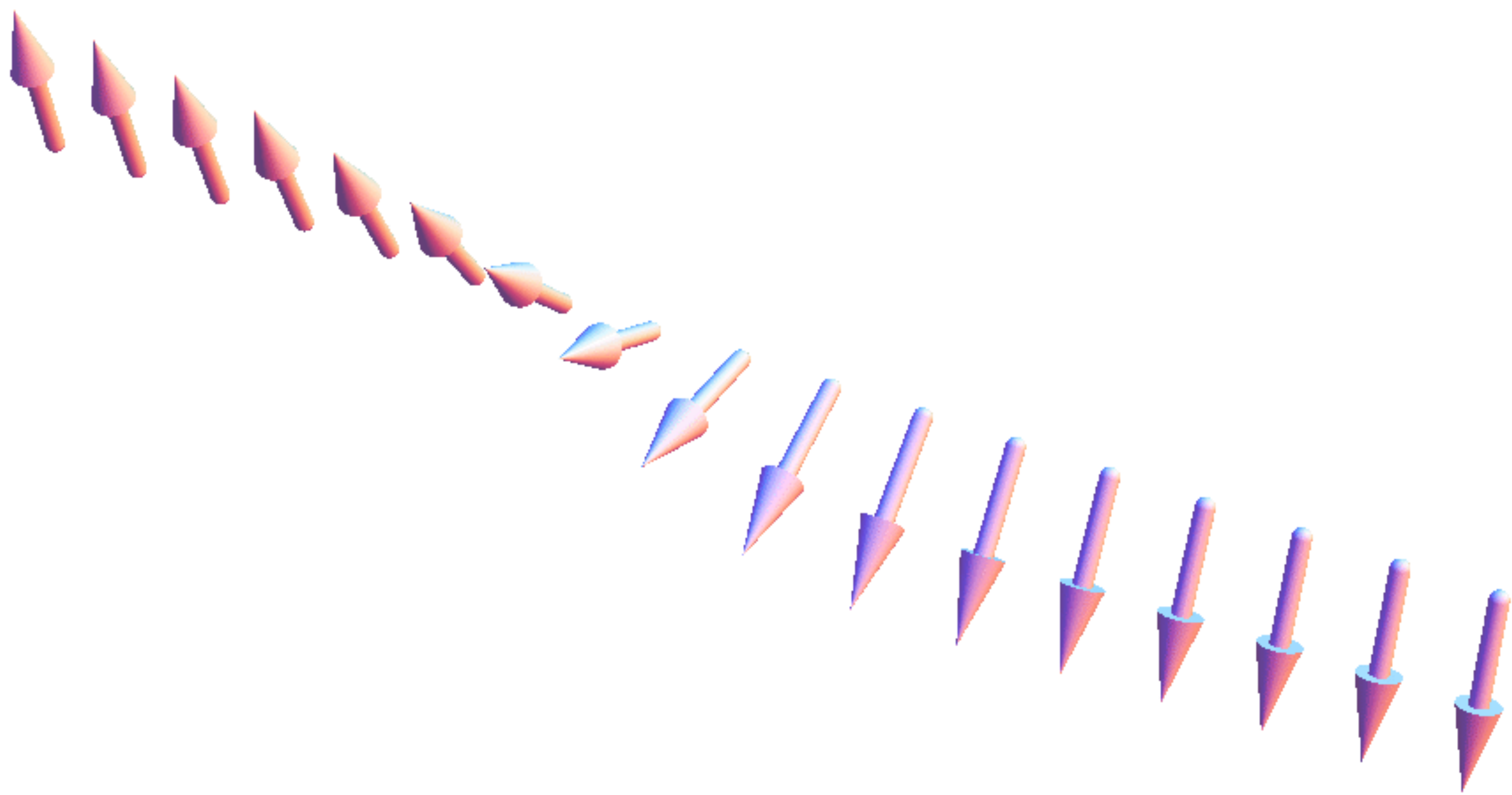


Off resonance



U=2, *W*=4, flat DOS
T=0, half filling
E=0.15, Ω=0.8Δ

On resonance



U=2, W=4, flat DOS
T=0, half filling
E=0.15, Ω=Δ

Third harmonic generation (THG)

- Current

$$\boldsymbol{j} = -e \sum_{\boldsymbol{k}\sigma} \boldsymbol{v}_{\boldsymbol{k}-e\boldsymbol{A}(t)} \langle c^{\dagger}_{\boldsymbol{k}\sigma} c_{\boldsymbol{k}\sigma} \rangle$$

$$= \sum_{\boldsymbol{k}} (-e\, \boldsymbol{v}_{\boldsymbol{k}-e\boldsymbol{A}(t)} + e\, \boldsymbol{v}_{\boldsymbol{k}+e\boldsymbol{A}(t)}) \sigma^z_{\boldsymbol{k}}$$

$$\approx 2e^2 \boldsymbol{A}(t) \sum_{\boldsymbol{k}} \alpha_1 \epsilon_{\boldsymbol{k}} \delta\sigma^z_{\boldsymbol{k}}$$

$$\epsilon_{\boldsymbol{k}} \delta\sigma^z_{\boldsymbol{k}} = \Delta \delta\sigma^x_{\boldsymbol{k}}$$

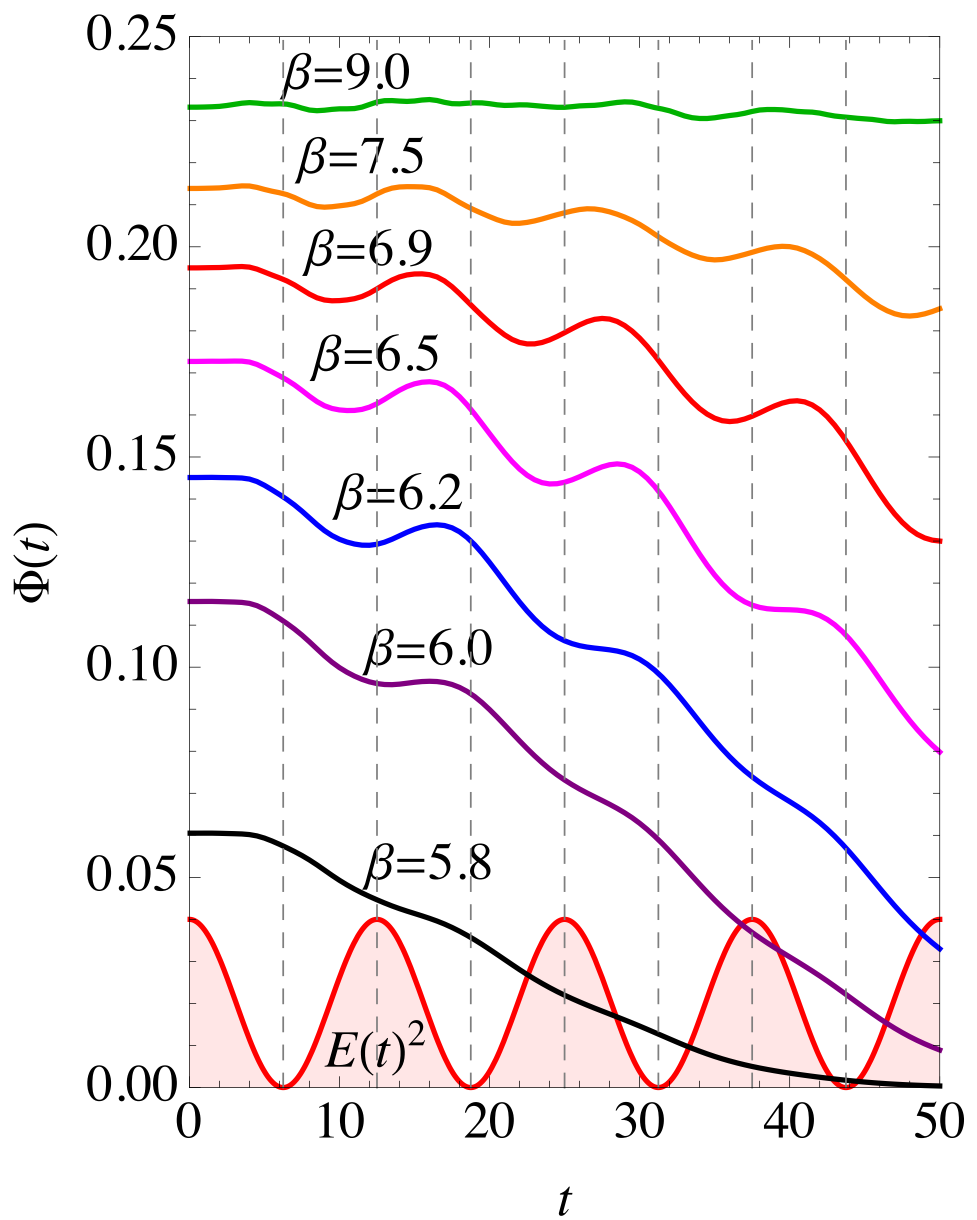
$$= 2e^2 \boldsymbol{A}(t) \sum_{\boldsymbol{k}} \alpha_1 \Delta \delta\sigma^x_{\boldsymbol{k}} = 2\alpha_1 e^2 \boldsymbol{A}(t) \Delta U^{-1} \delta\Delta(t)$$

- Divergent enhancement of THG!

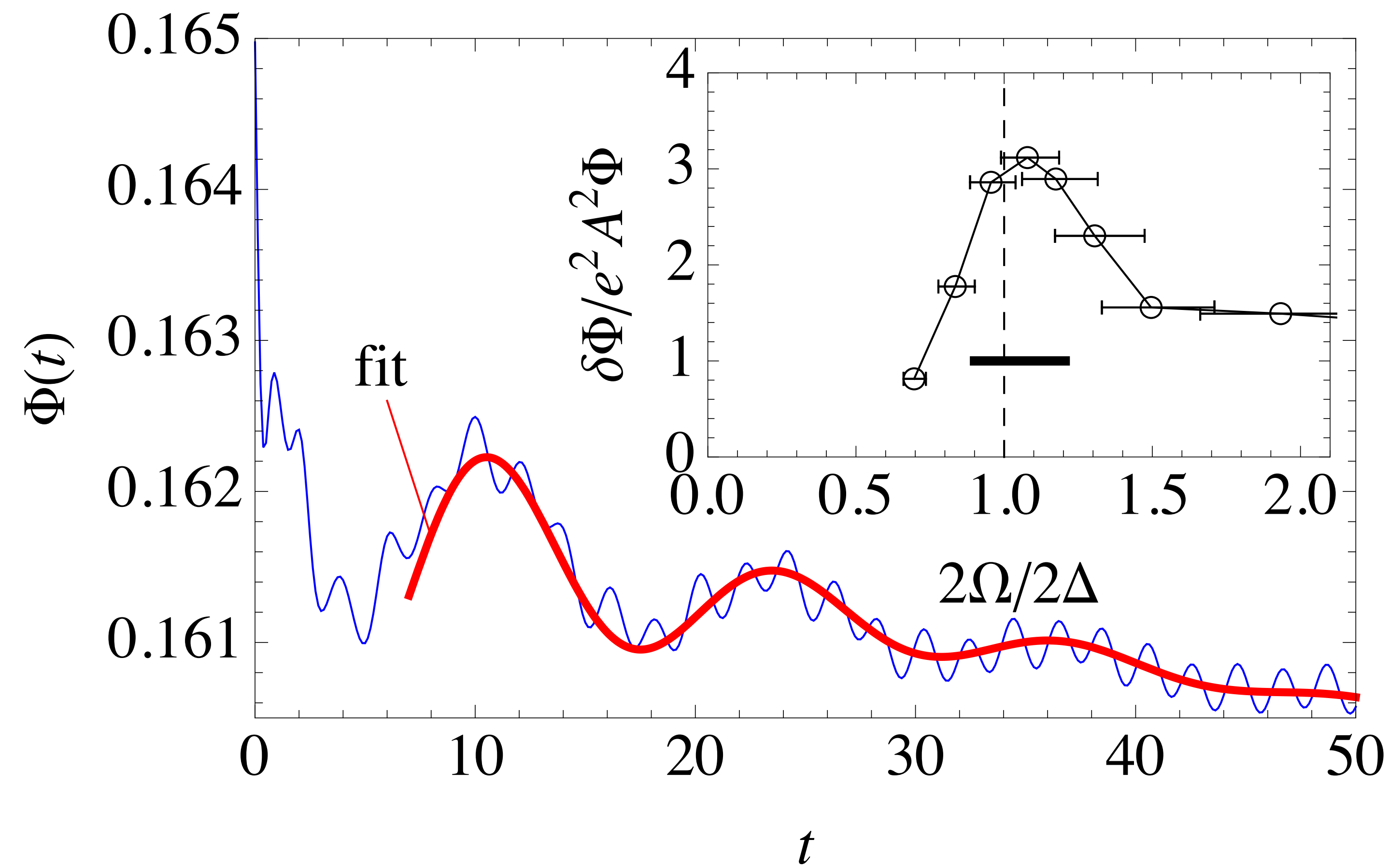
$$\boldsymbol{j}^{(3)}(t) \propto \delta\Delta(t) \boldsymbol{A}(t)$$

Coulomb scattering

- Nonequilibrium DMFT calculation [Aoki, Tsuji et al., Rev. Mod. Phys. (2014)] of the attractive Hubbard model.



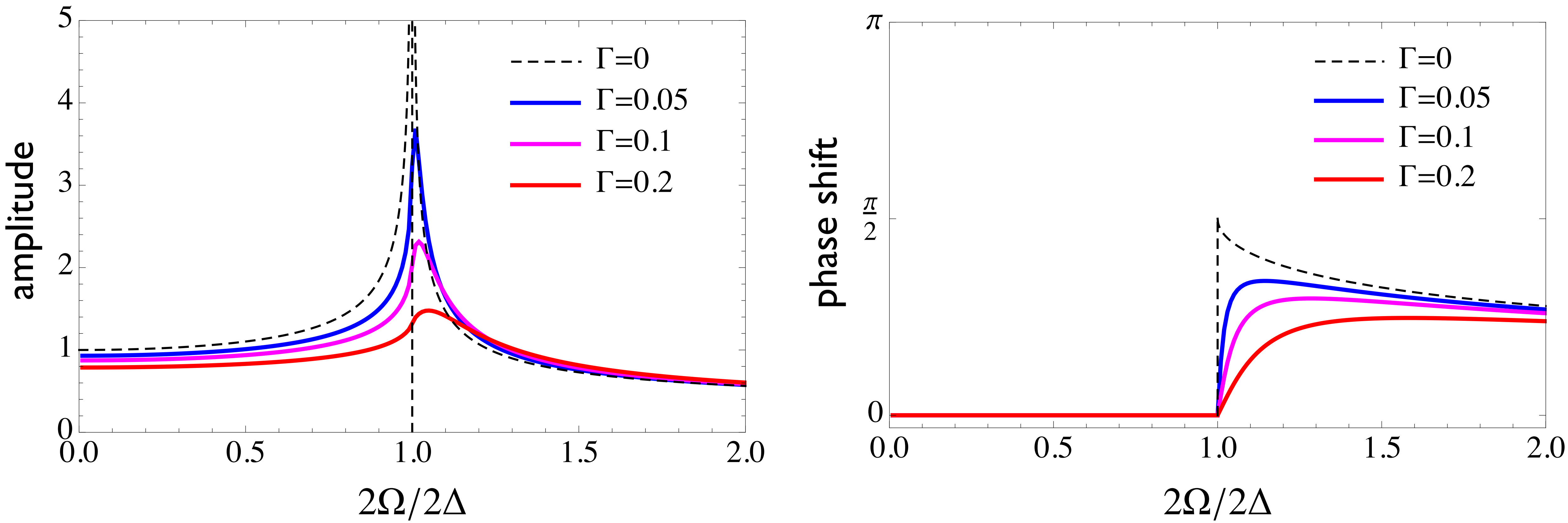
U=3.5, *A*=0.15, Ω=2π/25



U=3.5, Ω=2π/25

Impurity scattering

- Non-magnetic impurities → 2Δ does not change (Anderson's theorem)
- Calculation based on Abrikosov-Gor'kov theory ($\lambda \to 0$, T=0):



Future directions

- Can one further pursue an analogy between pseudospin and spin?
  — e.g. NMR, ESR, spintronics, ...

- Can one control xy pseudomagnetic fields in addition to z?

- Nonlinear pseudospin dynamics beyond perturbative regime (A^2, A^4, ...)

- What is the dominant pseudospin relaxation process?

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