

Topological Kondo Insulator SmB_6

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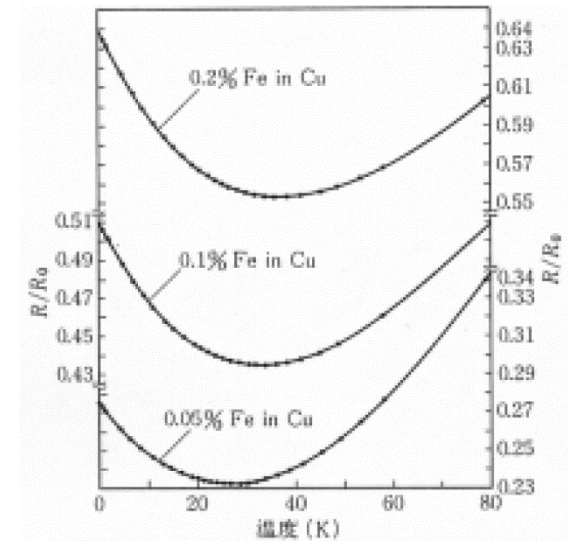
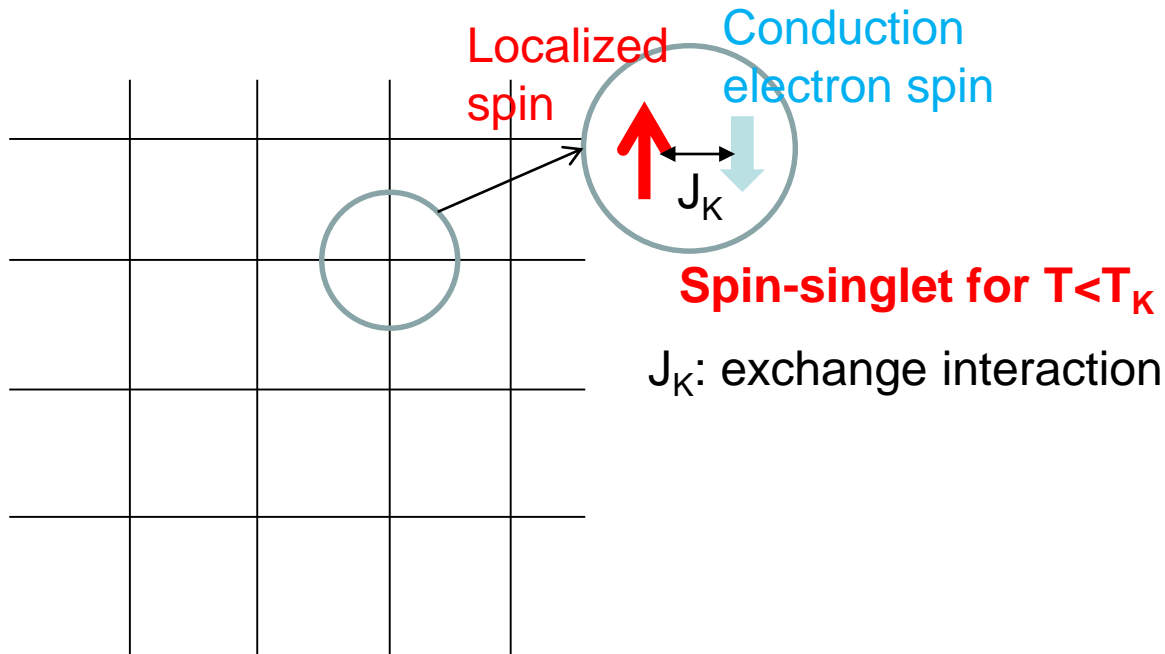
Kondo effect

Kondo lattice (strong correlation):

Lattice of localized spin

+

Sea of conduction electrons



Logarithmic increase of resistivity below T_K

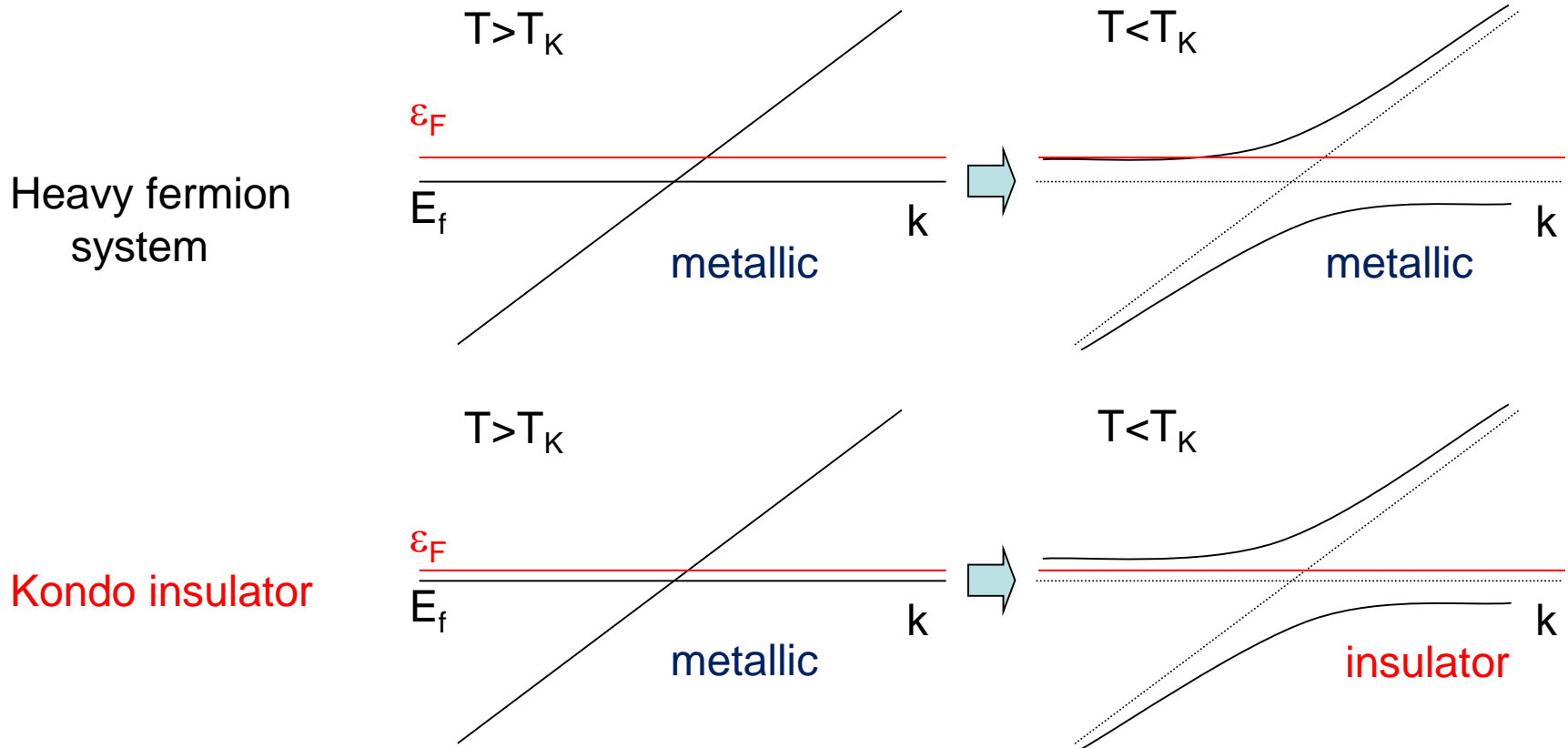
Due to the coherence of Kondo states, heavy fermion are formed in the Kondo lattice

Heavy fermion system and Kondo insulator

Another picture from weak correlation (Anderson lattice)

Renormalization of hybridization
between conduction and localized electrons
(instead of exchange interaction)

Anderson lattice
↓ Strong coupling limit
Kondo lattice



Introduction

PRL 104, 106408 (2010)

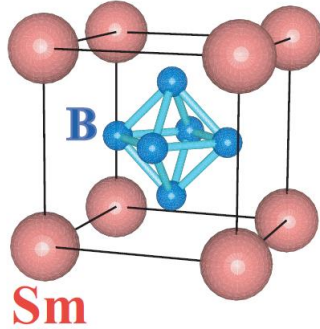
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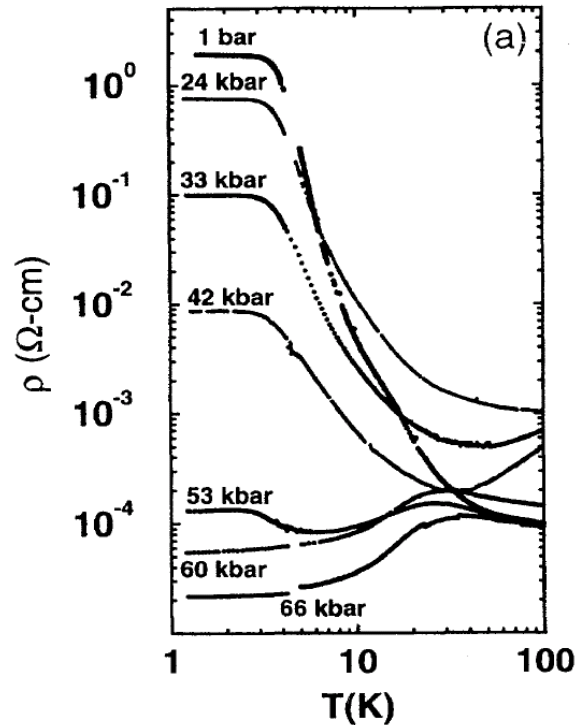
SmB₆: Kondo insulator

$$\Delta_{\text{gap}} = 15 \text{ meV}$$

point group : O_h



in-gap state below 5K



Cooley et al. PRL 74 1629 (1995)

Topological Kondo Insulators

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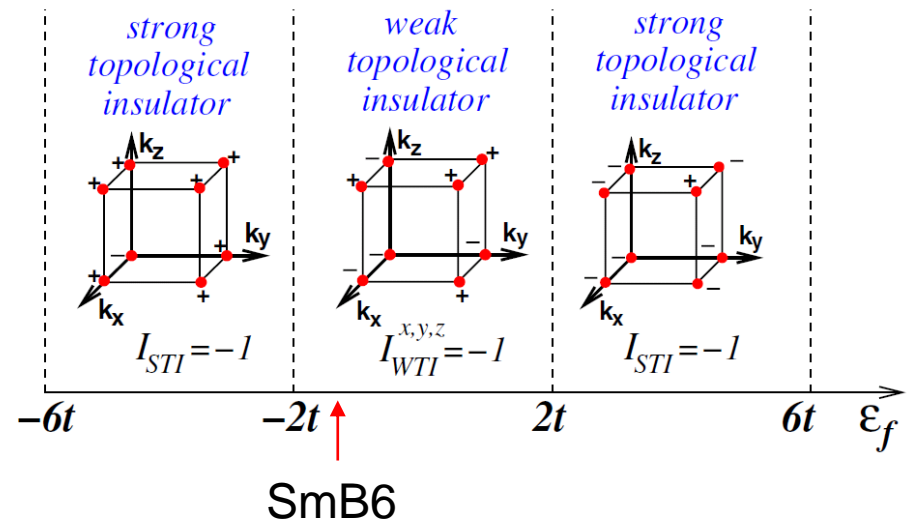
(Received 22 December 2009; published 12 March 2010)

periodic Anderson model (s+f)

f: $j_z = -1/2$ and $+1/2$ (Γ_8 of O_h)

$$\hat{H} = \sum_{\mathbf{k}, \alpha} \xi_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha} + \sum_{j\alpha} [V c_{j\alpha}^\dagger f_{j\alpha} + \text{H.c.}]$$

$$+ \sum_{j\alpha} \left[\varepsilon_f^{(0)} n_{f,j\alpha} + \frac{U_f}{2} n_{f,j\alpha} n_{f,j\bar{\alpha}} \right]$$

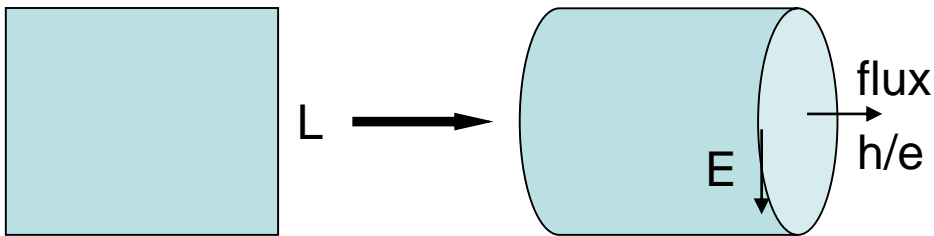


SmB₆

Topological Insulator

- two copies of integer quantum Hall system
- time reversal invariant
- large spin-orbit coupling
- metallic edge (surface) state

two-dimensional case (Kane & Mele PRL 95)

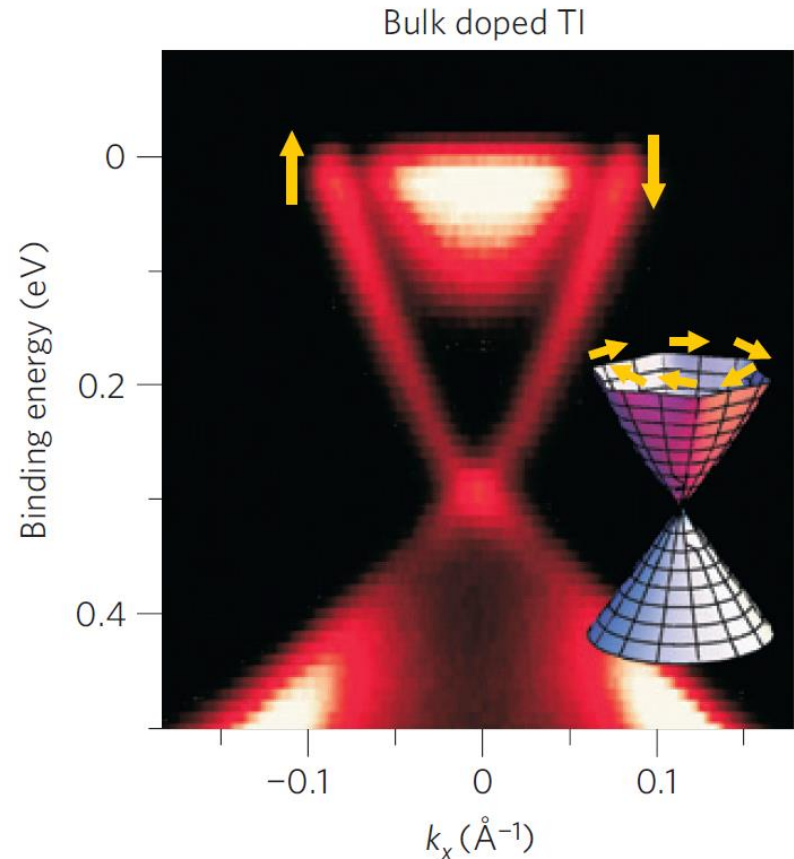


circumferential electric field E
can be induced by a threading flux

$$k \longrightarrow k + 2\pi/L$$

trivial insulator ($\nu_0=0$) no effect
gapped around E_F

topological insulator ($\nu_0=1$) spin Hall
particle-hole excitation around E_F on the edge
spin-direction is different between particle and hole



Dirac cone on a surface of Bi_2Se_3

Wray et al. Nature Phys. 7 32

ref.

Hasan & Kane RMP 82

Qi & Zhang RMP

Topological number in noninteracting system

In TRI compound (2D square lattice)

Fu and Kane: PRB **74** 195312

gauge invariance (phase of wave function)

time reversal symmetry

$$H(-\mathbf{k}) = \Theta H(\mathbf{k}) \Theta^{-1} \quad (\Theta = \sigma_y K)$$

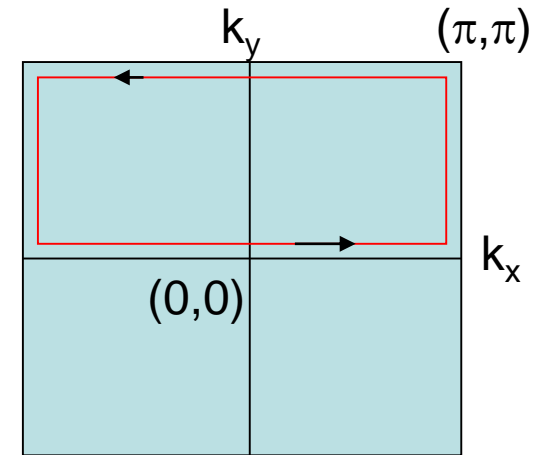
determine gauge potential $\mathcal{A}(k) = i \sum_n \langle u_{k,n} | \nabla_k | u_{k,n} \rangle \implies$ **topological #**

Chern # = 0 (cancellation in Kramers doublet)

topological # ν_0

calculated by choosing one from a doublet

$$\nu_0 = \frac{1}{2\pi} \left[\oint_{\partial\tau_{1/2}} d\ell \mathcal{A} - \int_{\tau_{1/2}} d\tau \mathcal{F} \right] \text{mod } 2 \quad \begin{array}{l} \text{field strength} \\ \mathcal{F} = d\mathcal{A} \end{array}$$



trivial case

$$\nu_0 = 0$$

\mathcal{A} is continuous

in Brillouin zone.

topological case

$$\nu_0 = \text{odd #}$$

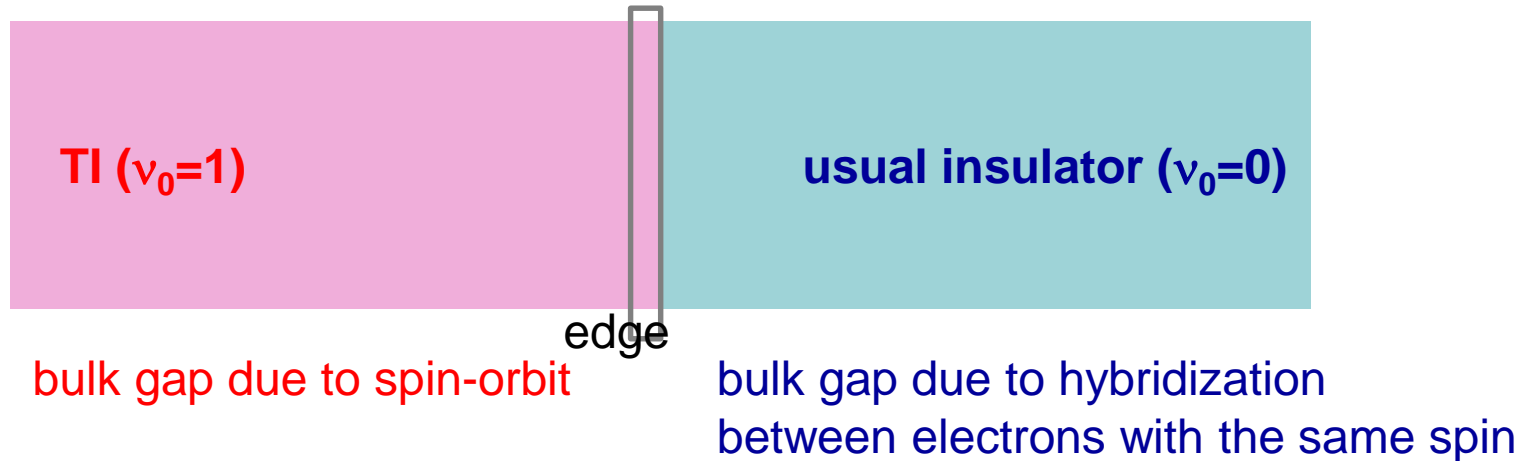
\mathcal{A} has one or three singular points

at $(0,0)$, $(\pi,0)$, $(0,\pi)$, or (π,π) .

Bulk-edge correspondence

electronic states in bulk \rightarrow topological number ν_0 $\nu_0=1$: topological
 $\nu_0=0$: trivial

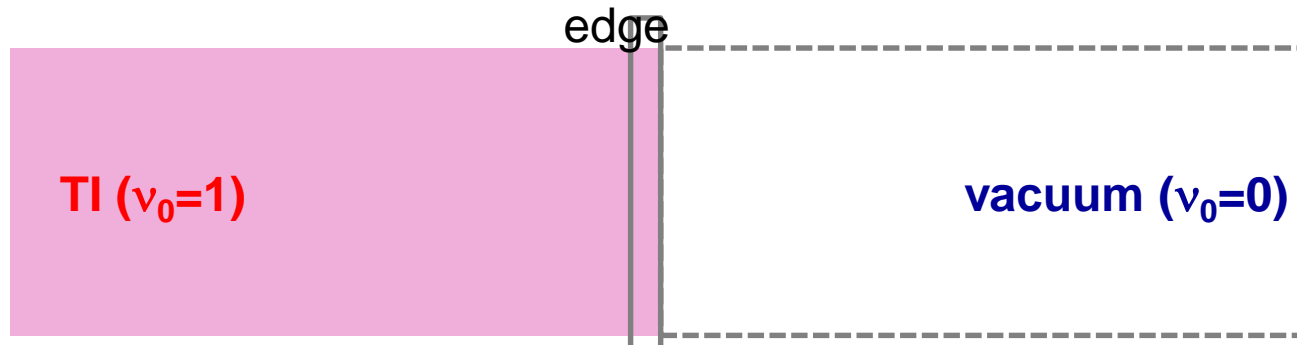
a junction between TI and usual insulator in real space



different origin of bulk gap

necessary close the gap to move to the other side!

gapless states must appear on the edge



relevant orbitals

dominant electronic states

$(4f)^5(5d)^1$ and $(4f)^6$

band structure calculation

Yanase and Harima:

Prog. Theor. Phys. Suppl. **108** 19, (1992).

Antonov et al. : PRB **66** 165209, (2002).

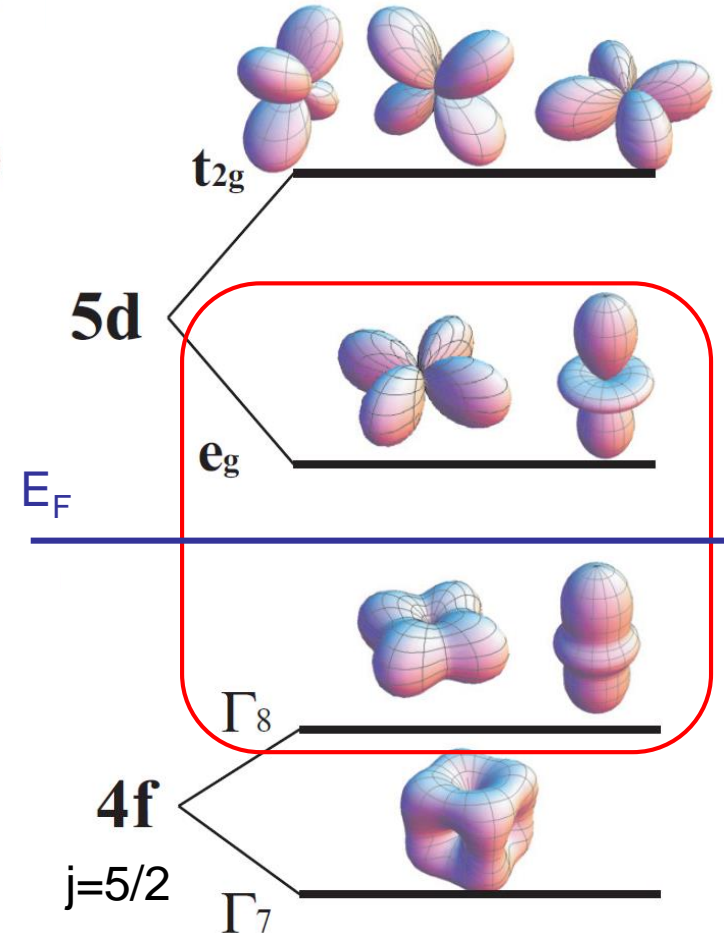
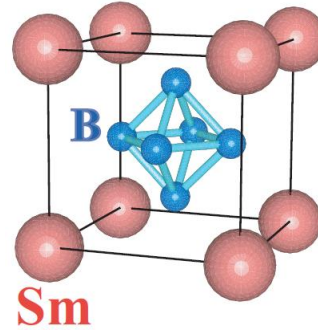
4f and 5d electrons are from Sm.

Sm³⁺: (4f)⁵

angular momentum: $J=5/2$

CEF ground state: Γ_8

**~ one hole in Γ_8 states
in j-j coupling scheme**



tight-binding

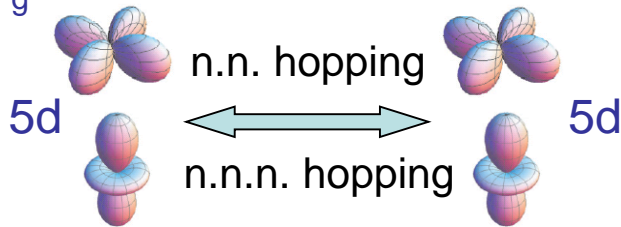
effective model

5d-electron term (NN+NNN)

$$H_c = \sum_{\mathbf{k}} \sum_{\xi, \xi'} \sum_{\sigma} \varepsilon_{\xi\xi'}^d(\mathbf{k}) c_{\mathbf{k}\xi\sigma}^\dagger c_{\mathbf{k}\xi'\sigma}$$

$$\hat{\varepsilon}^d(\mathbf{k}) = d_0^d(\mathbf{k})\hat{\tau}_0 + d_1^d(\mathbf{k})\hat{\tau}_z + d_2^d(\mathbf{k})\hat{\tau}_x$$

ξ : orbital in e_g
 σ : real spin



hybridization term (NN+NNN)

$$H_{hyb} \quad \text{The seed of singularity for } \mathcal{A}$$

$$= \sum_{\mathbf{k}} \sum_{\xi, \gamma} \begin{bmatrix} c_{\mathbf{k}\xi\uparrow}^\dagger & c_{\mathbf{k}\xi\downarrow}^\dagger \end{bmatrix} i\mathbf{V}_{\xi\gamma}(\mathbf{k}) \cdot \hat{\sigma} \begin{bmatrix} f_{\mathbf{k}\gamma+} \\ f_{\mathbf{k}\gamma-} \end{bmatrix} + h.c.,$$

$$\hat{V}^\alpha(\mathbf{k}) = V_0^\alpha(\mathbf{k})\hat{\tau}_0 + V_1^\alpha(\mathbf{k})\hat{\tau}_z + V_2^\alpha(\mathbf{k})\hat{\tau}_x$$

4f-electron term (NN+NNN)

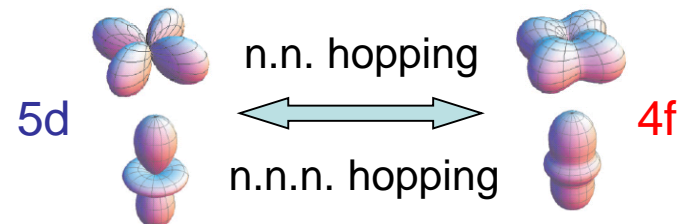
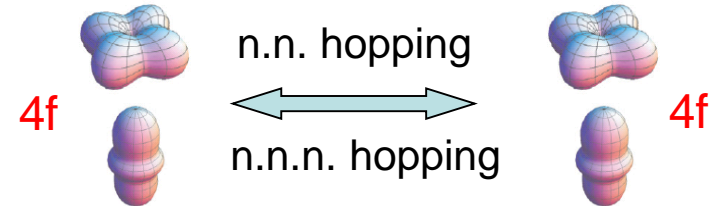
$$H_f^0 = \sum_{\mathbf{k}} \sum_{\gamma, \gamma'} \sum_{\tau, \tau'} \varepsilon_{\gamma\tau, \gamma'\tau'}^f(\mathbf{k}) f_{\mathbf{k}\gamma\tau}^\dagger f_{\mathbf{k}\gamma'\tau'}$$

$$\hat{\varepsilon}^f(\mathbf{k}) = d_0^f(\mathbf{k})\hat{\tau}_0\hat{\sigma}_0 + \sum_{n=1}^5 d_n^f(\mathbf{k})\Gamma^n$$

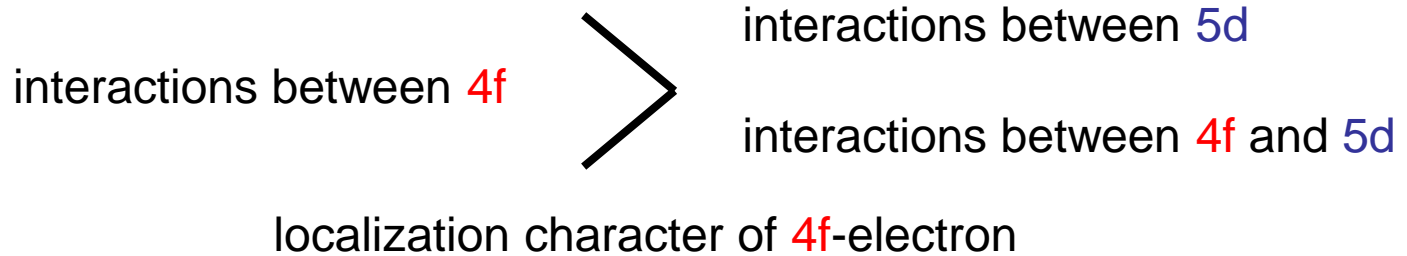
$$\Gamma^{1,2,3,4,5} = (\hat{\tau}_z\hat{\sigma}_0, \hat{\tau}_x\hat{\sigma}_0, \hat{\tau}_y\hat{\sigma}_x, \hat{\tau}_y\hat{\sigma}_y, \hat{\tau}_y\hat{\sigma}_z)$$

γ : orbital in Γ_8

τ : pseudo spin in a Kramers doublet



renormalization of interaction term



renormalization of interactions between 4f

- Gutzwiller projection
- slave boson

$$f_{\mathbf{k}\gamma\tau} \rightarrow \sqrt{z_{\mathbf{k}\gamma}} \tilde{f}_{\mathbf{k}\gamma\tau}$$

$$\varepsilon_{\gamma\tau,\gamma'\tau'}^f(\mathbf{k}) \longrightarrow z \varepsilon_{\gamma\tau,\gamma'\tau'}^f(\mathbf{k})$$

$$\mathbf{V}_{\xi\gamma}(\mathbf{k}) \longrightarrow \sqrt{z} \mathbf{V}_{\xi\gamma}(\mathbf{k})$$

map into effective Hamiltonian

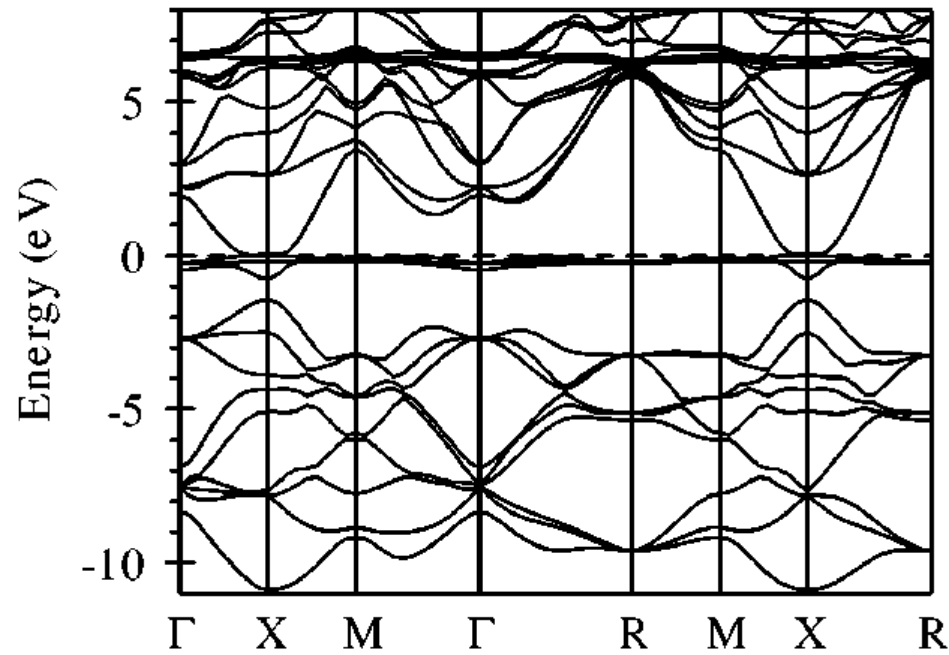
z : renormalization factor in Fermi liquid theory

electronic structure

Antonov et al.: PRB **66** 165209, (2002).

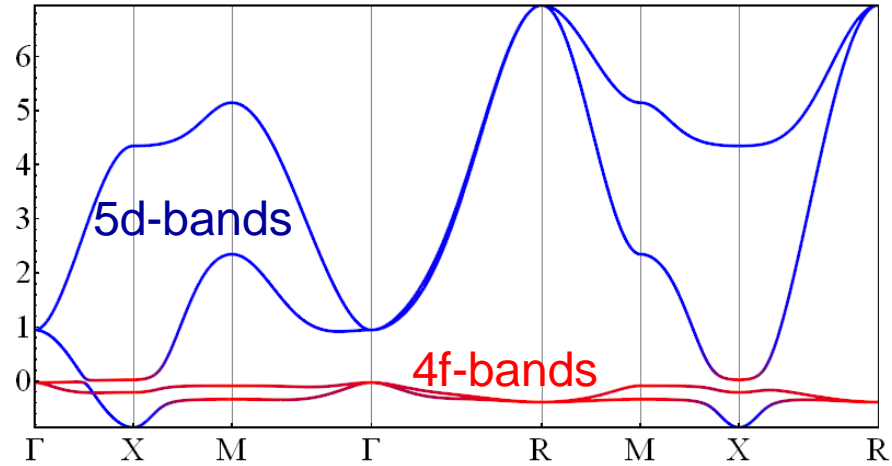
Sm²⁺

LSDA+U

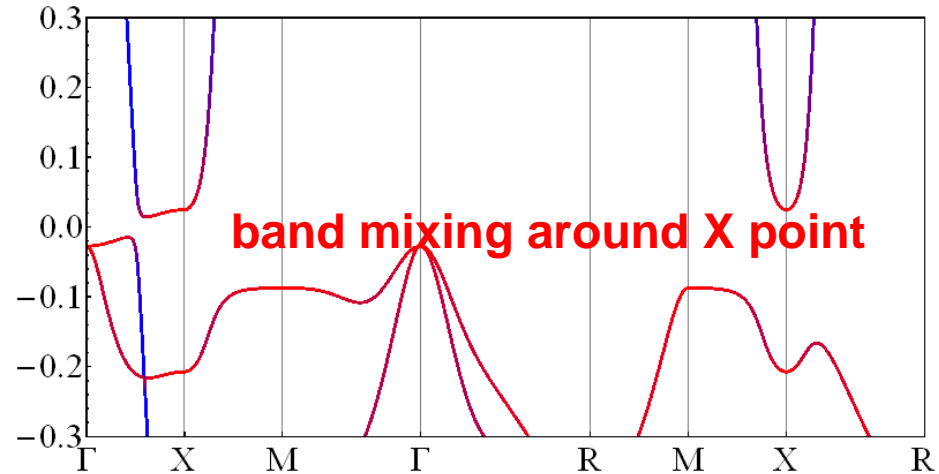


half-filling case provides insulator

Bulk Band Structure



Bulk Band Structure



topological index

$(\nu_0: \nu_1, \nu_2, \nu_3)$

$$(-1)^{\nu_0} = \prod_m \delta_m$$

$\nu_0=0$ WTI
 $\nu_0=1$ STI

$$(-1)^{\nu_k} = \prod_{n_k=1; n_j \neq k=0,1} \delta_{m=(n_1, n_2, n_3)}$$

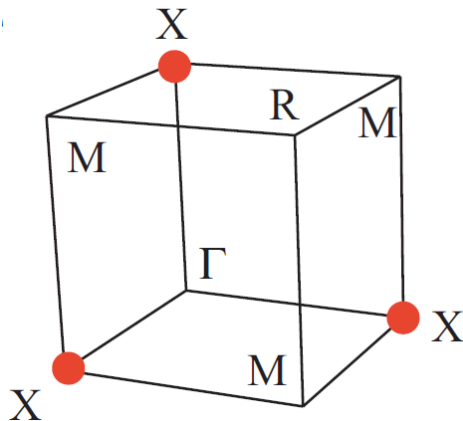
$$\delta_m = \prod_{E_n < \varepsilon_F} \delta_{n,m}$$

$\delta_{n,m}$: parity eigenvalue
of n-th two-fold degenerate band at \mathbf{k}_m^*

Fu and Kane: PRB 74 195312 (2006)
PRB 76 045302 (2007)

time reversal invariant point

$$-\mathbf{k}_m^* = \mathbf{k}_m^* + \mathbf{G}$$



our model
(1: 1,1,1)

$$E_-^d(\mathbf{k}_m^*) > E_+^f(\mathbf{k}_m^*) > E_-^f(\mathbf{k}_m^*) \quad \delta_m = +1$$

$$E_+^f(\mathbf{k}_m^*) > E_-^f(\mathbf{k}_m^*) > E_-^d(\mathbf{k}_m^*) \quad \delta_m = -1$$

parity operator

$$\hat{P} = \hat{s}_z \hat{\tau}_0 \hat{\sigma}_0$$

\hat{s}_α : 2X2 matrix between d- and f-electronic states

$$[\hat{H}(\mathbf{k}_m^*), \hat{P}] = 0$$

hybridization term is formed by s_x and s_y

$$\hat{P} \hat{s}_{x,y} \hat{P}^{-1} = -\hat{s}_{x,y}$$

hybridization term is irrelevant at \mathbf{k}^* for $\delta_{n,m}$

$E_\pm^d(\mathbf{k}_m^*)$ with even parity

$E_\pm^f(\mathbf{k}_m^*)$ with odd parity

$E_+^d(\mathbf{k}_m^*)$ is the highest energy

Necessary and sufficient conditions for TKI

$$H_{hyb} = \sum_{\mathbf{k}} \sum_{\xi, \gamma} \begin{bmatrix} c_{\mathbf{k}\xi\uparrow}^\dagger & c_{\mathbf{k}\xi\downarrow}^\dagger \end{bmatrix} \underline{i\mathbf{V}_{\xi\gamma}(\mathbf{k}) \cdot \hat{\sigma}} \begin{bmatrix} f_{\mathbf{k}\gamma+} \\ f_{\mathbf{k}\gamma-} \end{bmatrix} + h.c.,$$

$$\hat{V}^\alpha(\mathbf{k}) = V_0^\alpha(\mathbf{k})\hat{\tau}_0 + V_1^\alpha(\mathbf{k})\hat{\tau}_z + V_2^\alpha(\mathbf{k})\hat{\tau}_x$$

n	$V_n^x(\mathbf{k})$	$V_n^y(\mathbf{k})$	$V_n^z(\mathbf{k})$
0	$4V_{df}S_x$	$-4V_{df}S_y$	$4V_{df}S_z$
1	$2V_{df}S_x$	$-2V_{df}S_y$	$-4V_{df}S_z$
2	$-2\sqrt{3}V_{df}S_x$	$-2\sqrt{3}V_{df}S_y$	0

$$\mathbf{V}_{\xi\gamma}(\mathbf{k}) \cdot \hat{\sigma} \sim \mathbf{k}' \cdot \boldsymbol{\sigma} \text{ at } \mathbf{k}=\mathbf{k}^*+\mathbf{k}' \quad (|\mathbf{k}'| \ll \pi)$$

\mathbf{k}' -linear: different parities
 $\boldsymbol{\sigma}$: large spin-orbit coupling

The seed of singularity for \mathcal{A} at \mathbf{k}^*

Necessary condition for TKI

hybridization gap between d- and f-bands
 around $\mathbf{k}=\mathbf{k}^*$



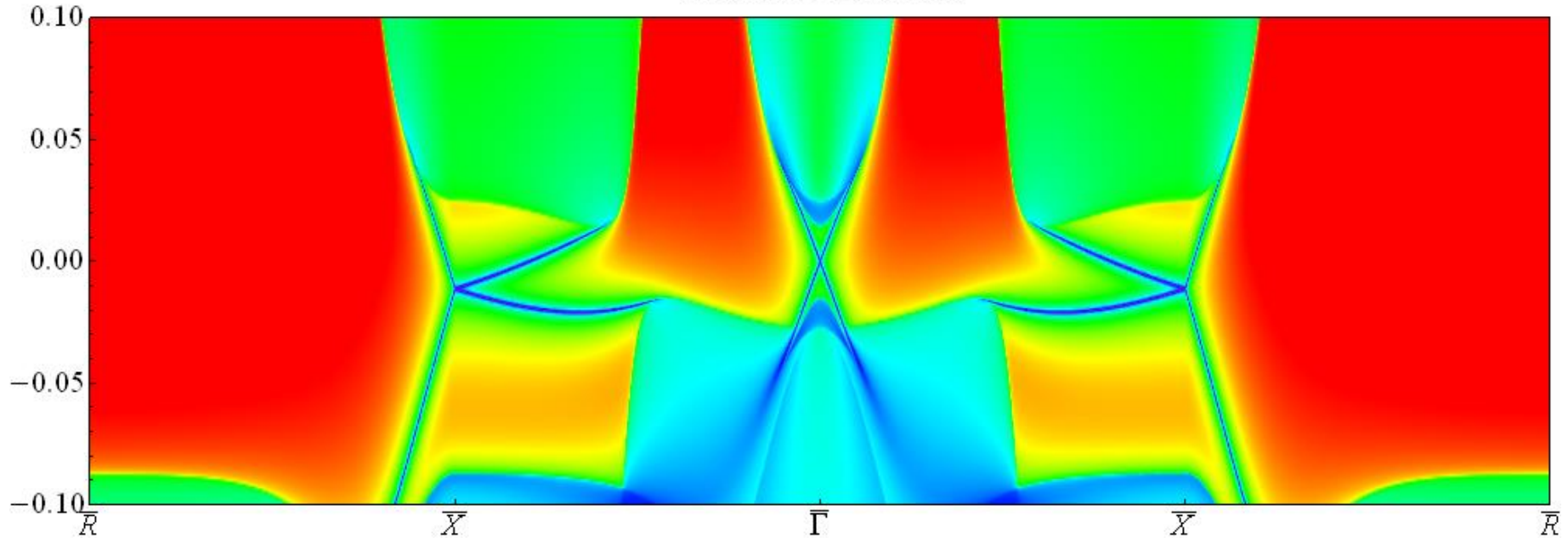
Nontrivial topological #

Determined by Hamiltonian without H_{hyb}

Sufficient condition for TKI

Spectral function of (001)-surface

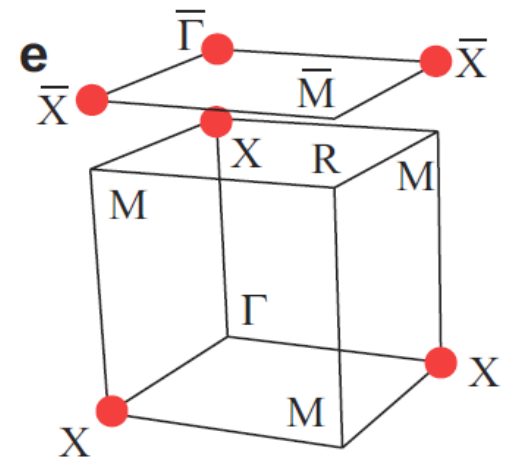
001 Surface Dispersion



There are Dirac cones at Γ and X points.

Intensity around X is stronger than that around Γ

There is a v.-H. singularity along Γ -X.

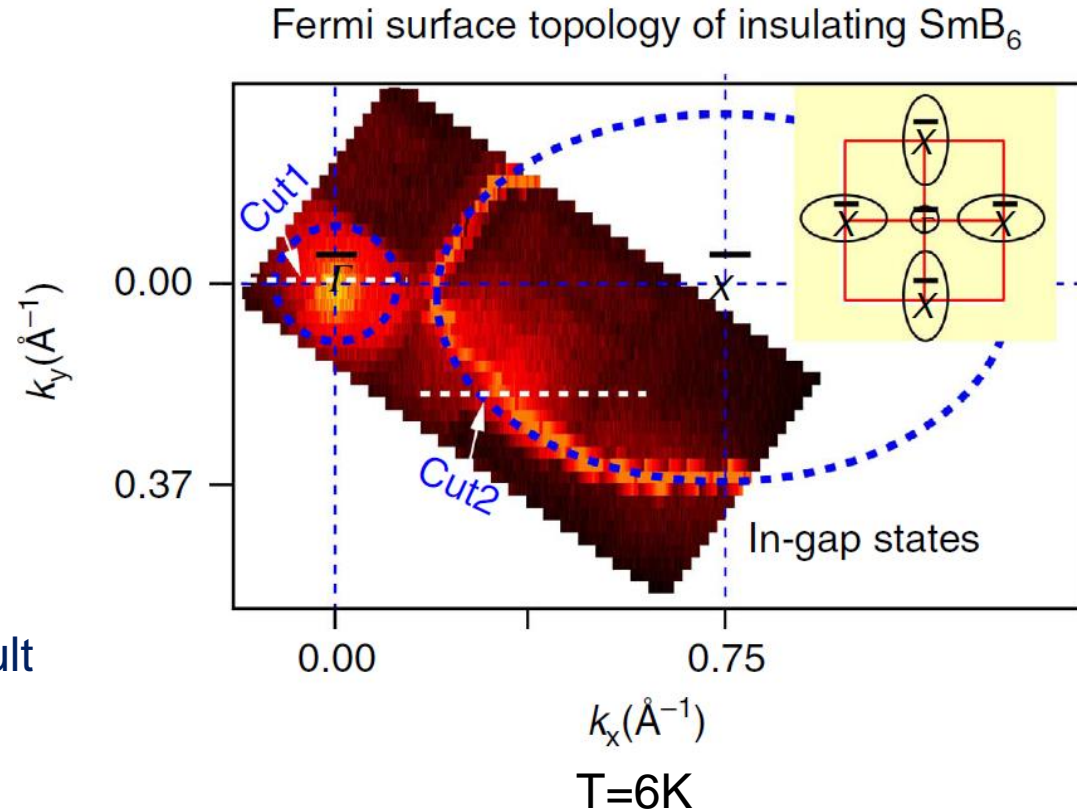


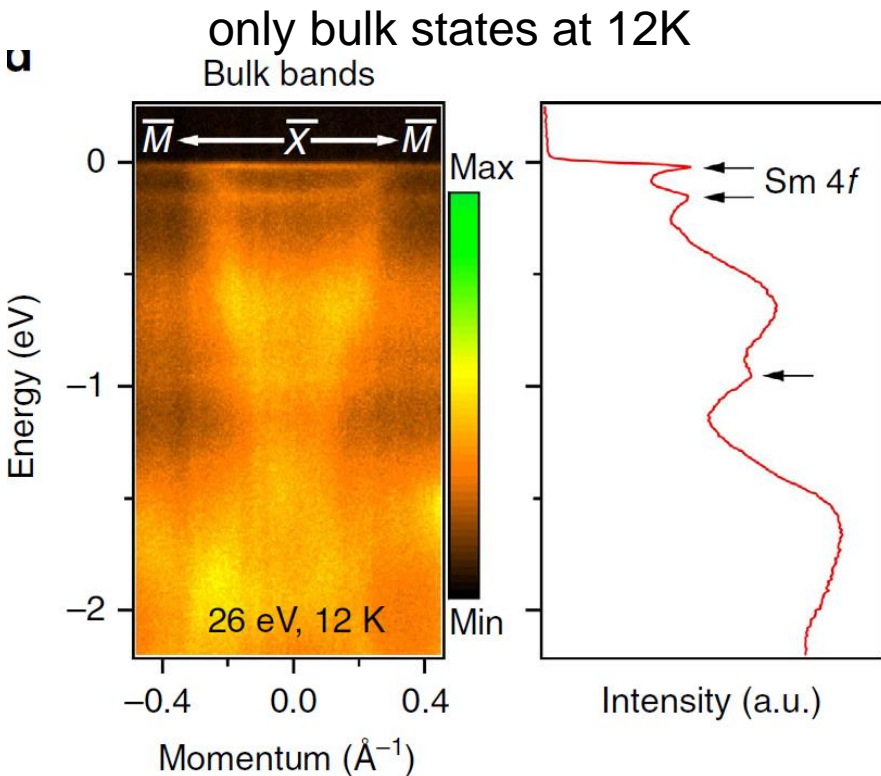
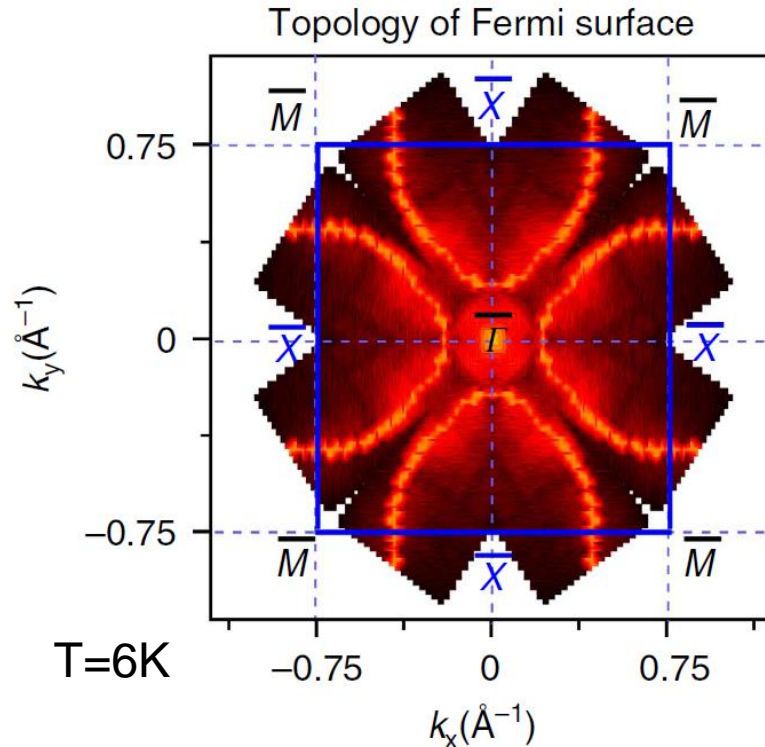
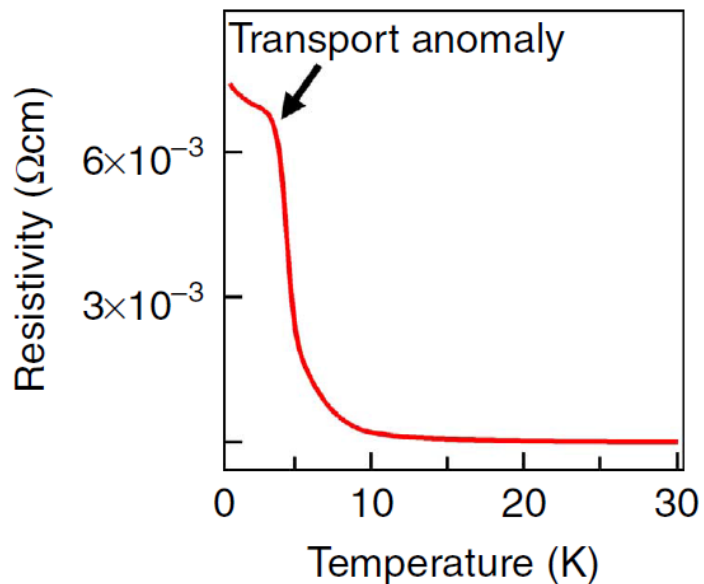
Recent ARPES measurement for (001) surface

M. Neupane et al. (M.Z.Hasan)
Nat. Commun. 4, 2991 (2013).

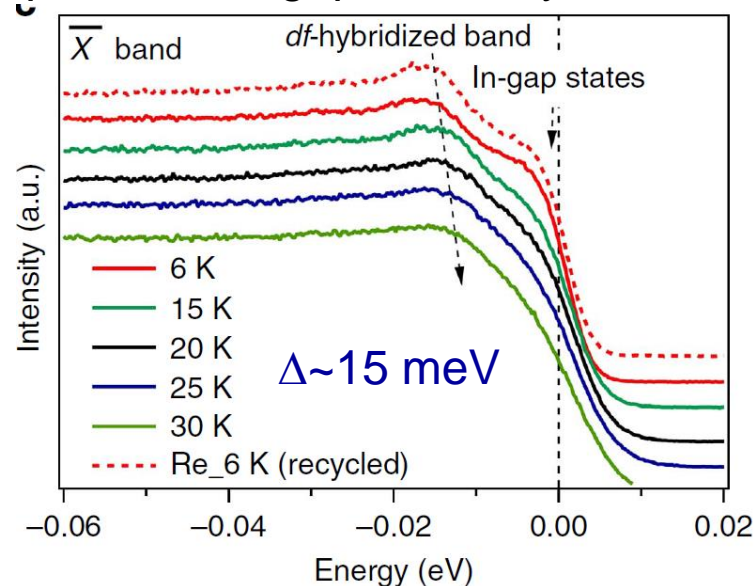
Fermi surface around Γ and X

topology of Fermi surface
is the same as the theoretical result





development of in-gap states by decreasing T



NMR in SmB₆

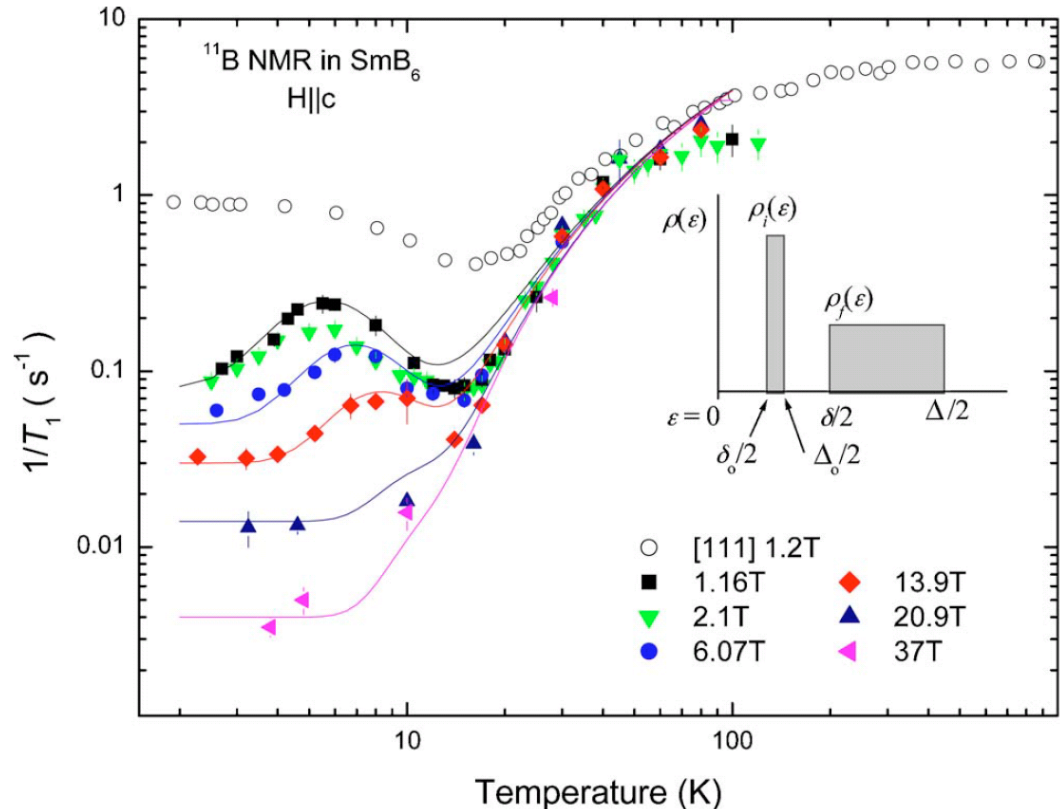
Caldwell et al. PRB **75** 075106 (2007)

$$\frac{1}{T_1} = \frac{\hbar k_B T}{\omega_n} \sum_q A^2 F(q) \chi''_{q, \omega_n}$$

- insulator behavior down to ~20K
- contribution from the **in-gap state** below 20K
- magnetic field suppresses the in-gap state

The magnetic field breaks TRS.

The metallic surface states of TI is protected by TRS.
The surface states should be gapped by the magnetic field



in-gap state~surface state of topological insulator

Why the value of $1/T_1 T$ is so large at the low temperature limit?

Summary

We study the possibility of the topological insulator for SmB_6 , calculating the topological index and spectral function of [001] surface, based on a realistic model.

The topological index is (1: 1,1,1).

In the spectral function, surface states appear with odd number of Dirac cones.

Based on the calculation, SmB_6 is a topological Kondo insulator.

Recent ARPES measurements show Fermi surfaces surrounding the Dirac points consistent with theoretical suggestions.

next step

experiment: measurements of Dirac points, spin texture

theory: clarification of electron correlation in the surface states