NHSCP 2014@ISSP
ISSP International Workshop
New Horizon of Strongly Correlated Physics
June 16 - July 4, 2014 (Symposium June 25-27)
ISSP, The University of Tokyo

Superfluid state in the multi-component fermionic optical lattice systems

Tokyo Tech. A. Koga

Y. Okanami et al. Phys. Rev. A **89**, 053622 (2014).

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Superfluid state in the multi-component fermionic optical lattice systems

Collaborators

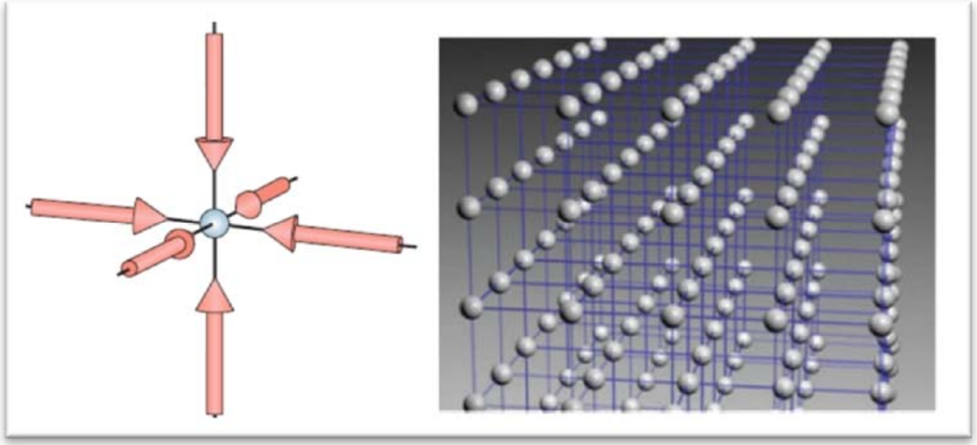
Tokyo Tech. Y. Okanami
N. Takemori

Fribourg P. Werner

Y. Okanami et al. Phys. Rev. A **89**, 053622 (2014).

Ultracold fermions

- Optical lattices (^{40}K, ^{6}Li, ^{171}Yb, ^{173}Yb)



I. Bloch, Nature Physics 1, 23 (2005)

✓ Number of fermions
✓ Confining potential
✓ Onsite interactions

Controllable

◆ **Fermi surface**
M. Köhl et al., Phys. Rev. Lett. 94, 080403 (2005)

◆ **Superfluid state**
J. K. Chin et al., Nature 443, 961 (2006)

◆ **Mott insulating state**
R. Jördens et al., Nature 455, 204 (2008)

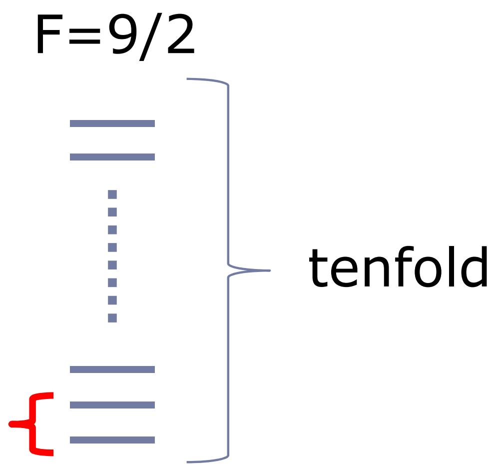
Quantum Simulator

Spin degrees of freedom

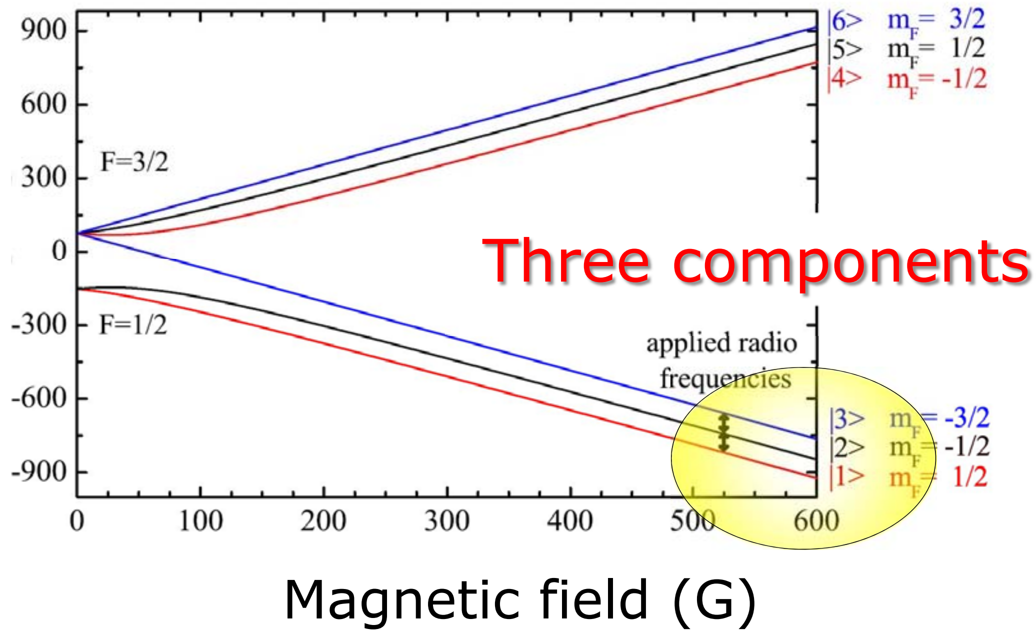
- Potassium ^{40}K (F=9/2, 7/2)
  - I=4
  - L=0
  - S=1/2

$$|F, F_z>:|9/2, -9/2>=|\uparrow>$$
$$|9/2, -7/2>=|\downarrow>$$

Two-component systems

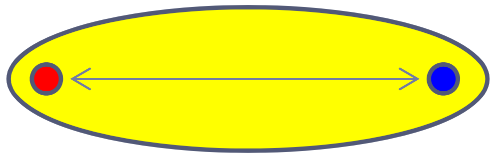


- Lithium ^{6}Li (F=3/2, 1/2)
  - I=1
  - L=0
  - S=1/2

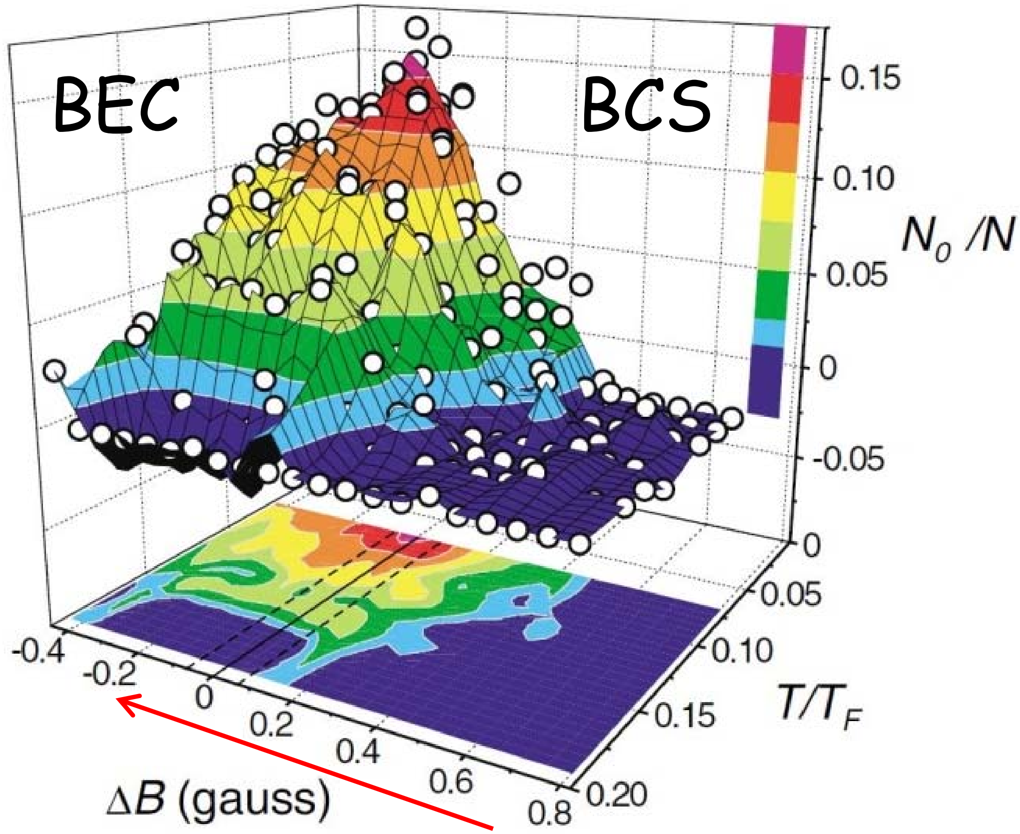


Superfluid state in ^{40}K (F=9/2)

- Two component fermions
  - BCS-BEC crossover



Cooper pair



$$|F, F_z>: |9/2, -9/2>= |\uparrow>$$
$$|9/2, -7/2>= |\downarrow>$$

Two-component systems

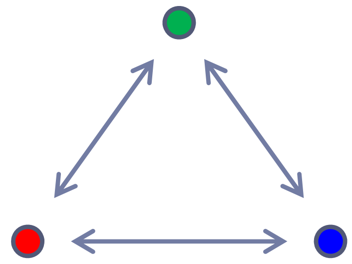
Attractive interaction

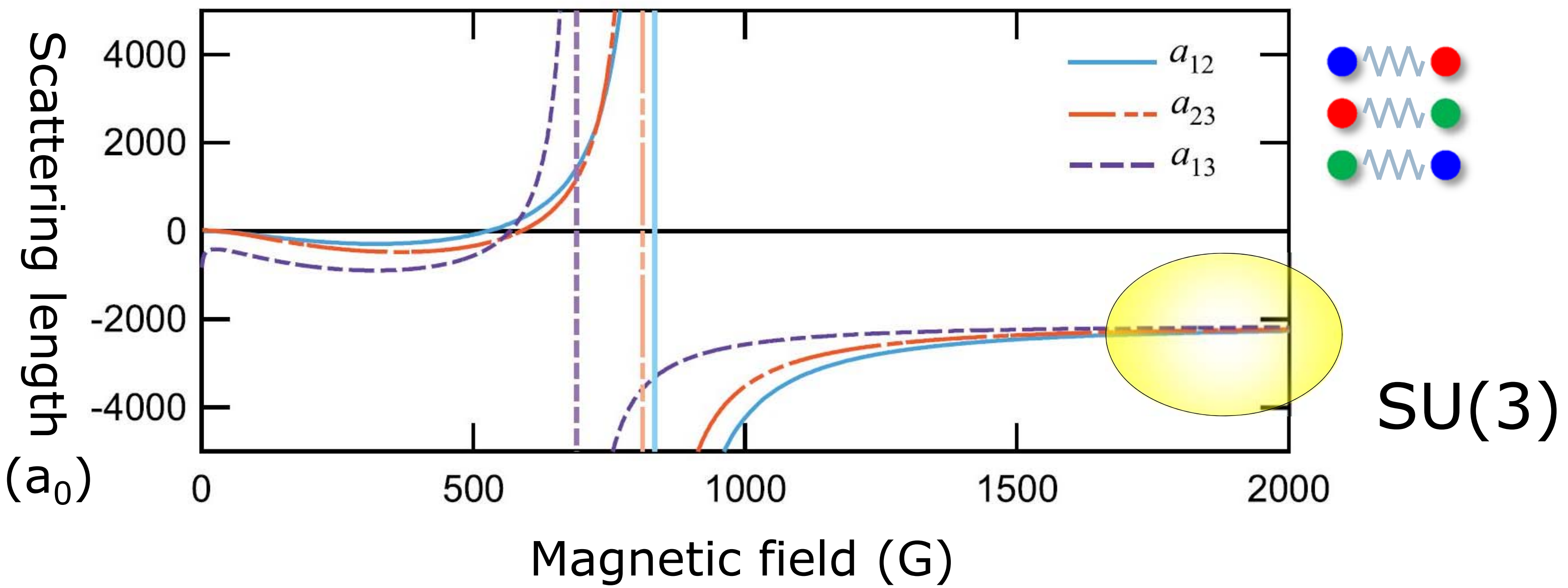
Feshbach resonance

C. A. Regal, et al., Phys. Rev. Lett. **92**, 040403 (2004).

Three-component fermions

- **Lithium ^{6}Li : S=1/2, I=1**
  - $|F, m_F\rangle = |1/2, 1/2\rangle, |1/2, -1/2\rangle, |3/2, -3/2\rangle$

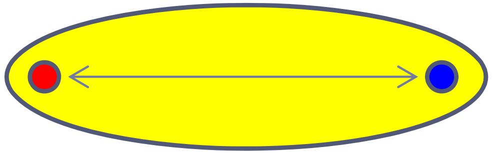




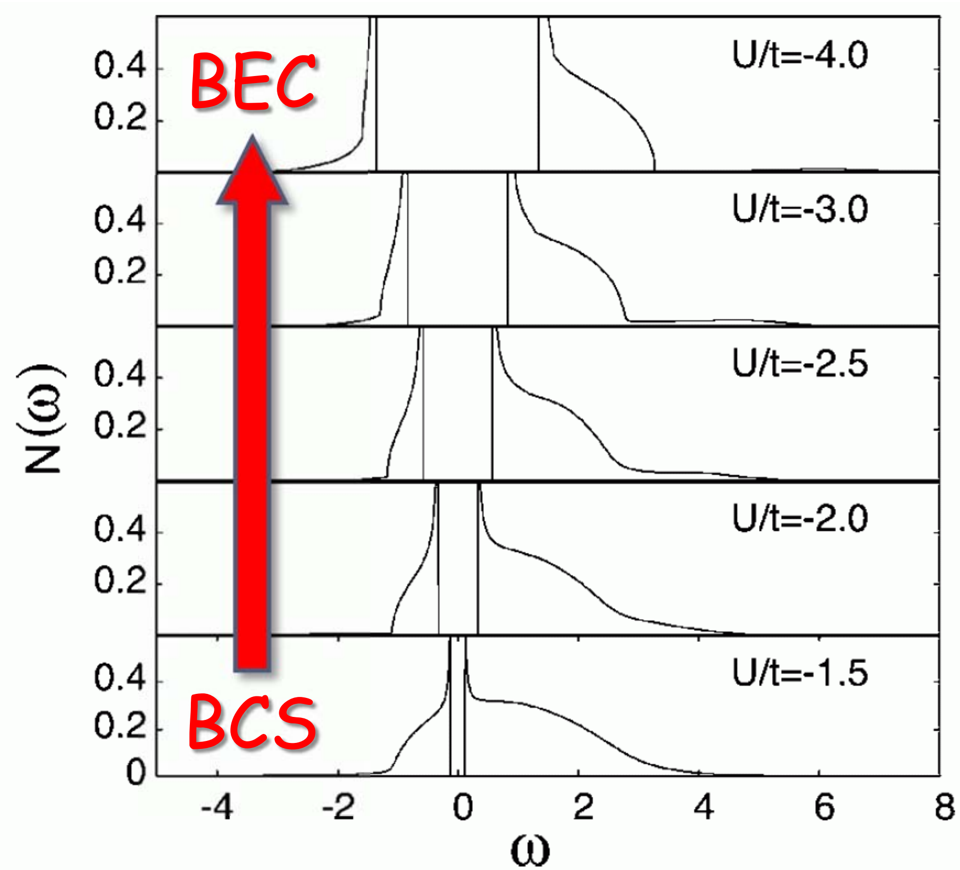
Ottenstein et al., Phys. Rev. Lett. 101, 203202 (2008)
J. H. Huckans et al., Phys. Rev. Lett. 102, 165302 (2009)

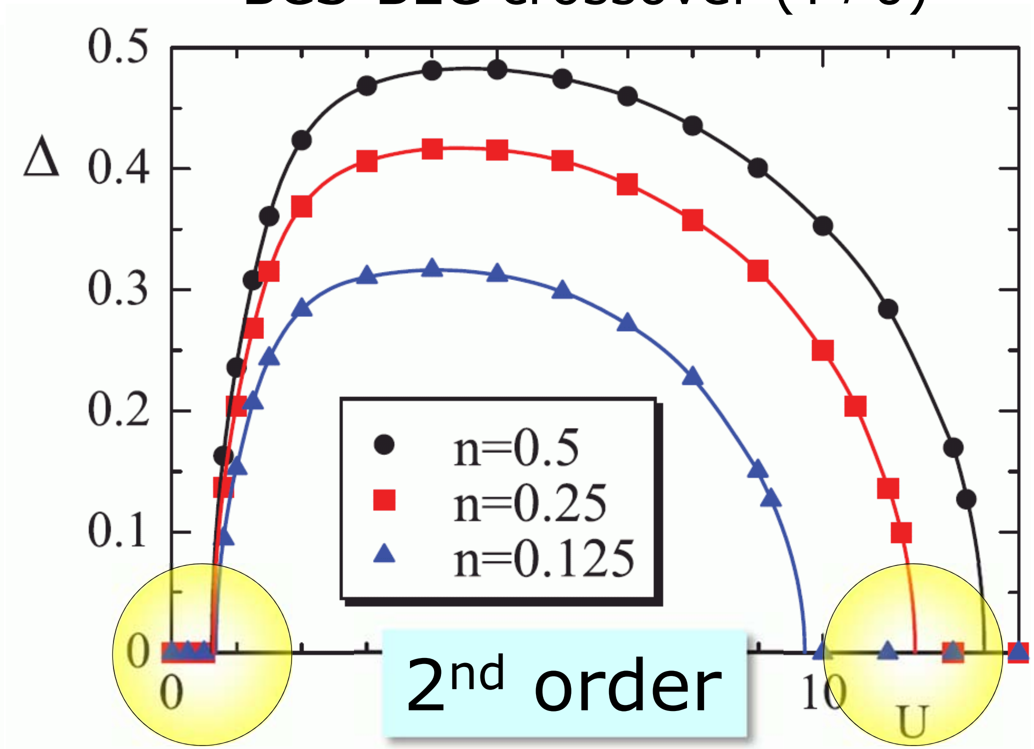
Two-component systems (DMFT)

- BCS-BEC crossover



BCS-BEC crossover (T≠0)





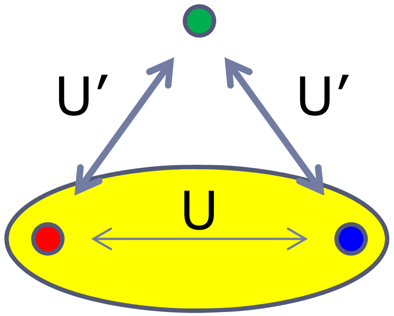
Attractive case only

A. Garg et al., PRB 72, 023517 (2005)
AK and P. Werner, PRA 84, 023638 (2011)

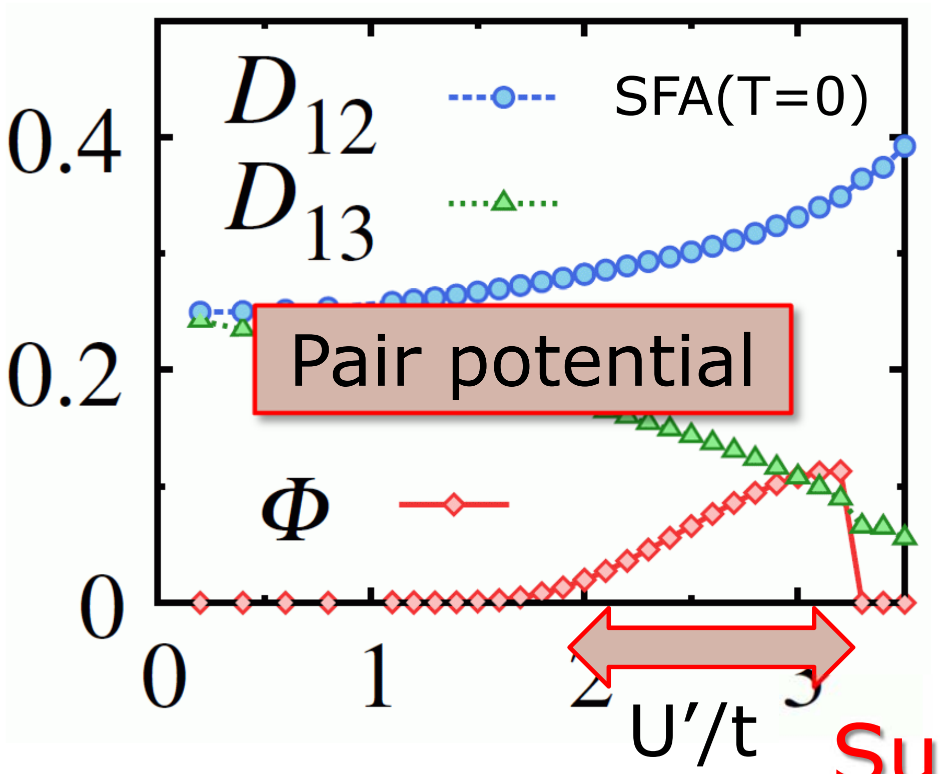
Three-component fermions (half filling)

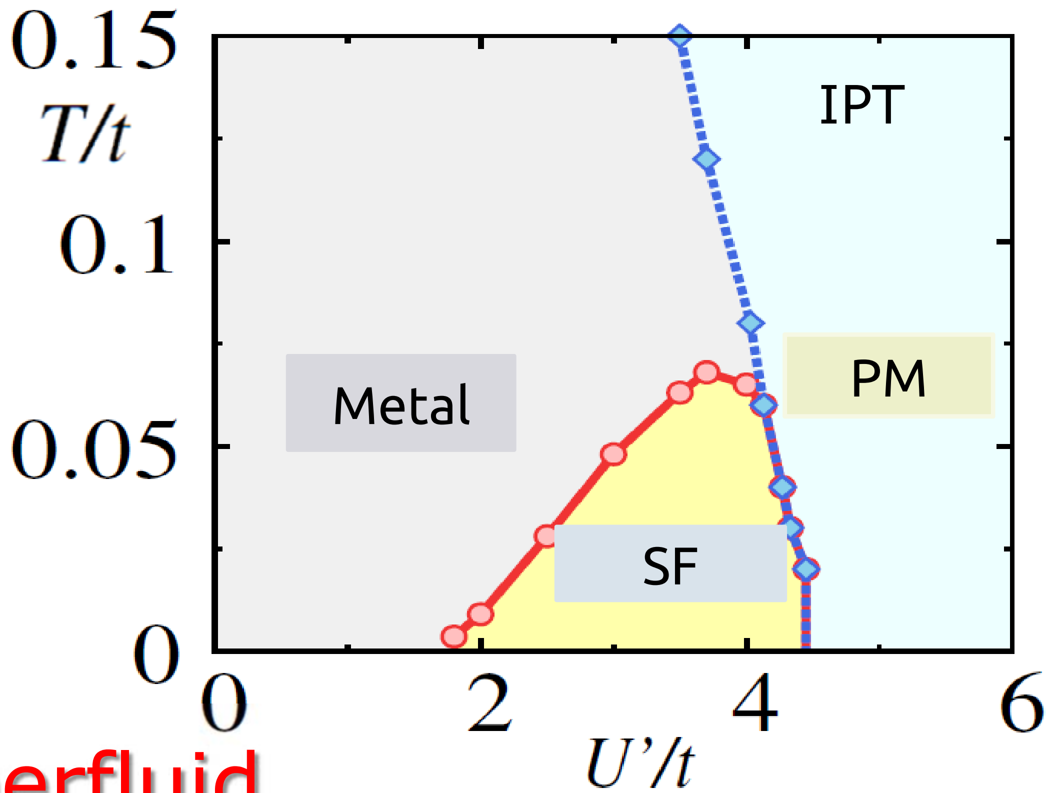
- Color superconductivity

U/U′=0.1



Repulsive interaction





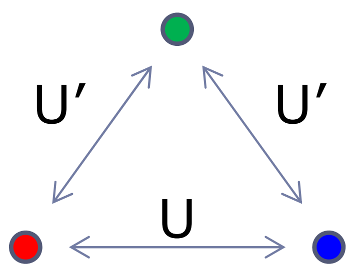
Superfluid

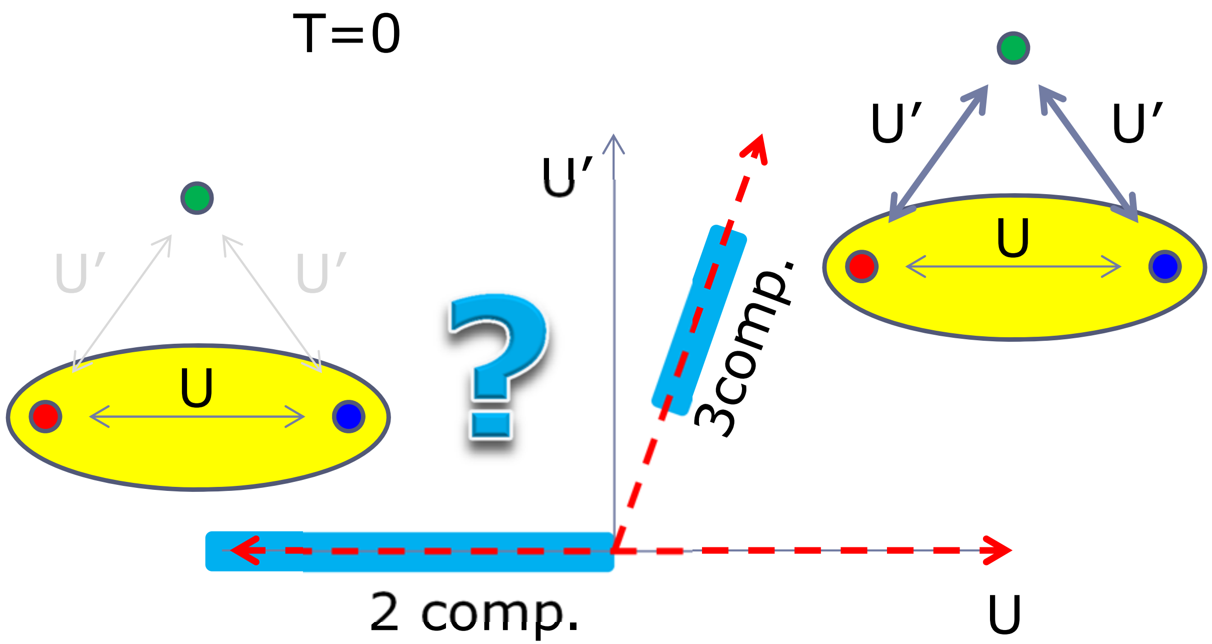
K. Inaba and S. Suga, Phys. Rev. Lett. 108, 255301 (2012)

Questions

- Superfluid state
  - 2 component system vs. 3 component system

Attractive Repulsive

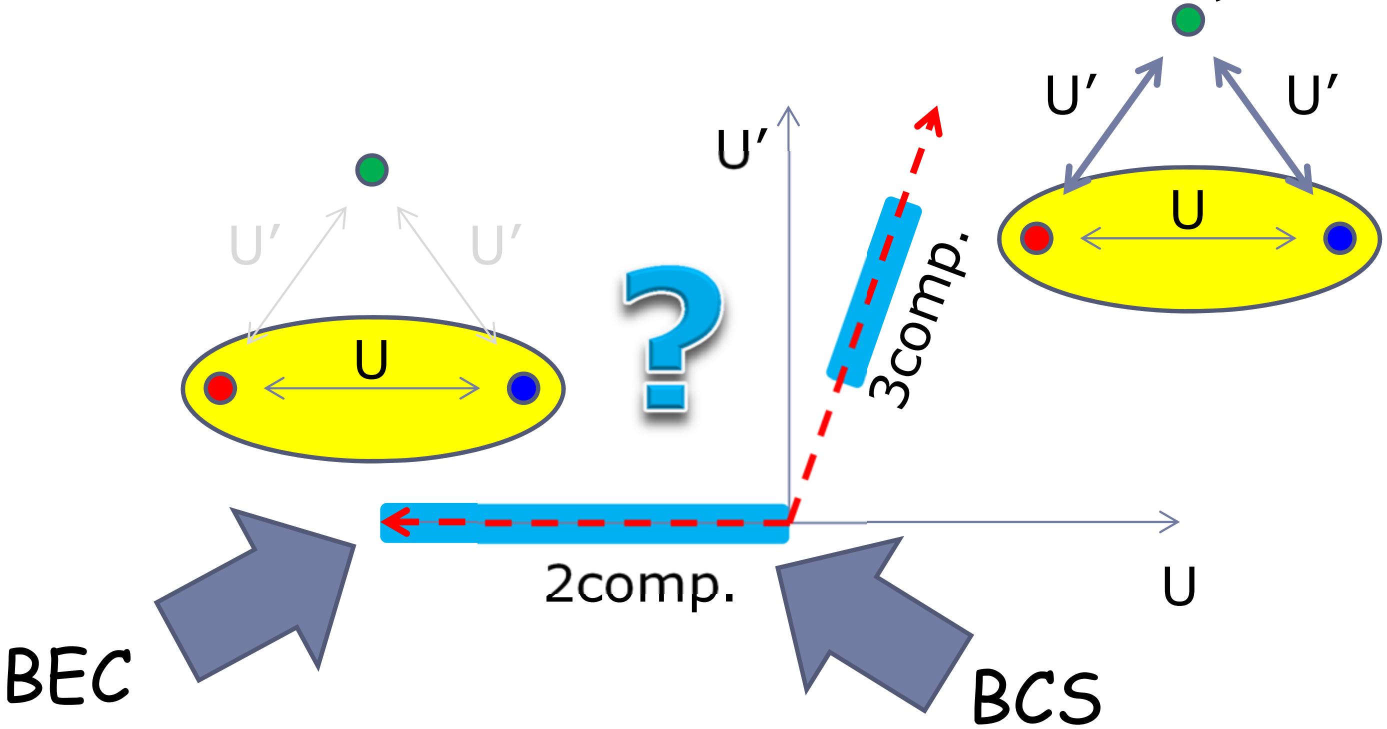




- 4 components ?

Contents

- Superfluid state in multicomponent systems

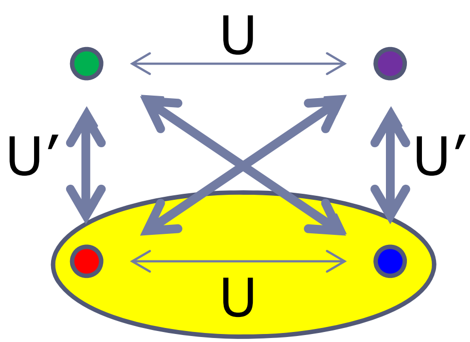


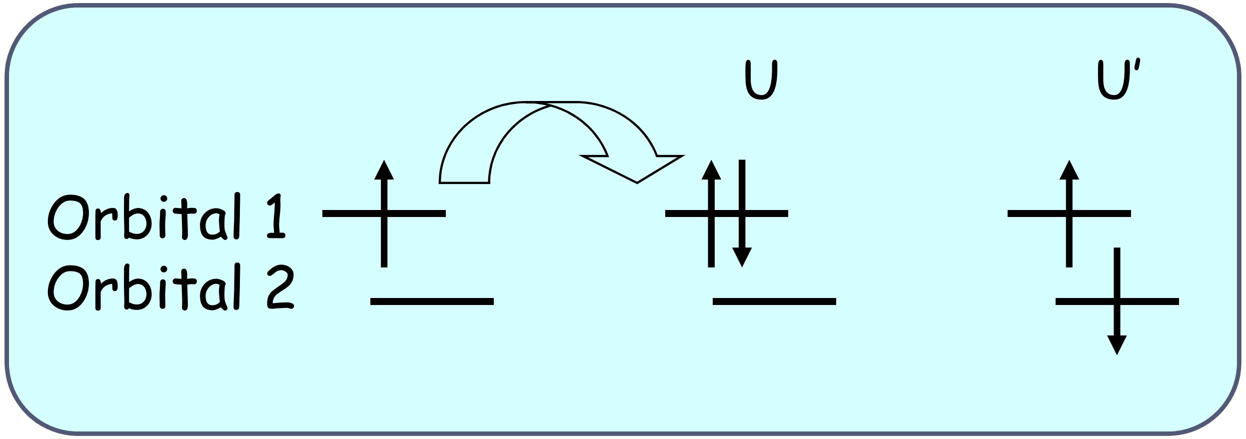
  - BCS-BEC crossover ?
  - Paramagnetic phase diagram ?

Okanami et al., Phys. Rev. A 89, 053622 (2014)

Contents

- Four component systems





S-wave superconductivity?
orbital degrees of freedom?
condensed matter?

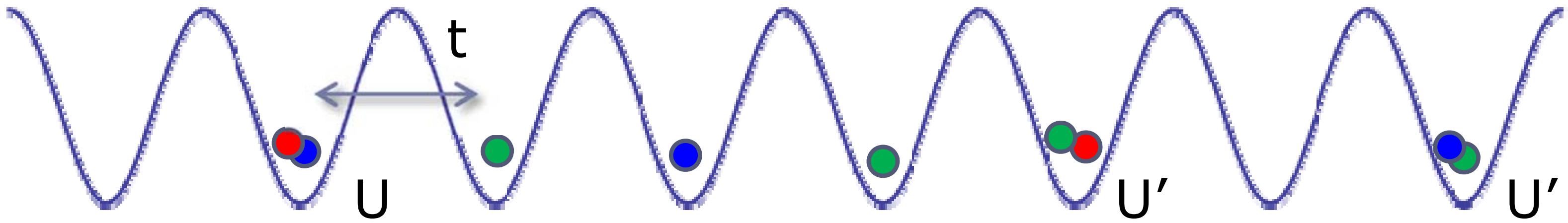
S-wave superfluid in degenerate Hubbard model ?

Model and Methods

Three component fermions

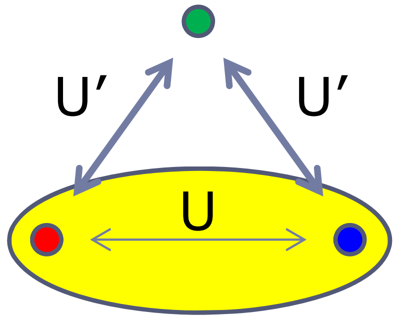
- Hubbard model

$$H = -t\sum_{<i,j>}\sum_{\alpha=1}^{3} c^\dagger_{i\alpha}c_{i\alpha} - \sum_i\sum_\alpha^{3} \mu_\alpha n_{i\alpha} + \sum_i \sum_{\alpha\neq\beta} U_{\alpha\beta} n_{i\alpha} n_{i\beta}$$



- Pair potential

$$\Delta = \langle c_{i1} c_{i2} \rangle$$

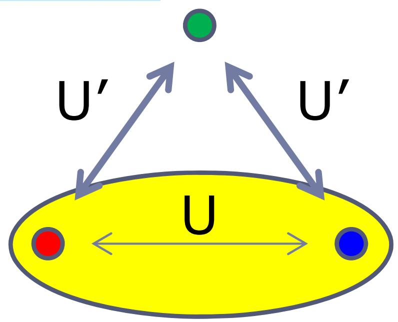


Frustration
No Density Wave & Magnetic order

K. Inaba and S. Suga, Phys. Rev. Lett. 108, 255301 (2012)

Simple static mean-field theory

- BCS theory



$$U n_1 n_2 \quad \rightarrow \quad U\Big(n_1\langle n_2\rangle + \langle n_1\rangle n_2 + \Delta c_2^\dagger c_1^\dagger + \Delta^* c_1 c_2\Big)$$

$$= \quad \frac{U}{2}\Big(n_1 + n_2\Big) + U\Big(\Delta c_2^\dagger c_1^\dagger + \Delta^* c_1 c_2\Big)$$

$$U' n_1 n_3 \rightarrow U'\Big(n_1\langle n_3\rangle + \langle n_1\rangle n_3\Big) = \frac{U'}{2}\Big(n_1 + n_3\Big)$$

Chemical potential

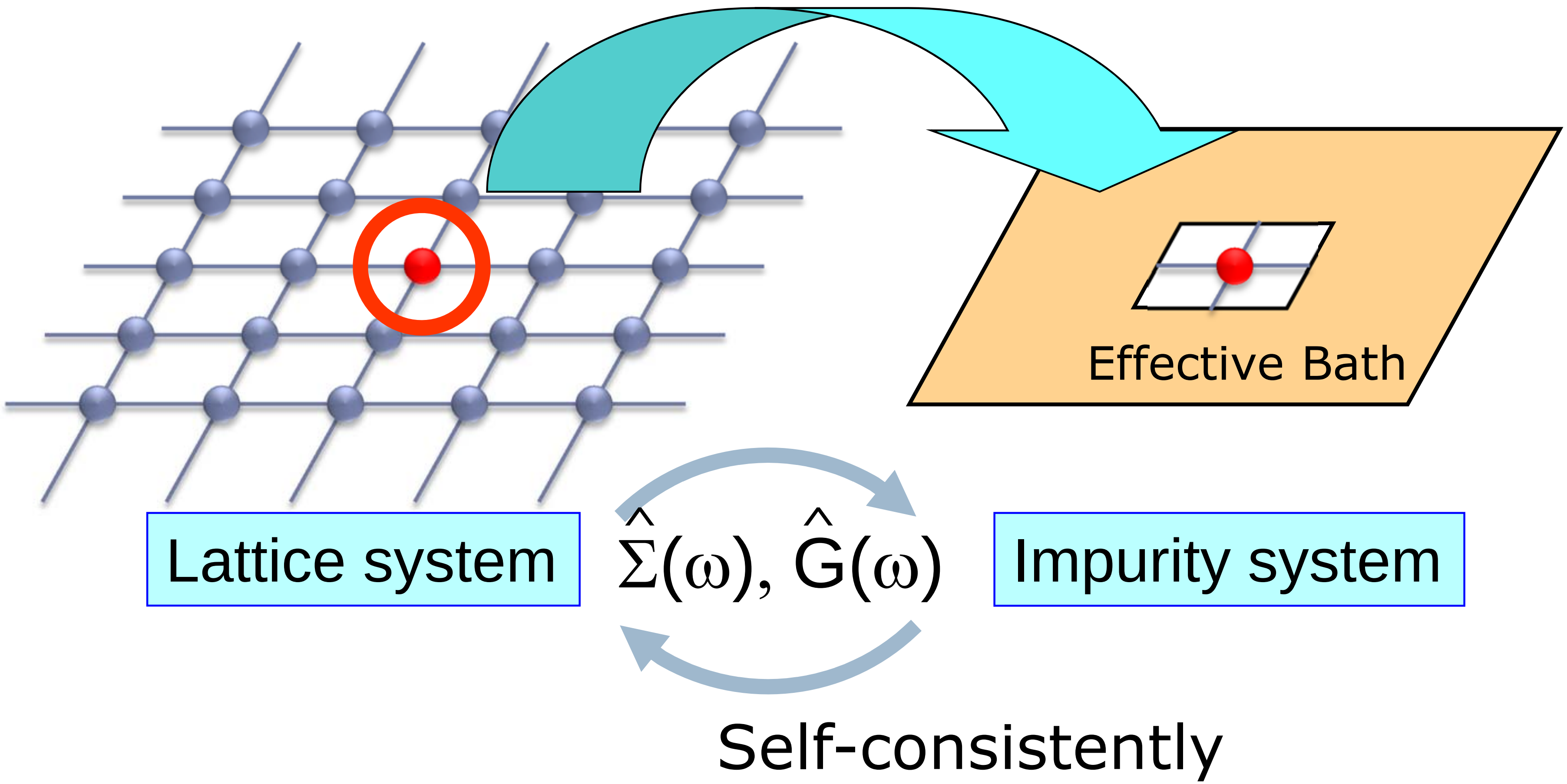
U′ no role for Superfluid

Static mean-field → Dynamical mean-field

Dynamical mean-field theory

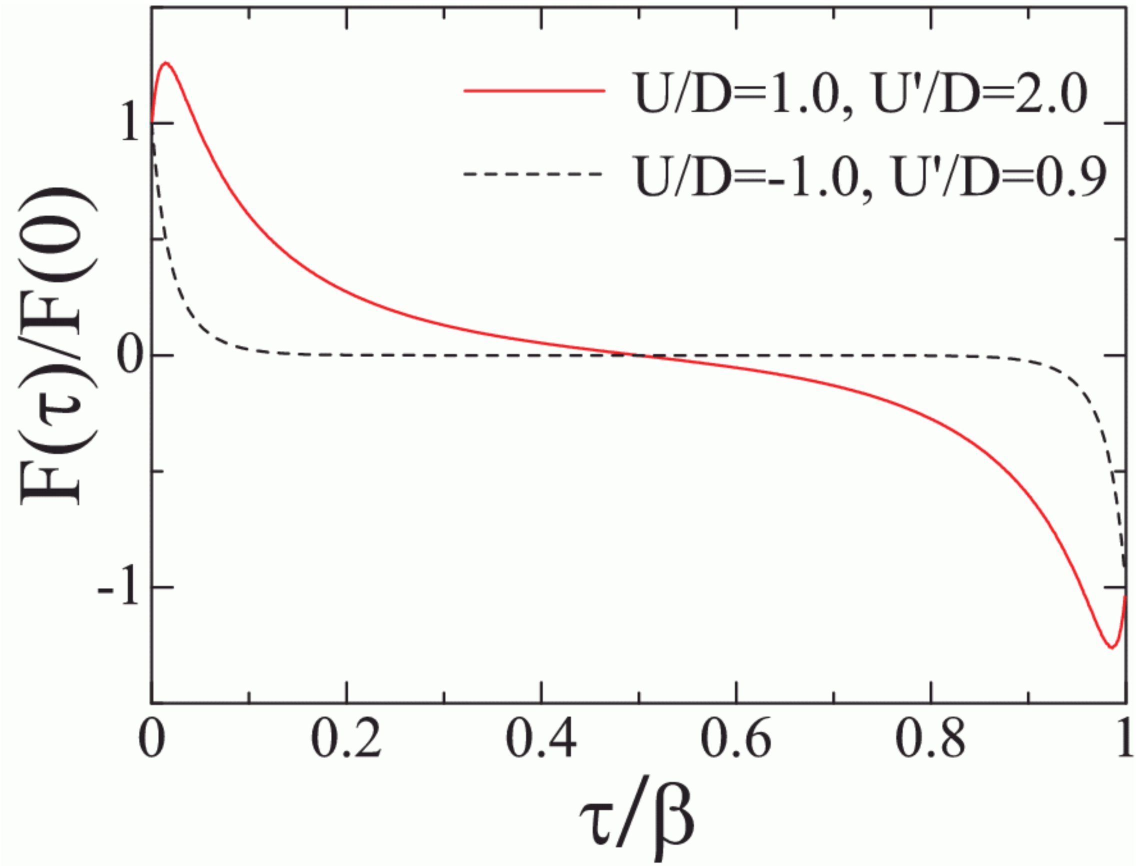
Pruschke, Jarrell, Freericks, Adv. Phys. **44**, 187 (1995)
Georges, Kotliar, Krauth, Rozenberg, Rev. Mod. Phys. **68**, 13 (1996)
Kotliar & Vollhardt, Physics Today 53, (2004)

Local particle correlations



Impurity solver

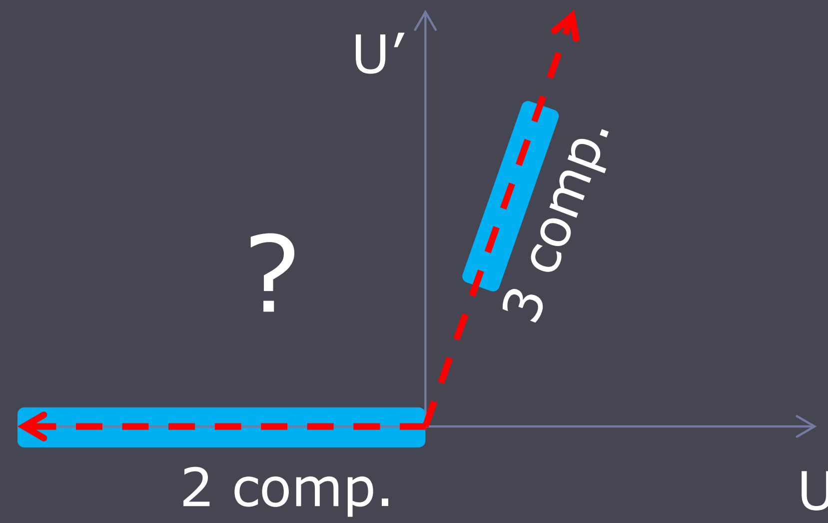
- **Continuous-Time QMC method (Nambu)**
  - Strong coupling approach



Treat SF directly

P. Werner, et al., Phys. Rev. Lett. **97**, 076405 (2006).
E. Gull, et al., Rev. Mod. Phys. **83**, 349 (2011).

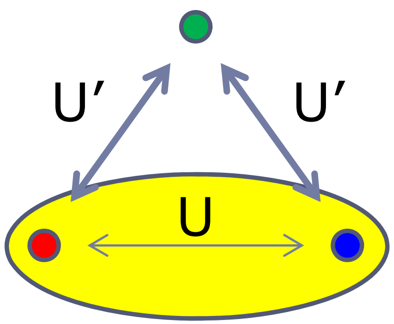
Three-component fermions

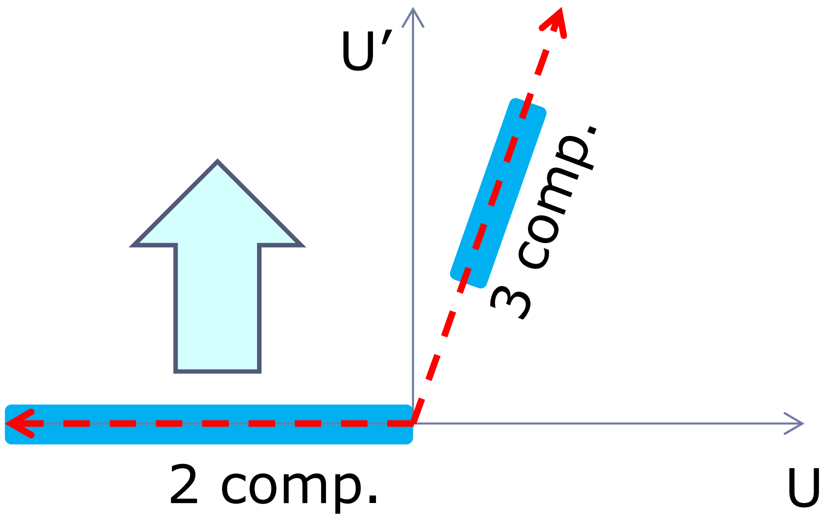


Three component fermions

- **Hubbard model**

$$H = -t \sum_{<i,j>} \sum_{\alpha=1}^{3} c_{i\alpha}^{\dagger} c_{i\alpha} - \sum_{i} \sum_{\alpha}^{3} \mu_{\alpha} n_{i\alpha} + \sum_{i} \sum_{\alpha \neq \beta} U_{\alpha\beta} n_{i\alpha} n_{i\beta}$$

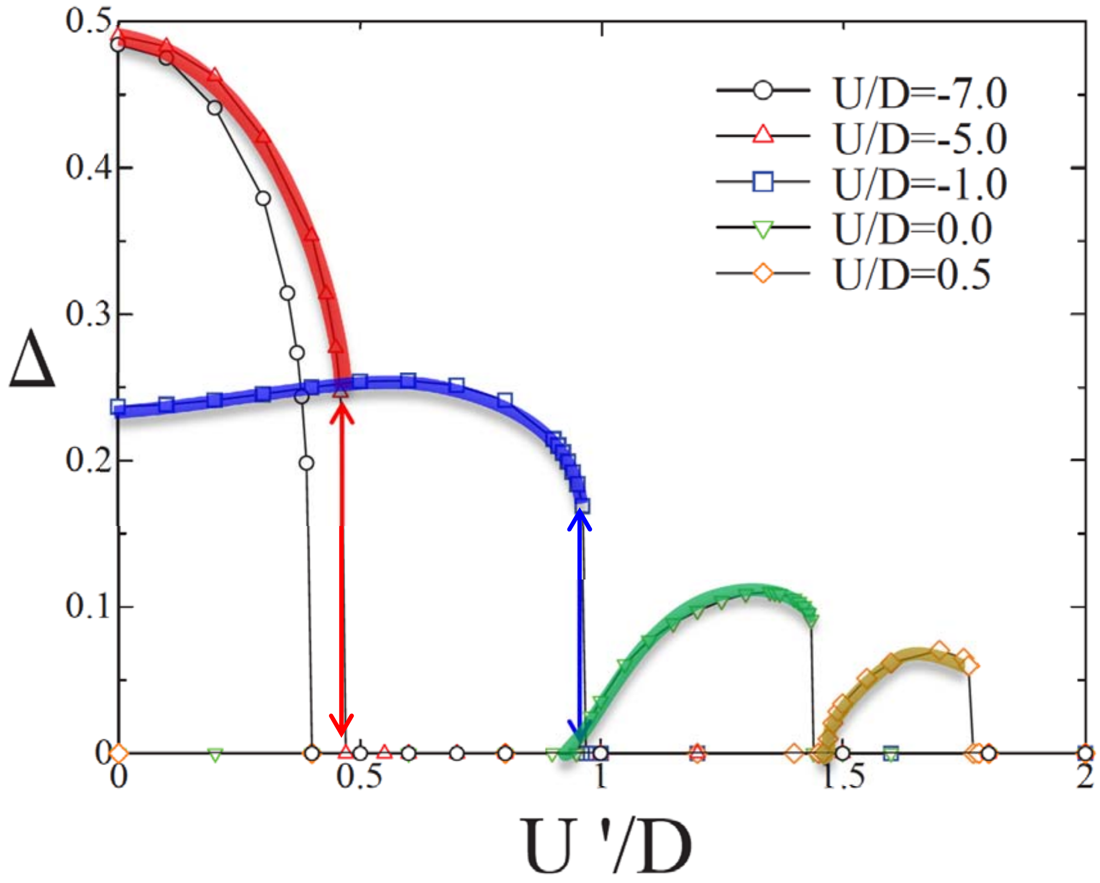


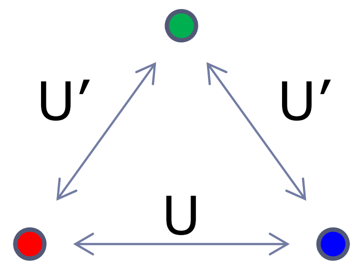


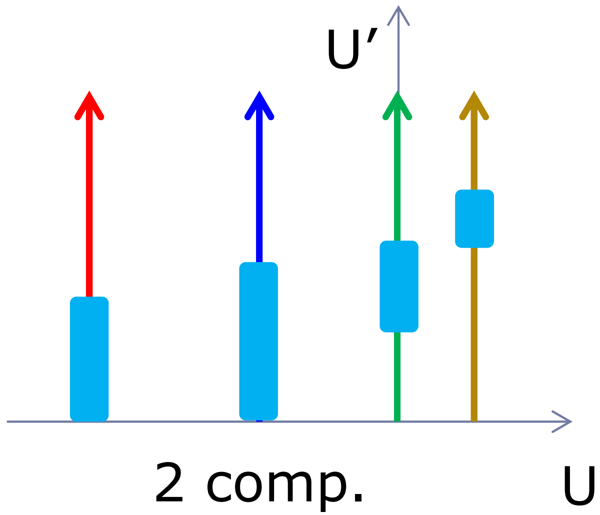
K. Inaba and S. Suga, Phys. Rev. Lett. 108, 255301 (2012)

Stability of superfluid state (T/D=0.03)

- Pair potential



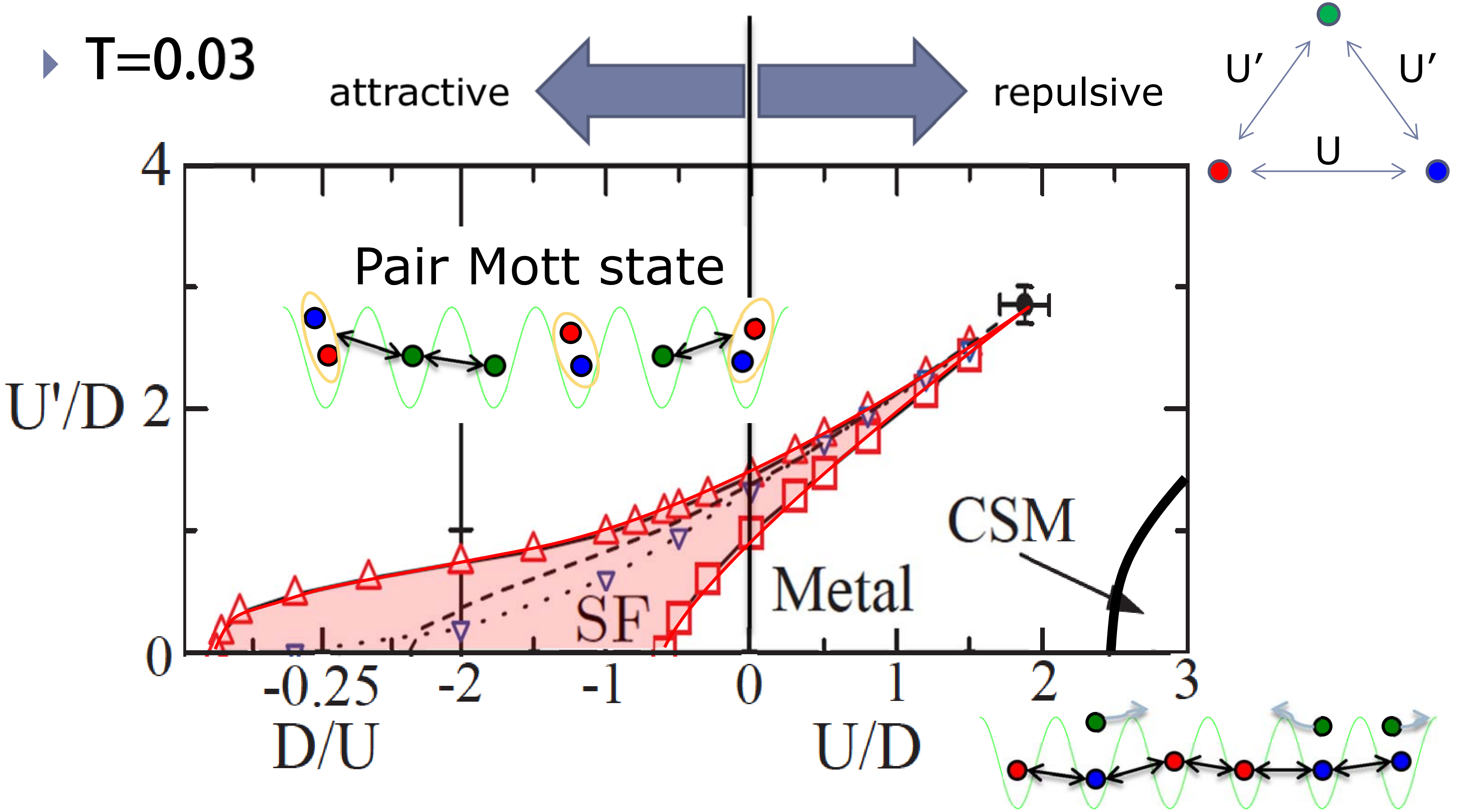




Y. Okanami et al., Phys. Rev. A 89, 053622 (2014)

Phase diagram

- T=0.03

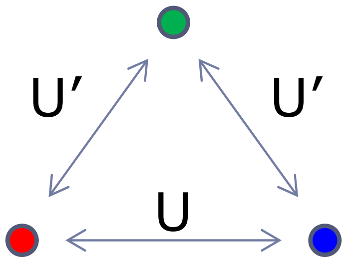


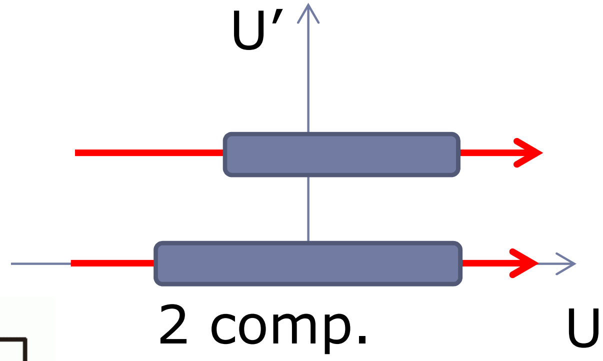
2 comp. & 3 comp.

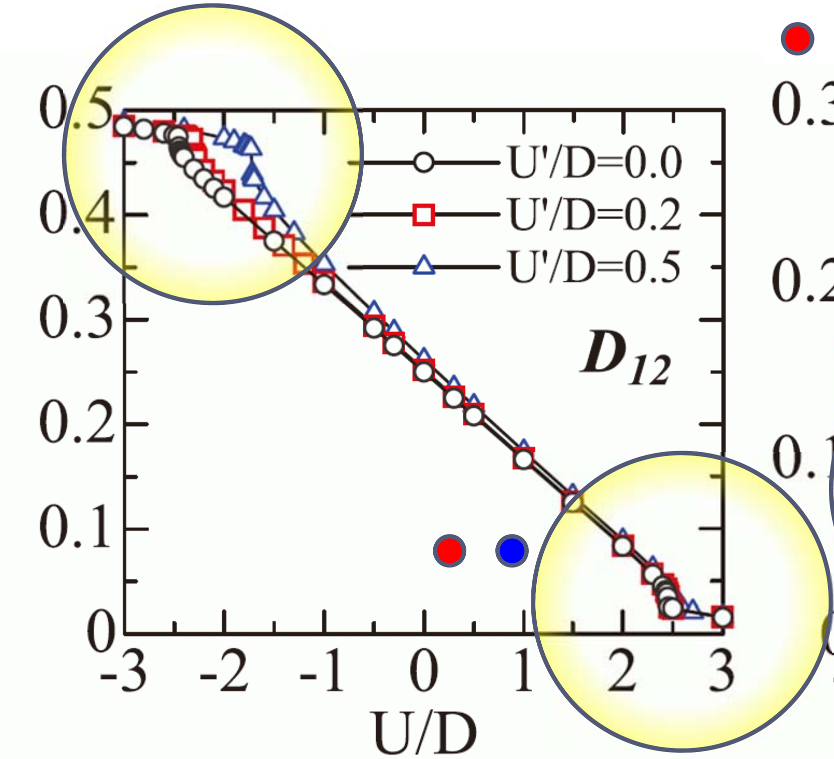
Y. Okanami et al., Phys. Rev. A 89, 053622 (2014)

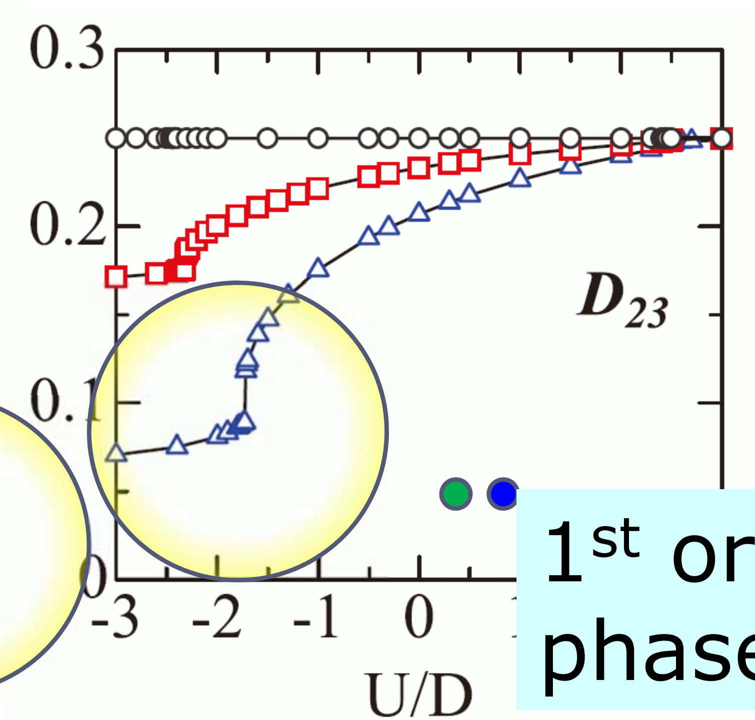
Double occupancy (Para)

- T/D=0.03



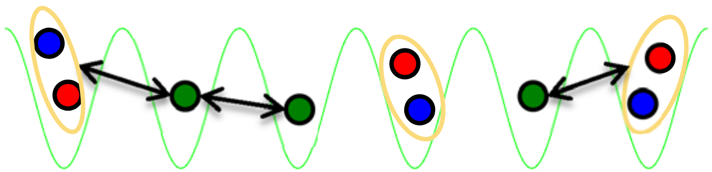




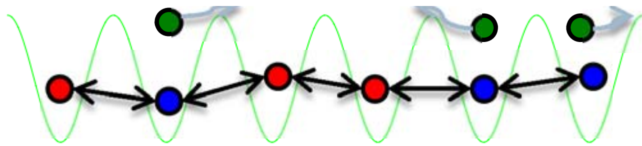


1st order
phase transitions

Pair Mott state

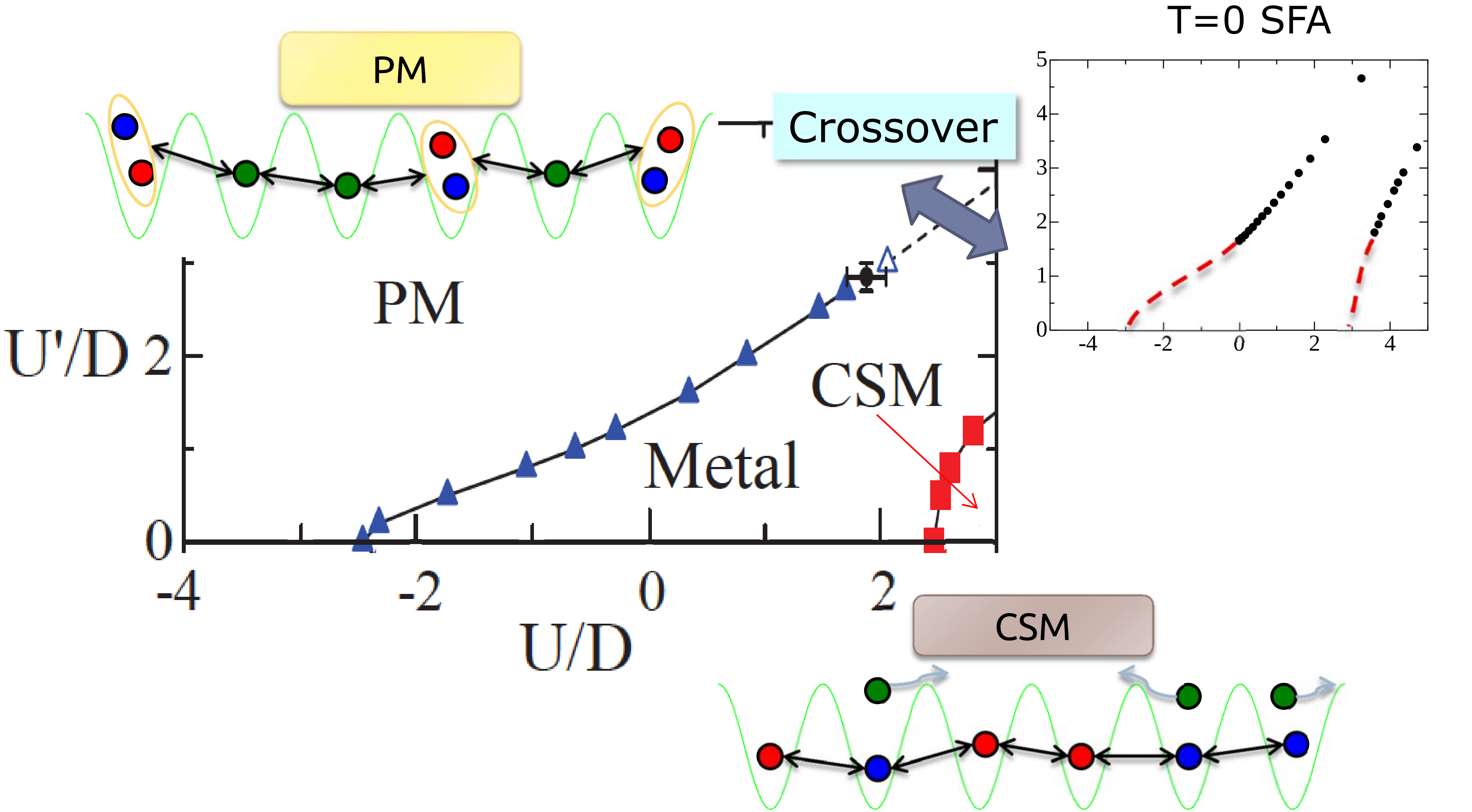


color Mott state



Phase diagram (para)

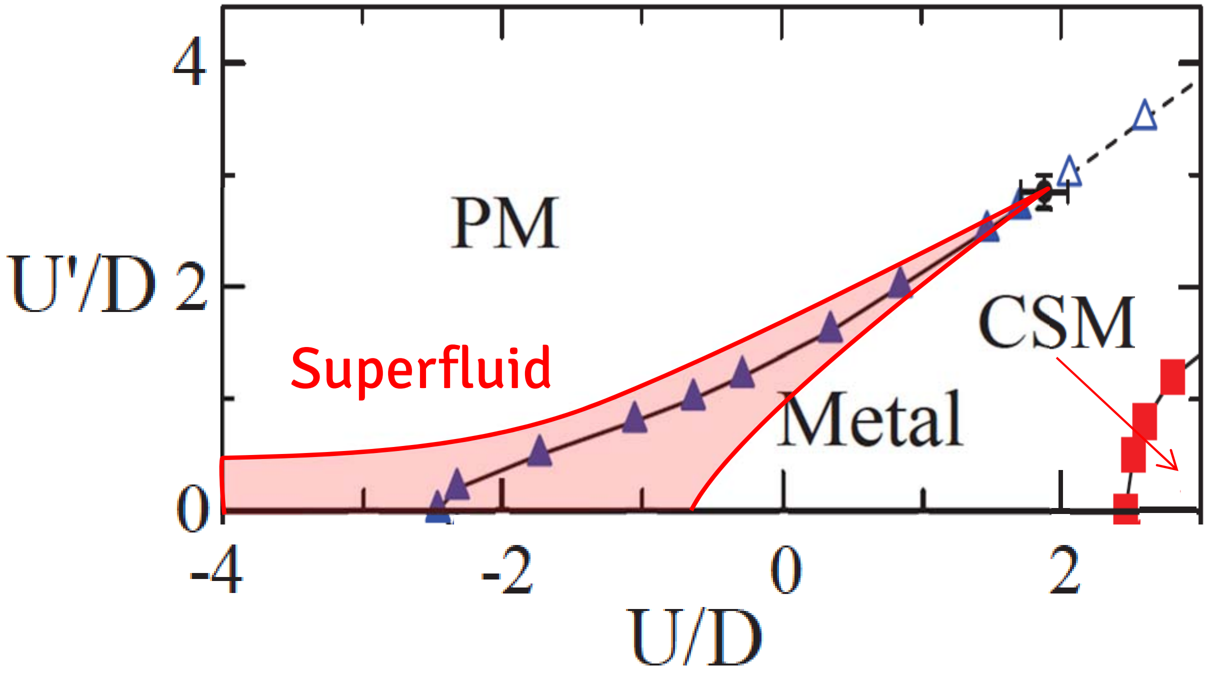
- T/D=0.03

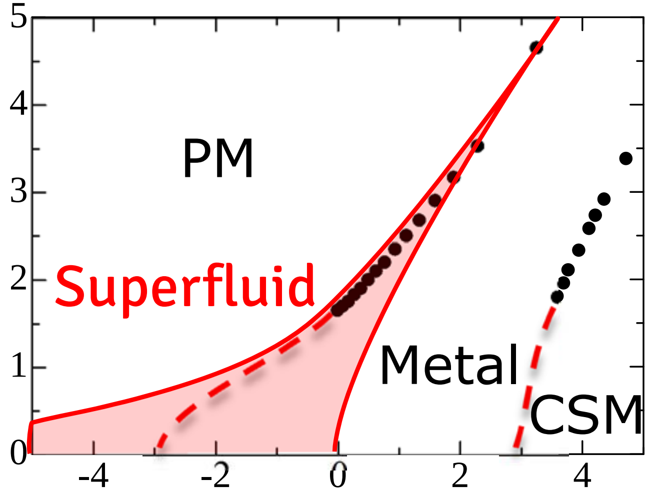


K. Inaba et al., Phys. Rev. A **82**, 051602 (2010).

Comparison (T/D=0.03)

- Stability of superfluid state





Superfluid

stable along the phase boundary between PM and metal

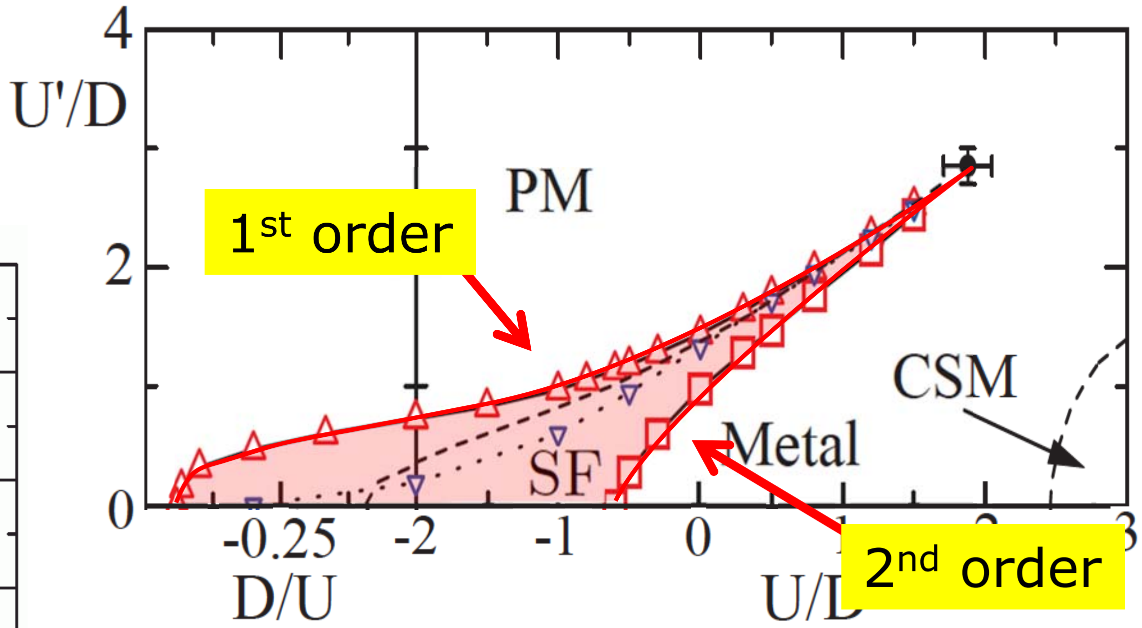
K. Inaba et al., Phys. Rev. A **82**, 051602 (2010).

BCS-BEC crossover

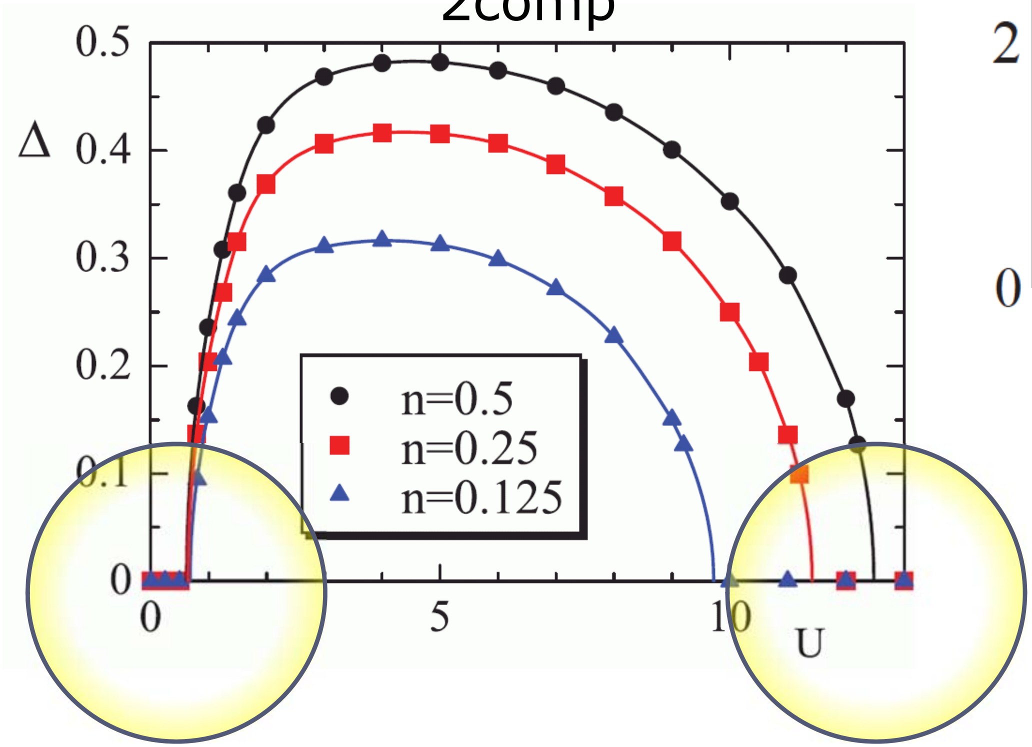
Attractive Hubbard model

- BCS-BEC crossover
  - BCS weak coupling
  - BEC strong coupling

3 components



2comp

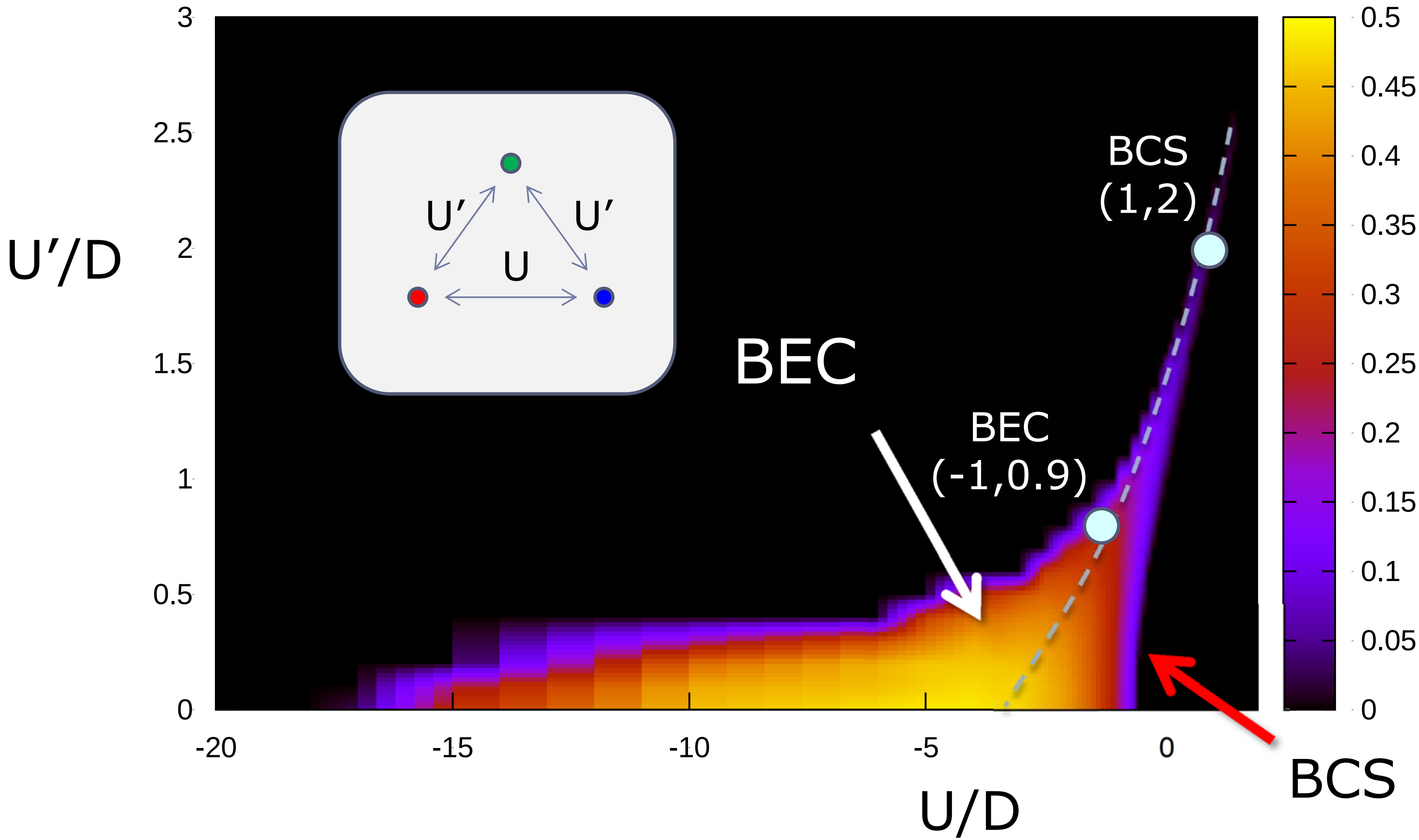


2^{nd} order phase transitions

Nature of BCS & BEC ?

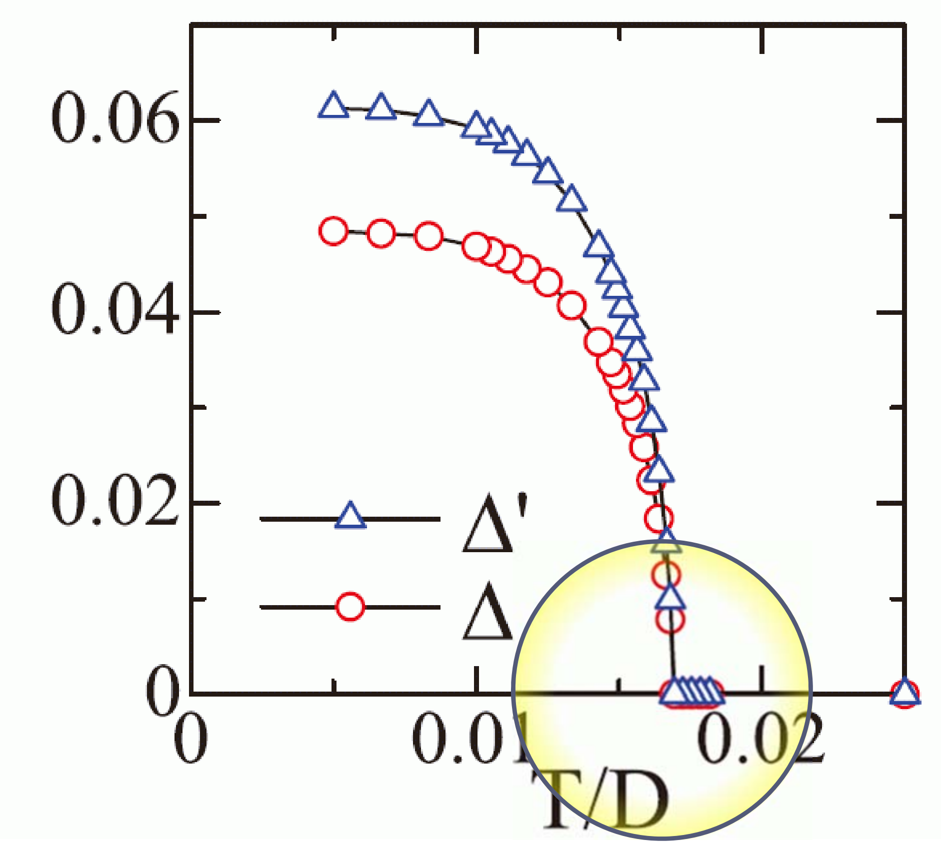
Pair potential (T/D=0.03)

- Density plot



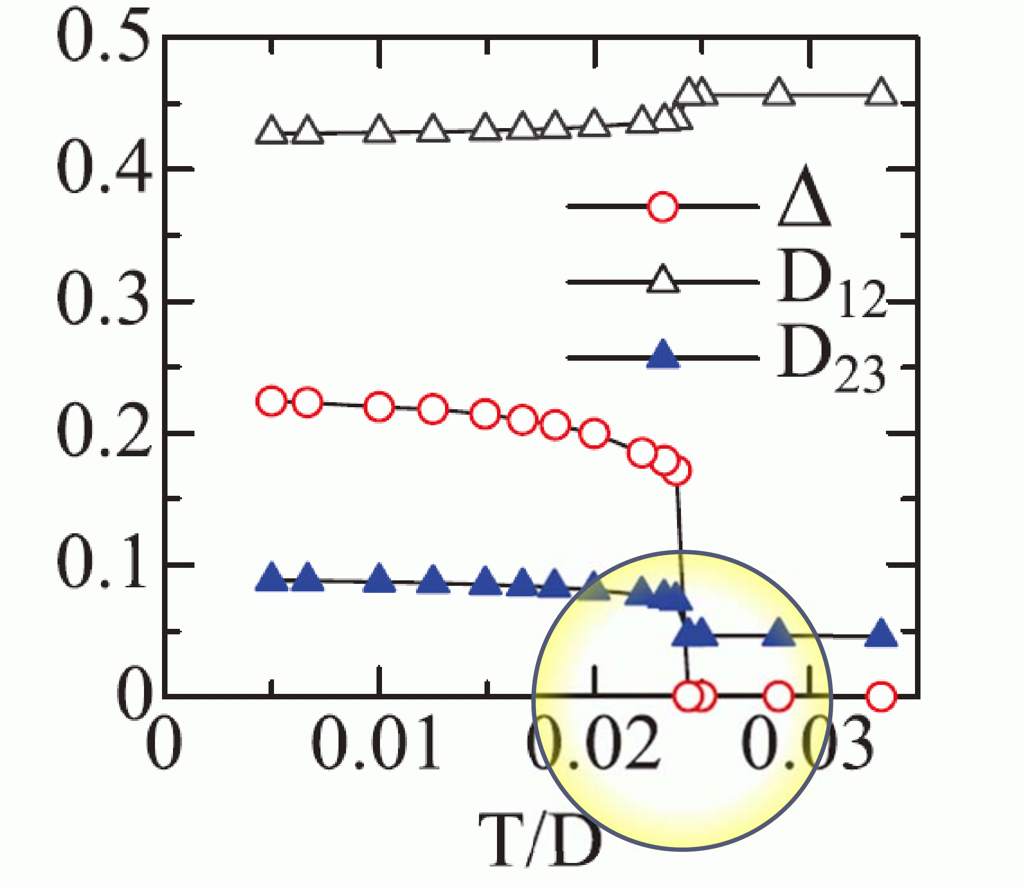
Phase transitions

- BCS region
  - U/D=1.0, U'/D=2.0



2nd order

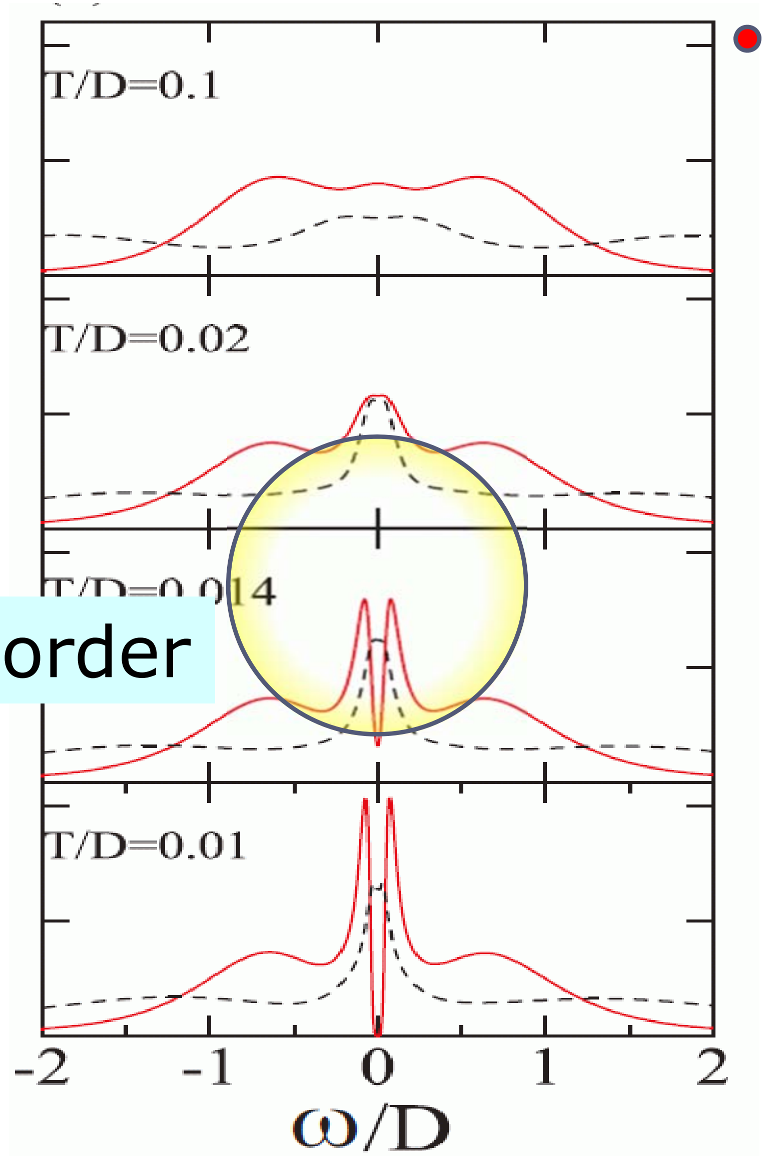
- BEC region
  - U/D=-1.0, U'/D=0.9



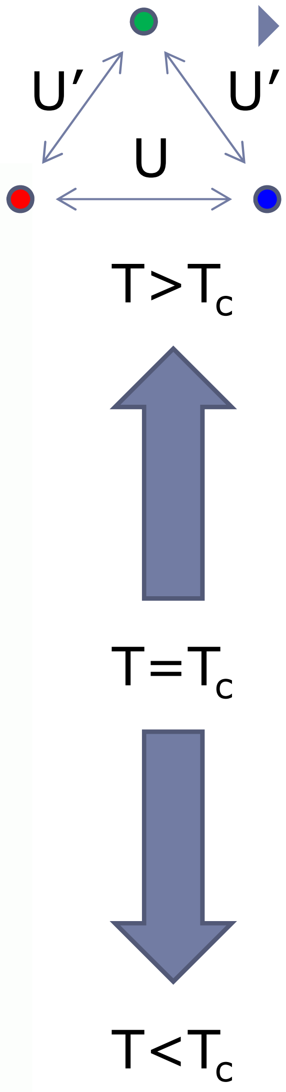
1st order

Density of states

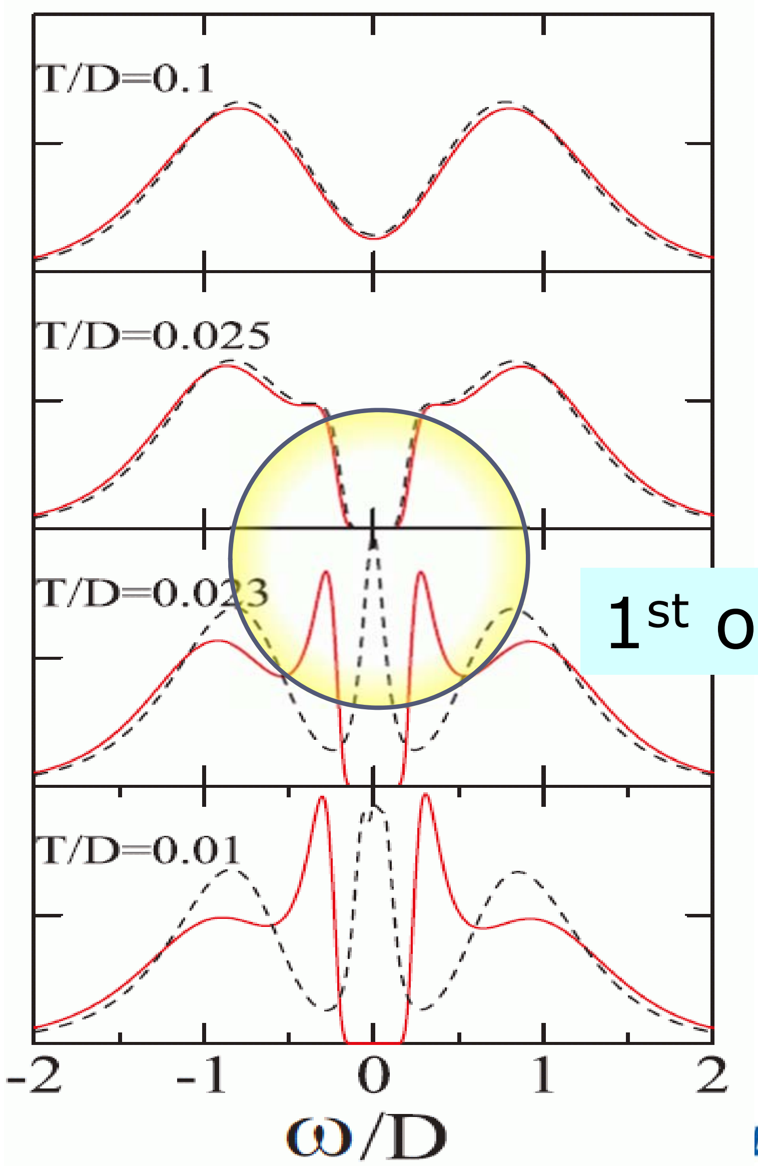
- BCS region
  - U/D=1.0, U'/D=2.0



2nd order



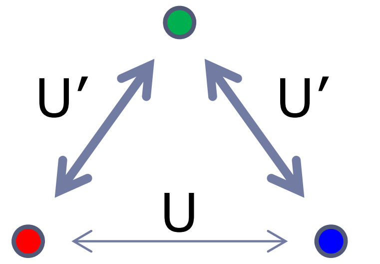
- BEC region
  - U/D=-1.0, U'/D=0.9

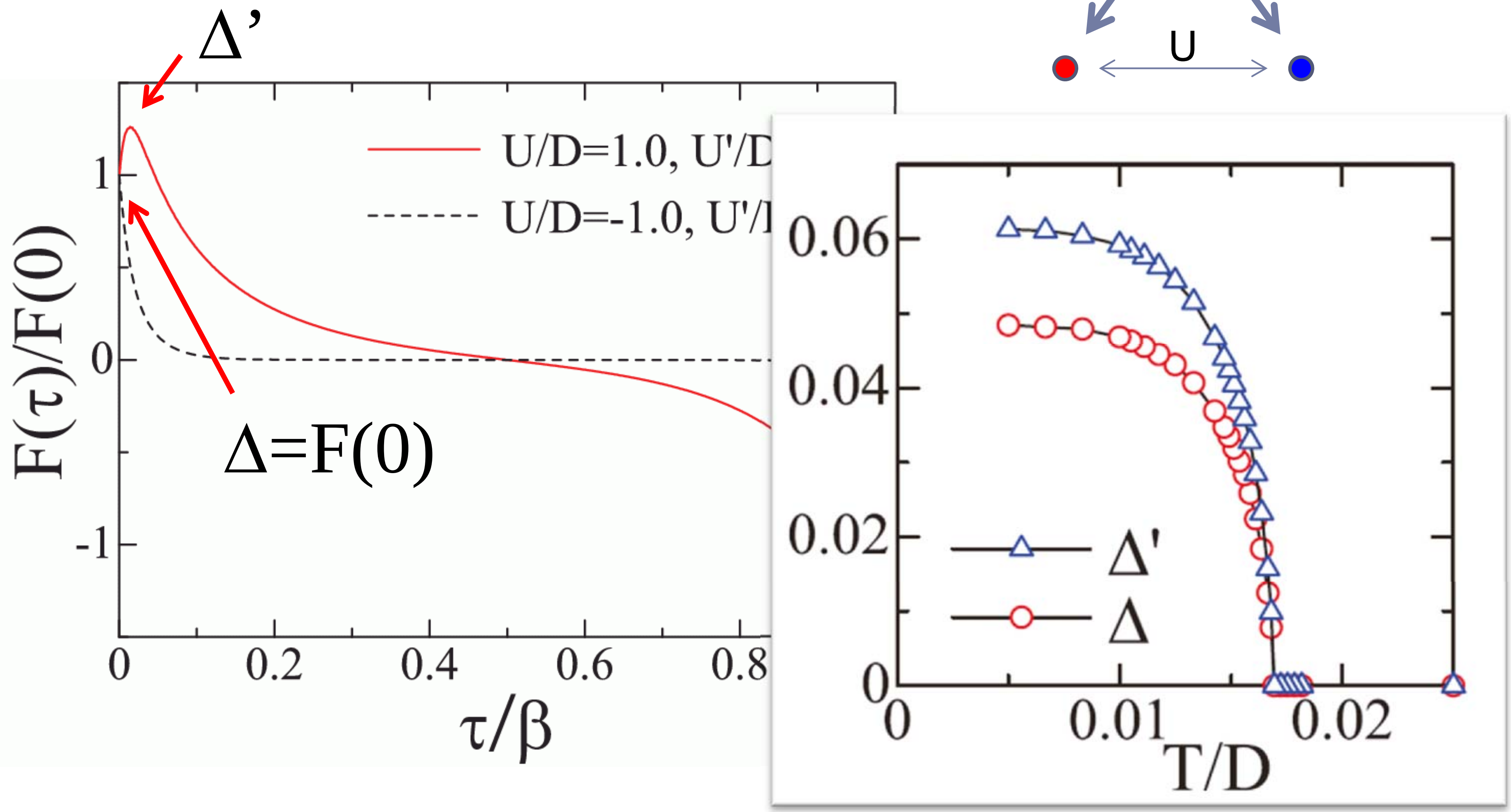


1st order

Effect of repulsive interaction

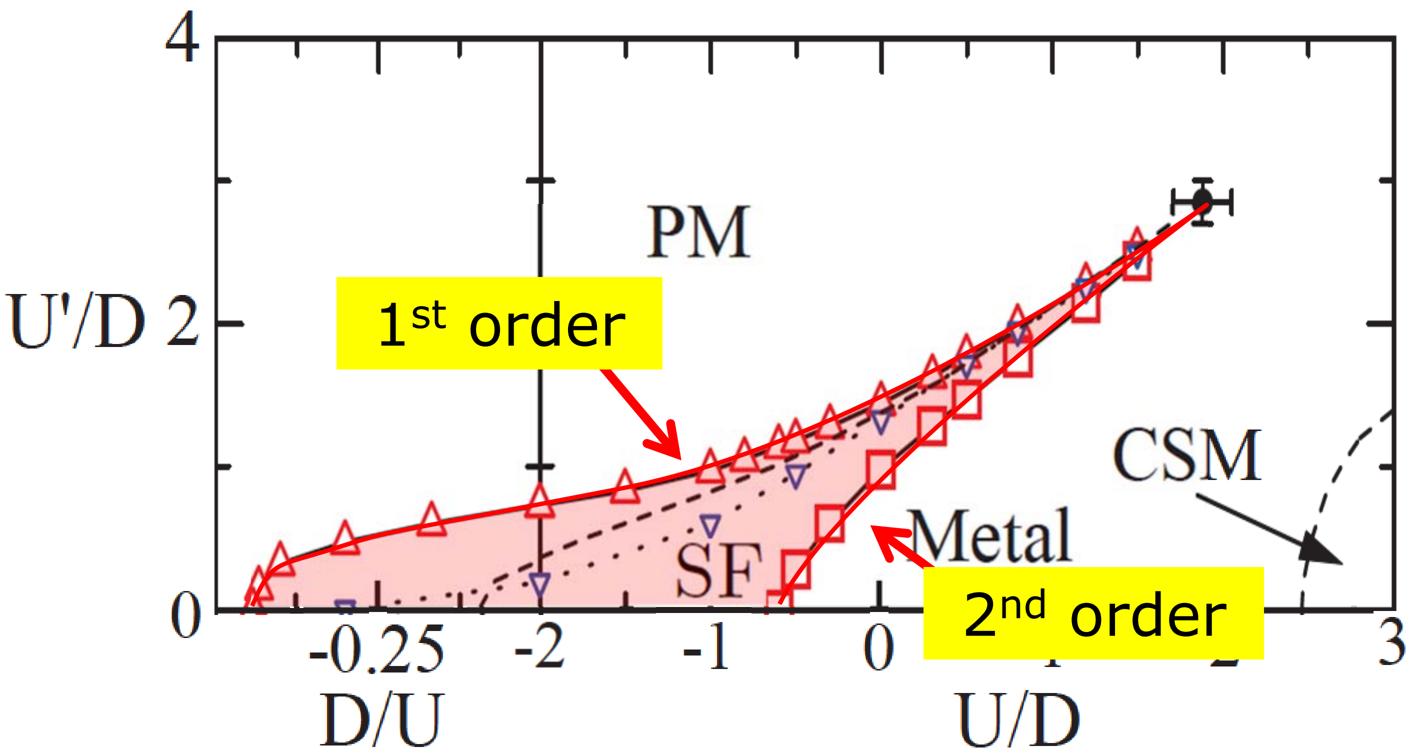
- U/D=1.0, U'/D=2.0





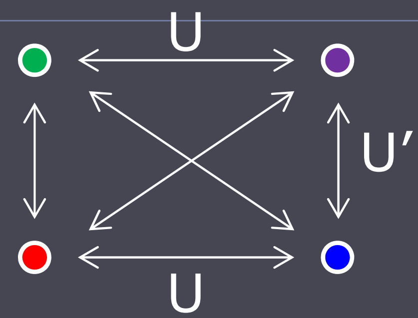
Summary (3 components)

- Superfluid state in three component fermions
  - Repulsive interaction
    - Competition PM & Metal



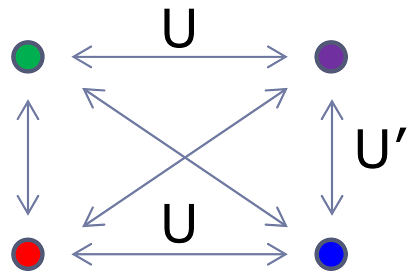
Dynamical correlations important!

Four component fermions



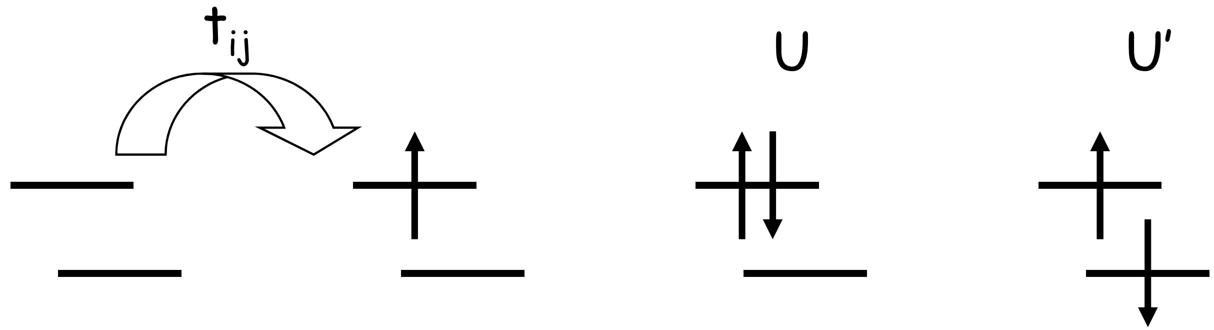
Four component fermions

- Degenerate Hubbard model (Mn, V, Ti, ⋯)



$$H = \sum_{\substack{<i,j> \\ \alpha,\sigma}} t_{ij} c^\dagger_{i\alpha\sigma} c_{i\alpha\sigma} + U \sum_{i\alpha} n_{i\alpha\uparrow} n_{i\alpha\downarrow} + U' \sum_{i,\sigma,\sigma'} n_{i1\sigma} n_{i2\sigma'}$$

Orbital 1
Orbital 2



Hund coupling J=0

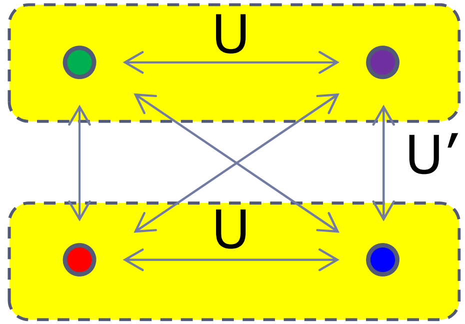
U,U' : independent

Effective Hamiltonian U=U'
$CeTIn_5$ (T=Co, Ir, Rh)
Takimoto et al., J. Phys. 14, 369 (2002)

Momoi & Kubo, Phys. Rev. B 58, 567 (1998)

Dynamical mean-field theory

- Green functions
  - Assumption　only intra-orbital pairs

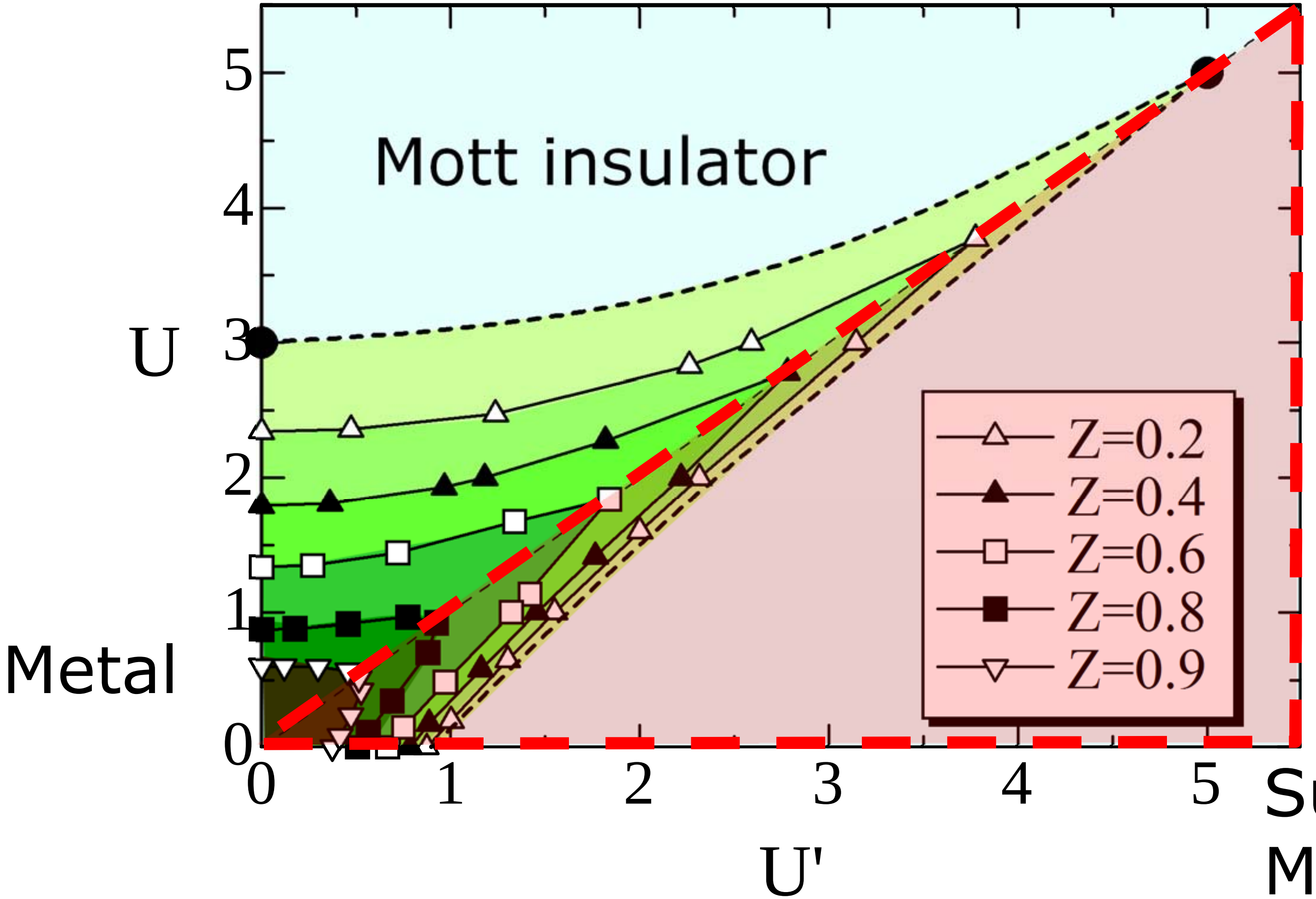


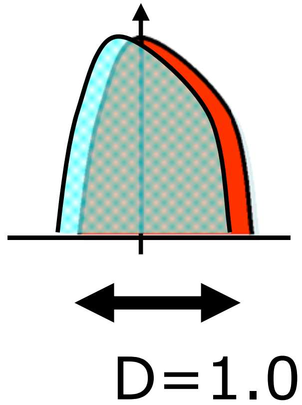
$$\hat{G}(\tau) = \begin{pmatrix} G_{1\uparrow}(\tau) & F_1(\tau) & & \\ F_1(\tau) & G_{1\downarrow}(-\tau) & & \\ & & G_{2\uparrow}(\tau) & F_2(\tau) \\ & & F_2(\tau) & G_{2\downarrow}(-\tau) \end{pmatrix}$$

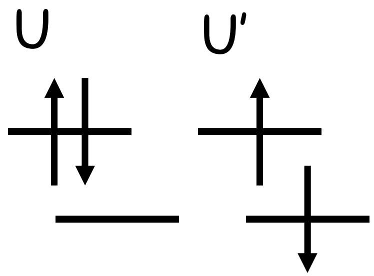
- Impurity solvers
  - Strong-coupling expansion CTQMC method

Phase diagram (para)

- Degenerate Hubbard model
  - Half filling







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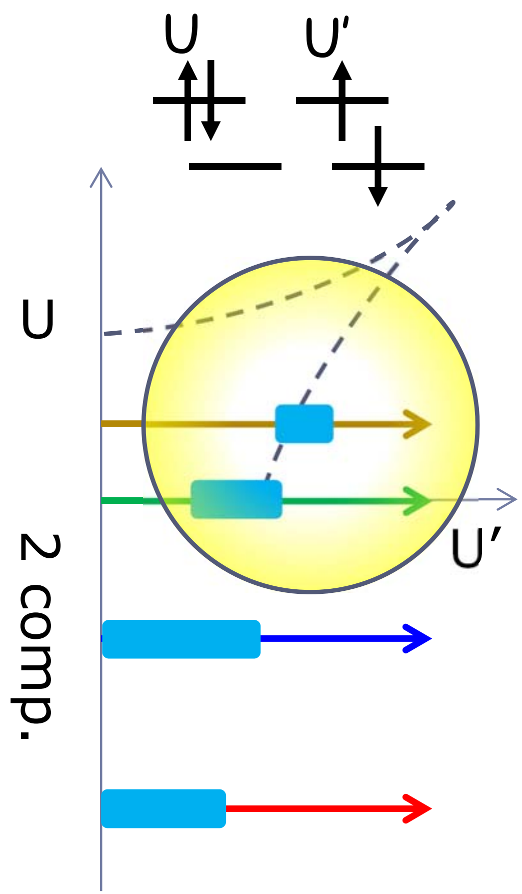
Superconductivity?
Mott transitions?

AK, Imai & Kawakami, Phys. Rev. B 66, 165107 (2002)

S-wave superconductivity (T=0.01D)

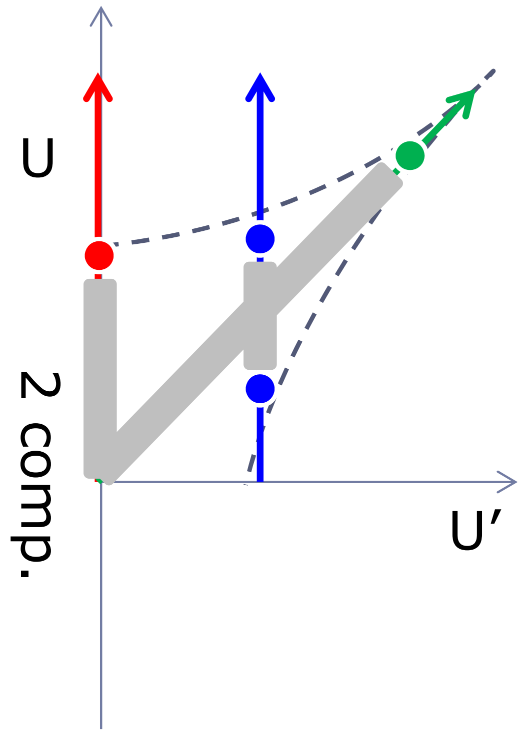
- Pair potential



Repulsive systems
s-wave superconductivity

Mott transitions (paramagnetic state)

- Renormalization factors (T=0.01D)



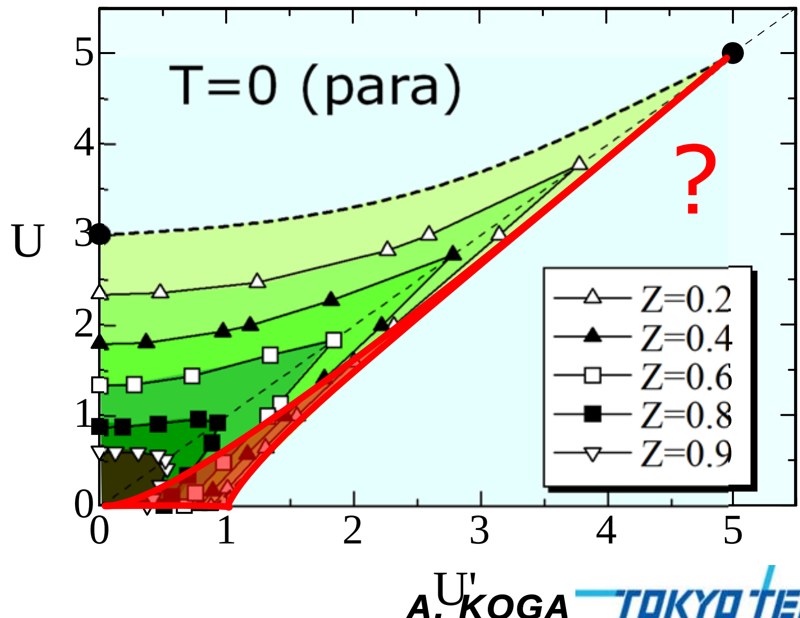
Mott transitions

Phase diagram

- Degenerate Hubbard model

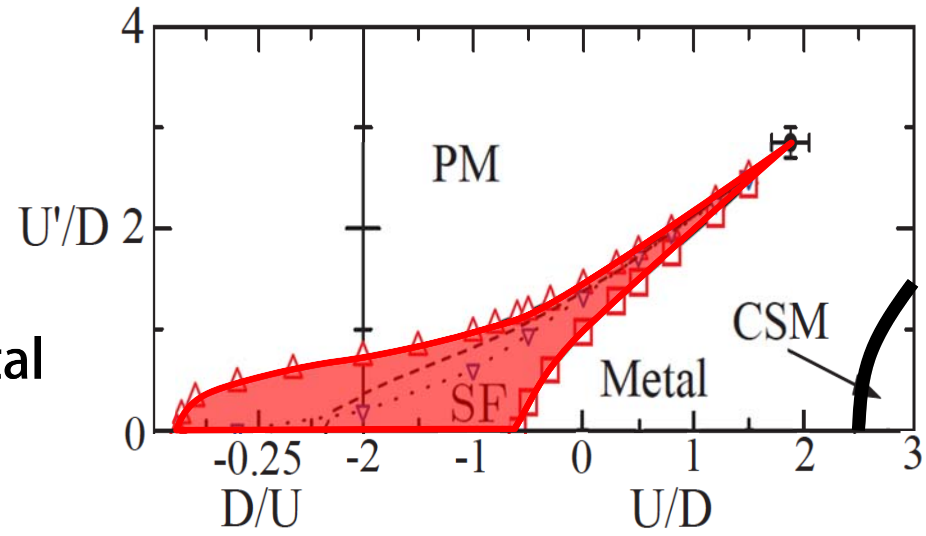
S-wave SC
repulsive interacting case

Between Metal & Mott states



Summary

- Superfluid state in multicomponent fermions
  - 3 comp.
    - 2comp. Vs 3comp.
    - Repulsive interaction
      - Competition PM & Metal



  - 4 comp. (2 orbitals)
    - Repulsion-induced SF

Dynamical correlations important!

Okanami et al., Phys. Rev. A **89**, 053622 (2014).