2014/06/20 (fri) @NHSCP2014

Odd-frequency superconductivity in two-channel Kondo lattice and its electromagnetic response

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Outline

1. Introduction

    odd-frequency SC

    two-channel Kondo systems

2. Phase diagram of two-channel Kondo lattice

    divergence of pairing susceptibility

    composite order parameters

    S. Hoshino and Y. Kuramoto: PRL 112 (2014) 167204

3. Mean-field theory for odd-frequency SC

    effective low-energy Hamiltonian

    Meissner kernel

    S. Hoshino: arXiv:1406.1983

4. Summary

History

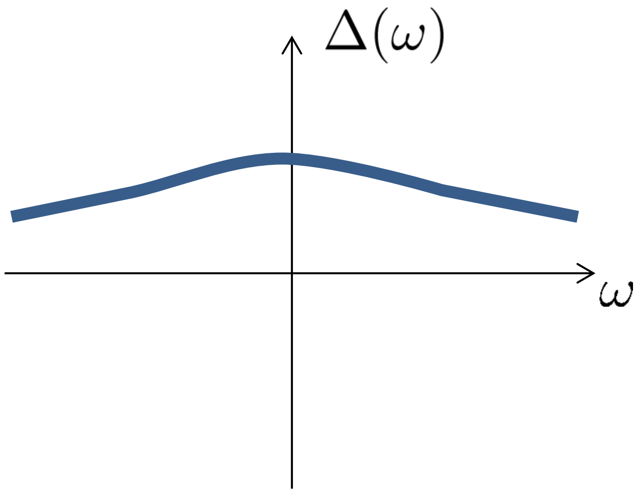
1911	zero resistivity	**Kamerlingh Onnes**
1923	lambda transition of ^{4}He	**Kamerlingh Onnes & Dana**
1930	superfluidity of ^{4}He	**Keesom**
1933	perfect diamagnetism	**Meissner & Ochsenfeld**
1957	microscopic theory	**Bardeen, Cooper, Schrieffer**
1962	quantum interference effect	**Josephson**
1972	superfluidity of ^{3}He	**Osheroff, Richardson, Lee**
1974	proposal of odd-freq. SC for ^{3}He	**Berezinskii**
1986	cuprate high-T_c superconductor	**Bednortz & Muller**
1992	revival of odd-freq. SC	**Balatsky & Abrahams**
	proposal in two-channel Kondo system	**Emery & Kivelson**

$time$ ↓

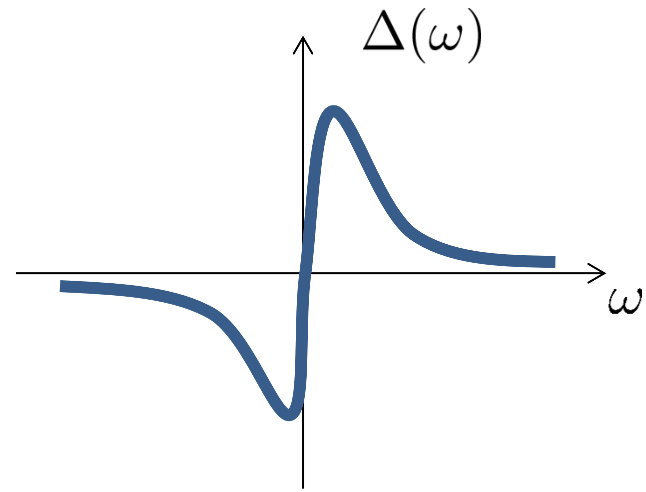
Pairing state with zero amplitude at equal time

$$\langle c^\dagger(t)c^\dagger\rangle = 0 \quad \text{at } t = 0$$

Berezinskii: JETP Lett. (1974)



even-frequency SC



odd-frequency SC

- nontrivial spin and space structures

  e.g.) [s-wave, spin-triplet], or [p-wave, spin-singlet], etc…

- finite density of states at chemical potential

Balatsky & Abrahams: PRB (1992)

Pairing state with zero amplitude at equal time

$$\langle c^\dagger(t) c^\dagger \rangle = 0 \quad \text{at } t = 0$$

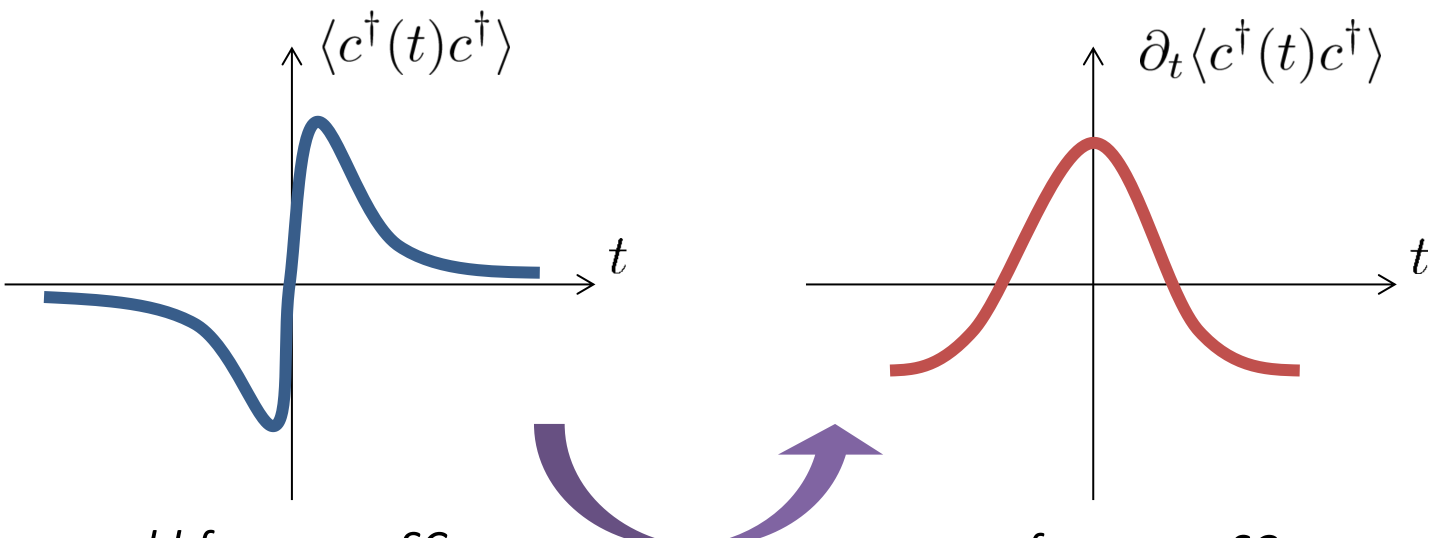
Berezinskii: JETP Lett. (1974)

- composite pair amplitude with even frequency

i.e.) derivative of odd frequency gives even frequency

Emery & Kivelson: PRB (1992)
Balatsky & Bonca: PRB (1993)

$$\mathrm{i}\frac{\partial}{\partial t}\langle c^\dagger(t) c^\dagger \rangle = \langle [c^\dagger(t), \mathcal{H}] c^\dagger \rangle \neq 0 \quad \text{at } t = 0$$



odd-frequency SC

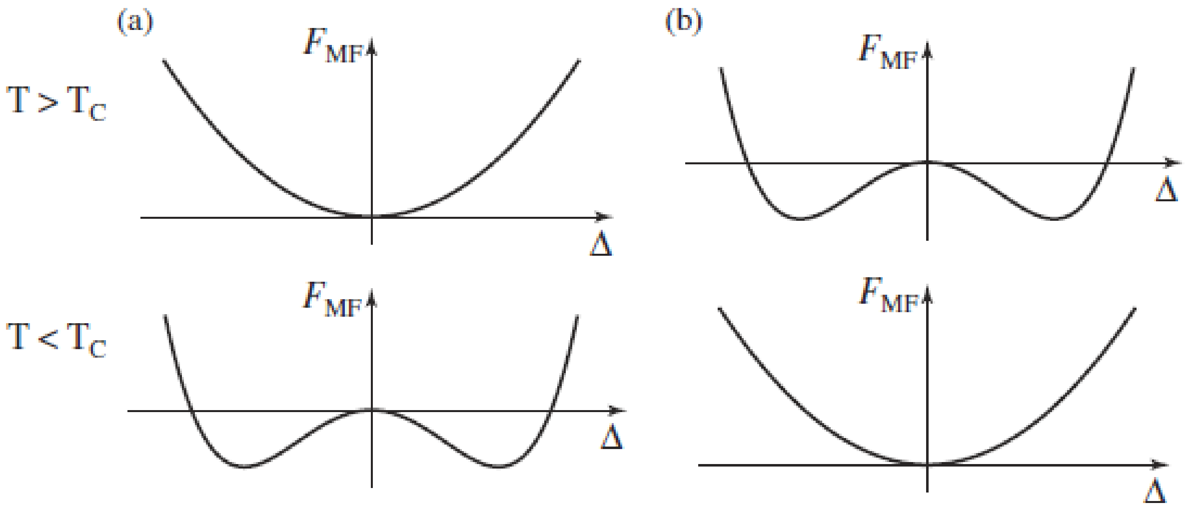
even-frequency SC

t=0: two-body quantity (composite)

Thermodynamic Stability

mean-field like approximation $V(\sigma_1 k_1, \sigma_2 k_2 | \sigma_3 k_3, \sigma_4 k_4)$

Heid: Z. Phys. B (1995)
Kusunose et al.: JPSJ (2011)



$\phi(\mathbf{k}, -\omega_n) = \phi(\mathbf{k}, \omega_n)$

$\phi(\mathbf{k}, -\omega_n) = -\phi(\mathbf{k}, \omega_n)$

Negative Meissner effect ⇔ negative superfluid weight

odd-frequency SC seems thermodynamically unstable…

Proposals to resolve thermodynamic unstability

Coleman et al: PRL (1993)
Heid: Z. Phys. B (1995)

(1) strong coupling corrections
(2) first-order phase transition
(3) inhomogeneous state

Abrahamas et al. PRB (1995)
Balatsky et al.: New J. Phys. (2009)

(4) composite-operator description

$$F_{12}^{\dagger}(\mathrm{i}\varepsilon_n) = +F_{21}(-\mathrm{i}\varepsilon_n)^*$$

effective MF Hamiltonian
(Lehmann representation can be used)

Belitz & Kirkpatrik: PRB (1999)
Solenov et al.: PRB (2009)
Kusunose et al.: JPSJ (2011)

(5) without Hermite relation

$$F_{12}^{\dagger}(\mathrm{i}\varepsilon_n) = -F_{21}(-\mathrm{i}\varepsilon_n)^*$$

No ordinary MF Hamiltonian

Possible Realizations

Berezinskii	^{3}He	JETP 1974
Emery & Kivelson	two-channel Kondo systems	PRB 1992, PRL 1993
Balatsky & Bonca	t-J model	PRB 1993
Coleman et al.	Kondo lattice	PRL 1993, PRB 1994
Zachar et al.	Kondo lattice	PRL 1996
Jarrell et al.	two-channel Kondo lattice	PRL 1997
Vojta & Dagotto	frustrated electron system	PRB 1999
Anders	two-channel Anderson lattice	PRB 2002
Fuseya et al.	AFM phase near QCP	JPSJ 2004
Yada et al.	Extended Hubbard model near SDW QCP	arXiv 2008
Shigeta et al.	Extended Hubbard model with spin-orbit coupling	PRB 2009
Hotta	FM phase near orbital order QCP	JPSJ 2009
Kusunose et al.	electron-phonon system	JPSJ 2011
Yanagi et al.	frustrated electron system	JPSJ 2012
Shigeta et al.	quasi-one dimensional system	JPSJ 2011, JPSJ 2013
Hoshino & Kuramoto	two-channel Kondo lattice	PRL 2014

New / Original

- アルカリ金属
- アルカリ土類金属
- 遷移元素
- ランタノイド
- アクチノイド
- 卑金属
- 非金属元素
- 希ガス

- C 固体
- Br 液体
- H 気体
- Tc 人工元素

	1 IA	2 IIA	3 IIIB	4 IVB	5 VB	6 VIB	7 VIIB	8	9 VIIIB	10	11 IB	12 IIB	13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA	18 VIIIA
1	1 H 水素 1.00794																	2 He ヘリウム 4.002602
2	3 Li リチウム 6.941	4 Be ベリリウム 9.012182											5 B ホウ素 10.811	6 C 炭素 12.0107	7 N 窒素 14.00674	8 O 酸素 15.9994	9 F フッ素 18.9984032	10 Ne ネオン 20.1797
3	11 Na ナトリウム 22.989770	12 Mg マグネシウム 24.3050											13 Al アルミニウム 26.981538	14 Si 珪素 28.0855	15 P リン 30.973761	16 S 硫黄 32.066	17 Cl 塩素 35.453	18 Ar アルゴン 39.948
4	19 K カリウム 39.0983	20 Ca カルシウム 40.078	21 Sc スカンジウム 44.955910	22 Ti チタン 47.867	23 V バナジウム 50.9415	24 Cr クロム 51.9961	25 Mn マンガン 54.938049	26 Fe 鉄 55.8457	27 Co コバルト 58.933200	28 Ni ニッケル 58.6934	29 Cu 銅 63.546	30 Zn 亜鉛 65.409	31 Ga ガリウム 69.723	32 Ge ゲルマニウム 72.64	33 As ヒ素 74.92160	34 Se セレン 78.96	35 Br 臭素 79.904	36 Kr クリプトン 83.798
5	37 Rb ルビジウム 85.4678	38 Sr ストロンチウム 87.62	39 Y イットリウム 88.90585	40 Zr ジルコニウム 91.224	41 Nb ニオブ 92.90638	42 Mo モリブデン 95.94	43 Tc テクネチウム (98)	44 Ru ルテニウム 101.07	45 Rh ロジウム 102.90550	46 Pd パラジウム 106.42	47 Ag 銀 107.8682	48 Cd カドミウム 112.411	49 In インジウム 114.818	50 Sn 錫 118.710	51 Sb アンチモン 121.760	52 Te テルル 127.60	53 I ヨウ素 126.90447	54 Xe キセノン 131.293
6	55 Cs セシウム 132.90545	56 Ba バリウム 137.327	57 to 71	72 Hf ハフニウム 178.49	73 Ta タンタル 180.9479	74 W タングステン 183.84	75 Re レニウム 186.207	76 Os オスミウム 190.23	77 Ir イリジウム 192.217	78 Pt プラチナ 195.078	79 Au 金 196.96655	80 Hg 水銀 200.59	81 Tl タリウム 204.3833	82 Pb 鉛 207.2	83 Bi ビスマス 208.98038	84 Po ポロニウム (209)	85 At アスタチン (210)	86 Rn ラドン (222)
7	87 Fr フランシウム (223)	88 Ra ラジウム (226)	89 to 103	104 Rf ラザホージウム (261)	105 Db ドブニウム (262)	106 Sg シーボーギウム (266)	107 Bh ボーリウム (264)	108 Hs ハッシウム (269)	109 Mt マイトネリウム (268)	110 Ds ダルムスタチウム (271)	111 Rg レントゲニウム (272)	112 Uub ウンウンビウム (285)	113 Uut ウンウントリウム (284)	114 Uuq ウンウンクアジウム (289)	115 Uup ウンウンペンチウム (288)	116 Uuh ウンウンヘキシウム (292)	117 Uus ウンウンセプチウム	

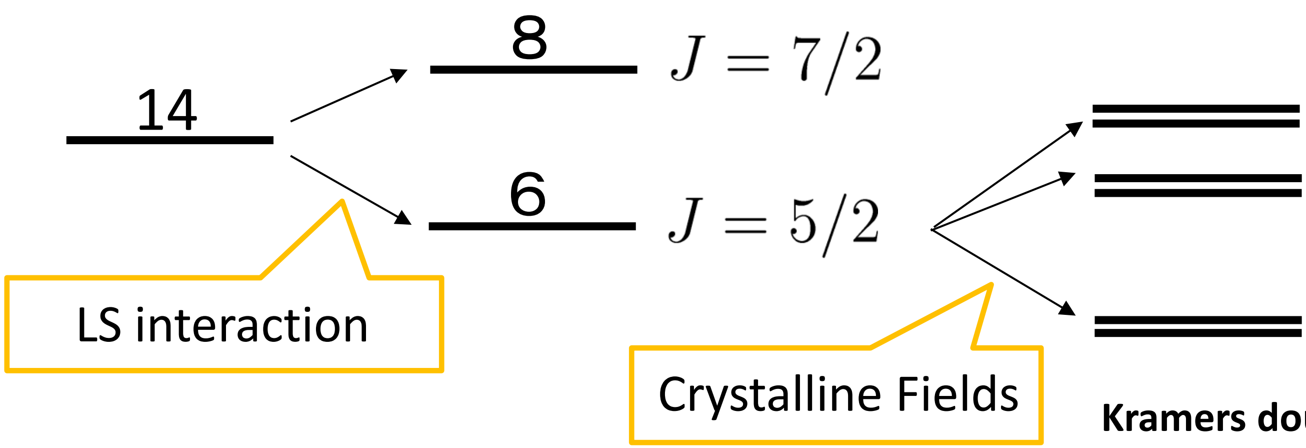
Atomic masses in parentheses are those of the most stable or common isotope.

57 La ランタン 138.9055	58 Ce セリウム 140.116	59 Pr プラセオジム 140.90765	60 Nd ネオジム 144.24	61 Pm プロメチウム (145)	62 Sm サマリウム 150.36	63 Eu ユウロピウム 151.964	64 Gd ガドリニウム 157.25	65 Tb テルビウム 158.92534	66 Dy ジスプロシウム 162.500	67 Ho ホルミウム 164.93032	68 Er エルビウム 167.259	69 Tm ツリウム 168.93421	70 Yb イッテルビウム 173.04	71 Lu ルテチウム 174.967
89 Ac アクチニウム (227)	90 Th トリウム 232.0381	91 Pa プロトアクチニウム 231.03588	92 U ウラン 238.02891	93 Np ネプツニウム (237)	94 Pu プルトニウム (244)	95 Am アメリシウム (243)	96 Cm キュリウム (247)	97 Bk バークリウム (247)	98 Cf カリホルニウム (251)	99 Es アインスタイニウム (252)	100 Fm フェルミウム (257)	101 Md メンデレビウム (258)	102 No ノーベリウム (259)	103 Lr ローレンシウム (262)

Note: The subgroup numbers 1-18 were adopted in 1984 by the International Union of Pure and Applied Chemistry. The names of elements 112-118 are the Latin equivalents of those numbers.

Kramers Doublet

Ce^{3+}(f^{1}) in solids

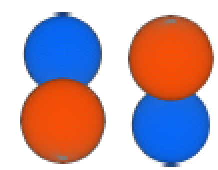


Kramers doublet
(protected by
time-reversal symmetry)

Localized Spin

$|\sigma = \uparrow\rangle$
$|\sigma = \downarrow\rangle$

$\boldsymbol{S}$



conduction electrons

$|\sigma = \uparrow\rangle$
$|\sigma = \downarrow\rangle$

Spin

$$\boldsymbol{s}_{\mathrm{c}} = \frac{1}{2}\sum_{\sigma\sigma'} c^{\dagger}_{\sigma}\boldsymbol{\sigma}_{\sigma\sigma'}c_{\sigma'}$$

c-f interaction

$$J\boldsymbol{S}\cdot\boldsymbol{s}_{\mathrm{c}}$$

➡ **Kondo effect**
(Fermi liquid)

Non-Kramers Doublet

localized f^2 configuration (Pr^{3+}, U^{4+}) in cubic crystal

${}_{14}C_2$ → 9 $J = 4$ → Γ_3

Coulomb int. + LS int.

Crystalline Field

Non-Kramers doublet

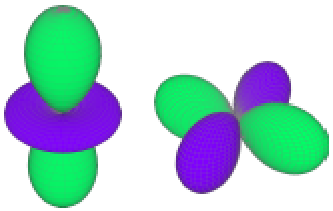
(orbital, quadrupole)

Pseudo-spin

$|\sigma = \uparrow\rangle$

$|\sigma = \downarrow\rangle$

$\boldsymbol{S}$



conduction electron Γ_8

$|\sigma = \uparrow\rangle$ $\otimes$ $|\alpha = 1\rangle$

$|\sigma = \downarrow\rangle$ $\otimes$ $|\alpha = 2\rangle$

pseudo-spin: $\boldsymbol{s}_c$

channel

c-f interaction

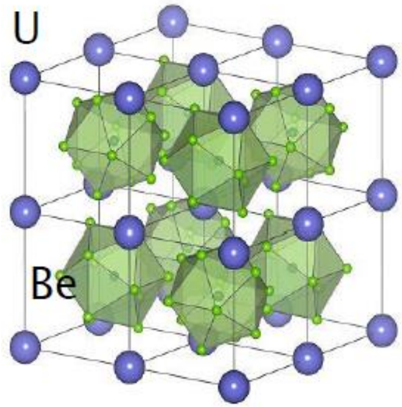
$$J\boldsymbol{S} \cdot (\boldsymbol{s}_{c1} + \boldsymbol{s}_{c2})$$

→ **2ch Kondo effect (Non-Fermi liquid)**

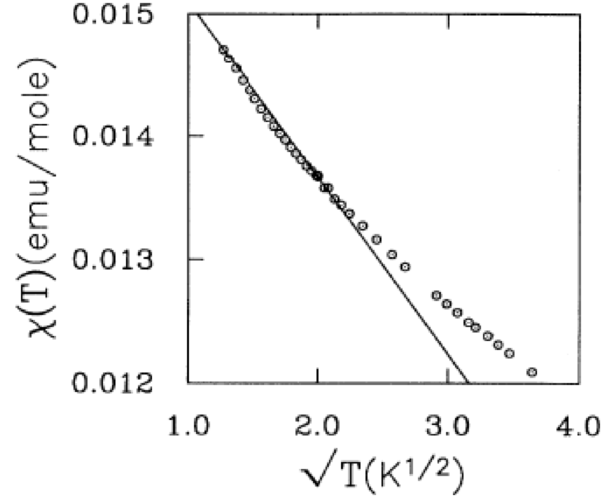
UBe_{13}

Possible two-channel Kondo system

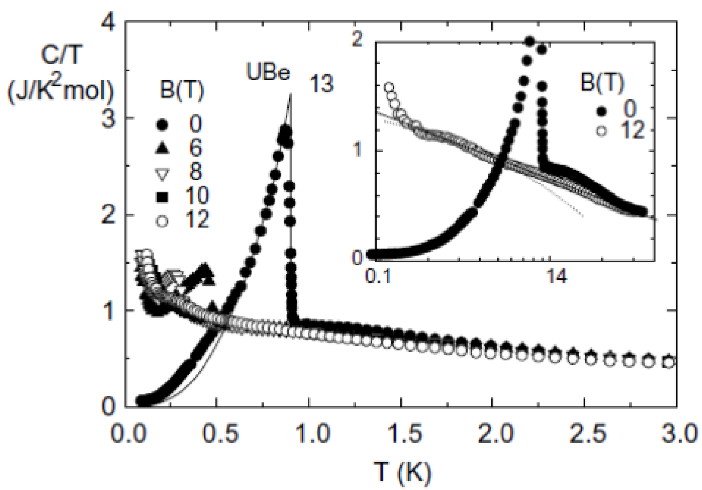
Cox, PRL (1987)



McElfresh et al., PRB (1993)



Gegenwart et al., Physica C (2004)



Superconducting state directly from Non-Fermi Liquid

Mayer et al., Z. Phys. B (1986)

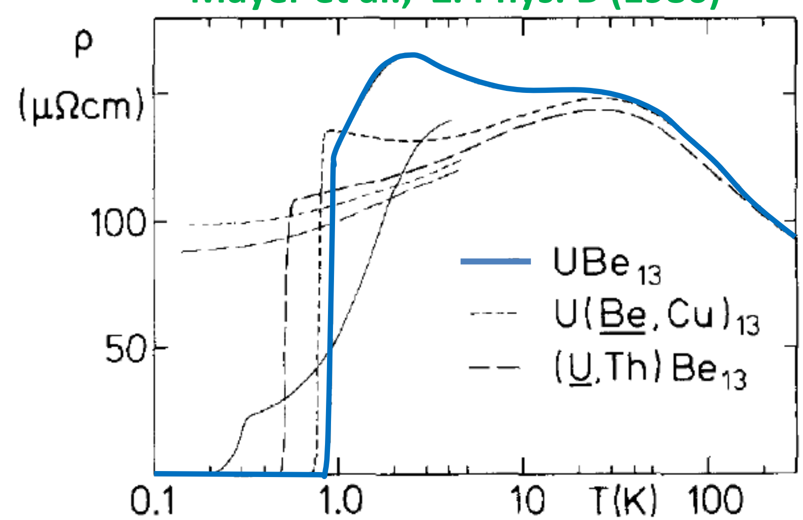
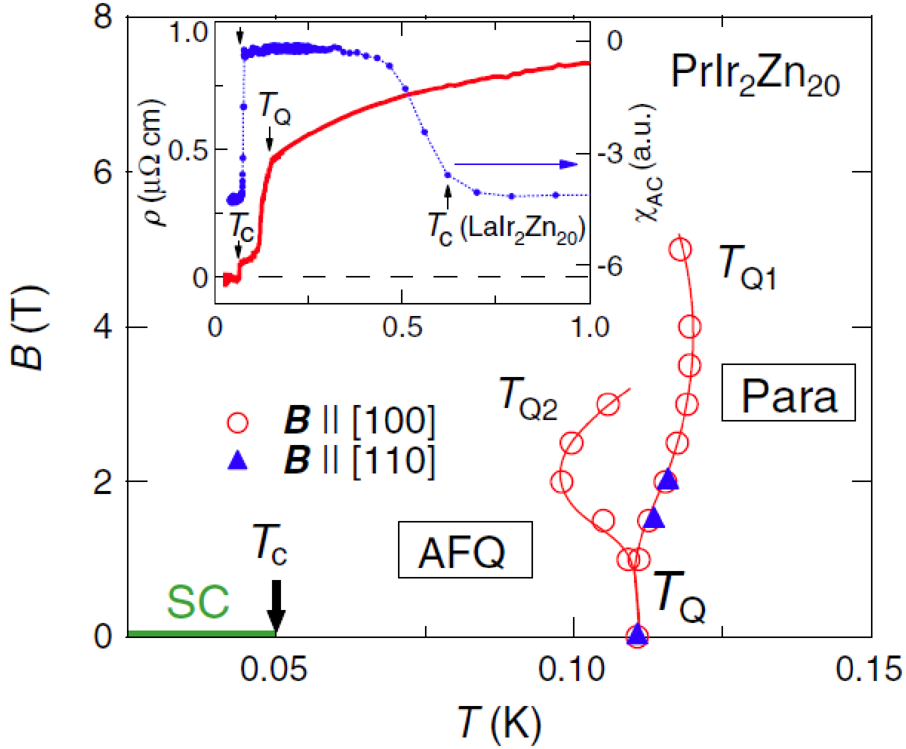


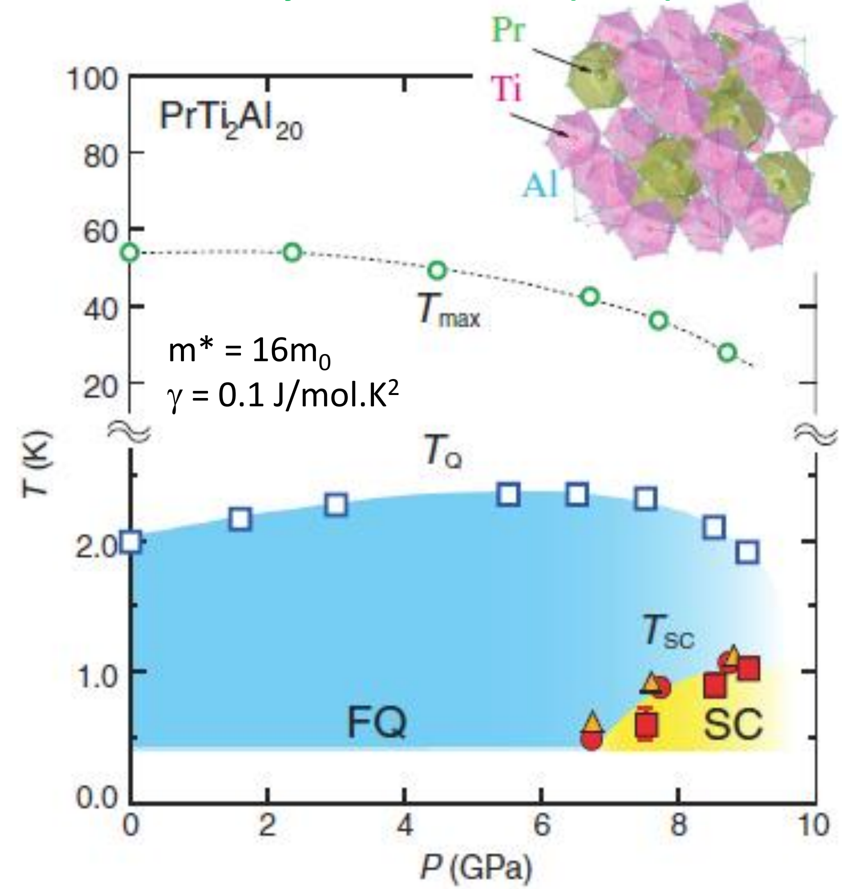
Fig. 19. Resistivity as a function of temperature, $\rho(T)$ for UBe_{13}, $U_{0.97}Th_{0.03}Be_{13}$ and $UBe_{12.94}Cu_{0.06}$ on a logarithmic T-scale for $B=0$ ($T \leq 300$ K) resp. $B=10$ T ($T \leq 4.2$ K, thin lines).

Pr1-2-20 compounds

Onimaru et al.: PRL (2011)



Sakai & Nakatsuji: JPSJ (2012)
Matsubayashi et al.: PRL (2013)



Pr-based non-Kramers doublet system:
superconductivity inside quadrupolar ordered state

Purpose

Superconductivity in Pr- and U-based compounds with non-Kramers doublet

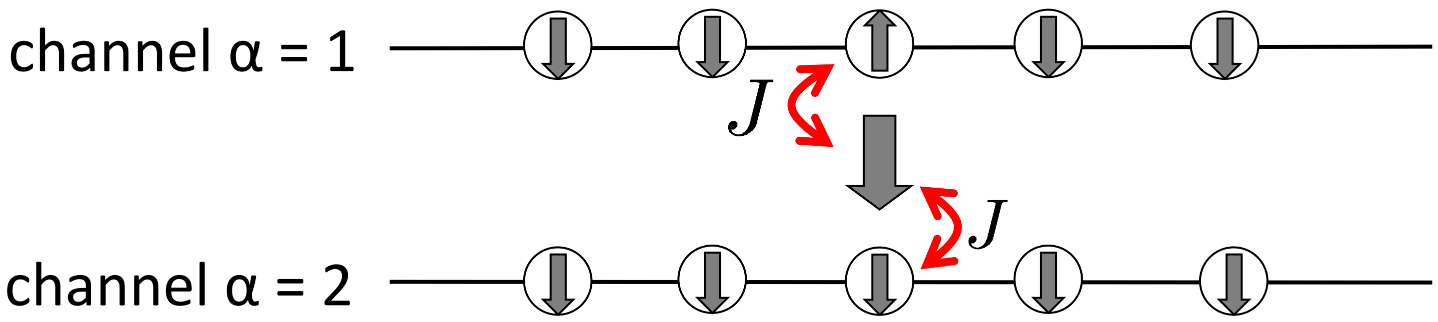
Two-channel Kondo effect induces superconductivity?

2. Odd-frequency superconductivity in two-channel Kondo lattice

Impurity Two-Channel Kondo Model

$$\mathcal{H} = \sum_{\boldsymbol{k}\alpha\sigma} \varepsilon_{\boldsymbol{k}} c^{\dagger}_{\boldsymbol{k}\alpha\sigma} c_{\boldsymbol{k}\alpha\sigma} + J \sum_{\alpha} \boldsymbol{S} \cdot \boldsymbol{s}_{c\alpha}$$

Nozieres & Blandin (1980)
Cox: PRL (1987)
Cox & Zawadowski: Adv. Phys. (1998)



fluctuation between channels
⇒ localized spins cannot be fully screened

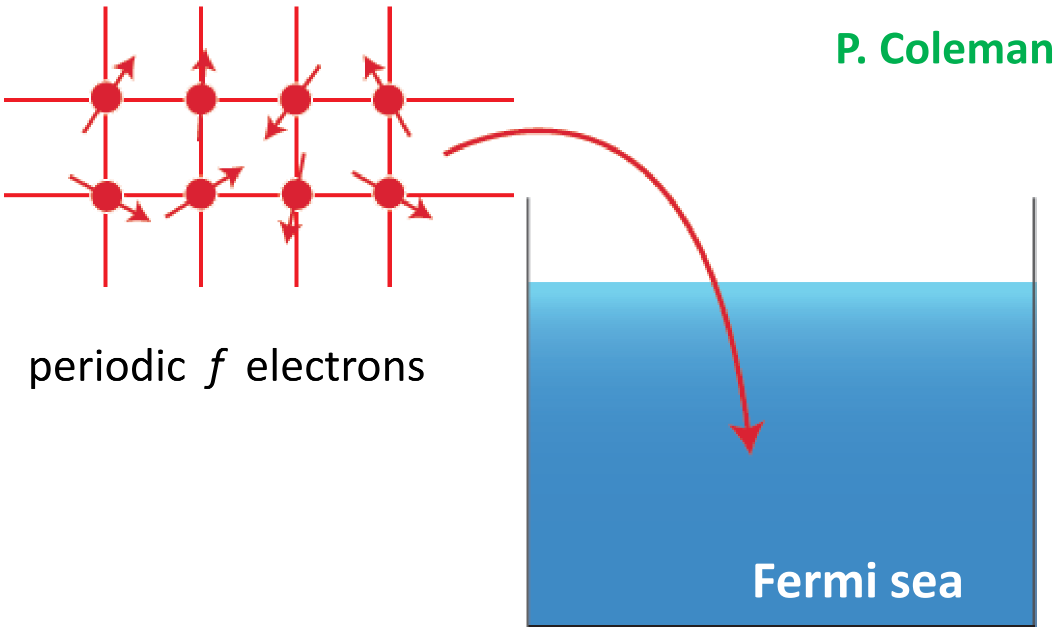
residual entropy at T=0 : **S = ½ ln 2**

Enhanced odd-frequency susceptibility at impurity site

Emery & Kivelson:PRB (1992)

Two-Channel Kondo Lattice

P. Coleman: Nature Mater. (2012)



$$\mathcal{H} = \sum_{\boldsymbol{k}\alpha\sigma} (\varepsilon_{\boldsymbol{k}} - \mu) c^{\dagger}_{\boldsymbol{k}\alpha\sigma} c_{\boldsymbol{k}\alpha\sigma} + J \sum_{i\alpha} \boldsymbol{S}_i \cdot \boldsymbol{s}_{ci\alpha}$$

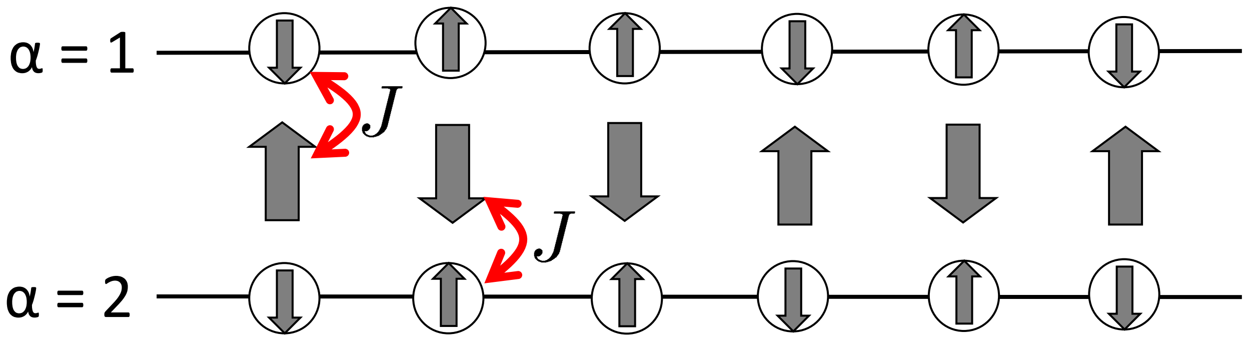
lattice

Spontaneous Symmetry Breaking

Two-Channel Kondo Lattice

$$\mathcal{H} = \sum_{\boldsymbol{k}\alpha\sigma} (\varepsilon_{\boldsymbol{k}} - \mu) c^\dagger_{\boldsymbol{k}\alpha\sigma} c_{\boldsymbol{k}\alpha\sigma} + J \sum_{i\alpha} \boldsymbol{S}_i \cdot \boldsymbol{s}_{ci\alpha}$$

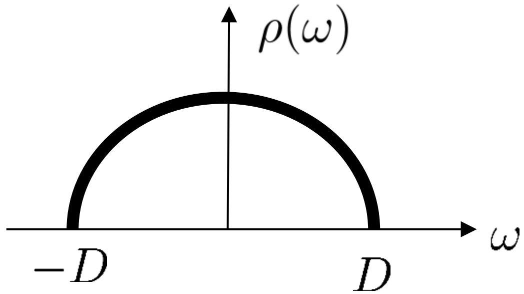
Jarrell et al: PRL (1996, 1997)



DMFT+CTQMC

- Kuramoto: Springer (1985)
- Metzner& Vollhardt: PRL (1989)
- Muller-Hartmann: Z. Phys. (1989)
- Georges et al: Rev. Mod. Phys. (1996)
- Rubtsov et al: JETP Lett. (2004)
- Werner et al: PRL (2006)
- Otsuki et al: JPSJ (2007)
- Gull et al: Rev. Mod. Phys. (2011)

bipartite lattice with
semi-circular DOS



Local correlations are fully incorporated.
(⇔ local self energy)

parameters

n_c; D=1, J, T

Possible Superconductivities

	space	channel	spin	time
CsSs (1)	s-wave	singlet	singlet	**odd**
CsSt (3)	s-wave	singlet	triplet	even
CtSs (3)	s-wave	triplet	singlet	even
CtSt (9)	s-wave	triplet	triplet	**odd**

$$\boldsymbol{q} = \boldsymbol{0} \quad (\text{uniform}), \quad \boldsymbol{q} = \boldsymbol{Q} \quad (\text{staggered})$$

Even-frequency susceptibility

$$\chi_{\boldsymbol{q}}^{\text{even}} = \frac{1}{\beta} \sum_{nn'} \chi_{\boldsymbol{q}}(\mathrm{i}\varepsilon_n, \mathrm{i}\varepsilon_{n'}) = \int_0^\beta \langle O_\mathrm{e}(\tau) O_\mathrm{e}^\dagger \rangle \mathrm{d}\tau > 0$$

Odd-frequency susceptibility

$$\chi_{\boldsymbol{q}}^{\text{odd}} = \frac{1}{\beta} \sum_{nn'} g_n g_{n'} \chi_{\boldsymbol{q}}(\mathrm{i}\varepsilon_n, \mathrm{i}\varepsilon_{n'}) \qquad (g_n = \mathrm{sgn}\, \varepsilon_n)$$

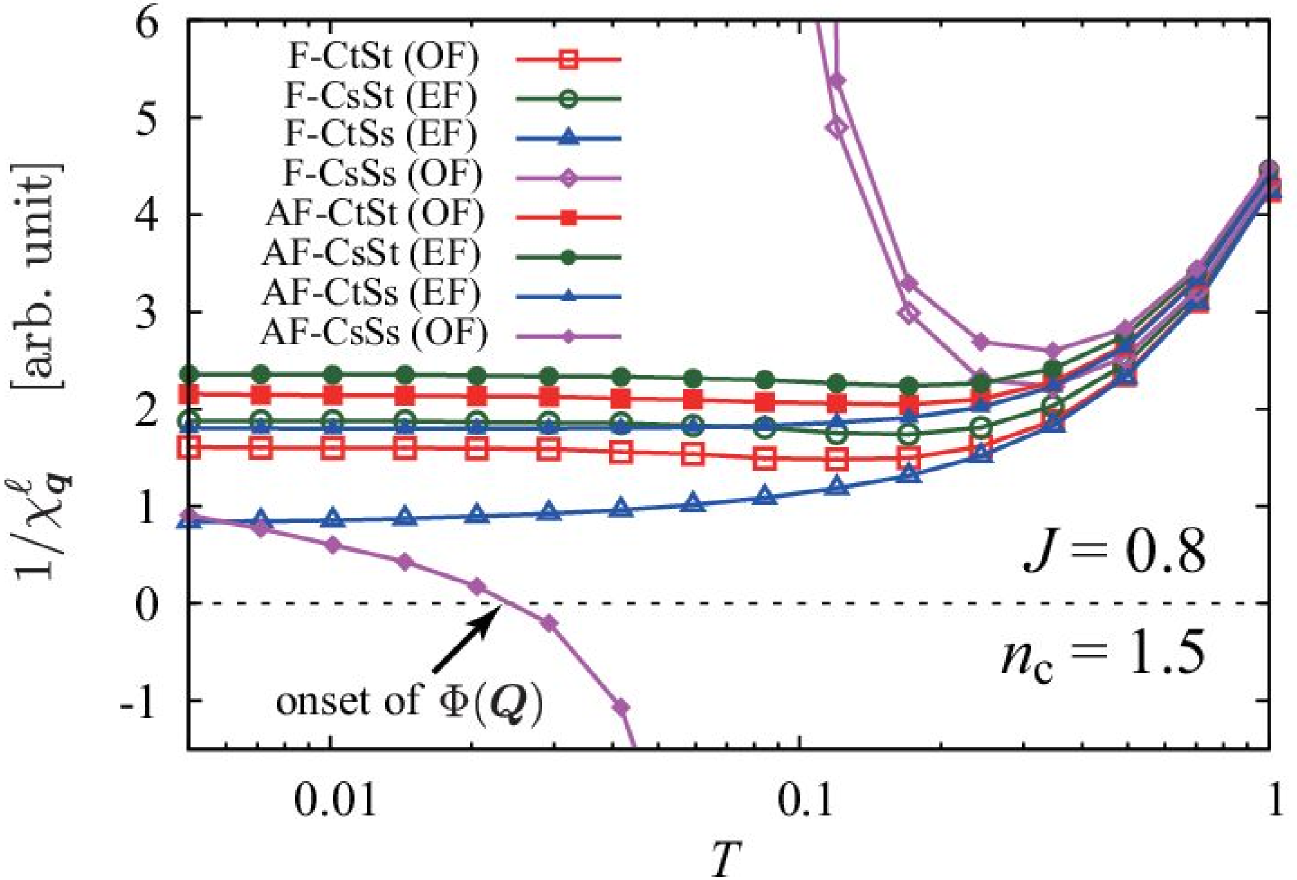
Jarrell et al.: PRL (1997), Anders: PRB (2002)

*Odd-frequency susceptibility is **not** positive definite.* **SH et al.: PRL (2011)**

■ DMFT calculations **Jarrell et al.: PRL (1997)**

However, no divergent susceptibility.

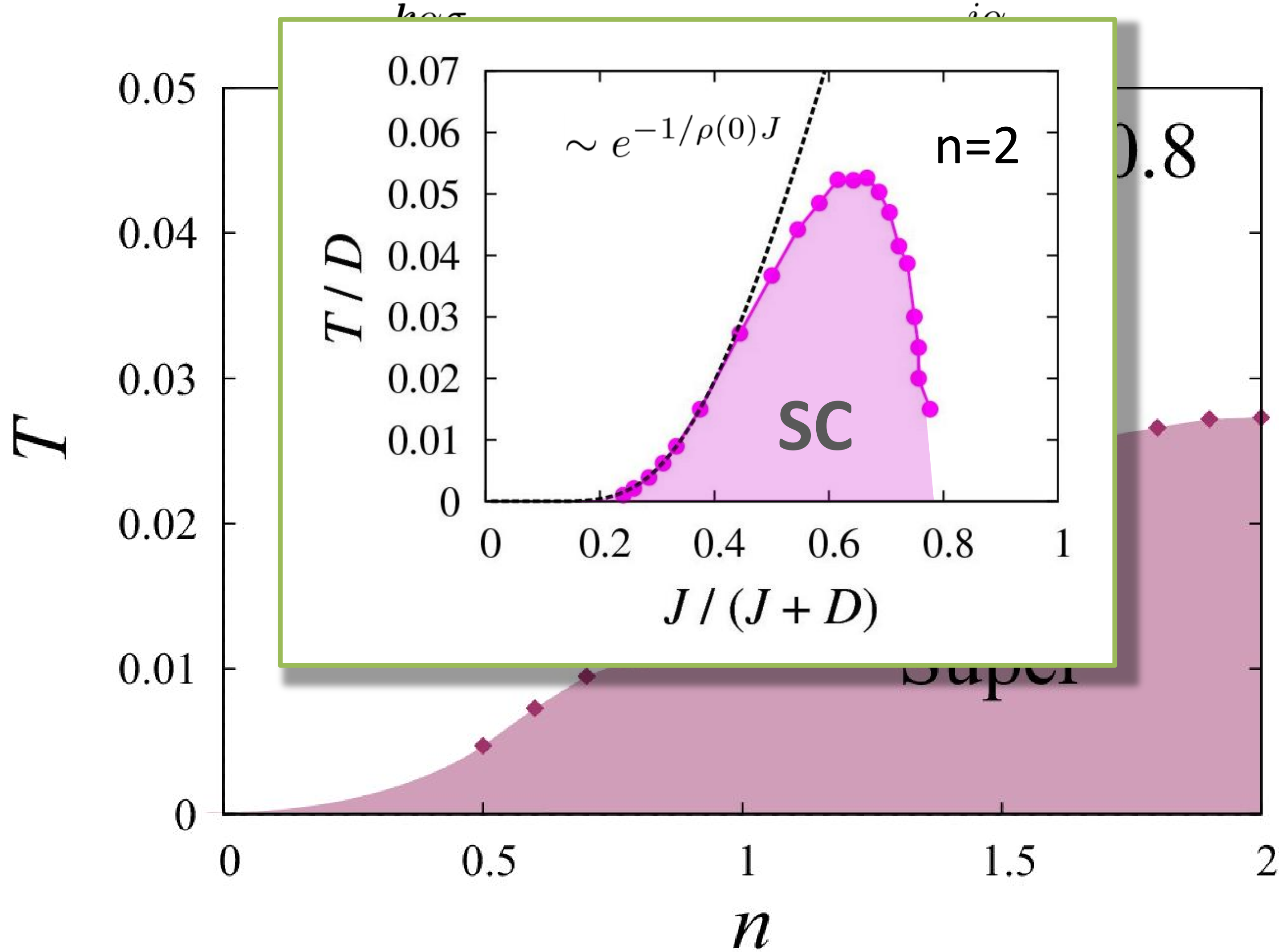
Pairing Susceptibilities



Odd-frequency (OF) superconductivity with staggered ordering vector (AF), Channel-singlet, Spin-singlet (CsSs)

Phase Diagram

$$\mathcal{H} = \sum_{\boldsymbol{k}\alpha\sigma} (\varepsilon_{\boldsymbol{k}} - \mu) c^{\dagger}_{\boldsymbol{k}\alpha\sigma} c_{\boldsymbol{k}\alpha\sigma} + J \sum_{i\alpha} \boldsymbol{S}_i \cdot \boldsymbol{s}_{ci\alpha}$$



Order Parameter for AF-CsSs Phase

	space	channel	spin	time
CsSs (1)	s-wave	singlet $c_1c_2 - c_2c_1$	singlet $c_\uparrow c_\downarrow - c_\downarrow c_\uparrow$	odd

Odd-frequency order parameter

$$O_{\mathrm{CsSs}}(\boldsymbol{Q},\tau) = \sum_{i\alpha\alpha'\sigma\sigma'} c^\dagger_{i\alpha\sigma}(\tau)\epsilon_{\alpha\alpha'}\epsilon_{\sigma\sigma'}c^\dagger_{i\alpha'\sigma'}\mathrm{e}^{\mathrm{i}\boldsymbol{Q}\cdot\boldsymbol{R}_i} \qquad (O_{\mathrm{CsSs}}(\boldsymbol{Q},0)=0)$$

$$\begin{pmatrix}\alpha = 1,2 \\ \sigma = \uparrow,\downarrow\end{pmatrix} \qquad \epsilon = \mathrm{i}\sigma^y = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad \mathrm{e}^{\mathrm{i}\boldsymbol{Q}\cdot\boldsymbol{R}_i} = \begin{cases} +1 & (i\in \mathrm{A}) \\ -1 & (i \in \mathrm{B}) \end{cases}$$

Even-frequency order parameter

$$\left.\frac{\partial O_{\mathrm{CsSs}}(\boldsymbol{Q},\tau)}{\partial\tau}\right|_{\tau=0} = \phi_{\mathrm{c}}(\boldsymbol{Q}) + J\Phi(\boldsymbol{Q})$$

$$\Phi(\boldsymbol{Q})^\dagger = \sum_{i\alpha\alpha'\sigma\sigma'} c^\dagger_{i\alpha\sigma}\epsilon_{\alpha\alpha'}[\boldsymbol{S}_i\cdot(\boldsymbol{\sigma}\epsilon)_{\sigma\sigma'}]c^\dagger_{i\alpha'\sigma'}\mathrm{e}^{\mathrm{i}\boldsymbol{Q}\cdot\boldsymbol{R}_i}$$

Composite pair amplitude

$$\phi_{\mathrm{c}}(\boldsymbol{Q})^\dagger = \sum_{\boldsymbol{k}\alpha\alpha'\sigma\sigma'} \varepsilon_{\boldsymbol{k}} c^\dagger_{\boldsymbol{k}\alpha\sigma}\epsilon_{\alpha\alpha'}\epsilon_{\sigma\sigma'}c^\dagger_{-\boldsymbol{k}-\boldsymbol{Q},\alpha'\sigma'}$$

non-local order parameter

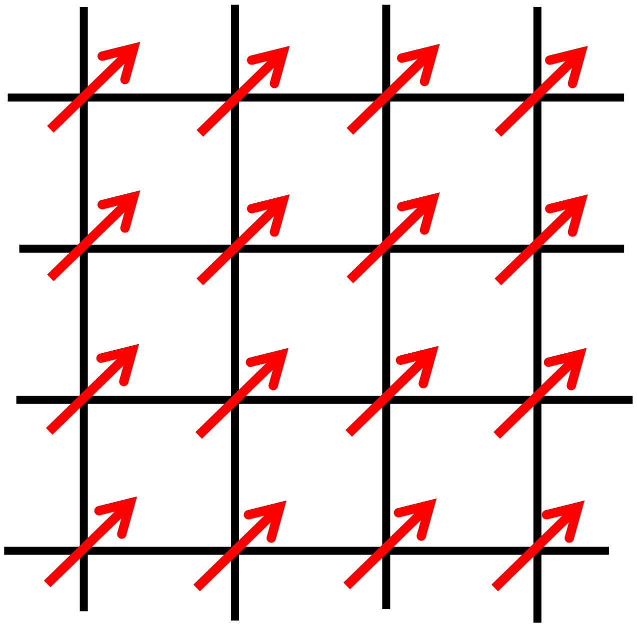
(Note: self energy is local in DMFT)

cf. η-pairing : CN Yang: PRL (1989)

Illustration for Staggered Pairing

BCS-type pairing

phase coherence

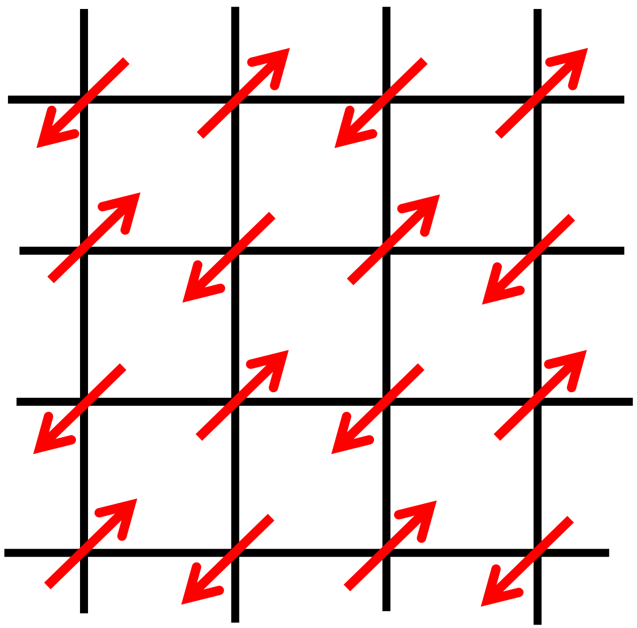


$$\phi_i^\dagger = \sum_{\sigma\sigma'} \epsilon_{\sigma\sigma'} \langle c_{i\sigma}^\dagger c_{i\sigma'}^\dagger \rangle$$

$$\epsilon = \mathrm{i}\sigma^y = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

staggered pairing

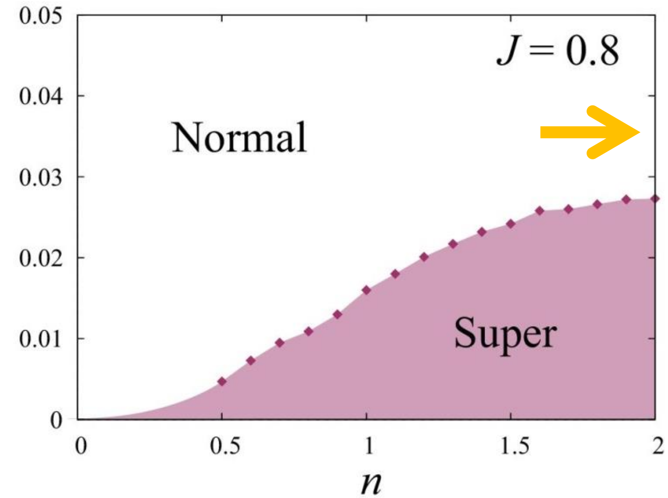
staggered alignment of phase



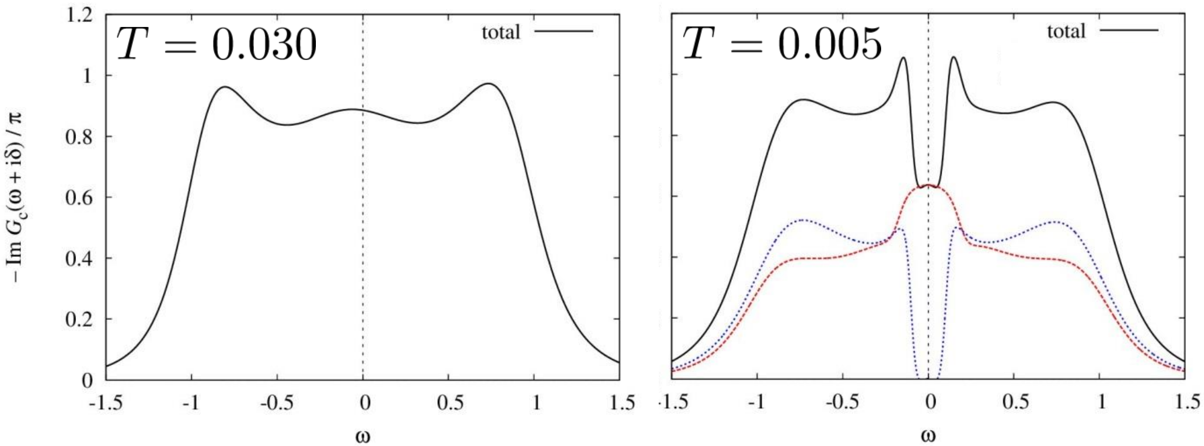
$$\Phi_i^\dagger = \sum_{\alpha\alpha'\sigma\sigma'} \epsilon_{\alpha\alpha'} \langle \boldsymbol{S}_i \cdot (\boldsymbol{\sigma}\epsilon)_{\sigma\sigma'} c_{i\alpha\sigma}^\dagger c_{i\alpha'\sigma'}^\dagger \rangle e^{\mathrm{i}\boldsymbol{Q}\cdot\boldsymbol{R}_i}$$

- gauge degree of freedom couples to lattice geometry
- no internal current with staggered phase

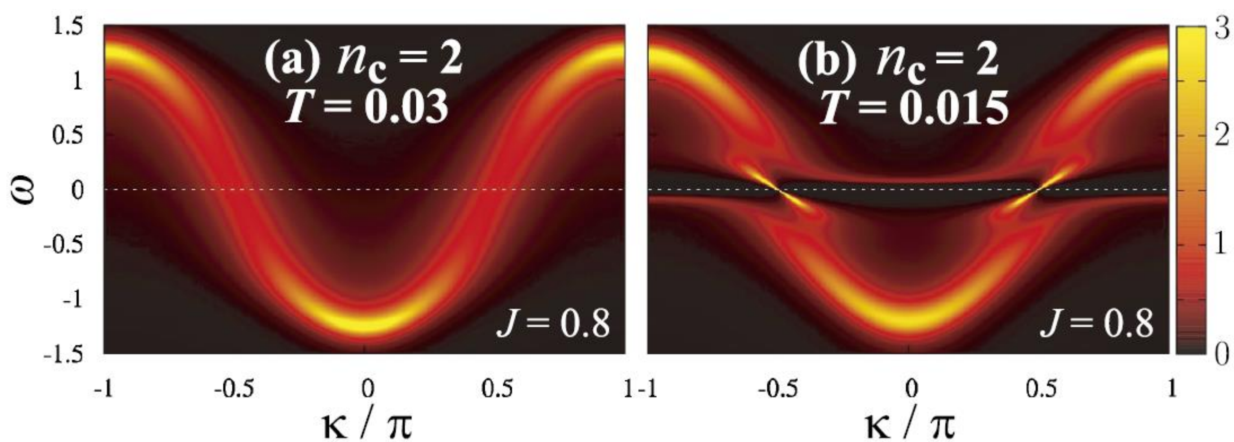
Single-Particle Spectrum at n=2



$T > T_c$ $\quad$ $T < T_c$

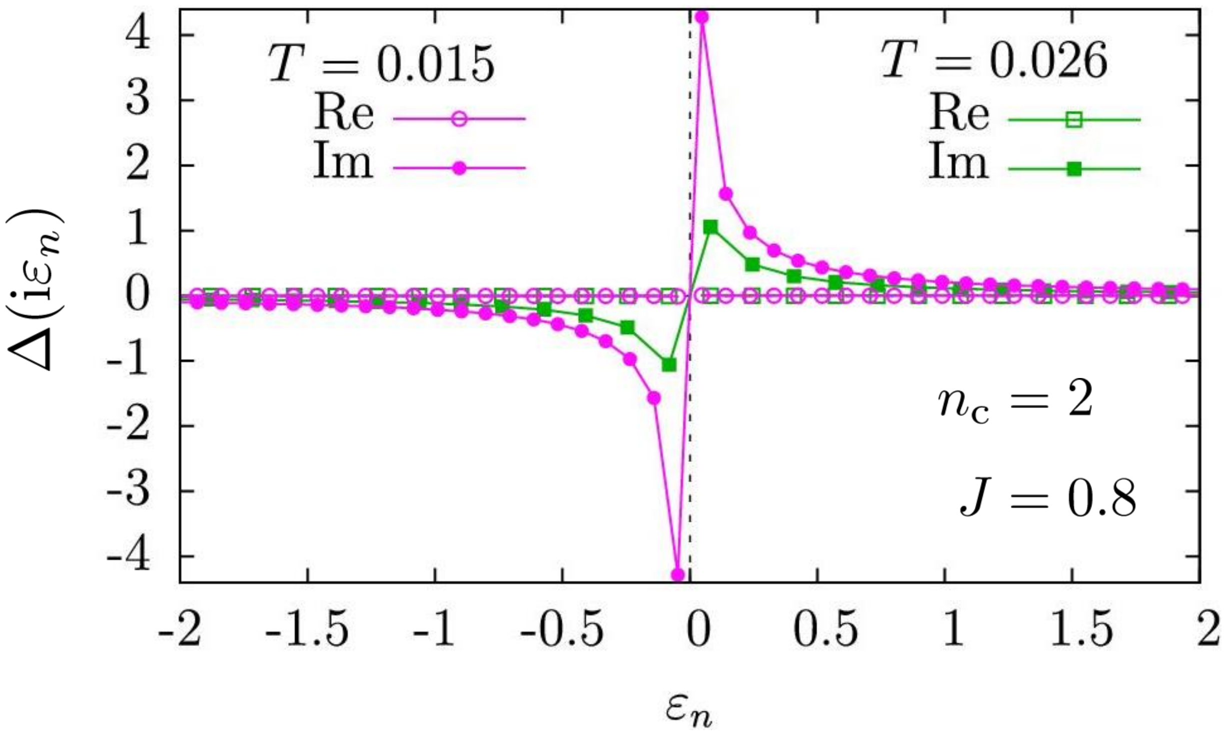


- Density of States remains inside ordered state
- superconductivity from non-Fermi liquid



$$\varepsilon_{\boldsymbol{k}} = -D\cos(\kappa)$$

Anomalous Local Self Energy



$$\Delta(\mathrm{i}\varepsilon_n \to 0) \sim \frac{1}{\mathrm{i}\varepsilon_n}$$

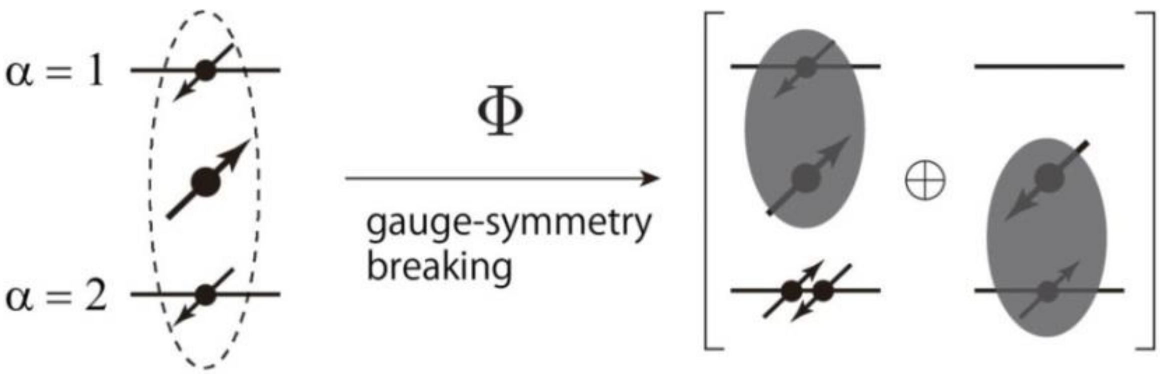
$$\Delta(\tau) \sim \mathrm{sgn}\,\tau$$

cf. EF pairing

$$\Delta(\mathrm{i}\varepsilon_n) \sim \Delta$$

$$\Delta(\tau) \sim \delta(\tau)$$

Schematic Local Picture at Half Filling



Non-Fermi Liquid

$$\Phi(\boldsymbol{Q})^{\dagger} = \sum_{i\alpha\alpha'\sigma\sigma'} c^{\dagger}_{i\alpha\sigma}\epsilon_{\alpha\alpha'}[\boldsymbol{S}_i \cdot (\boldsymbol{\sigma}\epsilon)_{\sigma\sigma'}]c^{\dagger}_{i\alpha'\sigma'}\mathrm{e}^{\mathrm{i}\boldsymbol{Q}\cdot\boldsymbol{R}_i}$$

Localized spins are fully screened by gauge-symmetry breaking

3. Mean-field description of odd-frequency superconductivity

Proposals to resolve thermodynamic unstability

Coleman et al: PRL (1993)
Heid: Z. Phys. B (1995)

(1) strong coupling corrections
(2) ~~first order phase transition~~
(3) inhomo

Abrahamas et
Balatsky et al

(4) compos

$$F^{\dagger}_{12}(\mathrm{i}\varepsilon_n) = +F_{21}(-\mathrm{i}\varepsilon_n)^*$$

ltonian
ntation can be used)

Possibility of mean-field description?
Sign of the Meissner kernel?

Belitz & Kirkpatrik: PRB (1999)
Solenov et al.: PRB (2009)
Kusunose et al.: JPSJ (2011)

(5) without Hermite relation

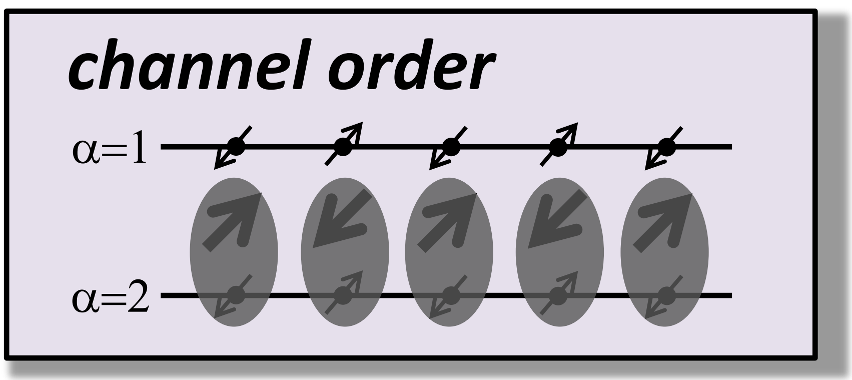
$$F^{\dagger}_{12}(\mathrm{i}\varepsilon_n) = -F_{21}(-\mathrm{i}\varepsilon_n)^*$$

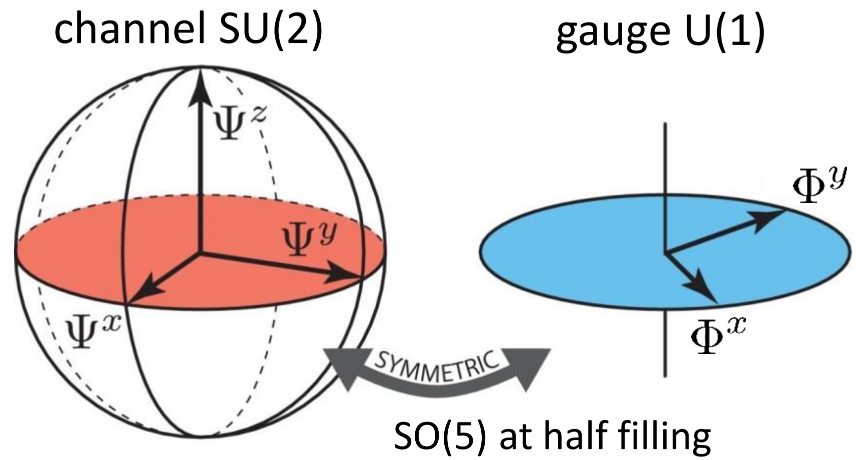
No ordinary MF Hamiltonian

Effective One-Body Model

Affleck et al.: PRB (1992)

superconductivity and channel orders are degenerate at half filling





$$\mathcal{H}_{\text{F-channel}} = \sum_{\boldsymbol{k}\alpha\sigma} \varepsilon_{\boldsymbol{k}} c^{\dagger}_{\boldsymbol{k}\alpha\sigma} c_{\boldsymbol{k}\alpha\sigma} + V \sum_{\boldsymbol{k}\sigma} (f^{\dagger}_{\boldsymbol{k}\sigma} c_{\boldsymbol{k}2\sigma} + \text{h.c.})$$

V : effective one-body mean field
$f_{\boldsymbol{k}\sigma}$: pseudofermion that describes low-energy behavior of localized spin

Effective One-Body Model

$$\mathcal{H}_{\text{F-channel}} = \sum_{\boldsymbol{k}\alpha\sigma} \varepsilon_{\boldsymbol{k}} c^\dagger_{\boldsymbol{k}\alpha\sigma} c_{\boldsymbol{k}\alpha\sigma} + V \sum_{\boldsymbol{k}\sigma} (f^\dagger_{\boldsymbol{k}\sigma} c_{\boldsymbol{k}2\sigma} + \text{h.c.})$$

$$\boldsymbol{Q} = (\pi, \pi, \pi)$$

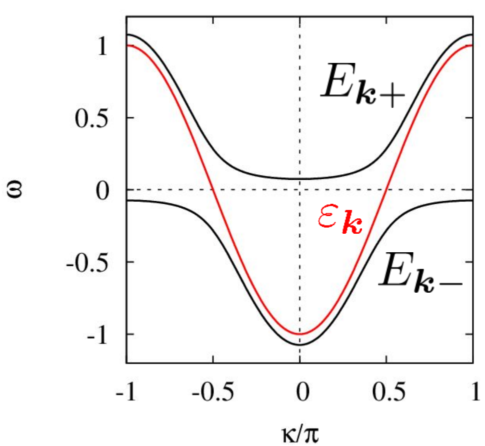
SO(5) rotation →

$$\mathcal{H}_{\text{SC}} = \sum_{\boldsymbol{k}\alpha\sigma} \varepsilon_{\boldsymbol{k}} c^\dagger_{\boldsymbol{k}\alpha\sigma} c_{\boldsymbol{k}\alpha\sigma} + V \sum_{\boldsymbol{k}} \left(\mathrm{e}^{\mathrm{i}\theta/2} f^\dagger_{\boldsymbol{k}\uparrow} c_{\boldsymbol{k}1\uparrow} + \mathrm{e}^{\mathrm{i}\theta/2} f^\dagger_{\boldsymbol{k}\downarrow} c_{\boldsymbol{k}1\downarrow} + \text{h.c.} \right) + V \sum_{\boldsymbol{k}} \left(\mathrm{e}^{-\mathrm{i}\theta/2} f^\dagger_{\boldsymbol{k}\uparrow} c^\dagger_{-\boldsymbol{k}-\boldsymbol{Q},2\downarrow} - \mathrm{e}^{-\mathrm{i}\theta/2} f^\dagger_{\boldsymbol{k}\downarrow} c^\dagger_{-\boldsymbol{k}-\boldsymbol{Q},2\uparrow} + \text{h.c.} \right)$$

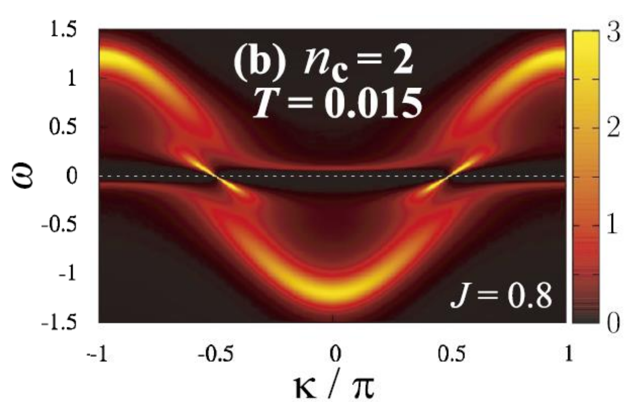
Anomalous local self energy of conduction electrons

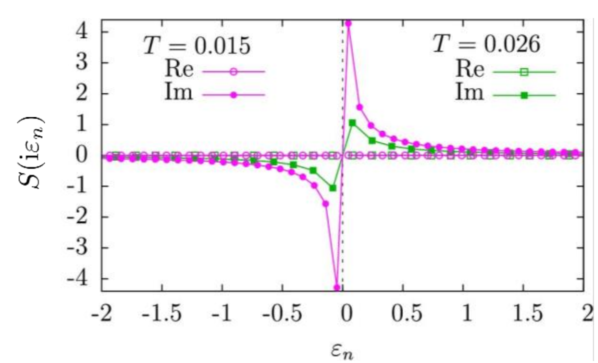
$$\Delta_i(\omega) = \frac{V^2 \mathrm{e}^{\mathrm{i}(\theta + \boldsymbol{Q}\cdot\boldsymbol{R}_i)}}{\omega}$$

staggered odd-frequency SC



DMFT results





Mean-Field Approximation

MFA for ordinary Kondo lattice: **G. -M. Zhang & L. Yu: PRB (2000)**

1. pseudofermion representation of localized spin

$$\boldsymbol{S}_i = \frac{1}{2}\sum_{\sigma\sigma'} f^\dagger_{i\sigma}\boldsymbol{\sigma}_{\sigma\sigma'} f_{i\sigma'} \qquad \sum_{\sigma} f^\dagger_{i\sigma} f_{i\sigma} = 1 \qquad \sum_{\sigma}\langle f^\dagger_{i\sigma} f_{i\sigma}\rangle = 1$$

$$J\sum_{\alpha}\boldsymbol{S}_i\cdot\boldsymbol{s}_{ci\alpha} = \tfrac{1}{2}J\sum_{\mu\alpha\sigma\sigma'\sigma''\sigma'''}\sigma^{\mu}_{\sigma\sigma'}\sigma^{\mu}_{\sigma''\sigma'''} f^\dagger_{i\sigma} f_{i\sigma'} c^\dagger_{i\alpha\sigma''} c_{i\alpha\sigma'''}$$

2. mean-field decoupling for present odd-frequency SC

$$J\boldsymbol{S}_i\cdot\boldsymbol{s}_{ci1} \to \sum_{\sigma}\left(V_1 f^\dagger_{i\sigma} c_{i1\sigma} + V_1^* c^\dagger_{i1\sigma} f_{i\sigma}\right)$$

$$J\boldsymbol{S}_i\cdot\boldsymbol{s}_{ci2} \to \mathrm{e}^{\mathrm{i}\boldsymbol{Q}\cdot\boldsymbol{R}_i}\sum_{\sigma\sigma'}\epsilon_{\sigma\sigma'}\left(W_2 f^\dagger_{i\sigma} c^\dagger_{i2\sigma'} + W_2^* c_{i2\sigma'} f_{i\sigma}\right)$$

Mean-Field Approximation

$$J\boldsymbol{S}_i \cdot \boldsymbol{s}_{ci1} \to \sum_{\sigma} \left(V_1 f^\dagger_{i\sigma} c_{i1\sigma} + V_1^* c^\dagger_{i1\sigma} f_{i\sigma} \right)$$

$$J\boldsymbol{S}_i \cdot \boldsymbol{s}_{ci2} \to \mathrm{e}^{\mathrm{i}\boldsymbol{Q}\cdot\boldsymbol{R}_i} \sum_{\sigma\sigma'} \epsilon_{\sigma\sigma'} \left(W_2 f^\dagger_{i\sigma} c^\dagger_{i2\sigma'} + W_2^* c_{i2\sigma'} f_{i\sigma} \right)$$

<u>hybridization processes</u>

No pairing among c electrons

Combination between V and W is necessary for ***pairing among conduction electrons***

Three-Dimensional Lattice

$$\varepsilon_{\boldsymbol{k}} = -2t(\cos k_x + \cos k_y + \cos k_z)$$

$$\varepsilon_{\boldsymbol{k}+\boldsymbol{Q}} = -\varepsilon_{\boldsymbol{k}}$$

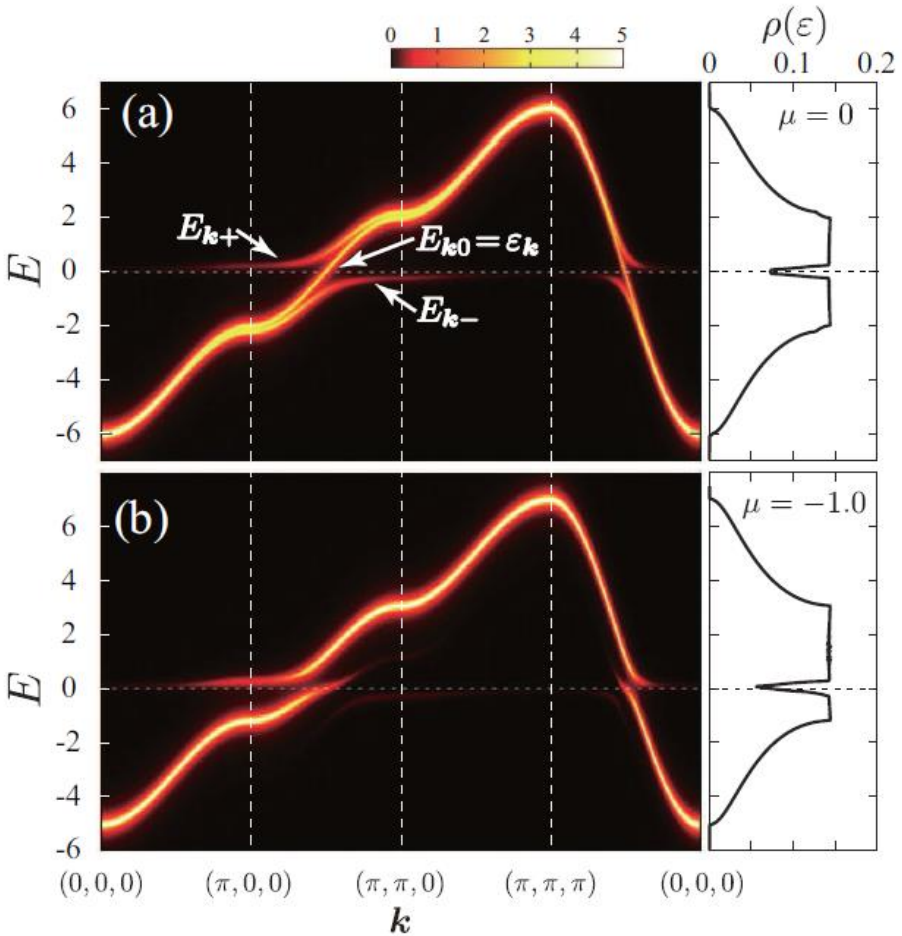
$$E_{\boldsymbol{k}0} = \varepsilon_{\boldsymbol{k}},$$

$$E_{\boldsymbol{k}\pm} = \frac{1}{2}\left(\varepsilon_{\boldsymbol{k}} \pm \sqrt{\varepsilon_{\boldsymbol{k}}^2 + 8|V|^2}\right)$$

$$\gamma_{\boldsymbol{k}\sigma 0} = \frac{1}{\sqrt{2}}\left(\mathrm{e}^{\mathrm{i}\theta/2} c_{\boldsymbol{k}1\sigma} + \mathrm{e}^{-\mathrm{i}\theta/2} \sum_{\sigma'} \epsilon_{\sigma\sigma'} c^{\dagger}_{-\boldsymbol{k}-\boldsymbol{Q},2\sigma'}\right)$$

$$\gamma_{\boldsymbol{k}\sigma\pm} = u_{\boldsymbol{k}\pm} f_{\boldsymbol{k}\sigma} + v_{\boldsymbol{k}\pm} \bar{\gamma}_{\boldsymbol{k}\sigma 0}$$

$$\bar{\gamma}_{\boldsymbol{k}\sigma 0} = \frac{1}{\sqrt{2}}\left(\mathrm{e}^{\mathrm{i}\theta/2} c_{\boldsymbol{k}1\sigma} - \mathrm{e}^{-\mathrm{i}\theta/2} \sum_{\sigma'} \epsilon_{\sigma\sigma'} c^{\dagger}_{-\boldsymbol{k}-\boldsymbol{Q},2\sigma'}\right)$$



Meissner Kernel in Lattice System

DJ Scalapino et al.: PRL (1992)

Peierls phase (simple cubic in 3D)

$$\mathcal{H}_{\rm kin} = -t \sum_{i\alpha\sigma} \sum_{\mu=x,y,z} \left[e^{-{\rm i}eA_\mu(\boldsymbol{R}_i)} c^\dagger_{i\alpha\sigma} c_{i+\delta_\mu,\alpha\sigma} + {\rm h.c.} \right]$$

$$J_\mu(\boldsymbol{R}_i) = -\frac{\partial \mathcal{H}}{\partial A_\mu(\boldsymbol{R}_i)}$$

$$= -{\rm i}et \sum_{\alpha\sigma} c^\dagger_{i\alpha\sigma} c_{i+\delta_\mu,\alpha\sigma} - e^2 t A_\mu(\boldsymbol{R}_i) \sum_{\alpha\sigma} c^\dagger_{i\alpha\sigma} c_{i+\delta_\mu,\alpha\sigma} + {\rm h.c.} + O(A^2)$$

paramagnetic current

diamagnetic current

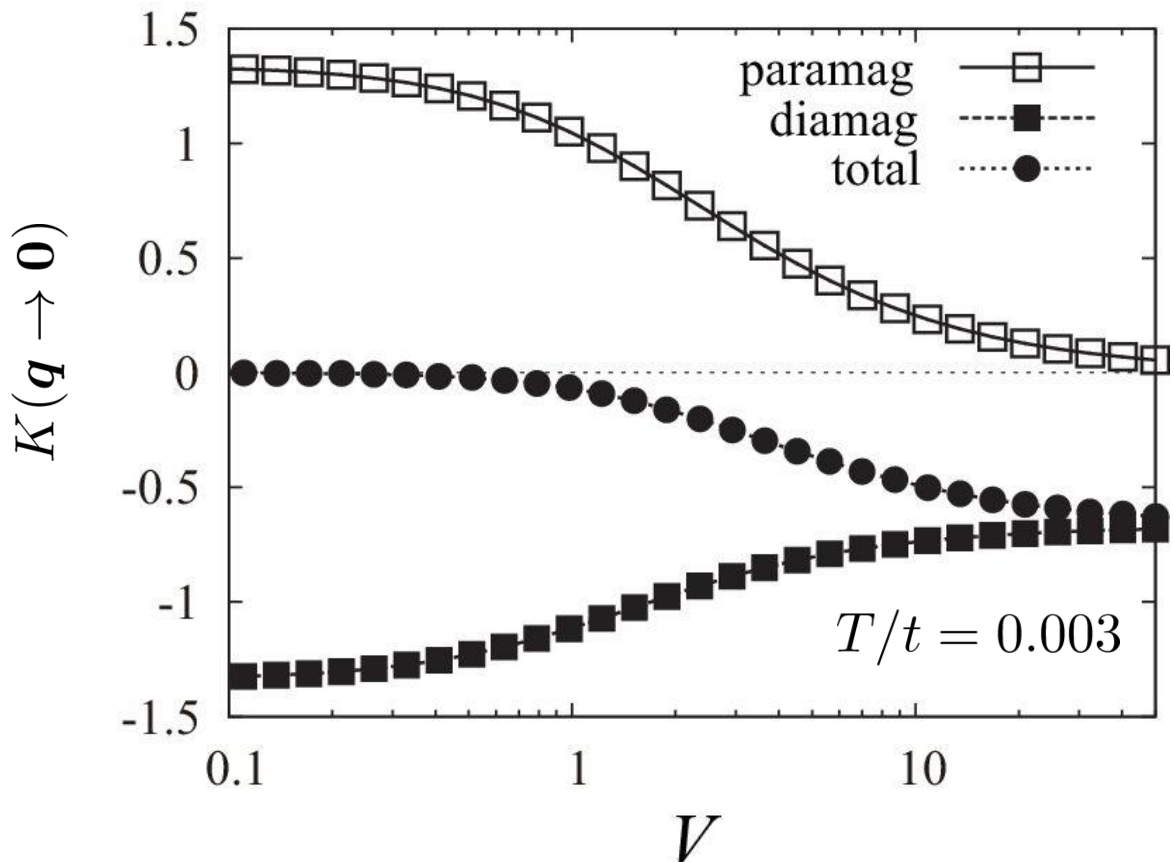
$$J_\mu(\boldsymbol{q}) = [K^{\rm para}(\boldsymbol{q}) + K^{\rm dia}(\boldsymbol{q})] A_\mu(\boldsymbol{q})$$

$$K^{\rm para}(\boldsymbol{q}) = \int_0^\beta {\rm d}\tau \langle j^{\rm p}_\mu(\boldsymbol{q},\tau) j^{\rm p}_\mu(-\boldsymbol{q}) \rangle$$

Meissner Kernel

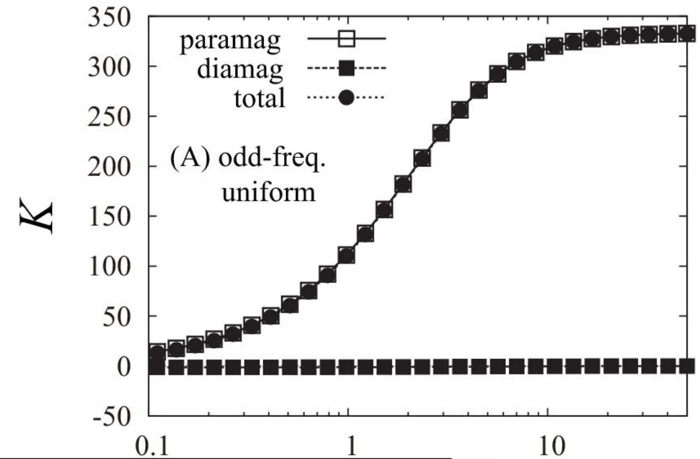
3D simple cubic with hopping t=1
half filling

$$J_\mu(\boldsymbol{q}) = [K^{\mathrm{para}}(\boldsymbol{q}) + K^{\mathrm{dia}}(\boldsymbol{q})]A_\mu(\boldsymbol{q})$$



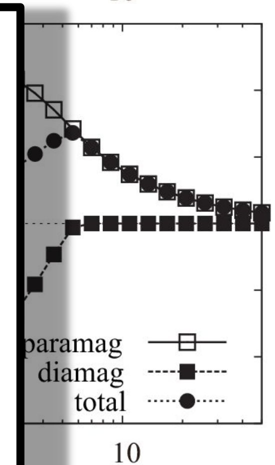
"*weak*" positive Meissner effect

Related Models



(A) uniform

$\Delta_i(\omega) =$



	uniform $(\boldsymbol{q} = \boldsymbol{0})$	staggered $(\boldsymbol{q} = \boldsymbol{Q})$
EF pairing	positive	negative
OF pairing	negative	positive

(positive: ordinary physical Meissner effect)

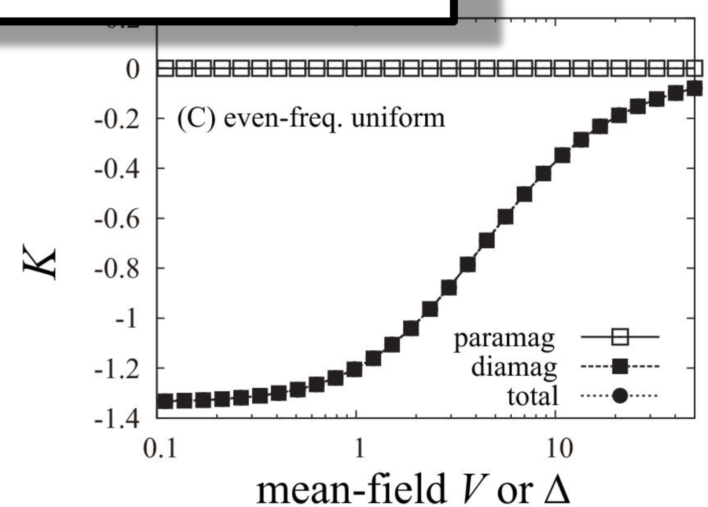
(B) stagger

$\Delta_i(\omega) =$

(C) uniform even-freq. pairing

$\Delta_i(\omega) = \Delta$

Positive Meissner effect



Proposals to resolve thermodynamic unstability

Coleman et al: PRL (1993)
Heid: Z. Phys. B (1995)

~~(1) strong coupling corrections~~

~~(2) first order phase transition~~

(3) inhomogeneous state

Abrahamas et al. PRB (1995)
Balatsky et al.: New J. Phys. (2009)

~~(4) composite operator description~~

$$F^{\dagger}_{12}(\mathrm{i}\varepsilon_n) = +F_{21}(-\mathrm{i}\varepsilon_n)^*$$

***two-channel Kondo lattice belongs to the case* (3)**

can be used)

Belitz & Kirkpatrik: PRB (1999)
Solenov et al.: PRB (2009)
Kusunose et al.: JPSJ (2011)

~~(5) without Hermite relation~~

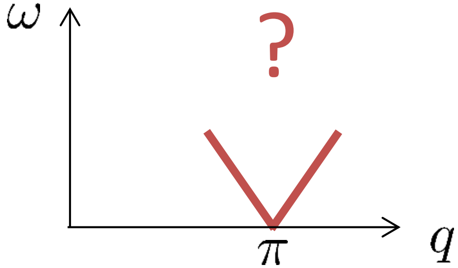
$$F^{\dagger}_{12}(\mathrm{i}\varepsilon_n) = -F_{21}(-\mathrm{i}\varepsilon_n)^*$$

No ordinary MF Hamiltonian

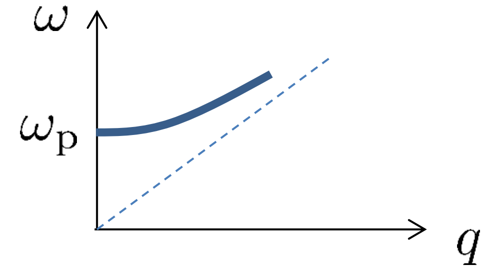
Remaining theoretical issues

Goldstone mode?

long-ranged Coulomb interaction
(charged-particle system)



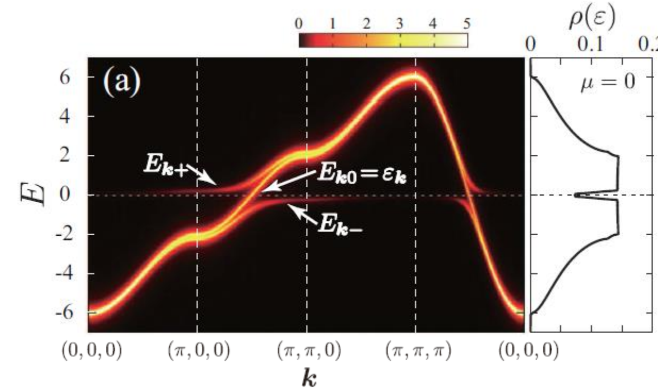
staggered pairing



uniform pairing

instability inside staggered pairing state ?

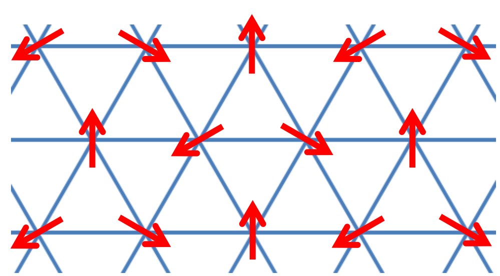
No entropy in ground state
but
possible Fermi surface instability
even inside of pairing state



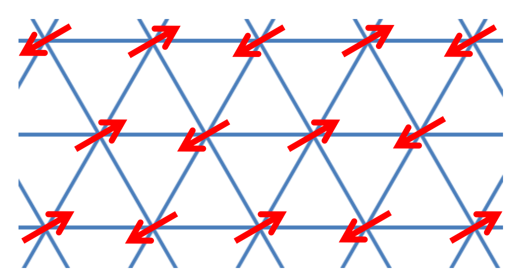
geometrically frustrated lattices ?

internal current state?

$$I \propto \sin \Delta\theta$$



120 Neel



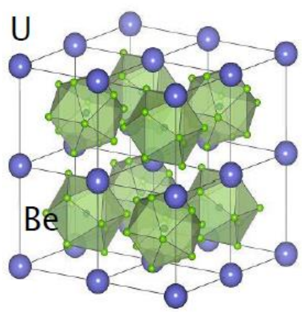
stripe

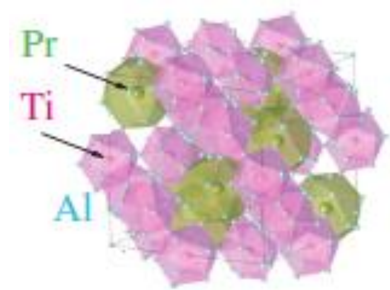
Relevance to real materials…

- Two-channel Kondo lattice
  - → A model for Pr- or U-based non-Kramers doublet systems
- Staggered OF superconductivity directly from non-Fermi liquid

Improvements

- f-electron charge degrees of freedom
  Two-channel Anderson lattice
  Anders: PRL (1997)
- anisotropic exchange interactions
- realistic conduction-band structures
- spatial correlations





LDA
+
DMFT

CDMFT

Summary

Odd-frequency superconductivity with staggered ordering vector in two-channel Kondo lattice

- DMFT+CTQMC approach

  Divergent pairing susceptibility and phase diagram.
  Composite order parameters and wave functions.

  S. Hoshino and Y. Kuramoto: PRL 112 (2014) 167204

- Effective low-energy model

  Hermitian mean-field Hamiltonian.
  Electron-pairing through hybridization with pseudofermions.
  Positive but "weak" Meissner effect.

  S. Hoshino: arXiv:1406.1983