Pairing Symmetry of Superfluid in Three-Component Repulsive Fermionic Atoms in Optical Lattices

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Introduction

Cold atoms in optical lattices: high controallability

➡ Quantum simulator

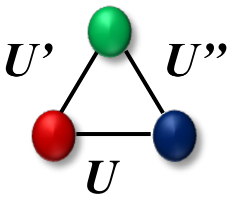
Repulsively interacting three-component fermionic atoms in optical lattices

Three-component fermionic gases realized in ^{6}Li atoms.

Ottenstein et al, PRL **101**, 203202 (2008); Huckans et al, PRL **102**, 165302 (2009)

Three kinds of internal degrees of freedom:

Three kinds of repulsions:



Model

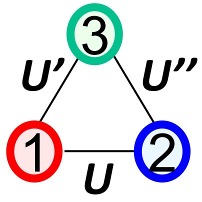
$$H = -t \sum_{\langle i,j \rangle} \sum_{\alpha=1,2,3} c_{i\alpha}^{+} c_{j\alpha} + \frac{1}{2} \sum_{i} \sum_{\alpha \neq \beta} U_{\alpha\beta} n_{i\alpha} n_{i\beta} - \sum_{i,\alpha} \mu_{\alpha} n_{i\alpha}, \quad U_{\alpha\beta} > 0$$

According to the experiments for ^{6}Li atoms

balanced population: $n_1 = n_2 = n_3 = n$,

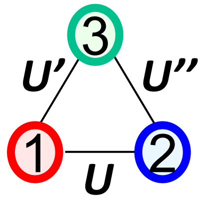
Half filling: n=1/2, total atom number N=3/2

$U'', U', U > 0$



Properties at half Filling: N=3/2

DMFT at T=0



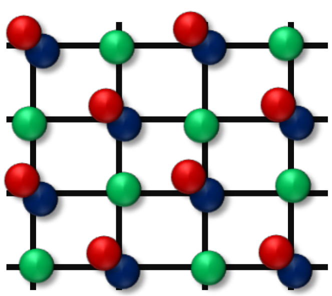
Inaba, Miyatake, and Suga, PRA **80**, 051602(R) (2010)

Miyatake, Inaba, and Suga, PRA **81**, 051602(R) (2010)

- Two Mott states in 'color paramagnetic' sector

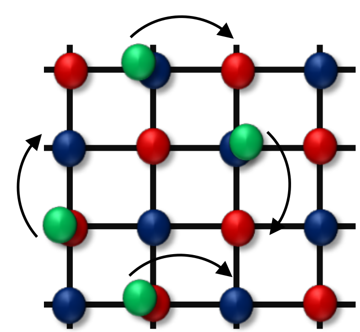
For $U'', U' \gg U$

paired Mott insulator (PMI)



For $U'', U' \ll U$

color-selective Mott state (CSM)



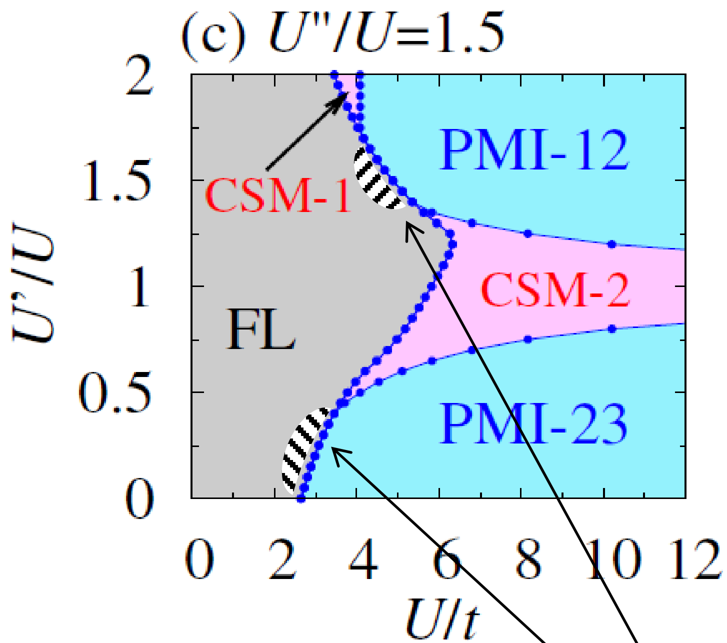
- the ground state

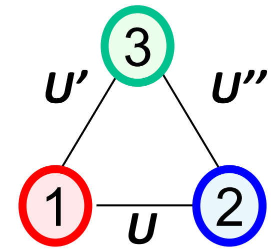
color-density wave (CDW)

color selective AF (CSAF)

Phase diagram for PMI, CSM, and FL at T=0

HF





Pair fluctuations are enhanced

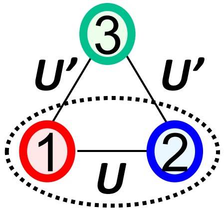
- in the FL close to PMI transition point,
- close to HF .

Superfluid is expected to appear there.

Superfluid state

Inaba and Suga, PRL **108**, 255301 (2012)

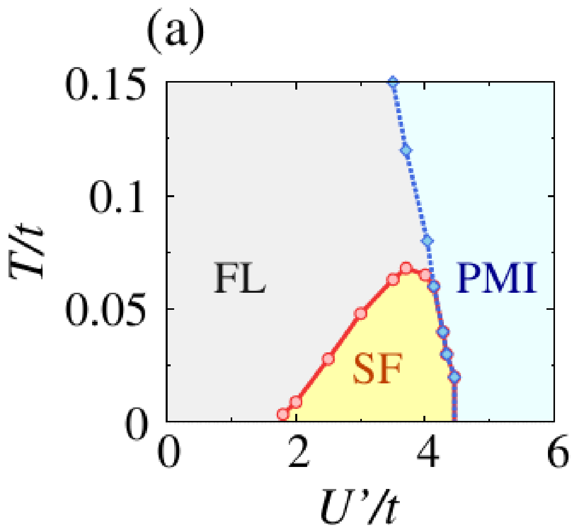
･Half filling in ‘color paramagnetic’ sector



Self-energy functional approach

U’’=U’ ≫ U

n=1/2
U/U’=0.1



FL: Fermi liquid
SF: superfluid
PMI: paired Mott insulator

SF at HF has been obtained with DMFT + CTQMC.

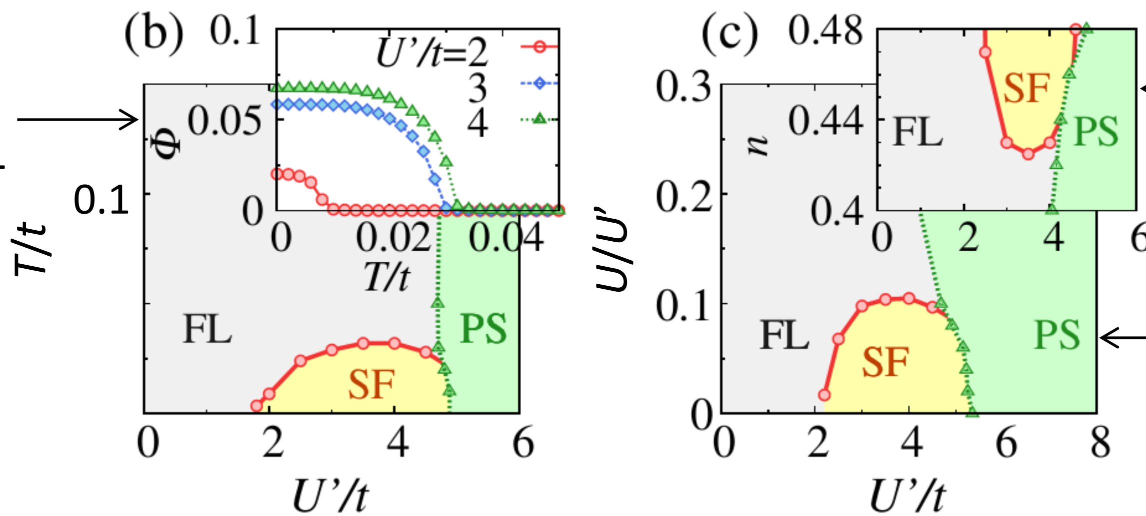
Okanami, Takemori, Koga, arXiv: 1401.5610

Prof. Koga’s talk at 10:30 in June 23 (Mon.)

▪Close to HF: DMFT + modified iterated-perturbation theory

Inaba and Suga, PRL **108**, 255301 (2012)

n=0.48
U/U'=0.1



U/U'=0.1
T/t=0.03

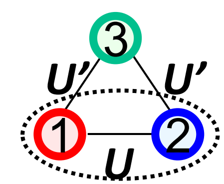
n=0.48
T/t=0.03

PS: phase separation into paired atoms and unpaired atoms

Superfluid appears for

▪*U/U'*<0.11: large difference in repulsions,

▪0.42<*n*≤0.5: close to HF (HF: *n*=0.5).



Effective attractive interaction is caused by density fluctuations of unpaired color-3 atoms.

Aim

Pairing symmetry of superfluid state in repulsively interacting three-component fermionic atoms in optical lattices.

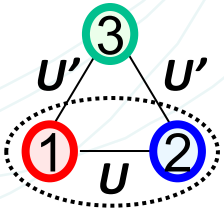
Model

$$H=-t\sum_{\langle i,j\rangle}\sum_{\alpha=1,2,3}c_{i\alpha}^{+}c_{j\alpha}+\frac{1}{2}\sum_{i}\sum_{\alpha\neq\beta}U_{\alpha\beta}n_{i\alpha}n_{i\beta}-\sum_{i,\alpha}\mu_{\alpha}n_{i\alpha},\quad U_{\alpha\beta}>0$$

balanced population: $n_1=n_2=n_3=n$, HF: $n=1/2$

$U' \gg U>0$

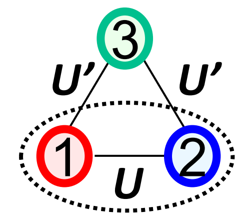
Cooper pairs: color-1 and 2 atoms

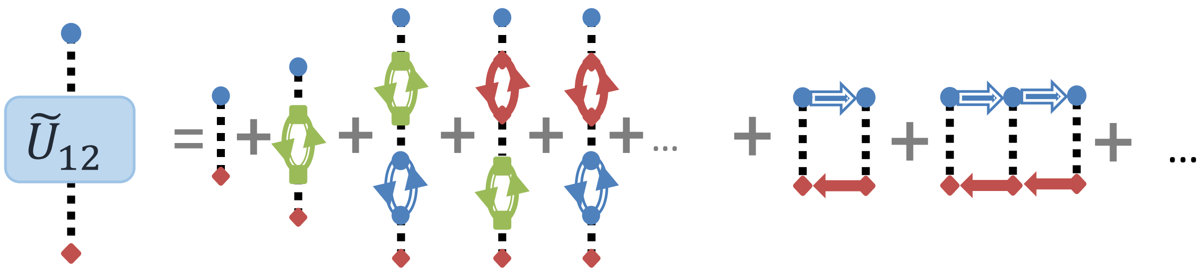


Eliashberg equation

Effective interaction $\widetilde{U}_{12}(\boldsymbol{q})$ between color-1 & 2 atoms

RPA diagrams and ladder diagrams





$$\widetilde{U}_{12}(\boldsymbol{q}) = \mathrm{U} + \frac{3}{2}U^2\chi_{\mathrm{s}}(\boldsymbol{q}) - \frac{1}{2}U^2\chi_{\mathrm{c}}(\boldsymbol{q})$$

$$\begin{cases} \chi_{\mathrm{s}}(\boldsymbol{q}) = \dfrac{\chi_{11}^{(0)}(\boldsymbol{q})}{1 - U\chi_{11}^{(0)}(\boldsymbol{q})} \\ \chi_{\mathrm{c}}(\boldsymbol{q}) = \dfrac{R^2\chi_{11}^{(0)}(\boldsymbol{q}) + 2\chi_{33}^{(0)}(\boldsymbol{q}) - 2U\chi_{11}^{(0)}(\boldsymbol{q})\chi_{33}^{(0)}(\boldsymbol{q})}{1 + U\chi_{11}^{(0)}(\boldsymbol{q}) - 2U'^2\chi_{11}^{(0)}(\boldsymbol{q})\chi_{33}^{(0)}(\boldsymbol{q})} \end{cases}$$

$\chi_{\mathrm{s}}(\boldsymbol{q})$ and $\chi_{\mathrm{c}}(\boldsymbol{q})$ have a possibility of divergence. ➡ competition

$$\lambda\Delta(\boldsymbol{k}) = -\frac{1}{N}\sum_{k'} \widetilde{U}_{12}(\boldsymbol{k}-\boldsymbol{k}')\frac{\tanh(\beta\xi_{k'}/2)}{2\xi_{k'}}\Delta(\boldsymbol{k}')$$

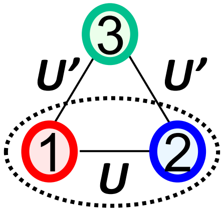
$\Delta(\boldsymbol{k})$: SF order parameter

λ : eigenvalue SF transition occurs at λ=1.

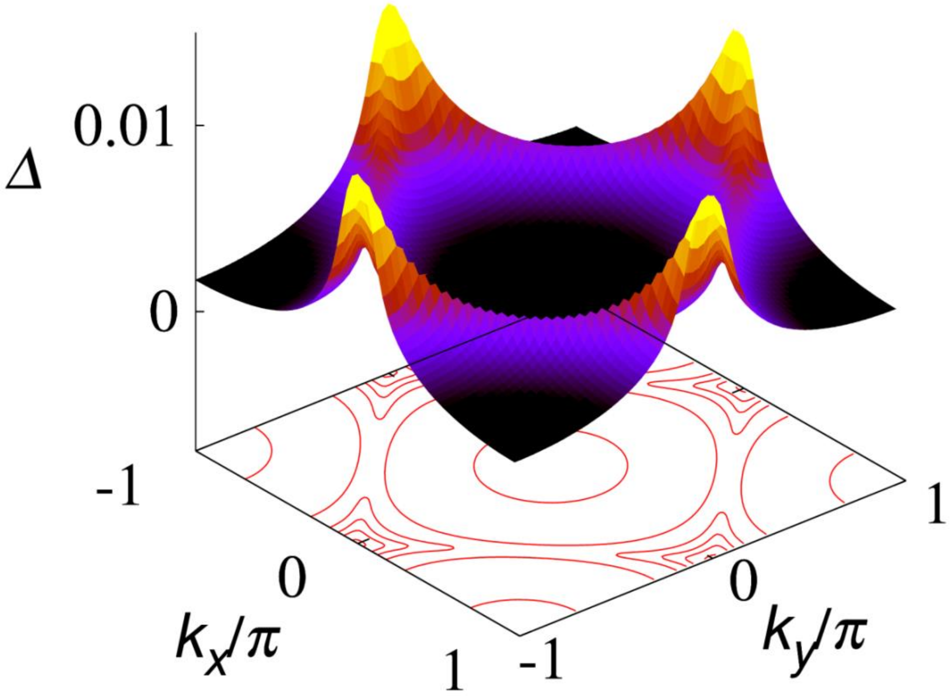
- Numerical diagonalization
- Iterative approximation

Results

$\Delta(\boldsymbol{k})$ in square optical lattices 1



***U/U'*=0.1**, U'/t=0.8, T/t=0.01 at n=0.49



Extended *s*-wave pairing

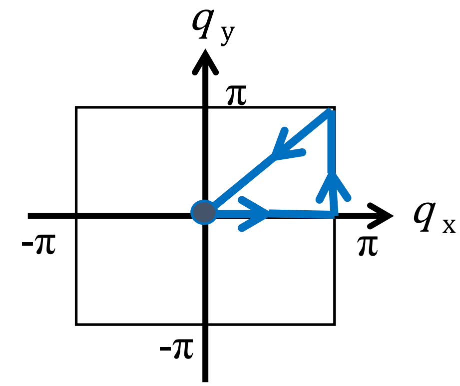
nodeless

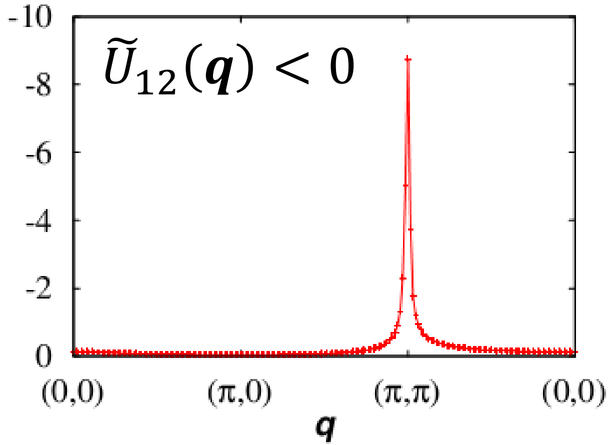
Extended *s*-wave SF, although SF is adjacent to PMI at HF.

$\widetilde{U}_{12}(\boldsymbol{q})$ in square optical lattices

$$\widetilde{U}_{12}(\boldsymbol{q}) = U + \frac{3}{2}U^2\chi_s(\boldsymbol{q}) - \frac{1}{2}U^2\chi_c(\boldsymbol{q})$$

***q*=*k*-*k*'**

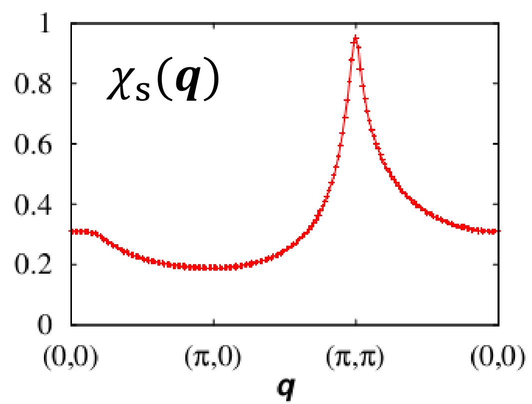


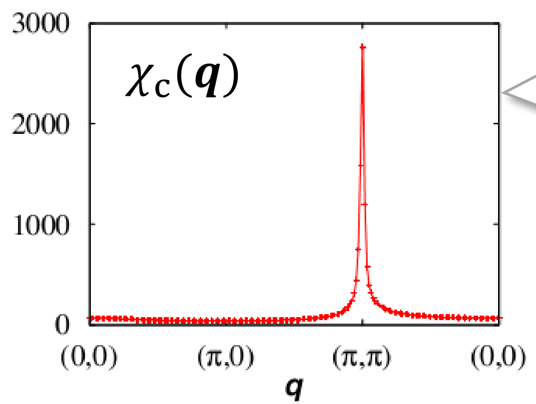


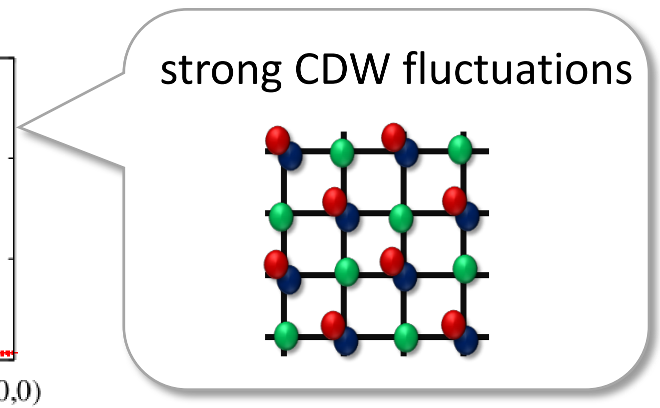
Peak in χ_c at ***q***=(π,π): strong CDW fluctuations close to HF

Large attractive peak in $\widetilde{U}_{12}(\boldsymbol{q})$

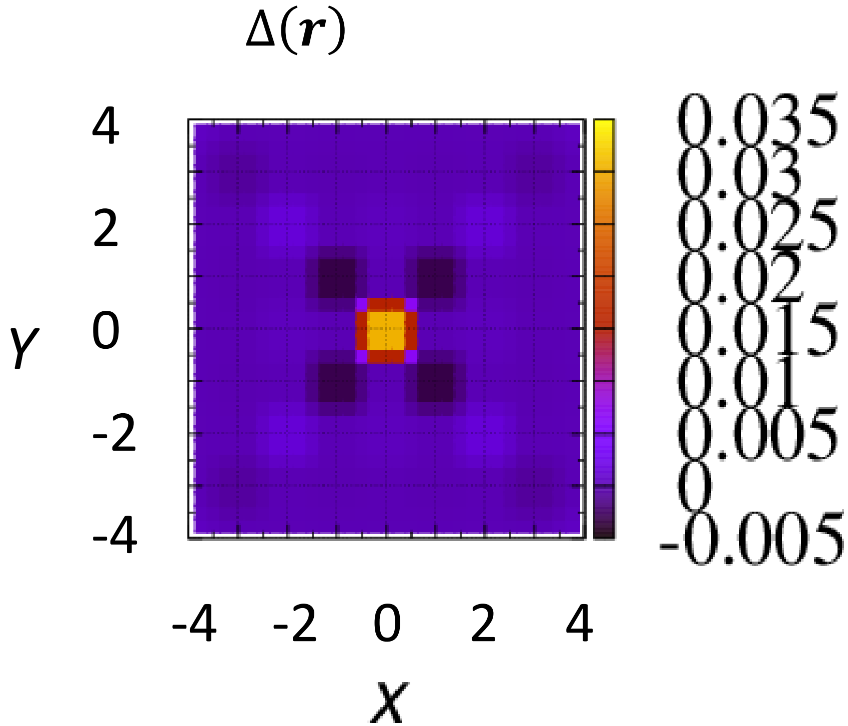
$U' \gg U$: CDW ground state at HF







$\Delta(\boldsymbol{r})$: Fourier component of $\Delta(\boldsymbol{k})$



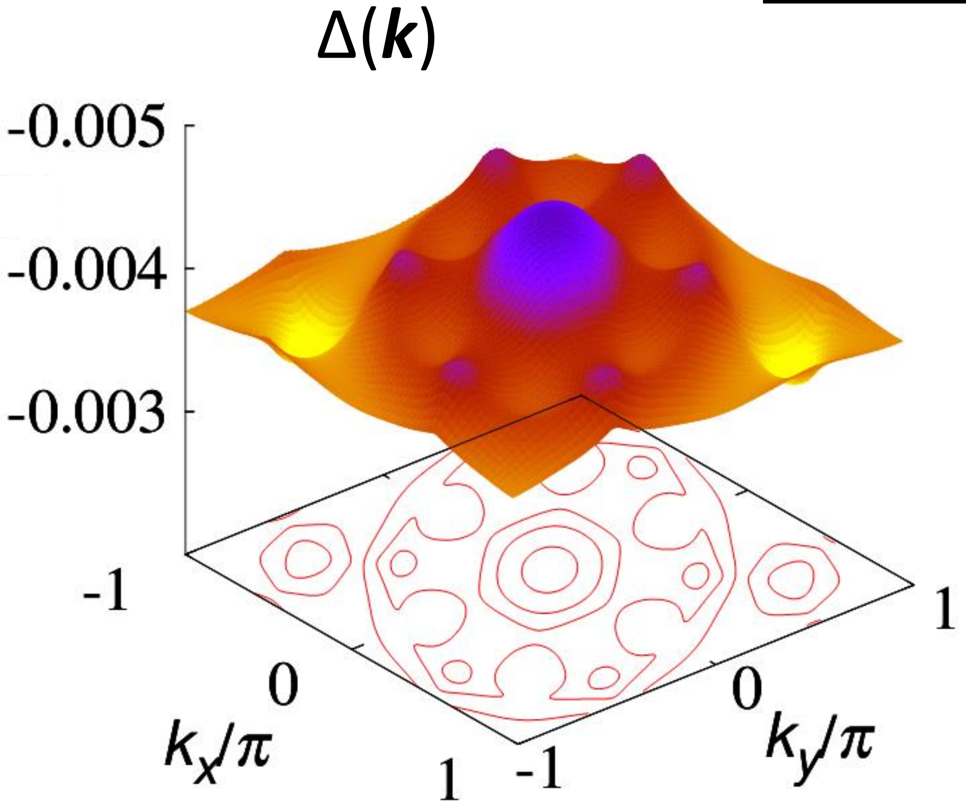
- Local component
  $\Delta_0 \approx 3.00 \times 10^{-2},\ r = (0,0)$
- Nonlocal components
  $\Delta_1 \approx -3.86 \times 10^{-3},$
  $r = (\pm 1, \pm 1), (\pm 1, \mp 1)$
  $\Delta_2 \approx 1.40 \times 10^{-3},$
  $r = (\pm 2, \pm 2), (\pm 2, \mp 2)$
  ...

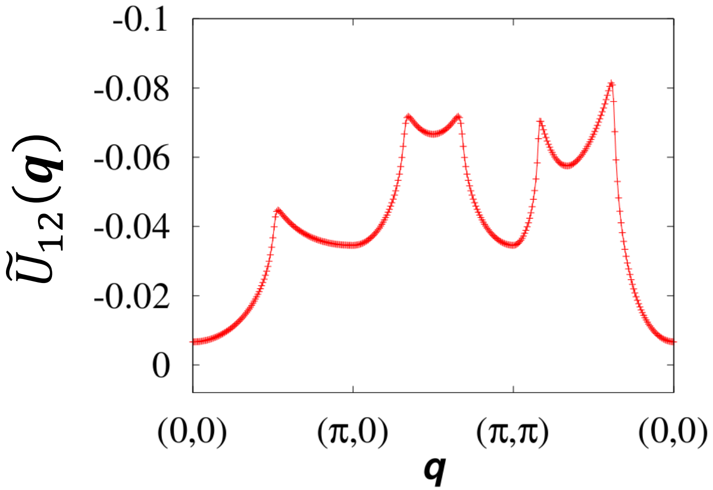
Local component Δ_0 is dominant, although the strong attractive peak caused by CDW fluctuations appears in $\widetilde{U}_{12}(\boldsymbol{q})$.

➡ Local correlation effects play an important role in this extended s-wave SF.

$\Delta(\boldsymbol{k})$ and $\widetilde{U}_{12}(\boldsymbol{q})$ in triangular optical lattices

<u>*U*/*U'*=0.1, *U'*/*t*=0.8, *T*/*t*=0.01 at *n*=0.49</u>



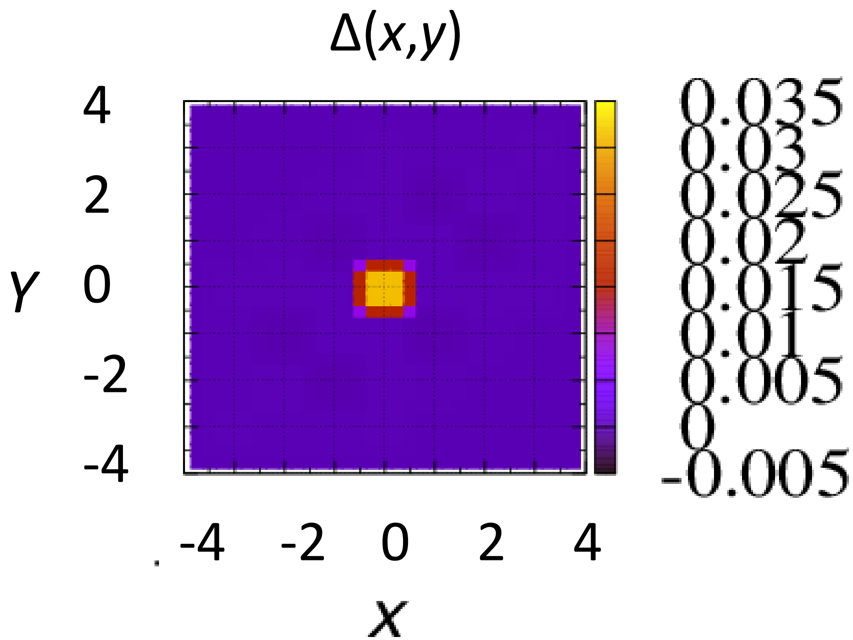


Extended *s*-wave pairing

No large attractive peak in $\widetilde{U}_{12}(\boldsymbol{q})$

CDW fluctuations are suppressed due to geometrical frustration.

$\Delta(\boldsymbol{r})$: Fourier component of $\Delta(\boldsymbol{k})$



- Local component

  $\Delta_0 \approx 3.12 \times 10^{-2},\ r = (0,0)$

- Nonlocal components

  $\Delta_1 \approx -5.06 \times 10^{-4},$

  $r = (\pm 2, \pm 1), (\pm 1, \mp 1), (\pm 1, \pm 2)$

  $\Delta_2 \approx -1.85 \times 10^{-4},$

  $r = (\pm 1, \pm 1), (\pm 1, 0), (0, \pm 1)$

  ⋯

Triangular OL: $\Delta_0 / \Delta_1 \approx 61.7$

Square OL: $\Delta_0 / \Delta_1 \approx 7.78$

Local component Δ_0 is more dominant in triangular OL.

Local correlation effects in triangular OL are more dominant.

➡ **DMFT picture** can be adequate in triangular OL.

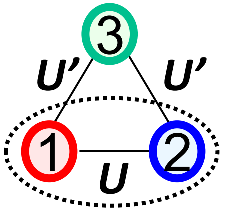
Ext. s-wave pairing symmetry for U/U' =0.1

In three-component ^{6}Li fermionic gases,
U/U' can be (somewhat) controllable.

Next issue

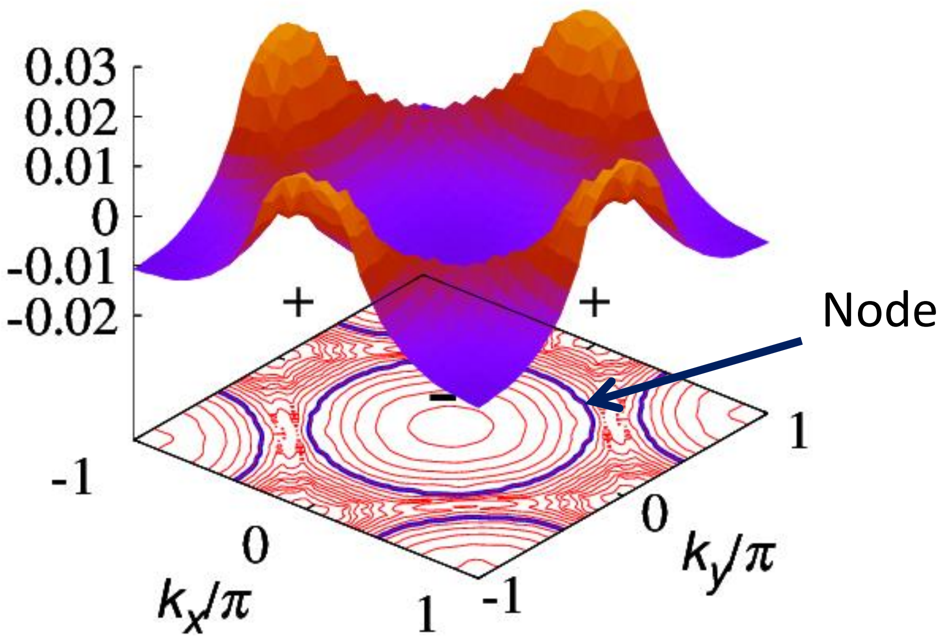
What happens for SF pairing symmetry,
when we change U/U' ?

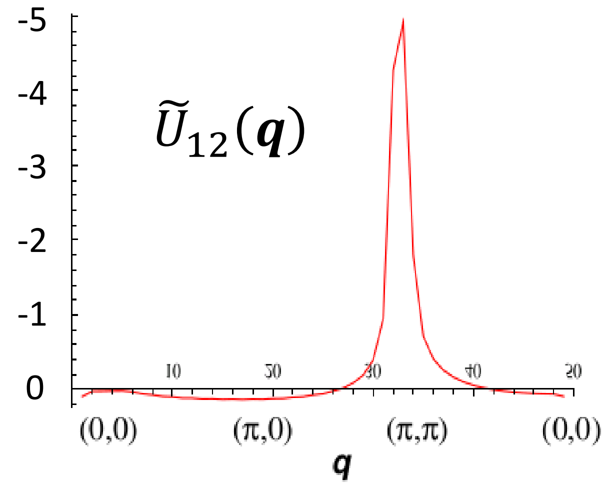
$\Delta(\boldsymbol{k})$ in square optical lattices 2



***U*/*U'*=0.2**, *U'*/*t*=0.8 ,*T*/*t*=0.01 at *n*=0.45

Nodal *s*-wave pairing





Attractive peak around ***q***=(*π*,*π*)

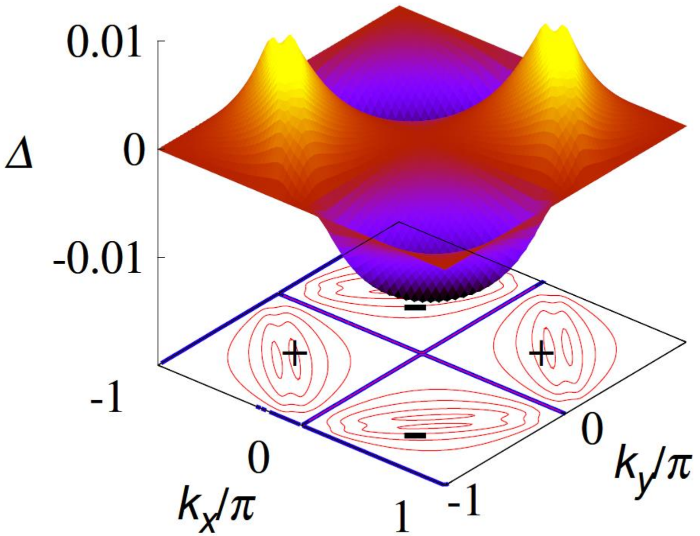
Repulsive for other ***q***

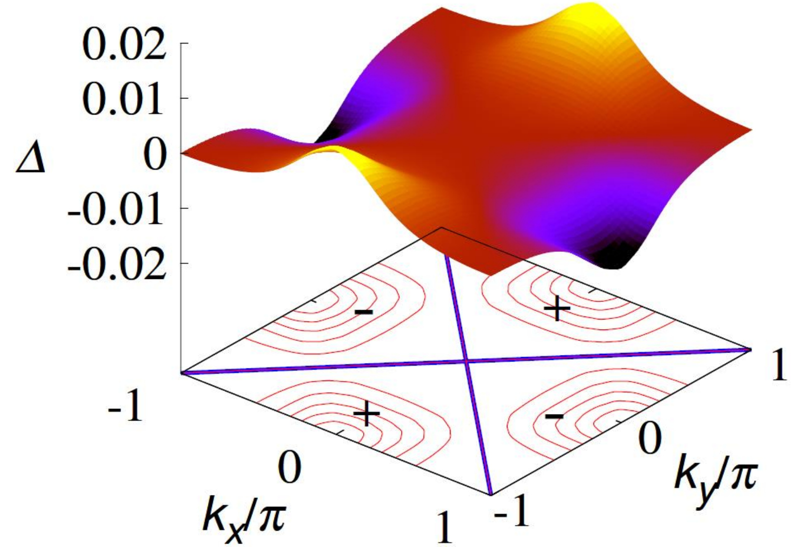
CDW fluctuations are reduced.

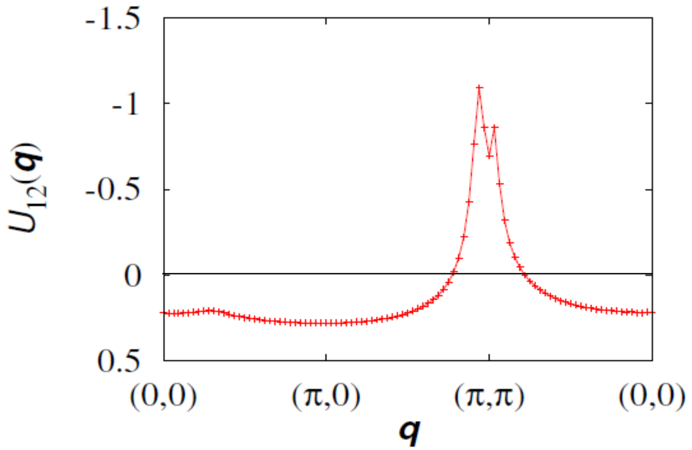
<u>U'/t=1.2, T/t=0.01 at n=0.45</u>

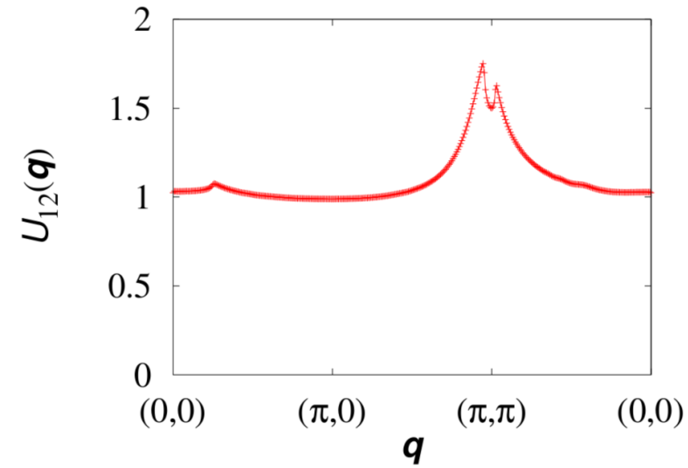
U/U'=0.4 d_{xy} **pairing**

U/U'=0.8 $d_{x^2-y^2}$ **pairing**



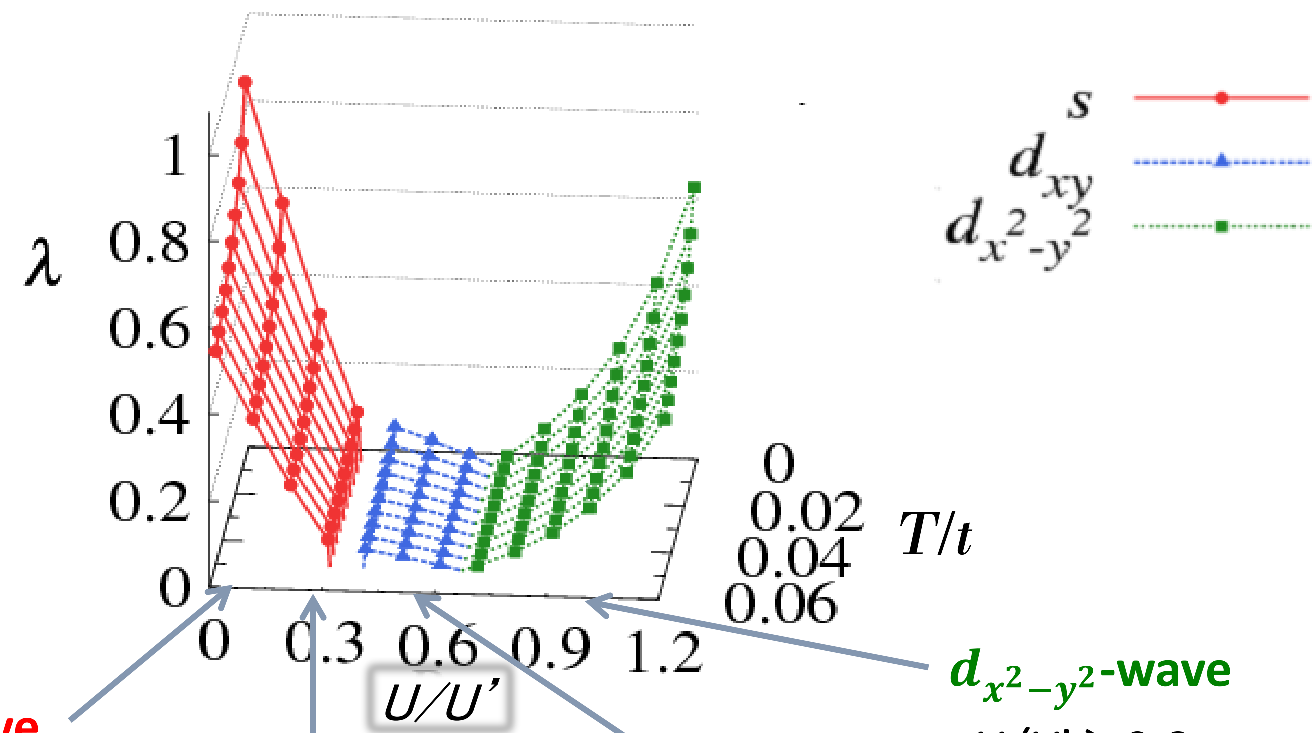






Dominant color AF fluctuations

Largest λ as a function of U/U' and T/t



Extended *s*-wave
$U/U' < 0.2$

Nodal *s*-wave
$U/U' \approx 0.2$

d_{xy}**-wave**
$U/U' \approx 0.3\text{-}0.6$

$d_{x^2-y^2}$**-wave**
$U/U' \gtrsim 0.8$

Extended *s*-wave pairing for **U/U'<0.1**
$d_{x^2-y^2}$-wave pairing for **U/U'>1.0**

can be observed in experiments.

Relation to experiments

Three-component fermionic atoms in optical lattices

^{6}Li atoms in optical lattices, ^{173}Yb-^{171}Yb mixture in optical lattices

Change in pairing symmetry can be probed with momentum-resolved one-particle excitation spectrums.

Stewart, Gaeber, and Jin, Nature **545**, 744 (2008)

	$\boldsymbol{k} = (0,0)$	$\boldsymbol{k} = (\pi, 0)$	$\boldsymbol{k} = \left(\frac{\pi}{2}, \frac{\pi}{2}\right)$
extended s wave	gap	gap	gap
nodal s wave	gap	gap	node
d_{xy} wave	node	node	gap
$d_{x^2-y^2}$ wave	node	gap	node

Summary

Pairing symmetry of superfluid state in repulsively interacting three-component fermionic atoms in optical lattices.

Extended s-wave SF appears for U/U'<0.2 close to HF in square and triangular optical lattices.

Local correlation effects play an important role.

Pairing symmetry changes with increasing U/U':

ext. s-wave ➡ nodal s-wave ➡ d_{xy}-wave ➡ $d_{x^2-y^2}$-wave

This phenomena can be probed in experiments.

Conclusion

Three-component repulsive fermionic atoms in optical lattices can be a quantum simulator for controlling SF pairing symmetry.