Wilsonian and large N theories of quantum critical metals

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Collaborators and References

R. Mahajan, D. Ramirez, S. Kachru, and SR, PRB **88**, 115116 (2013).

A. Liam Fitzpatrick, S. Kachru, J. Kaplan, and SR, PRB **88**, 125116 (2013).

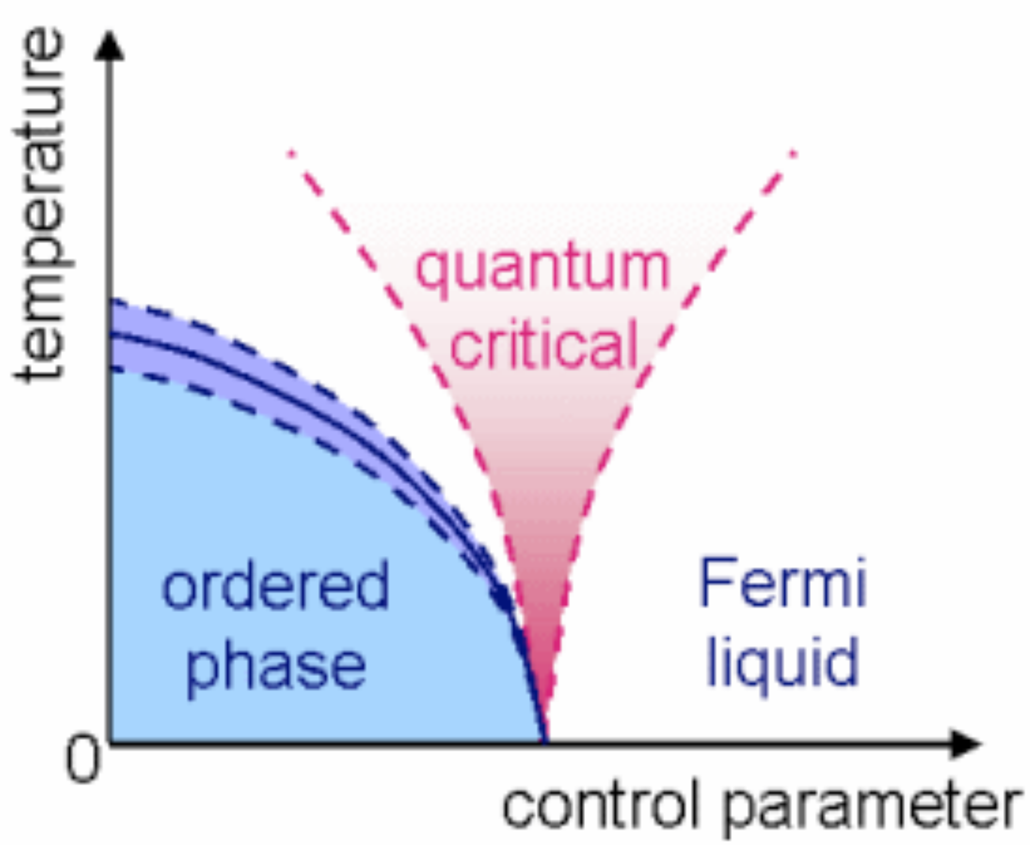
A. Liam Fitzpatrick, S. Kachru, J. Kaplan, and SR, PRB (2014).

A. Liam Fitzpatrick, S. Kachru, J. Kaplan, and SR, *to appear*.

With
Liam Fitzpatrick, Jared Kaplan, Shamit Kachru

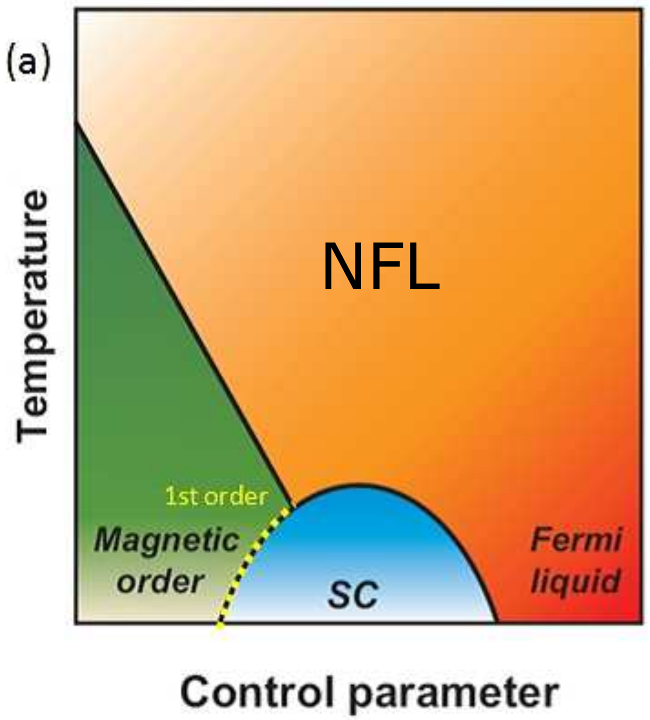
Breakdown of fermion quasiparticles

A recurring theme: Fermi liquid theory breaks down at a quantum phase transition.



NFL emanates from a critical point at T=0.

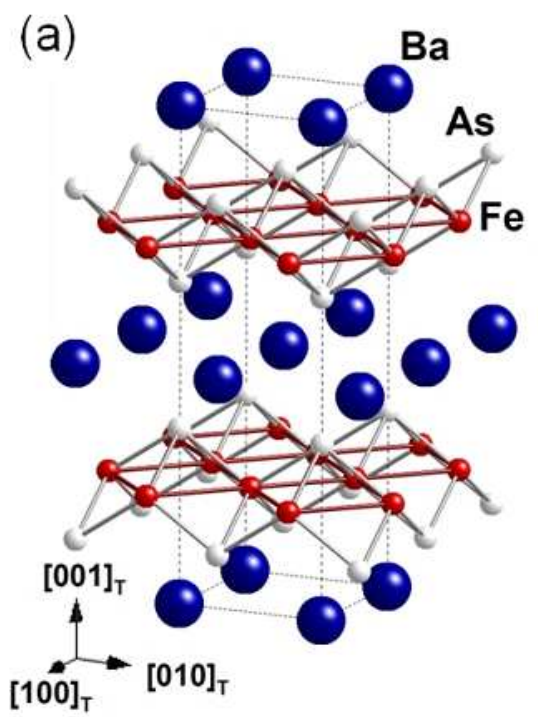
NFL can give way to higher T_c superconductivity.

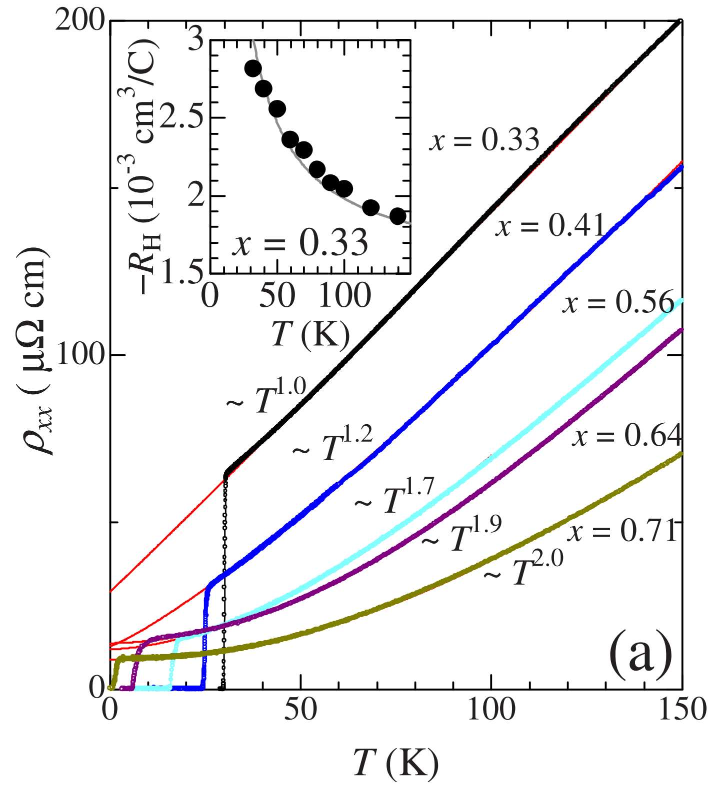


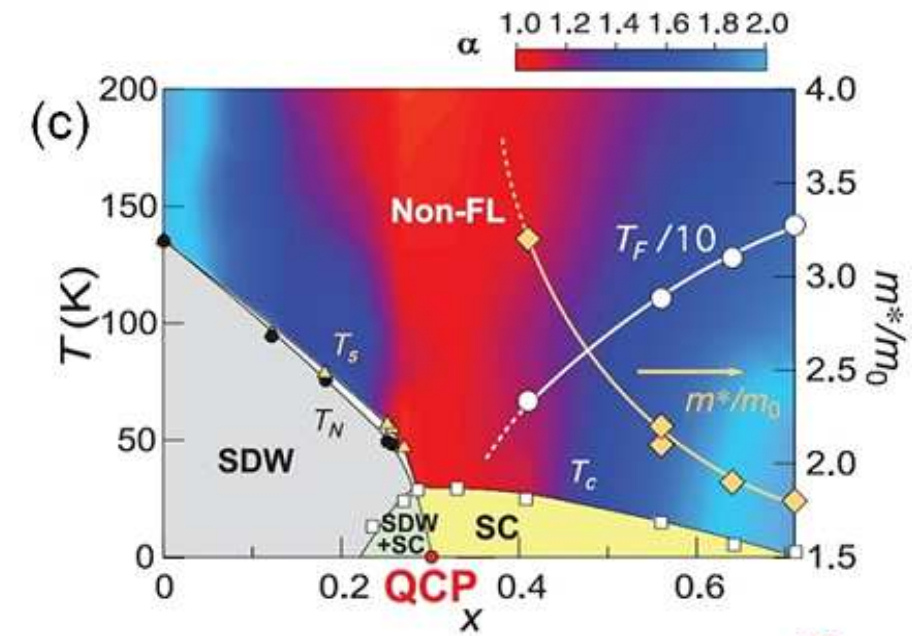
QCPs in metals: wide-open problem especially in d=2+1.

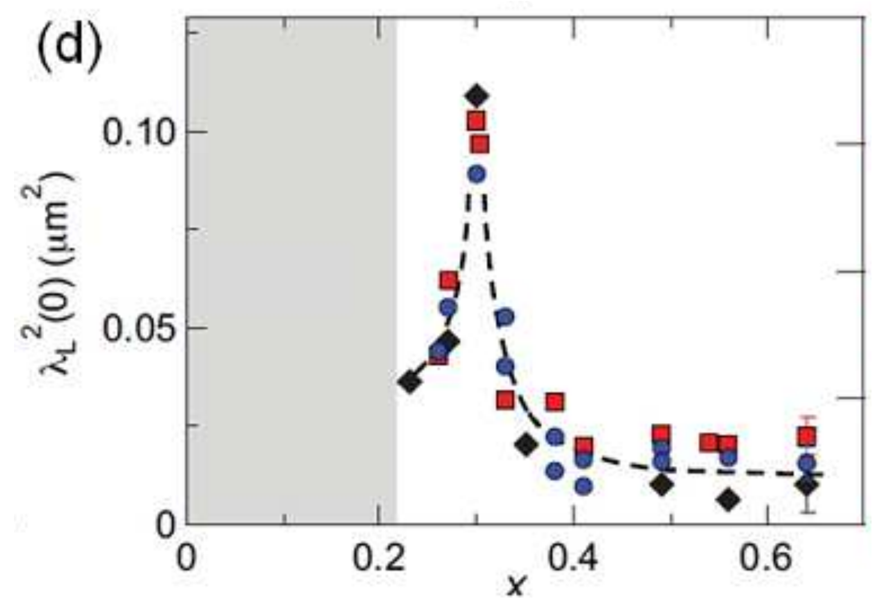
Example: Iron pnictides

$BaFe_2(As_{1-x}P_x)_2$









[PNAS 2009, PRB 2010]

Maximum superconducting T_c below the NFL.

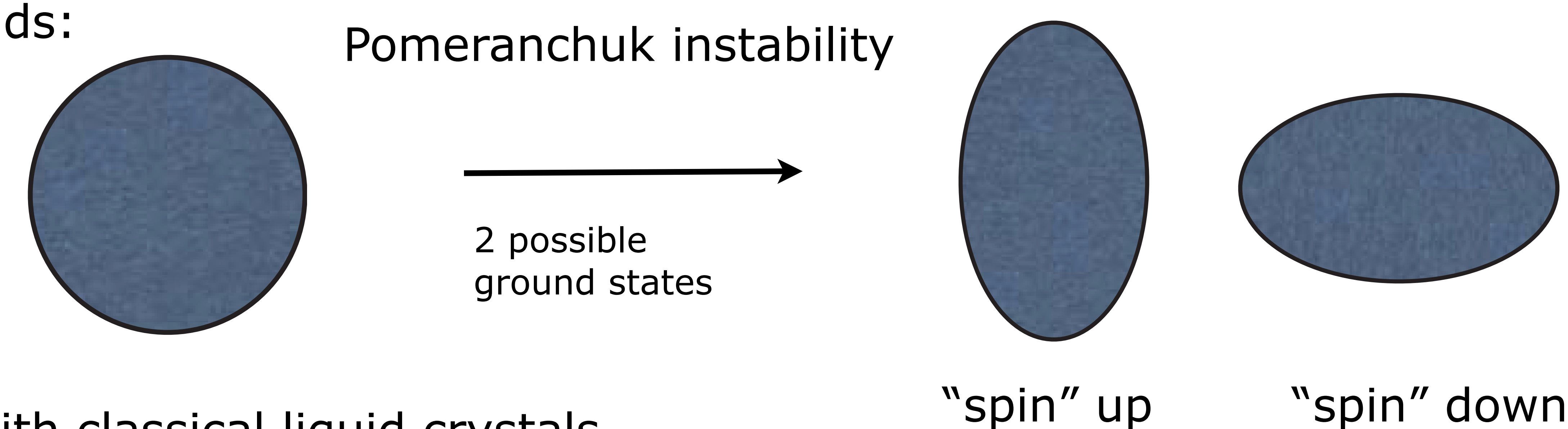
Superconductivity forms out of a non-Fermi liquid.

Talk by Prof. Shibauchi in the symposium.

Concrete model system

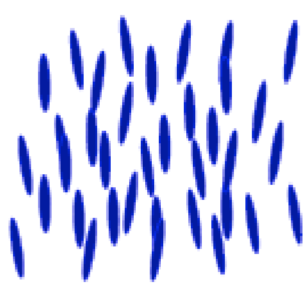
Ising nematic transition: breaking of point group symmetry.

Fermi liquids:



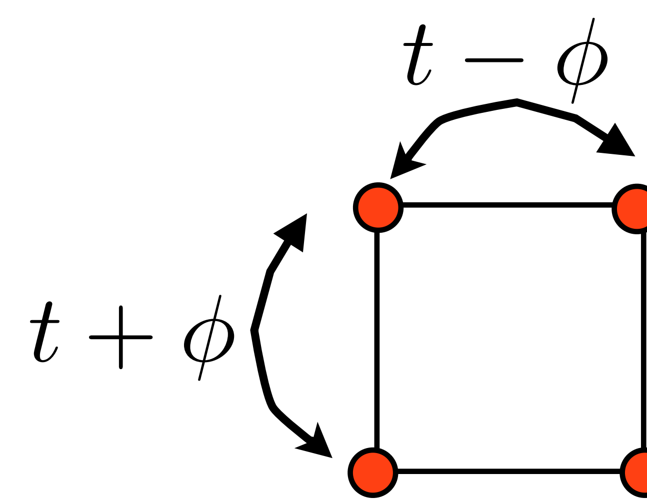
Analogy with classical liquid crystals

Isotropic



Nematic

order parameter: ϕ



The nematic state preserves lattice translation symmetry.

Effective theory: Fermion-boson problem

Starting UV action: $$S = S_\psi + S_\phi + S_{\psi-\phi}$$

S_ψ Landau Fermi liquid

S_ϕ Landau-Ginzburg-Wilson theory for order parameter.

$S_{\psi-\phi}$ Fermion-boson "Yukawa" coupling

Obtaining such an action: Start with electrons strongly interacting ("Hubbard model"). "Integrate out" high energy modes from lattice scale down to a new UV cutoff $\Lambda << E_F$.

Λ = Scale below which we can linearize the fermion Kinetic energy.

Effective theory: Fermion-boson problem

Starting UV action (in imaginary time):

$$\mathcal{S} = \int d\tau \int d^d x \; \mathcal{L} = S_\psi + S_\phi + S_{\psi-\phi}$$

$$\mathcal{L}_\psi = \bar{\psi}_\sigma \left[\partial_\tau + \mu - \epsilon(i\nabla)\right] \psi_\sigma + \lambda_\psi \bar{\psi}_\sigma \bar{\psi}_{\sigma'} \psi_{\sigma'} \psi_\sigma$$

Fermions

$$\mathcal{L}_\phi = m_\phi^2 \phi^2 + \left(\partial_\tau \phi\right)^2 + c^2 \left(\vec{\nabla}\phi\right)^2 + \frac{\lambda_\phi}{4!}\phi^4$$

bosons

$$S_{\psi,\phi} = \int \frac{d^{d+1}k d^{d+1}q}{(2\pi)^{2(d+1)}} g(k,q) \bar{\psi}(k) \psi(k+q) \phi(q),$$

"Yukawa" coupling

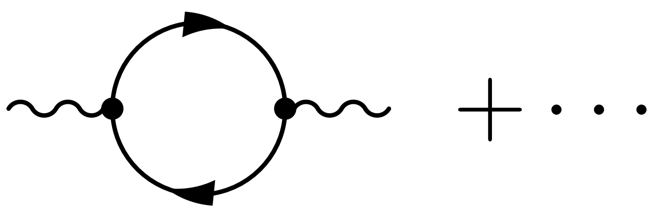
Ising nematic theory: $g(k,q) = g\left(\cos k_x - \cos k_y\right).$

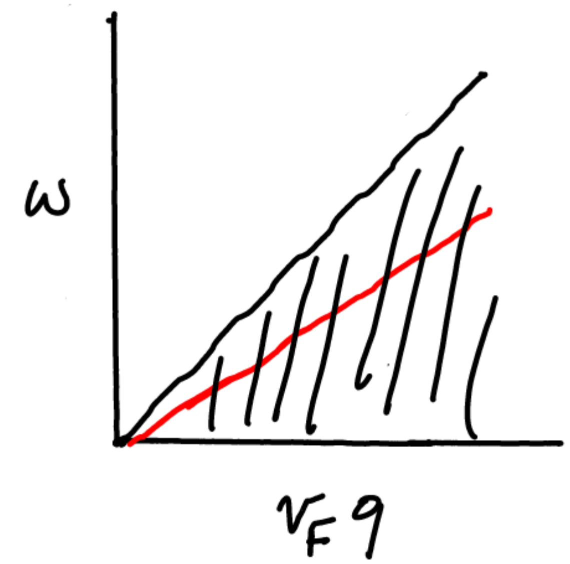
g=0: decoupled limit (Fermi liquid + ordinary critical point).

non-zero g: complex tug-of-war between bosons and fermions.

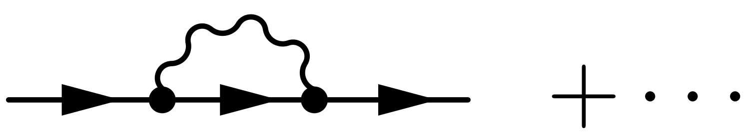
Tug-of-war between bosons and fermions

Non-zero g: Bosons can decay into particle-hole continuum -> overdamped bosons.





Non-zero g: Quasiparticle scattering enhanced due to bosons.



q.p. Scattering rate can exceed its energy.

Fermion propagators: poles become branch cuts.

Result: breakdown of Landau quasiparticle.

How to proceed???

Large N limits

Large N limits

Essence of the problem: dissipative coupling between bosons and fermions.

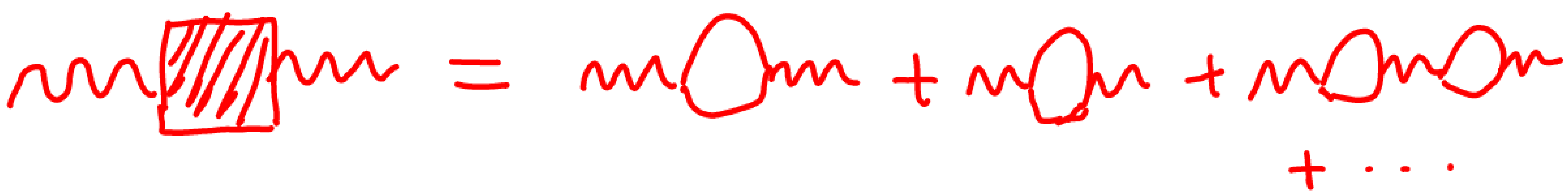
Large N limits: particles with many (N) flavors act as a dissipative "bath" while remaining degrees of freedom become overdamped.

e*.g*. Large number of fermion flavors (N_f). Boson can decay in many channels -> Overdamped bosons (NFL is subdominant). Mainstream (Hertz) theory captures the IR behavior in this regime.

e.g. Large number of boson flavors (N_b). Fermion can decay in many channels -> NFL is strongest effect (boson damping is subdominant).

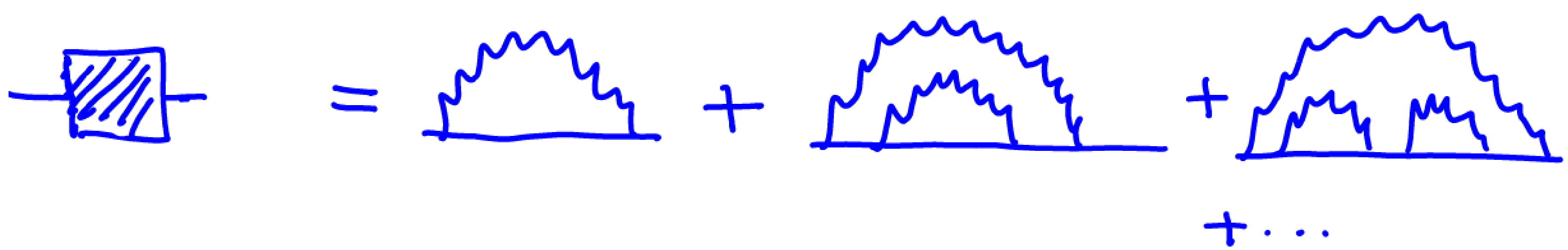
Large N limits

Large N_F:

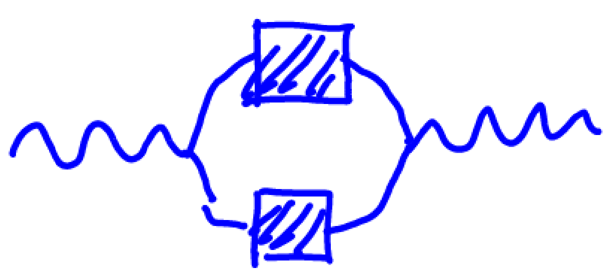


$\mathcal{O}(1/N_F)$:

Large N_B:



$\mathcal{O}(1/N_B)$:



Implementation of large N limits

$$\bar{\psi} \to \bar{\psi}^i_\alpha \qquad \alpha = 1 \cdots N_F$$

$$\psi \to \psi^\alpha_i \qquad i, j = 1 \cdots N_B$$

$$\phi \to \phi^j_i$$

$$g\bar{\psi}\psi\phi \to g\bar{\psi}^i_\alpha \psi^\alpha_j \phi^j_i$$ (repeated indices summed).

I will consider the case: $N_F = 1, N_B \to \infty.$

Large N_B action

$$\mathcal{L}_\psi = \bar{\psi}^i \left[\partial_\tau + \mu - \epsilon(i\nabla)\right] \psi_i + \frac{\lambda_\psi}{N_B} \bar{\psi}^i \psi_i \bar{\psi}^j \psi_j$$

$$\mathcal{L}_\phi = \mathrm{tr}\left(m_\phi^2 \phi^2 + \left(\partial_\tau \phi\right)^2 + c^2 \left(\vec{\nabla}\phi\right)^2 \right)$$

$$+ \frac{\lambda_\phi^{(1)}}{8N_B} \mathrm{tr}(\phi^4) + \frac{\lambda_\phi^{(2)}}{8N_B^2} (\mathrm{tr}(\phi^2))^2$$

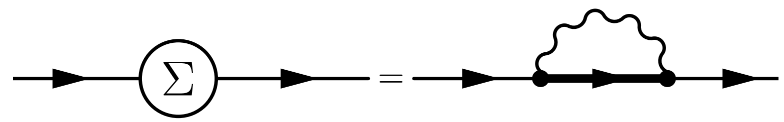
$$\mathcal{L}_{\psi,\phi} = \frac{g}{\sqrt{N_B}} \bar{\psi}^i \psi_j \phi_i^j \qquad\qquad i, j = 1 \cdots N_B$$

Impose an SO(N_B^2) symmetry: $\lambda_\phi^{(1)} = 0$

This symmetry is softly broken: *i.e.*, only at $\mathcal{O}(1/N_B^2)$.

Large N_B solution

$N_B \to \infty$:



$$\epsilon = 3 - d$$

Properties of the solution:

$$G(k,\omega) = \frac{1}{\omega^{1-\epsilon/2}} f\left(\frac{\omega}{k}; N_B\right)$$

1) Fermi velocity vanishes at infinite N_B.
2) Green function has branch cut spectrum.
3) Damping of order parameter is a $1/N_B$ effect.

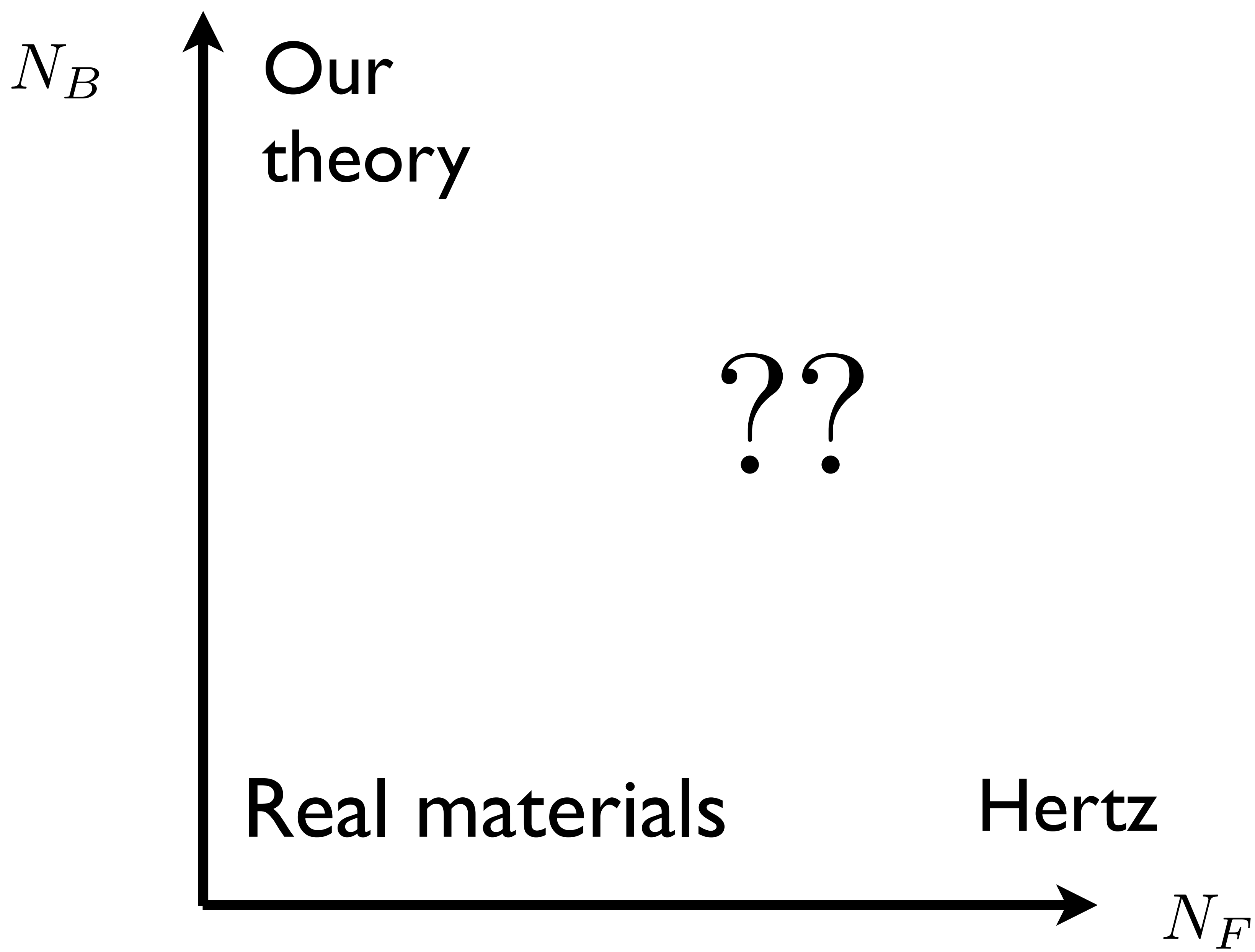
$$f\left(\frac{\omega}{k}; N_B \to \infty\right) = 1$$

The solution matches on to perturbation theory in the UV.

The theory can smoothly be extended to d=2. The theory describes infinitely heavy, incoherent fermionic quasiparticles.

We are currently investigating the strong CDW and superconducting instabilities of this system.

Scaling landscape



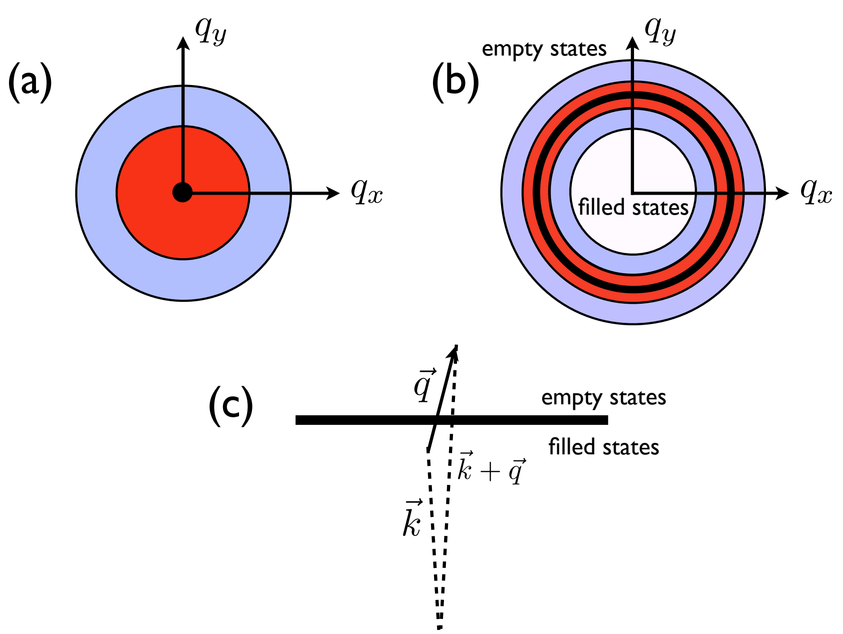
Moral of the story: there may be several distinct asymptotic limits with different scaling behaviors, dynamic crossovers in this problem.

Wilsonian RG analysis

Scaling near the upper-critical dimension

UV theory: decoupled Fermi liquid+ nearly free bosons (g=0).

Scaling must contend with vastly different kinematics of bosons and fermions.



Fermions: low energy = Fermi surface.

-> anisotropic scaling.

Bosons: low energy = point in k-space.

-> isotropic scaling.

Scale to preserve kinetic terms.

$$[g] = \frac{1}{2}(3-d)$$

Result: d=3 is the upper critical dimension.

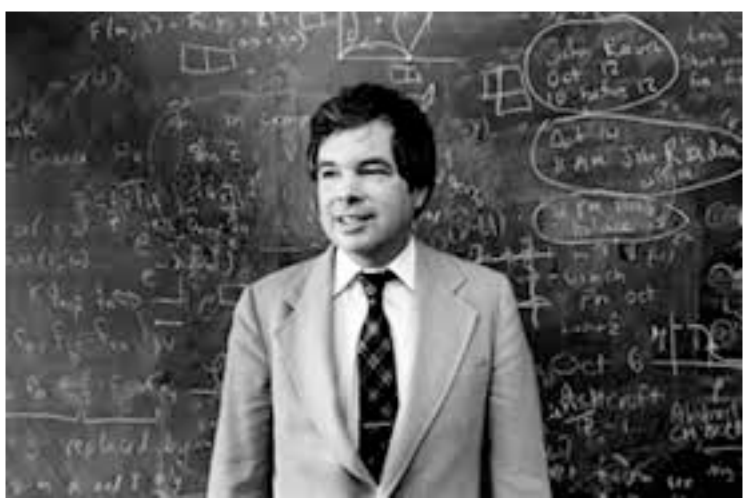
Renormalization group analysis

Integrate out modes with energy $\Lambda e^{-t} < E < \Lambda$

Integrate out modes with momenta $\Lambda_k e^{-t} < k < \Lambda_k$

$$\Lambda_k \propto \Lambda$$

$\Lambda =$ UV cutoff: scale below which fermion dispersion can be linearized (with a well-defined Fermi velocity).



K. G. Wilson

Following Wilson, we will integrate out only high-energy modes to obtain RG flows.

This is a radical departure from the standard approach to this problem.

Renormalization group analysis

RG flows at one-loop: $\epsilon = 3 - d$

$$\lambda\phi^4 \text{ term}: \quad \frac{d\lambda_\phi}{dt} = \epsilon\lambda_\phi - a\lambda_\phi^2 \quad a > 0$$

$$g\bar{\psi}\psi\phi \text{ term}: \quad \frac{dg}{dt} = \frac{\epsilon}{2}g - bg^3 \quad b > 0$$

Fermi velocity: $$\frac{dv}{dt} = -cg^2\mathcal{S}(v) \quad \mathcal{S}(v) \sim \text{sgn}(v)$$

Naive fixed point: $\lambda^* = \mathcal{O}(\epsilon)$

$$g^* = \mathcal{O}(\sqrt{\epsilon}) \quad v^* = 0$$

Properties of the naive fixed point

v/c: vanishes before the system reaches the fixed point!

This feature shuts down Landau damping.

Fermion 2-pt function takes the form:

$$G(\omega, k) = \frac{1}{\omega^{1-2\gamma_\psi}} f\left(\frac{\omega}{k_\perp}\right)$$

f(x) = scaling function

Consistent with the large N_B solution.

Is this too much of a good thing?? Infinitely heavy, incoherent fermions + non-mean-field critical exponents!

Introducing leading irrelevant couplings

$$\epsilon(\boldsymbol{k}) - \mu = v\ell + w\ell^2 + \cdots$$

w ~ band curvature

RG flow equations

$$\frac{dv}{dt} = -cg^2\mathcal{S}(v)$$

$$\mathcal{S}(v) \sim \mathrm{sgn}(v)$$

$$\frac{dw}{dt} = -w$$

w is dangerously irrelevant

w cannot be neglected below an emergent energy scale:

$$\mu_* \sim \Lambda e^{-\alpha v_0/g_0^2}, \quad \alpha \sim \mathcal{O}(1)$$

We don't know what happens below this scale (Lifshitz transition?)

Current and future work…

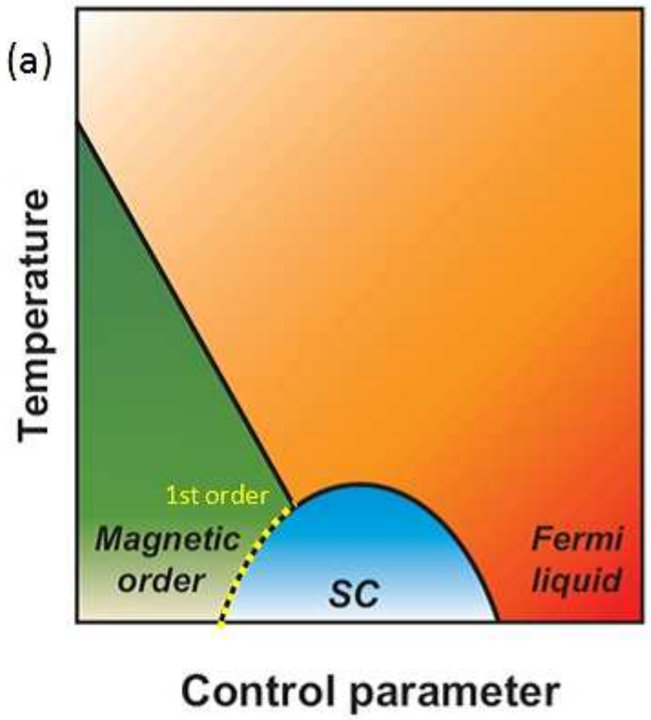
There are log-squared divergences in the Cooper channel in the vicinity of the quantum critical point. This reflects a much stronger superconducting tendency!

Break inversion and time-reversal symmetry: these effects are gone.

More detailed, systematic treatment is in progress.

Related work: Metlitski *et al*. 1403.3694.

Goal: to demonstrate enhanced superconductivity out of a non-Fermi liquid.



Summary and outlook

We studied a metal near a nematic quantum critical point and found non-Fermi liquid phenomena via 1) large N and 2)RG methods.

Both methods produce consistent results.

The fixed point corresponds to an infinitely heavy incoherent soup of fermions + order parameter fluctuations.

This fixed point is unstable, but it governs scaling laws over a broad range of energy/temperature scales.

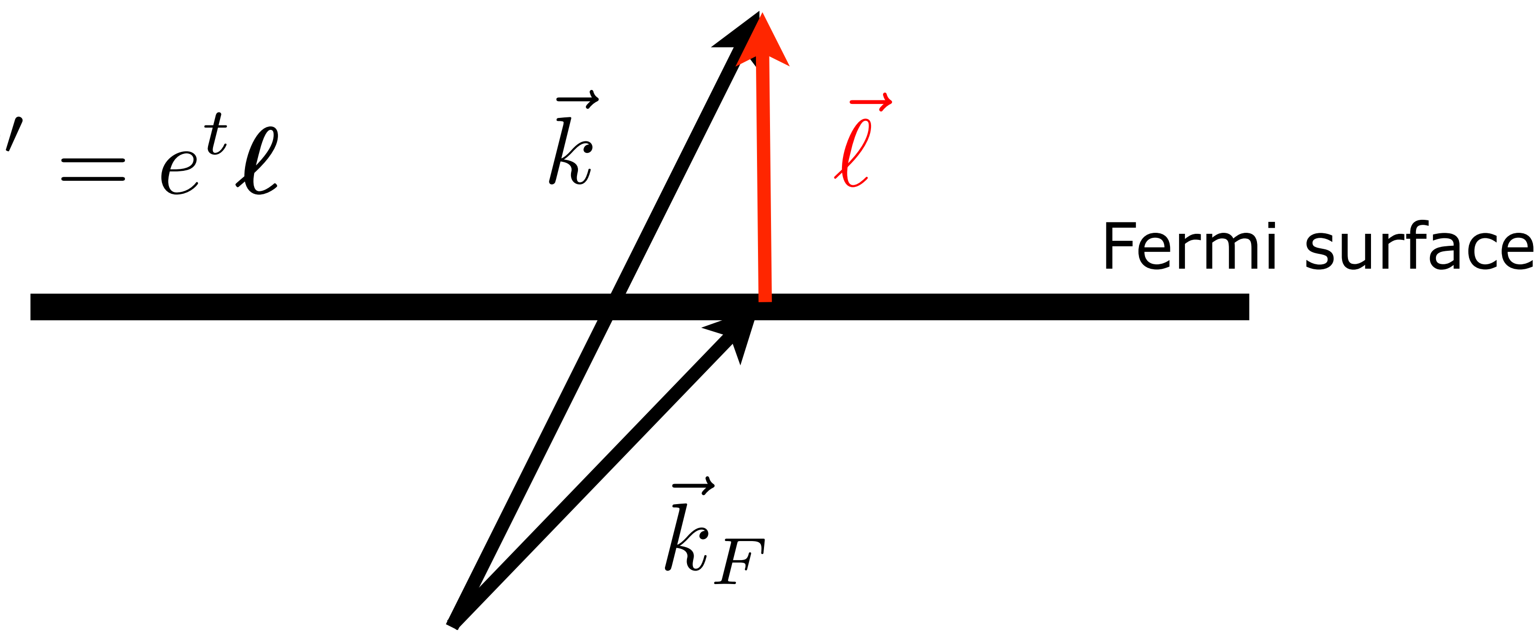
We are currently investigating experimental properties (eg. heat capacity) and superconducting/CDW instabilities in this regime.

Appendix

Scaling analysis

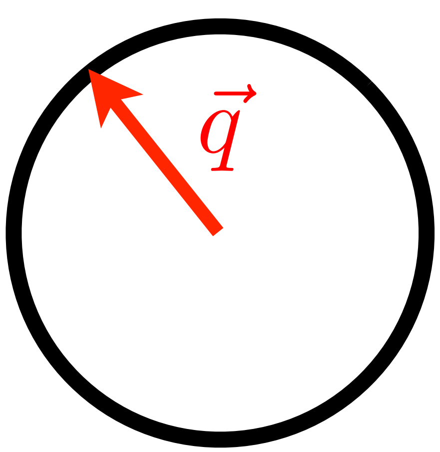
Fermions: only momentum component normal to Fermi surface scale with energy:

$$\omega' = e^t \omega, \boldsymbol{k}'_F = \boldsymbol{k}_F, \boldsymbol{\ell}' = e^t \boldsymbol{\ell}$$



BCS coupling: $\lambda'_\psi = \lambda_\psi$

Bosons: all components of momentum scale with energy:



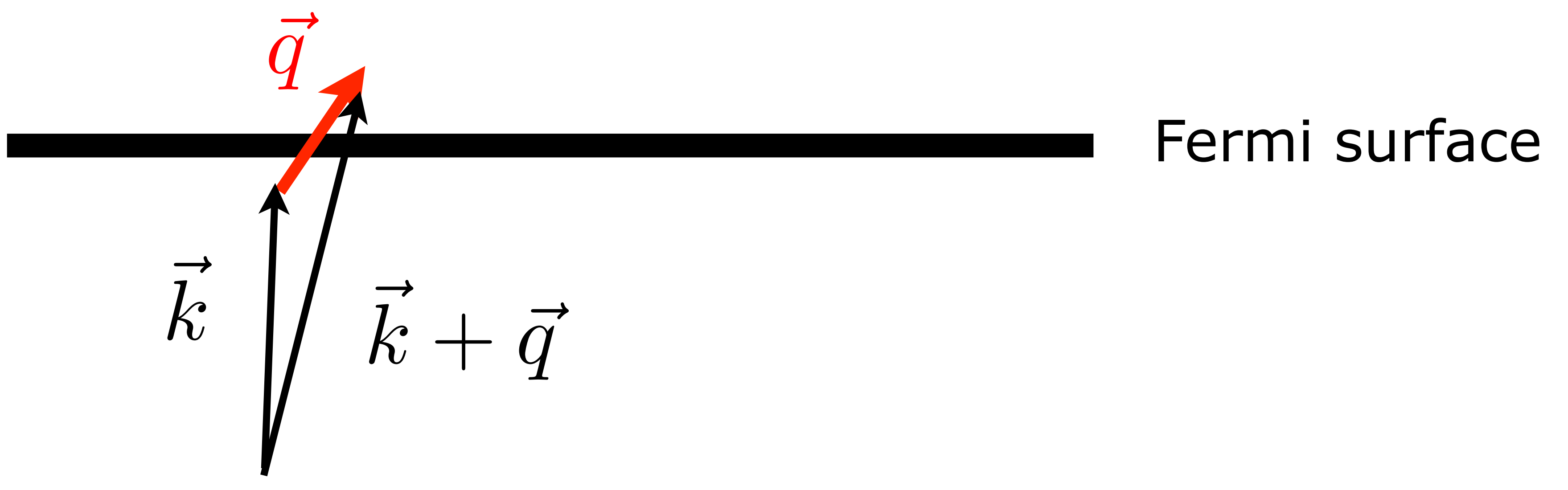
$$\omega' = e^t \omega, \boldsymbol{k}' = e^t \boldsymbol{k}$$

quartic term: $\lambda'_\phi = e^{(3-d)t} \lambda_\phi$

Scaling analysis

Boson-fermion coupling:

For small momentum transfer, the coupling is marginal in d=3.



This coupling becomes relevant when d < 3, as is true for boson interactions:

$$g' = e^{\frac{3-d}{2}t} g \qquad \lambda'_\phi = e^{(3-d)t}\lambda_\phi$$

This results in non-trivial fixed points in d<3.