



# Ultrafast laser control of the magnetic exchange interaction

ISSP Kashiwa, June 26, 2014

Martin Eckstein



## Collaborations:

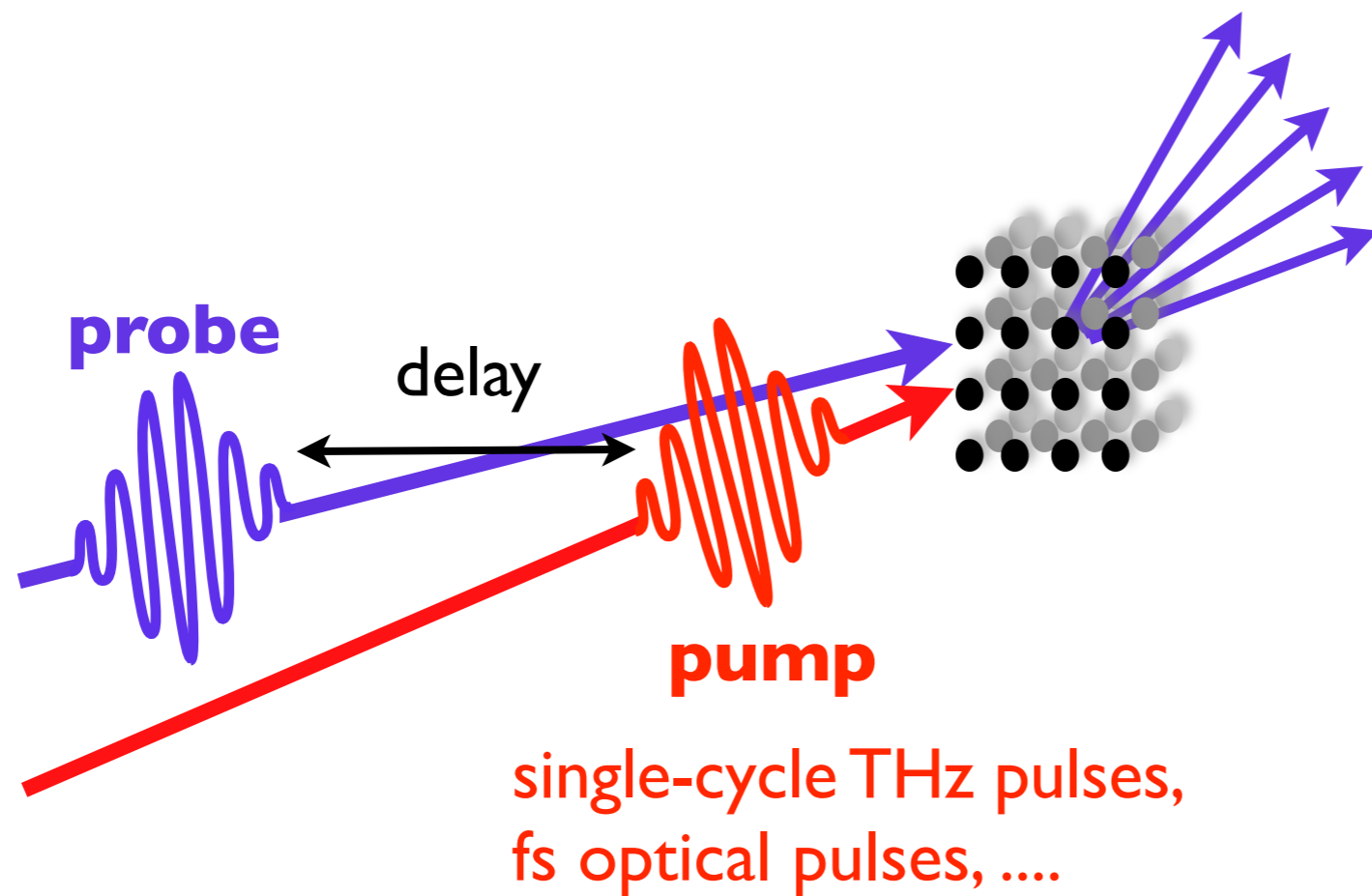


Johan Mentink Karsten Balzer

CFEL Hamburg

Philipp Werner    Uni. Fribourg  
Naoto Tsuji      Tokyo

# # Pump-probe experiments

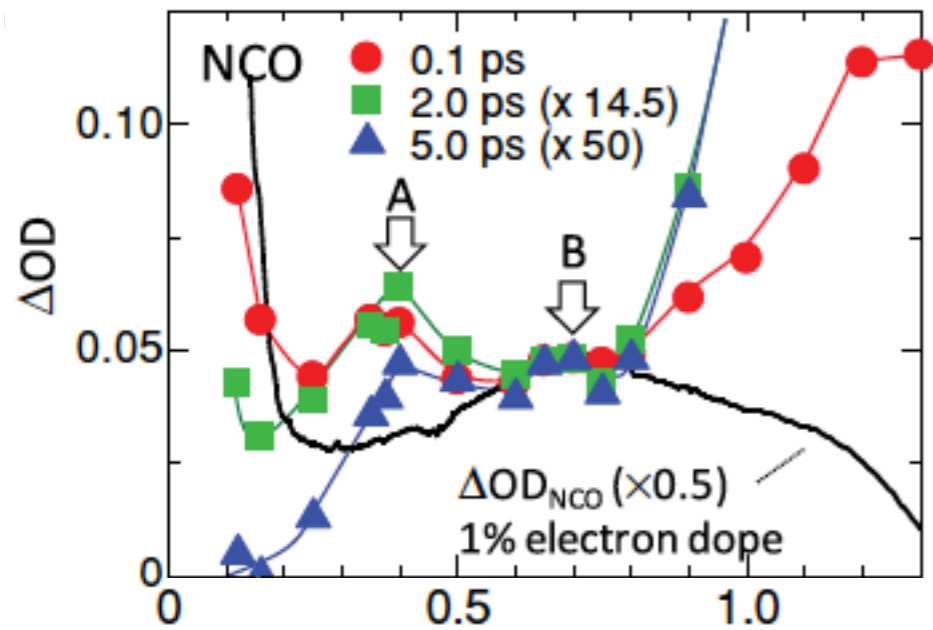


time-resolved ...

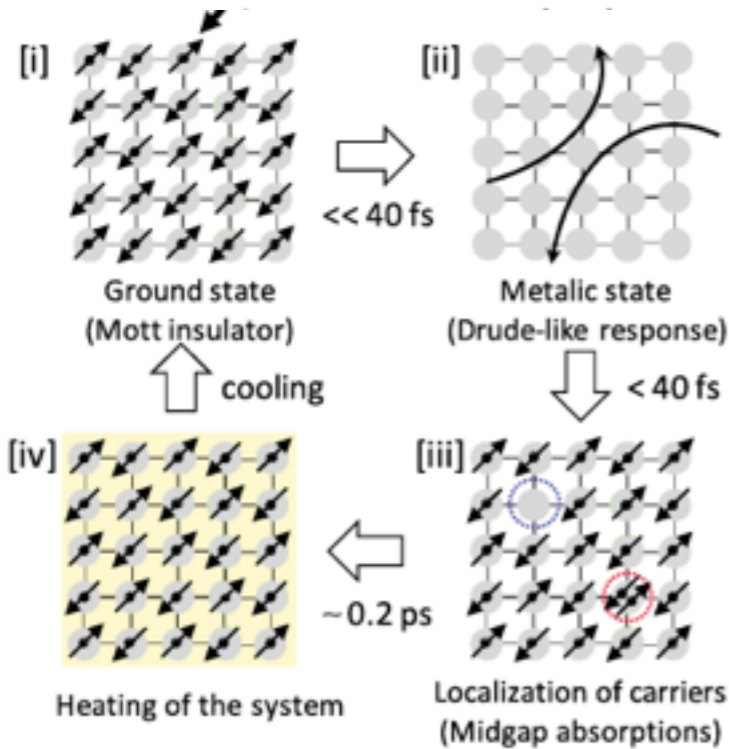
- photo-emission
- optical spectroscopy
- electron diffraction
- X-ray scattering
- RIXS

# # Many-body systems out of equilibrium

## Relaxation/Thermalization

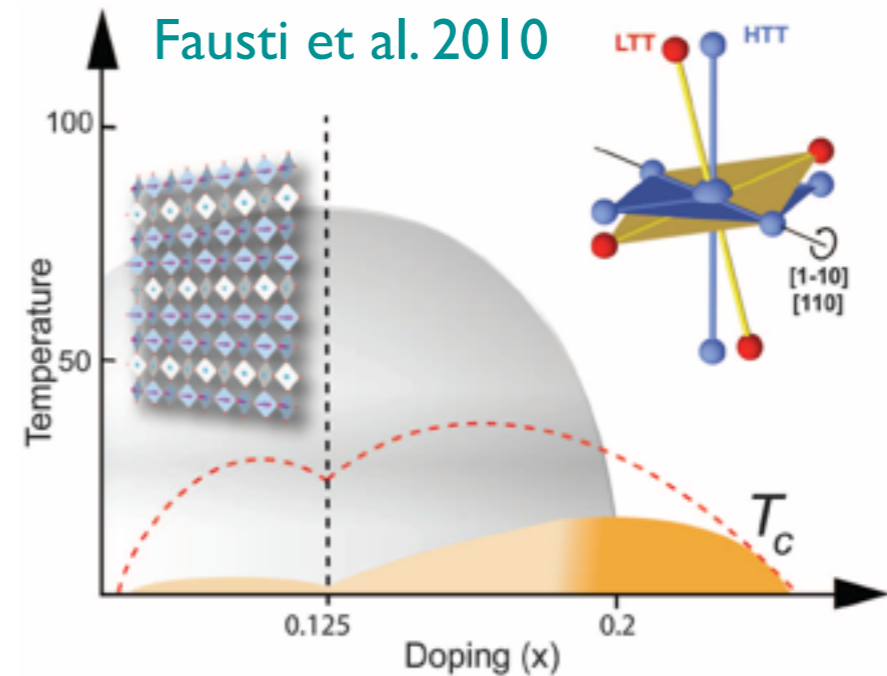


Okamoto et al, 2011



## „Control“

- ➔ Nonthermal transition pathways?
- ➔ Metastable states? Hidden states?
- ➔ Driven states?

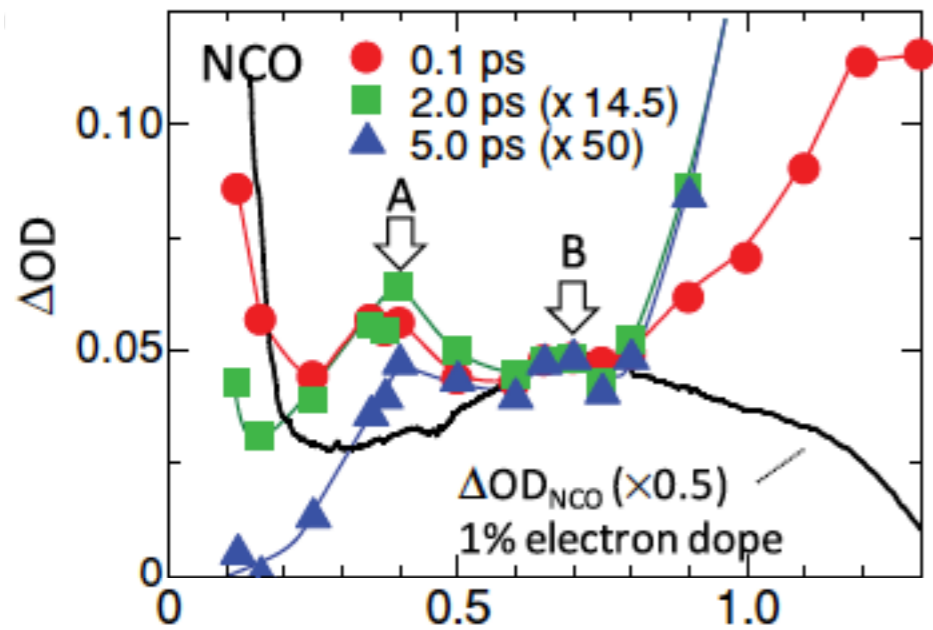


## Relaxation:

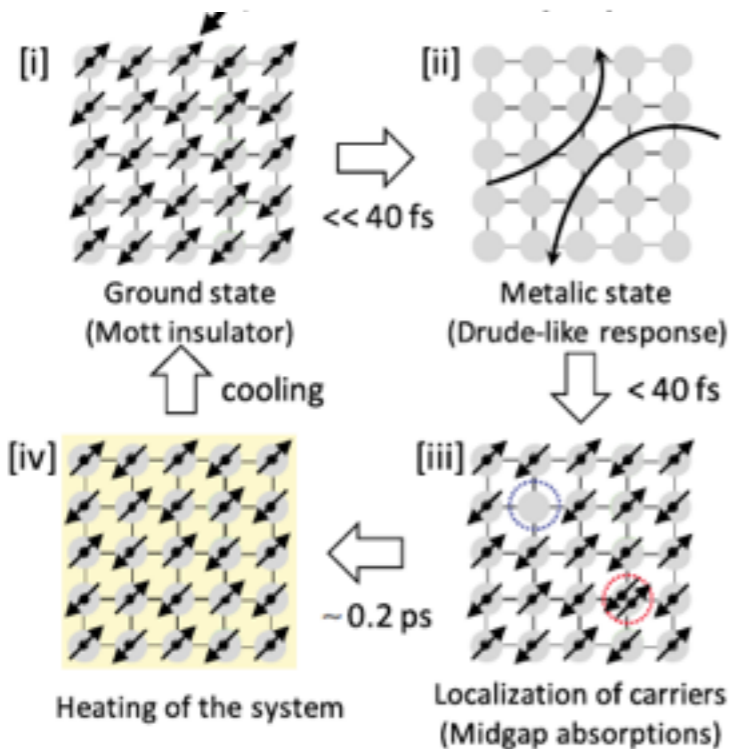
- ➔ Probing interactions?
- ➔ Nonequilibrium quasi-particles?

# # Many-body systems out of equilibrium

## Relaxation/Thermalization

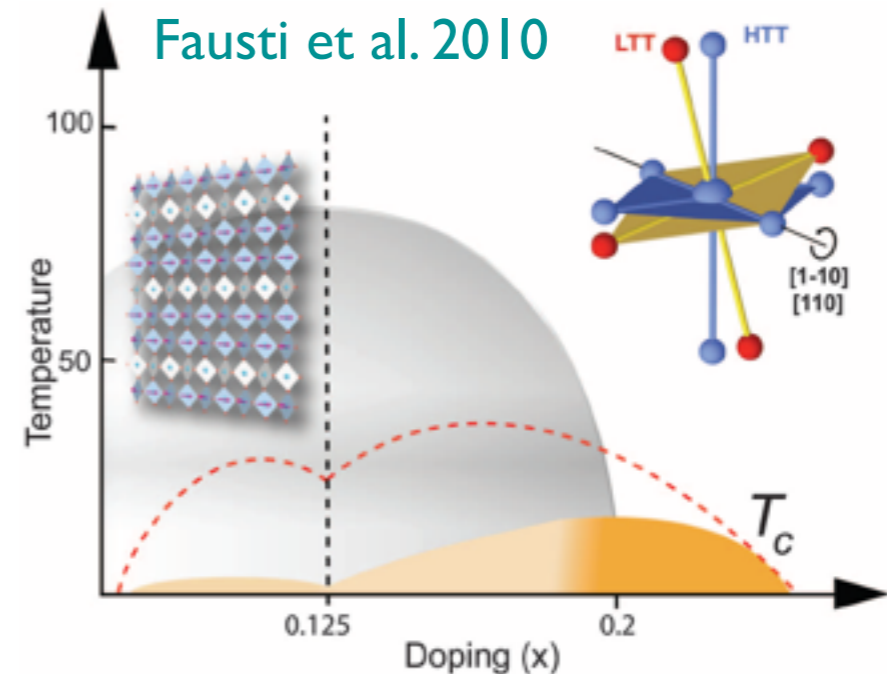


Okamoto et al, 2011



## „Control“

- ➔ Nonthermal transition pathways?
- ➔ Metastable states? Hidden states?
- ➔ Driven states?



## Optical control of magnetism?

Kirilyuk, Kimel & Rasing, RMP 2010

➔ Controlling exchange interactions?

LaMnO<sub>3</sub> Wall, Prabhakaran, Boothroyd & Cavalleri, 2009

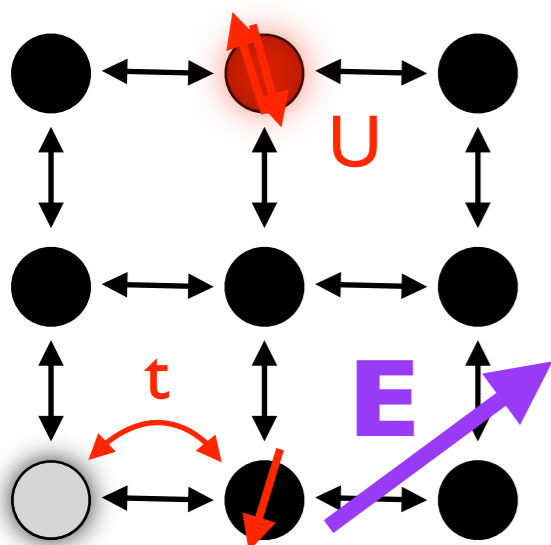
Eu<sub>1-x</sub>Gd<sub>x</sub>O Matsubara et al. arXiv 1304:2509

Relaxation:

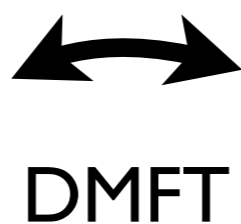
- ➔ Probing interactions?
- ➔ Nonequilibrium quasi-particles?

# #Nonequilibrium DMFT

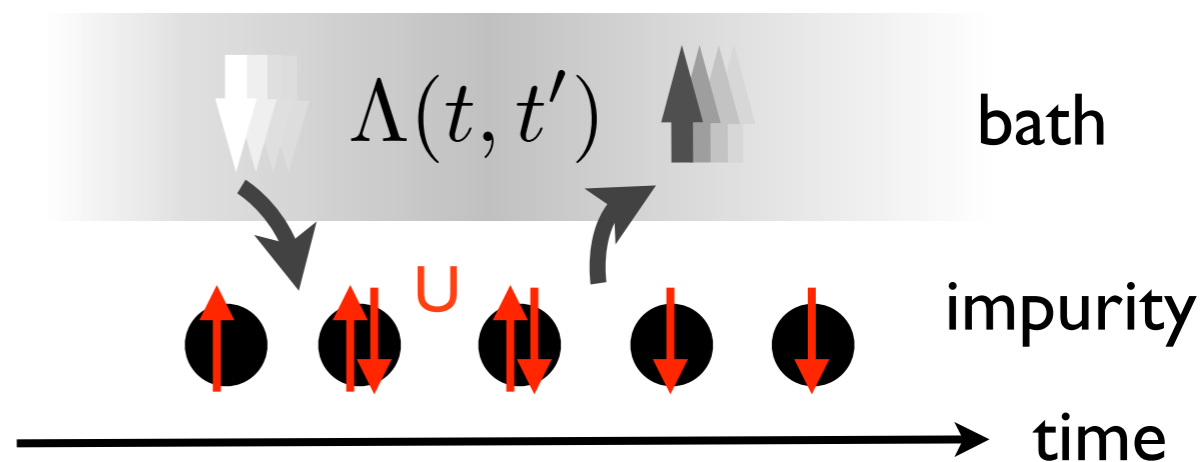
$$H = -t \sum_{\langle ij \rangle \sigma} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} \quad \text{+External field } E(t)$$



Metzner & Vollhardt 1989  
Georges et al, RMP 1996



$$\Sigma(\omega, \mathbf{k}) = \Sigma(\omega)$$

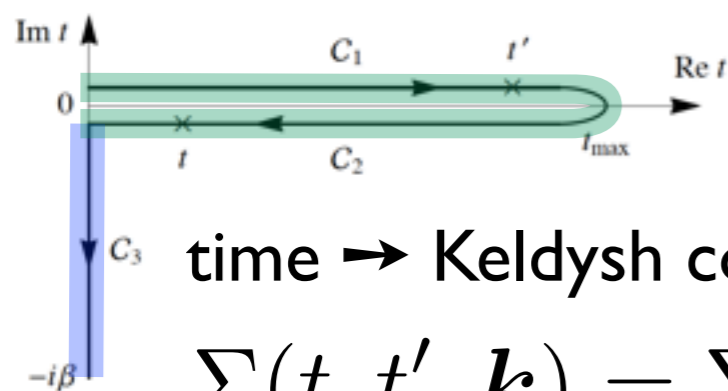


one „impurity problem“ per site,  
coupled by self-consistent bath

## → Nonequilibrium DMFT

Schmidt & Monien 2002

Freericks, Turkowski & Zlatic 2006



time  $\rightarrow$  Keldysh contour

$$\Sigma(t, t', \mathbf{k}) = \Sigma(t, t')$$

Impurity solver:

CTQMC,

Krylov,

*strong-coupling expansion (NCA)* Eckstein & Werner 2010

weak-coupling theory (IPT etc.) Tsuji & Werner 2013

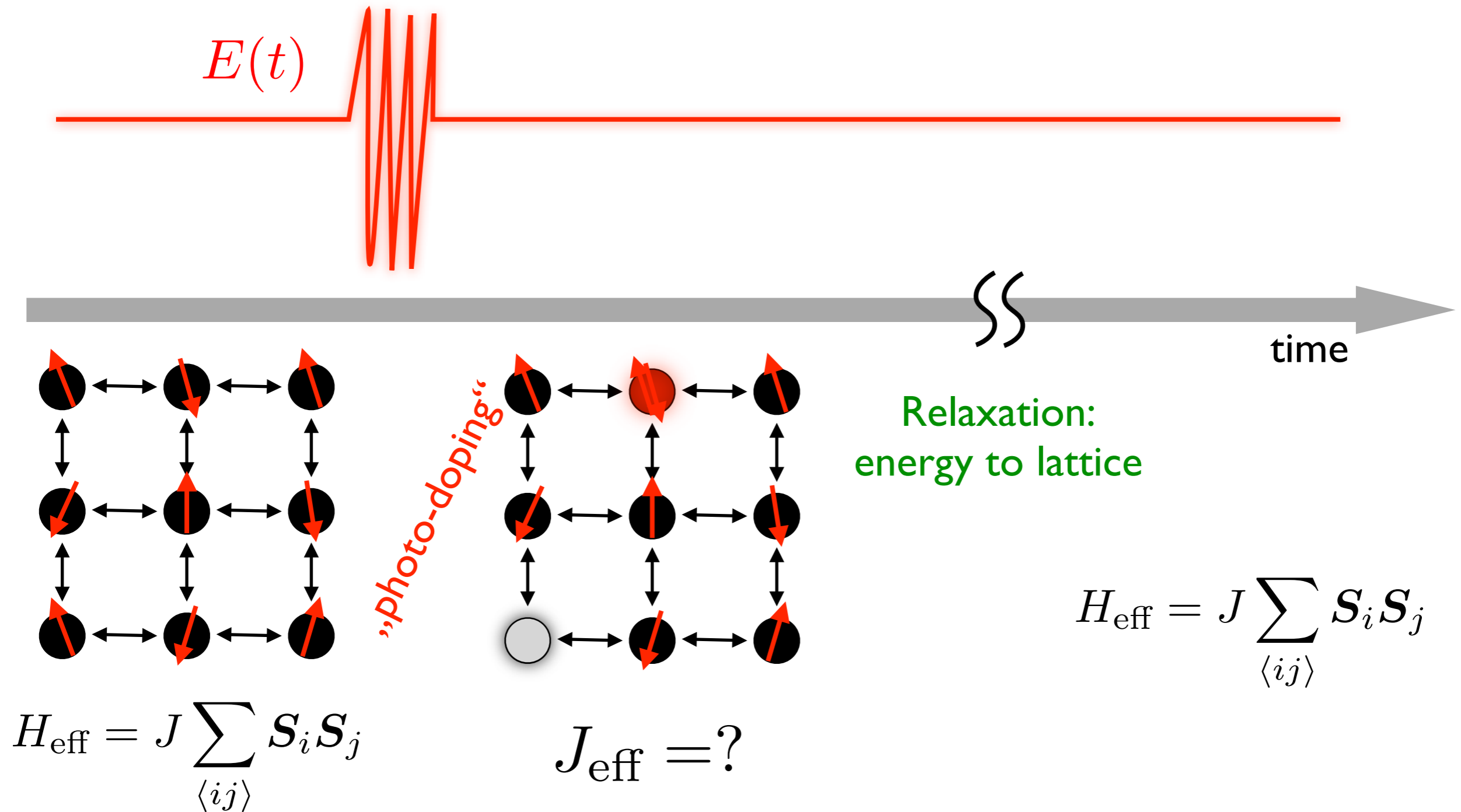
Werner, Oka Millis 2009

Gramsch, Balzer, Eckstein, Kollar 2013

$\Rightarrow$  talk by E. Gull

Review: Aoki, Tsuji, Eckstein, Kollar, Oka & Werner, RMP 2014

# # Modification of $J_{\text{ex}}$ by photo-doping



$$H_{\text{eff}} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

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$$J_{\text{eff}} = ?$$

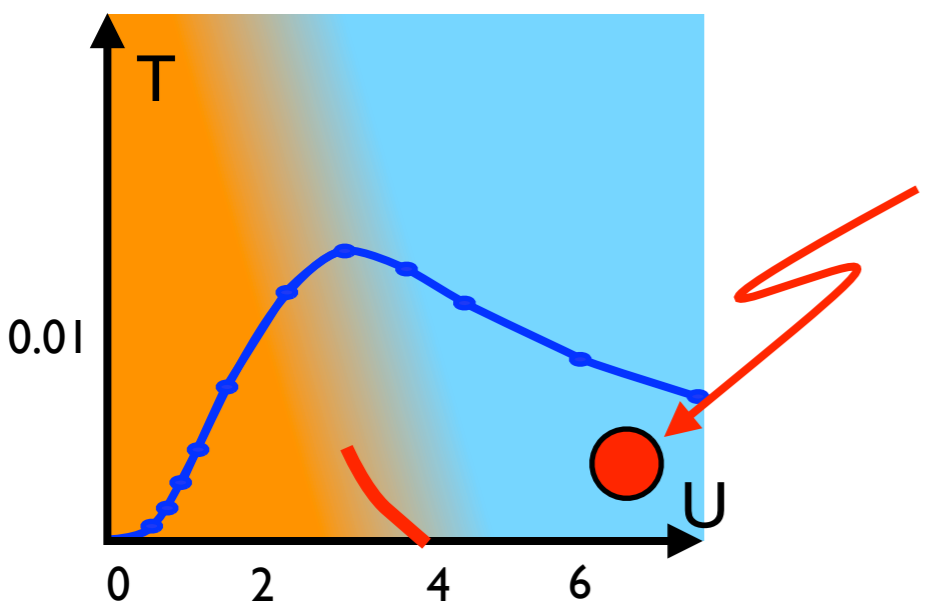
$$J_{\text{ex}} = t^2 / U$$

What is  $J_{\text{ex}}$  in a nonequilibrium state?

How fast can it be changed? (When is  $J_{\text{ex}}$  a useful concept at all?)

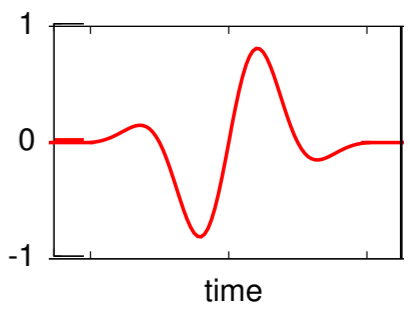
# # Photo-induced melting of the antiferromagnetic phase

see also Tsuji, Eckstein & Werner, 2013 ( $U \ll t$ ); Werner, Tsuji, Eckstein, 2012 ( $U \gg t$ )

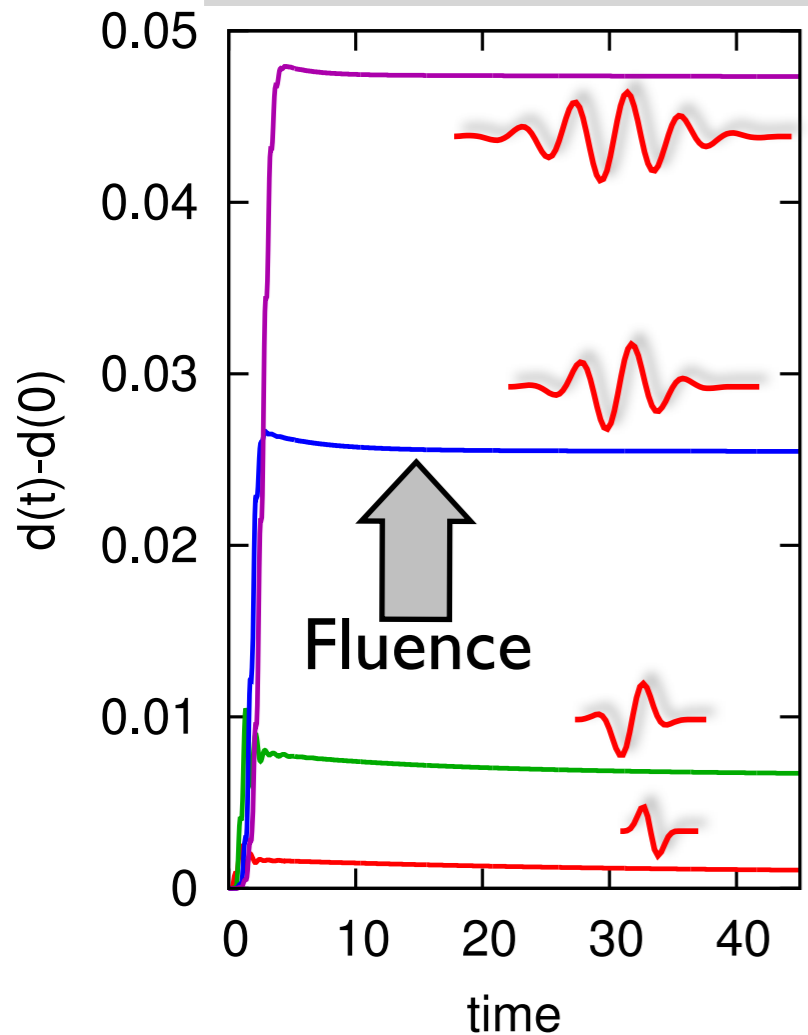


Laser excitation of *antiferromagnetic Mott insulator*  
 Pulse:  $\Omega = U \approx$  Mott gap  
 (hypercubic lattice,  $U=8$ )

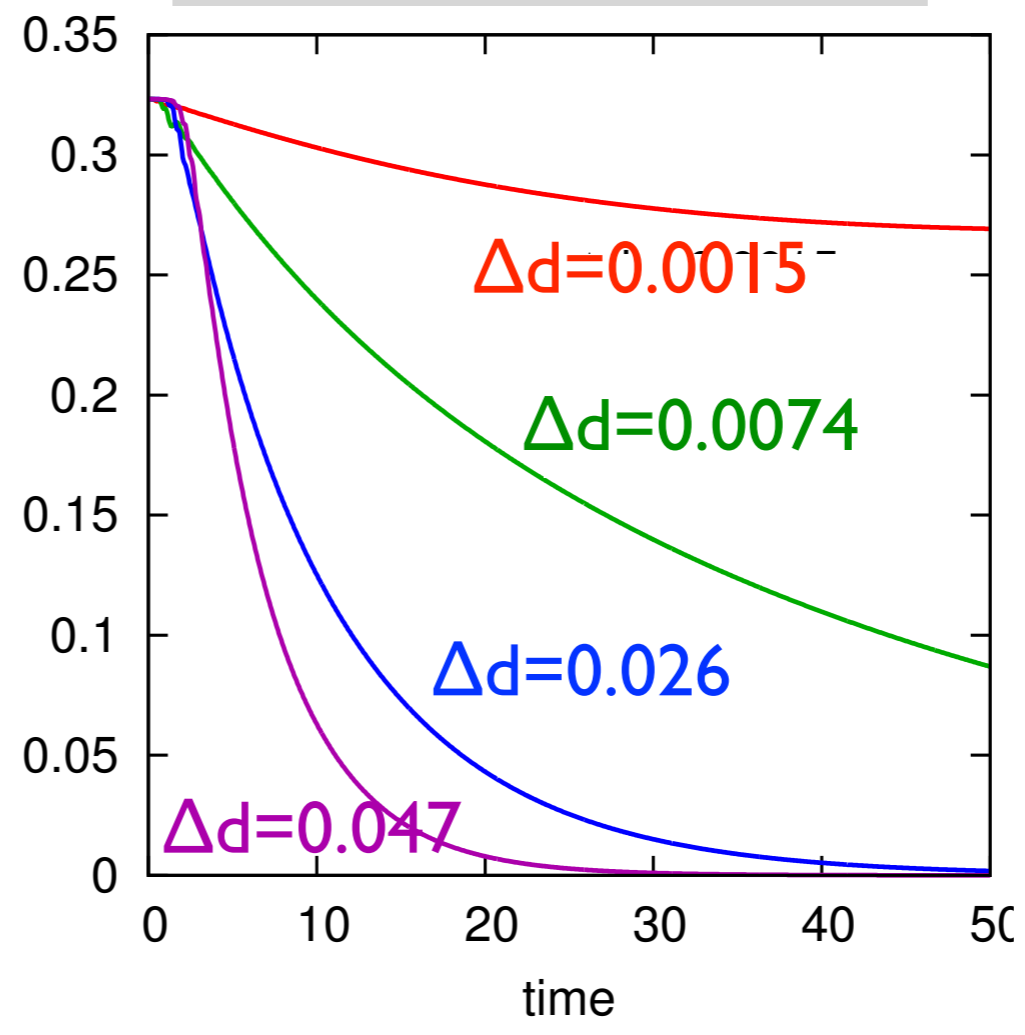
$E(t)$ : frequency  $\approx U$



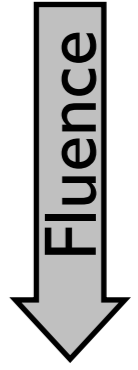
doublon density



AFM order parameter



non-thermal trapped state



ultrafast melting:

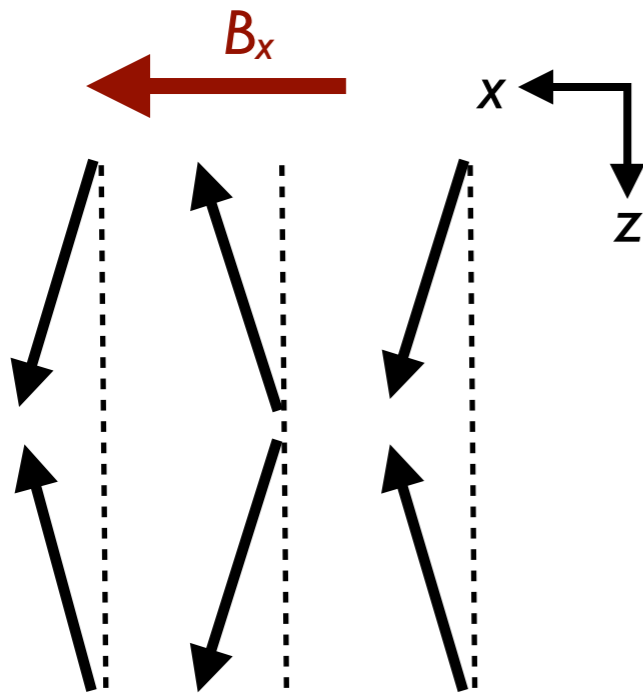


# # Rigid spin dynamics

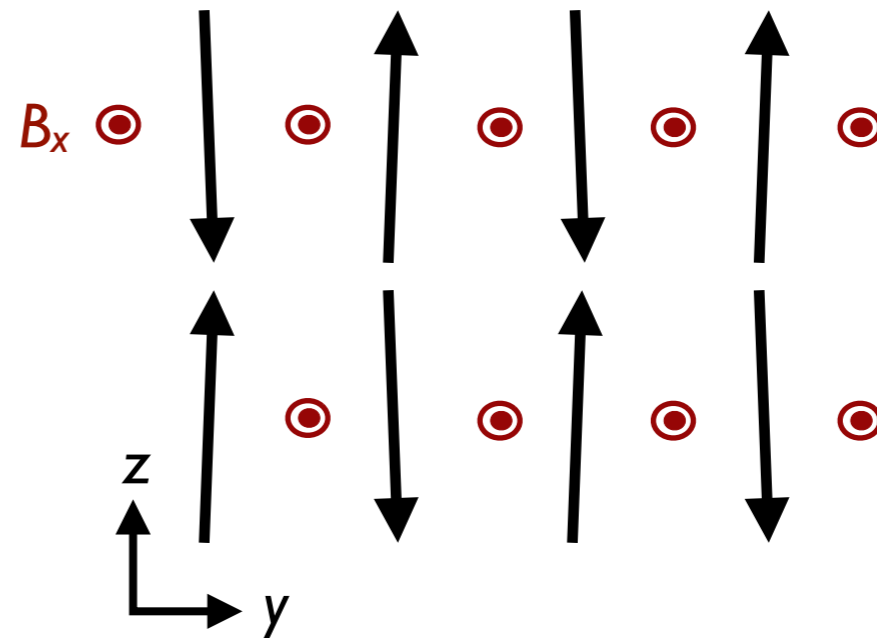
Direct „measurement“ of the exchange:

$$\frac{d\mathbf{S}}{dt} = -\gamma \mathbf{S} \times \left( J_{\text{ex}} \sum_{\text{neighbors } j} \mathbf{S}_j + \mathbf{B} \right) \quad \text{classical mean-field dynamics}$$

AFM in transverse B field: equilibrium



AFM in transverse B field: dynamics



$$S_x = -\frac{B_x}{2\tilde{J}_{\text{ex}}}$$

change of  $J_{\text{ex}} \Rightarrow$  precession around  $B_x$

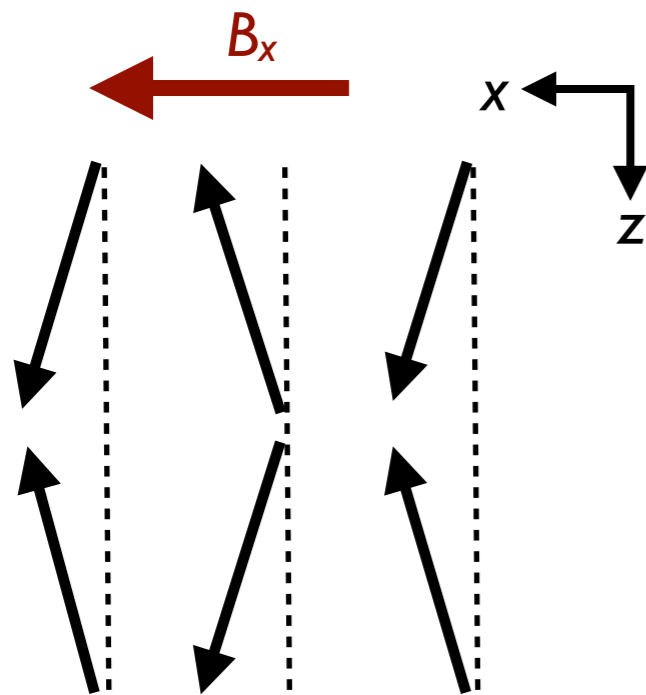
$$\frac{dS_x}{dt} = 0 \quad \frac{dS_y}{dt} = S_z(t) B_x \frac{\Delta J_{\text{ex}}}{J_{\text{ex}}}$$

# # Rigid spin dynamics

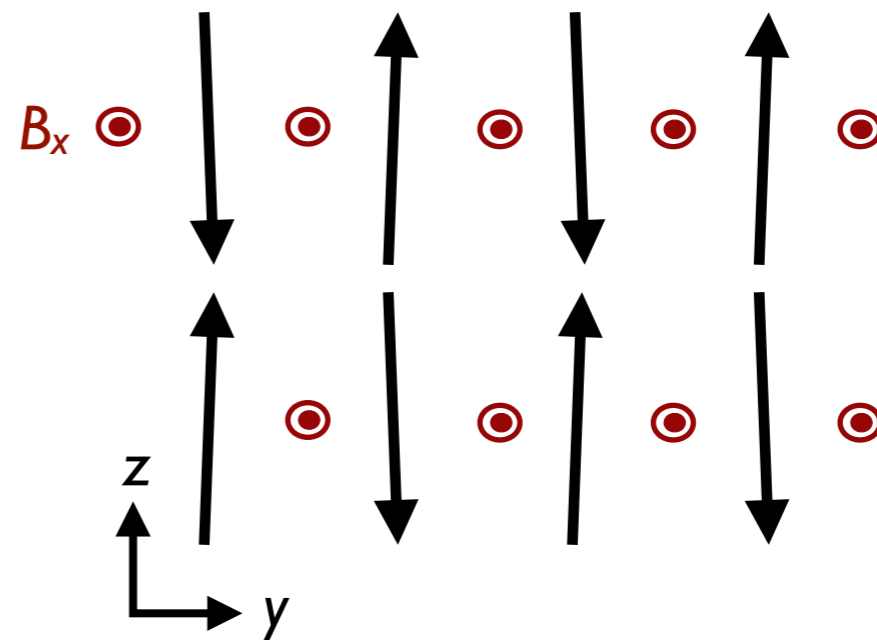
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AFM in transverse B field: equilibrium



AFM in transverse B field: dynamics



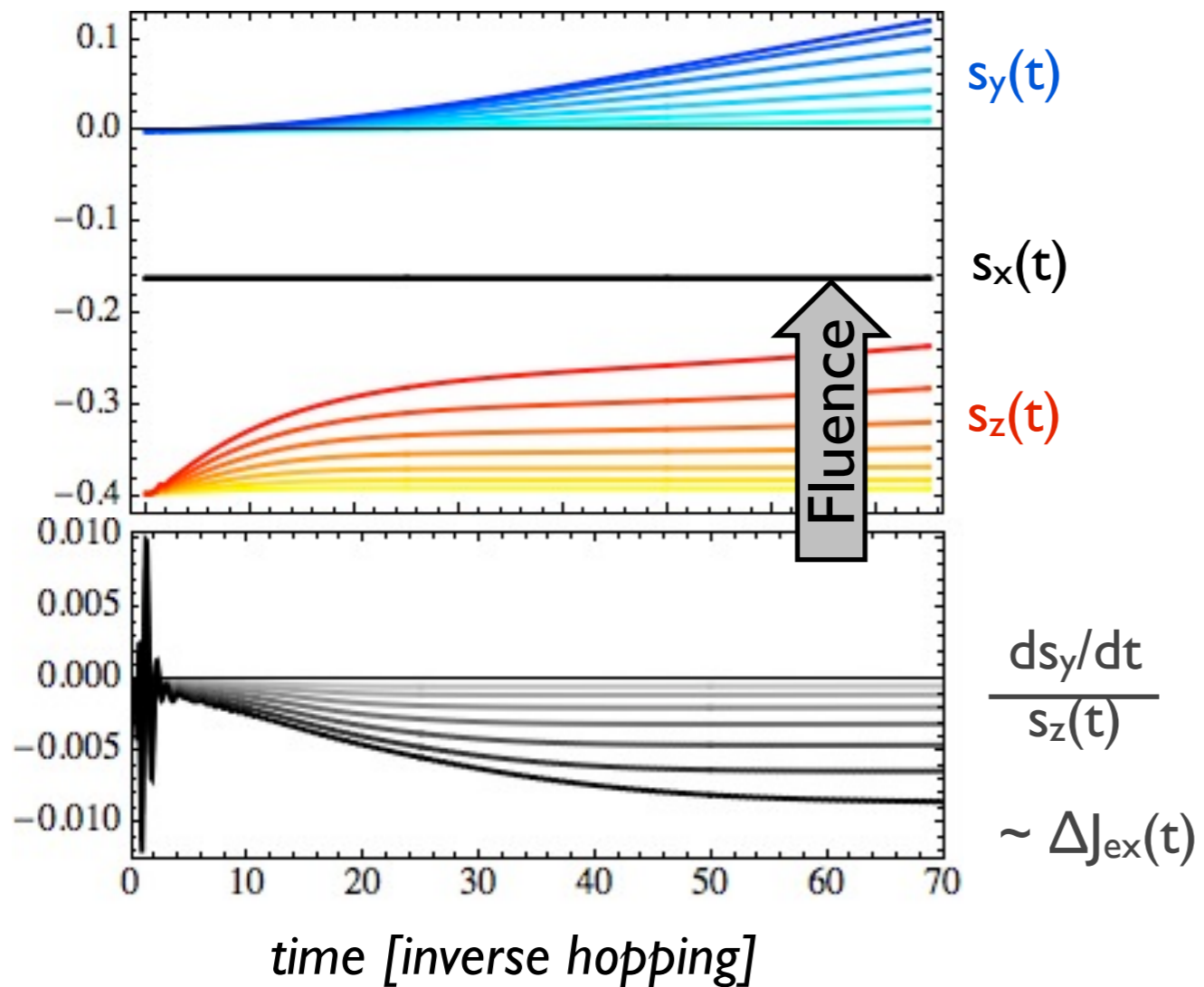
$$S_x = -\frac{B_x}{2\tilde{J}_{\text{ex}}}$$

change of  $J_{\text{ex}} \Rightarrow$  precession around  $B_x$

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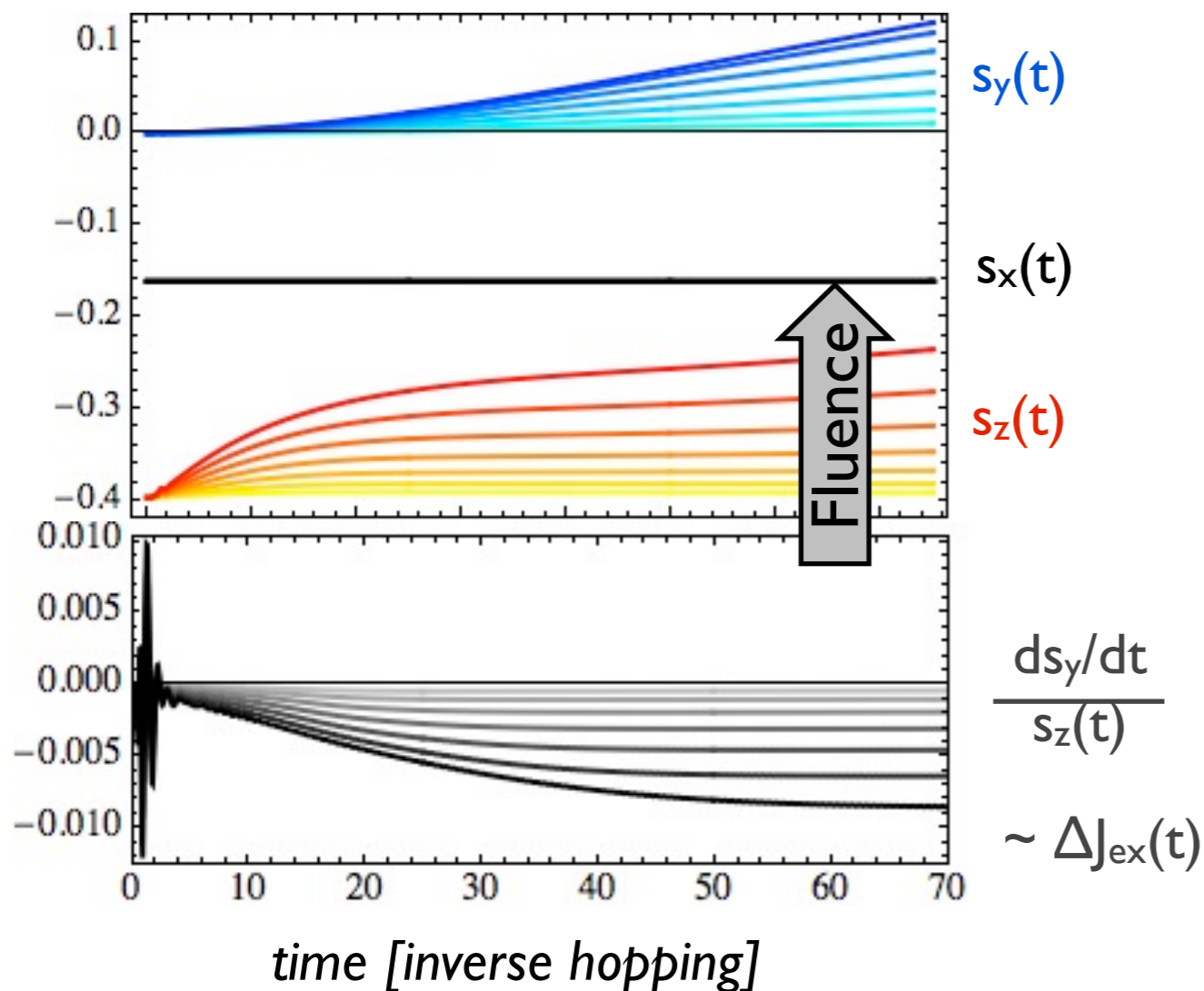
DMFT: AFM in transverse B field

Laser Pulse:  $\Omega = U \approx$  Mott gap  
(hypercubic lattice,  $U=8$ )



DMFT: AFM in transverse B field

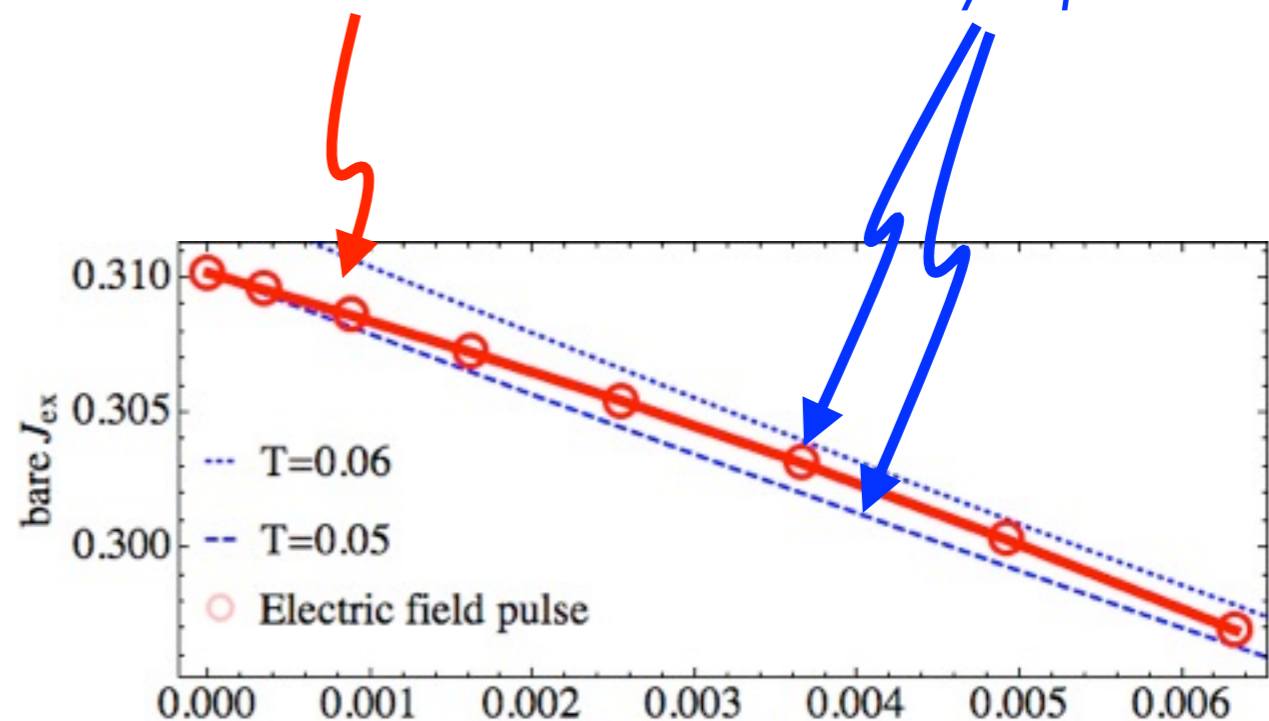
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Exchange  $J_{ex}$ :

photo-doped state

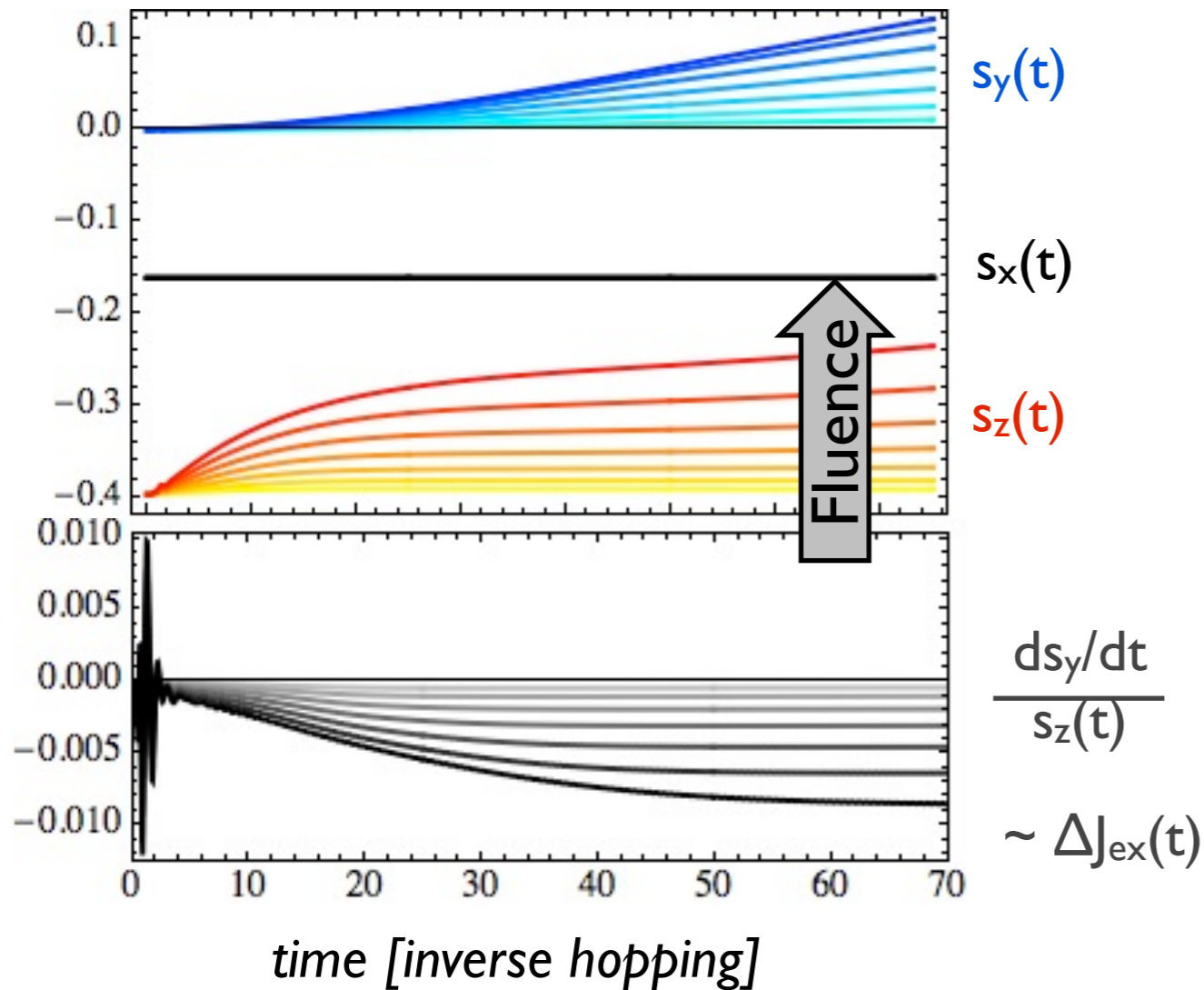
chemically-doped state



chemical doping  $n_{\uparrow} + n_{\downarrow} - 1$   
 photo-doping  $2[\langle n_{\uparrow} n_{\downarrow} \rangle - d(0)]$

DMFT: AFM in transverse B field

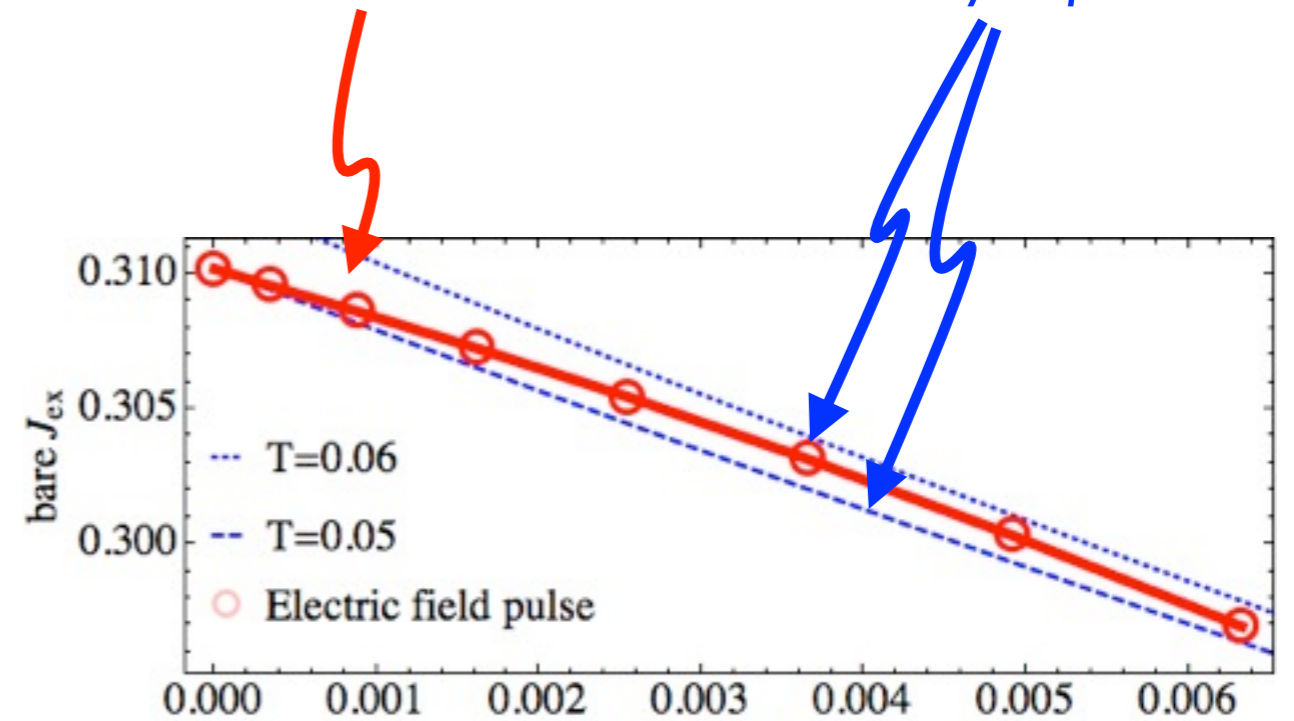
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Exchange  $J_{ex}$ :

photo-doped state

chemically-doped state



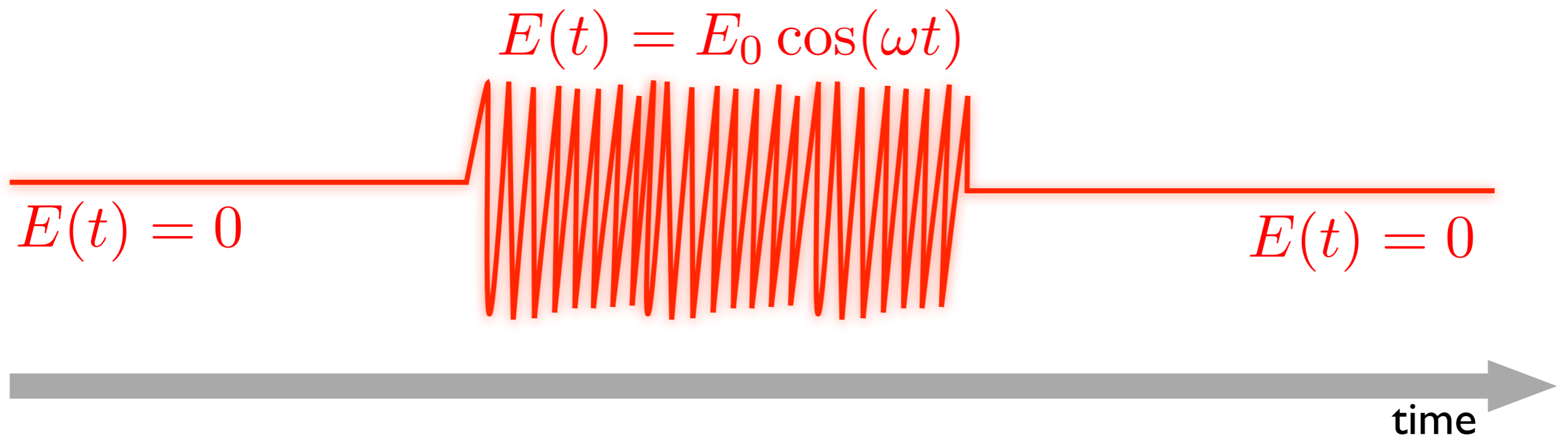
chemical doping  $n_{\uparrow} + n_{\downarrow} - 1$

photo-doping  $2[\langle n_{\uparrow}n_{\downarrow} \rangle - d(0)]$

Modification of  $J_{ex}$  like for chemical doping

Timescale=hopping  $< 1/J_{ex}$

# # Modification of $J_{ex}$ by in a driven state ??

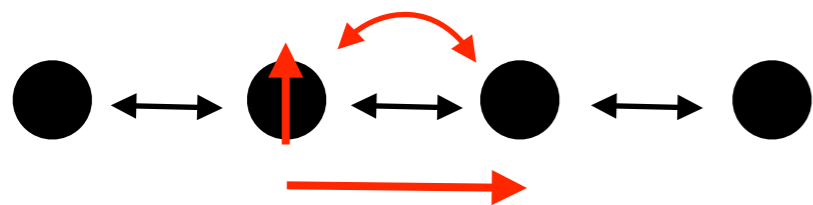


???

$$H_{\text{eff}} = J \sum_{\langle ij \rangle} \mathbf{S}_i \mathbf{S}_j$$
$$H_{\text{eff}} = -J \sum_{\langle ij \rangle} \mathbf{S}_i \mathbf{S}_j$$
$$H_{\text{eff}} = J \sum_{\langle ij \rangle} \mathbf{S}_i \mathbf{S}_j$$

Reversible control of  $J_{ex}$  under the off-resonant action of laser?

# # Coherent destruction of tunneling



$$E(t) = E_0 \cos(\omega t)$$

$$\epsilon(\mathbf{k}) \rightarrow \epsilon(\mathbf{k} + \mathbf{A}(t))$$

$$A(t) = \frac{E_0}{\omega} \cos(\omega t - \pi/2)$$

⇒ time-averaged dispersion:

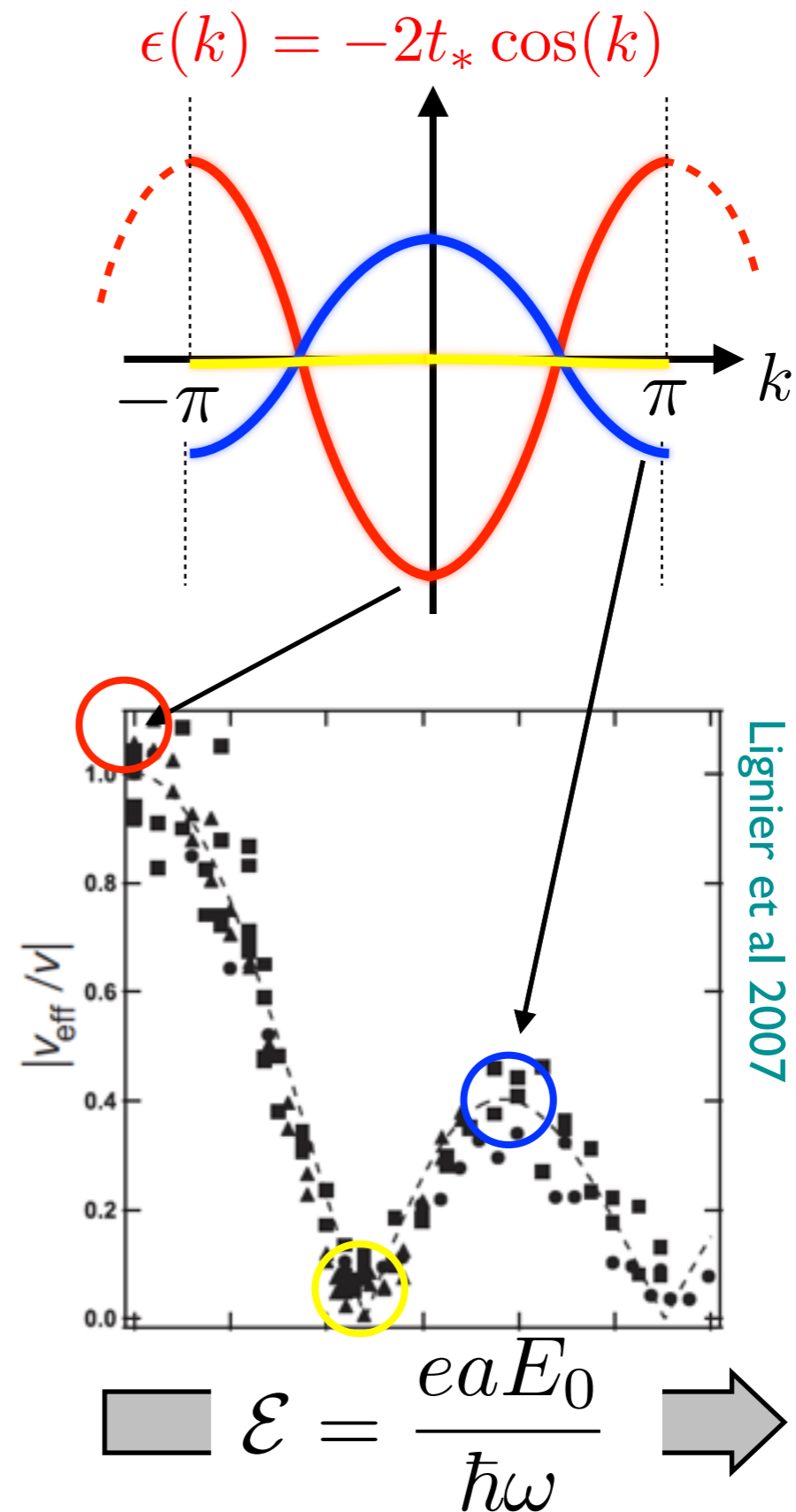
$$\begin{aligned} \bar{\epsilon}(k) &\equiv \frac{1}{T} \int_0^T dt \epsilon(k + A(t)) \\ &= J_0(\mathcal{E}) \epsilon(k) \end{aligned}$$

Grossmann & Haenggi 1992

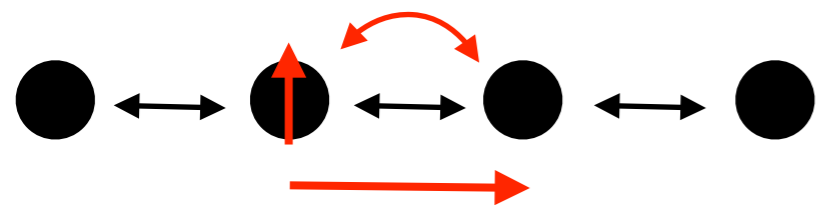
Struck et al. 2011

Tsuji et al. 2011

⇒ talk by T. Oka



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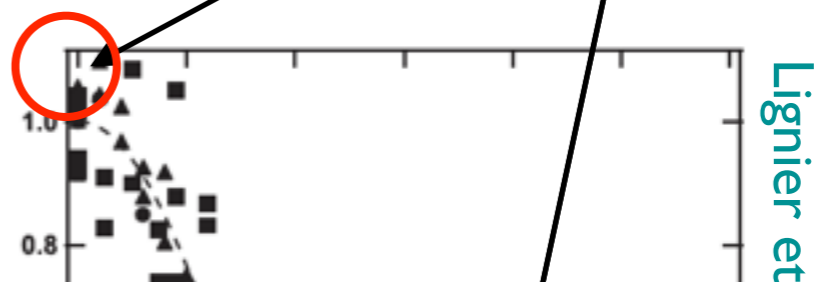
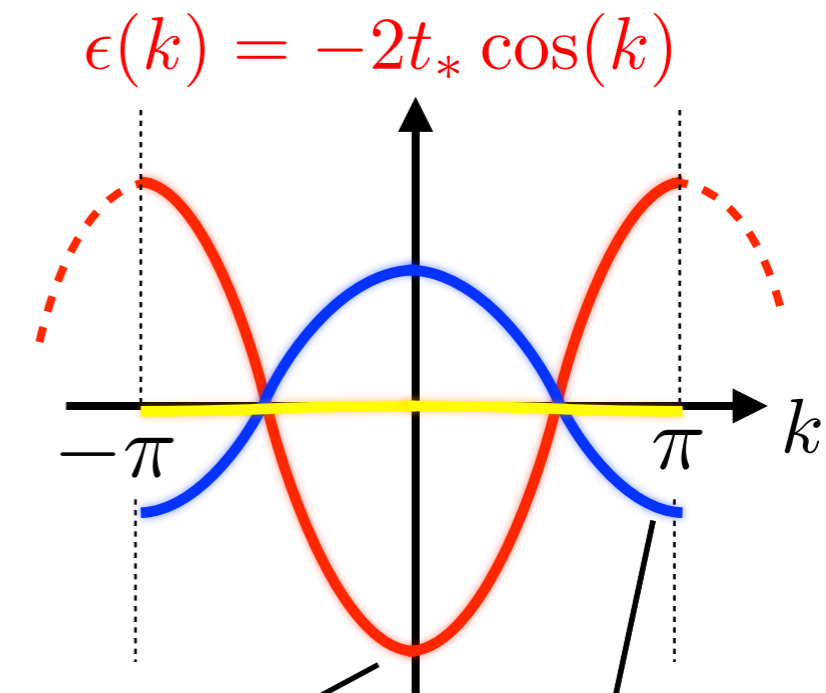
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Grossmann & Haenggi 1992

Struck et al. 2011

Tsuji et al. 2011

⇒ talk by T. Oka



$$J_{\text{ex}}(\mathcal{E}) = J_0(\mathcal{E})^2 J_{\text{ex}}(0)$$



$$\epsilon = \frac{eaE_0}{\hbar\omega}$$

Lignier et



# # Floquet theory

Schrödinger equation for time-periodic Hamiltonian

$$i\partial_t |\Psi(t)\rangle = H(t) |\Psi(t)\rangle$$

$$H(t + T) = H(t)$$

⇒

Solution via Floquet's theorem  
„Bloch theorem in time“

$$|\Psi(t)\rangle = e^{-i\epsilon_\alpha t} |\psi_\alpha(t)\rangle$$

$$|\psi_\alpha(t + T)\rangle = |\psi_\alpha(t)\rangle$$

„adiabatic principle“

Change amplitude/frequency slowly:  $|\dot{A}/A| \ll \omega$

adiabatic evolution into Floquet state

Floquet matrix:

$$|\psi_\alpha(t)\rangle = \sum_n e^{i\omega n t} |\psi_{\alpha,n}\rangle$$

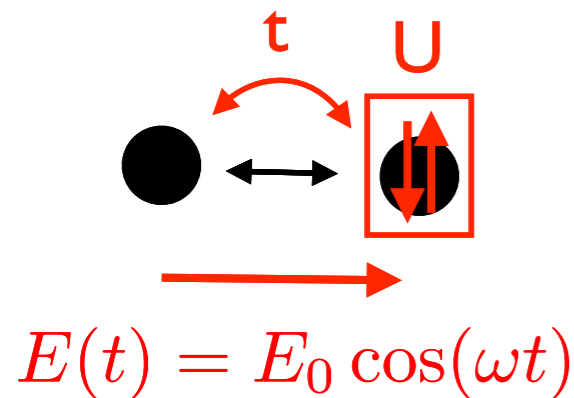
„component in nth Floquet sector“  
„n-photon absorbed state“

$$\begin{pmatrix} \ddots & \ddots & & & & & \\ \ddots & \hat{h}_{-1} & \hat{h}_0 + \omega & \hat{h}_1 & \hat{h}_2 & & \\ & \hat{h}_{-2} & \hat{h}_{-1} & \hat{h}_0 & \hat{h}_1 & & \\ & & \hat{h}_{-2} & \hat{h}_{-1} & \hat{h}_0 - \omega & \ddots & \\ & & & & \ddots & \ddots & \end{pmatrix} \begin{pmatrix} \vdots \\ \psi_1 \\ \psi_0 \\ \psi_1 \\ \vdots \end{pmatrix} = \epsilon \begin{pmatrix} \vdots \\ \psi_1 \\ \psi_0 \\ \psi_1 \\ \vdots \end{pmatrix}$$

$$\hat{h}_n = \frac{1}{T} \int_0^T dt H(t) e^{in\omega t}$$

# #Application to two-site Hubbard model

two-site Hubbard model:



$$|0\rangle|\uparrow\downarrow\rangle, |\uparrow\downarrow\rangle|0\rangle$$

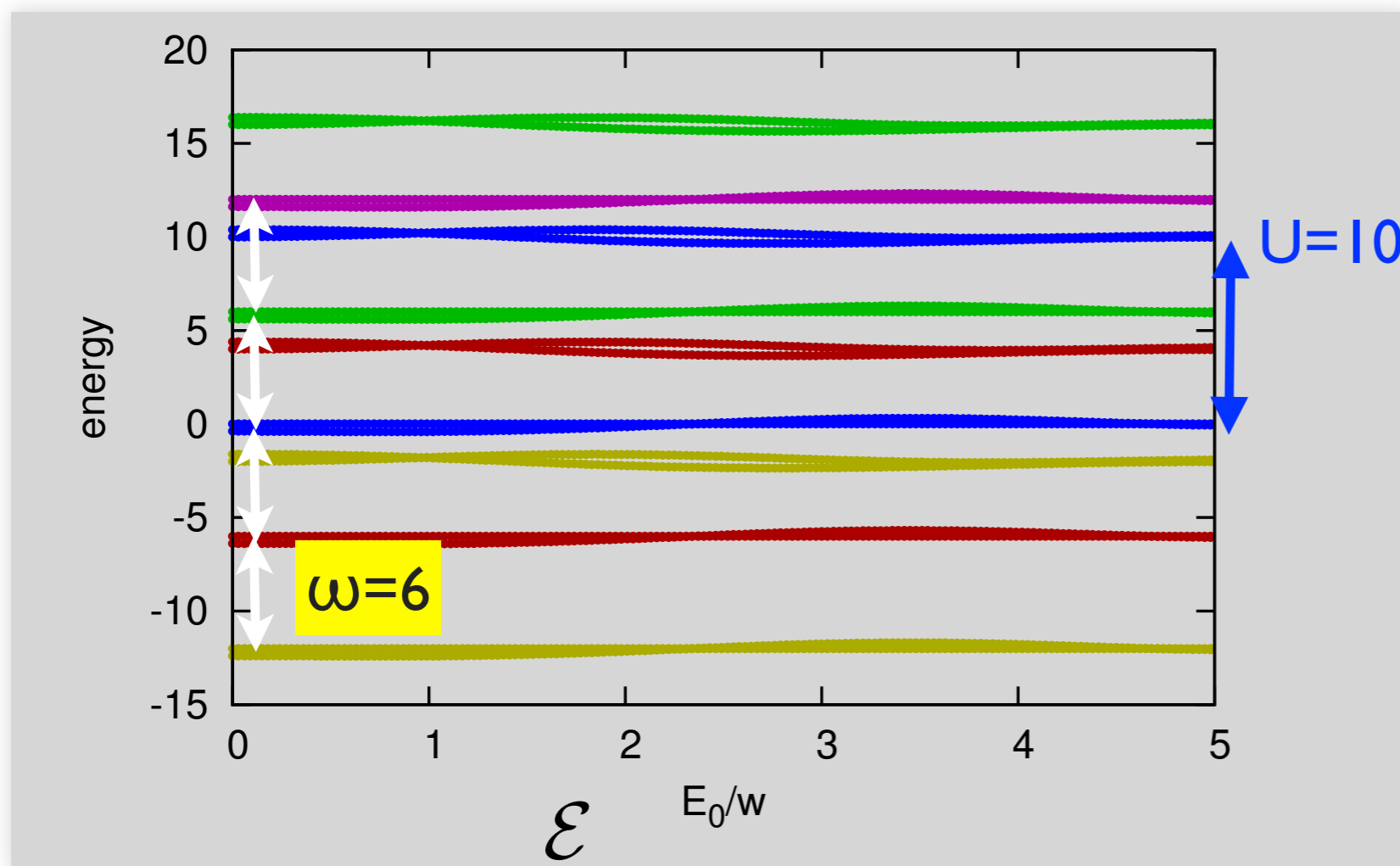
$$(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)/\sqrt{2}$$

$$(|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle)/\sqrt{2}$$

$$E \approx U$$

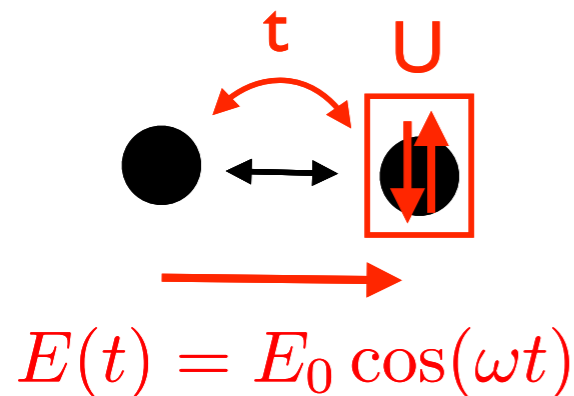
$$E_T = 0$$

$$E_S \approx -4t_*^2/U$$



# #Application to two-site Hubbard model

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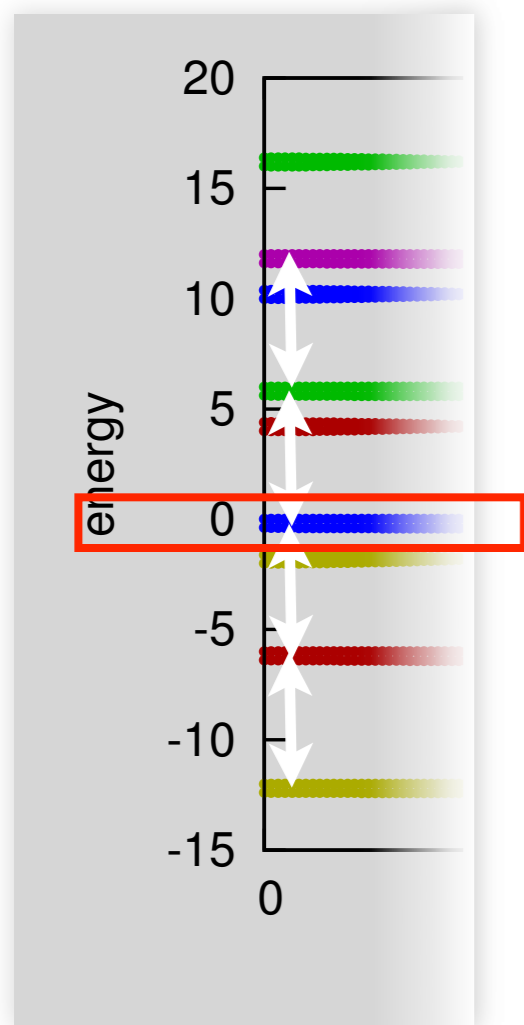
$$E \approx U$$

$$(|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle)/\sqrt{2}$$

$$E_T = 0$$

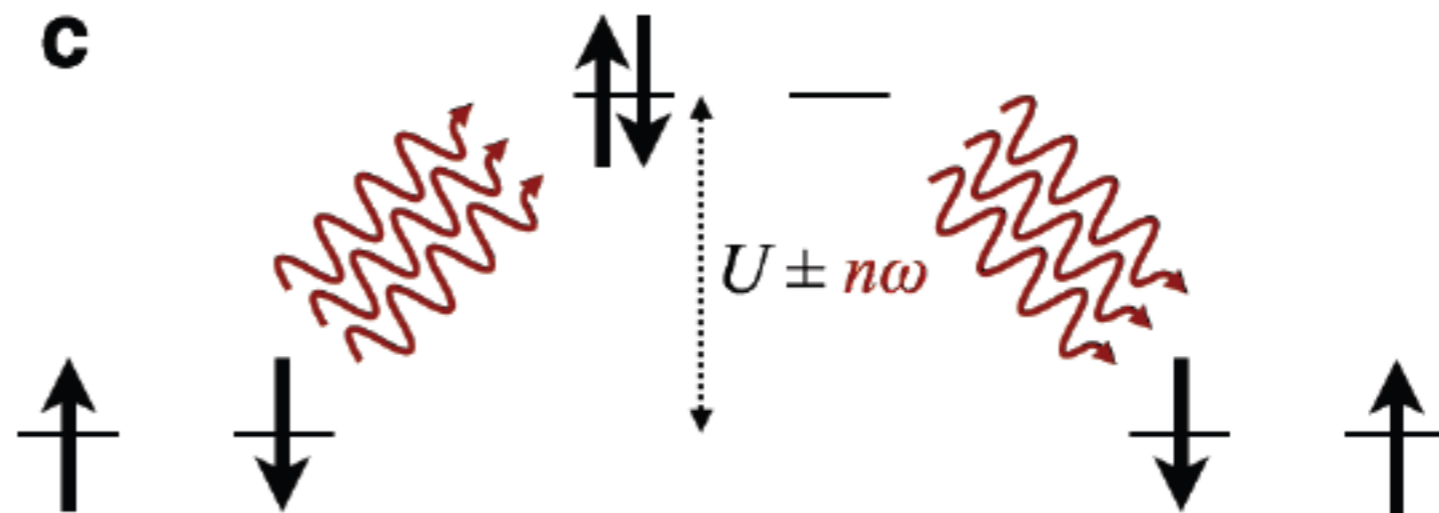
$$(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)/\sqrt{2}$$

$$E_S \approx -4t_*^2/U$$



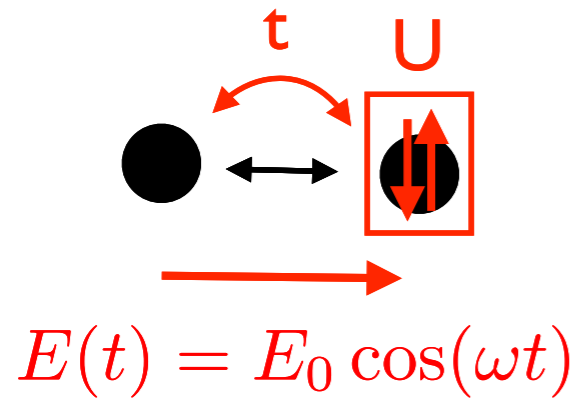
perturbative result  $\mathcal{E} \ll 1$

$$J_{ex} = \frac{2t_0^2}{U} + \Delta J_{ex} \quad \Delta J_{ex} = \frac{\mathcal{E}^2 t_0^2}{2} \left( \frac{1}{U + \omega} + \frac{1}{U - \omega} - \frac{2}{U} \right)$$



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two-site Hubbard model:



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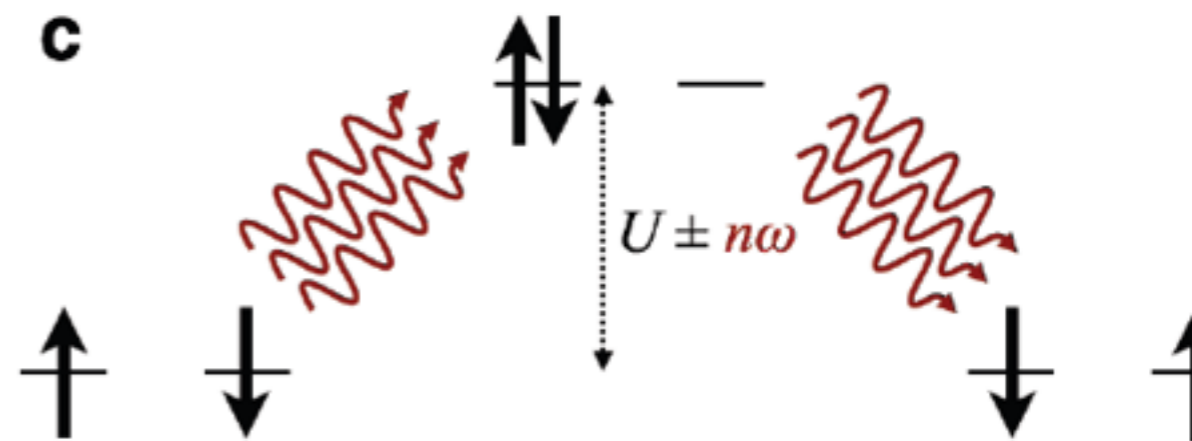
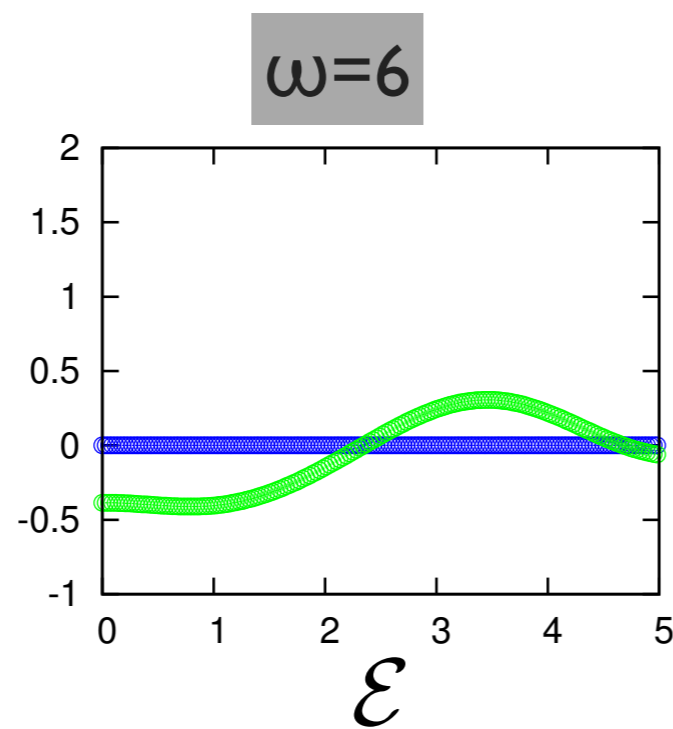
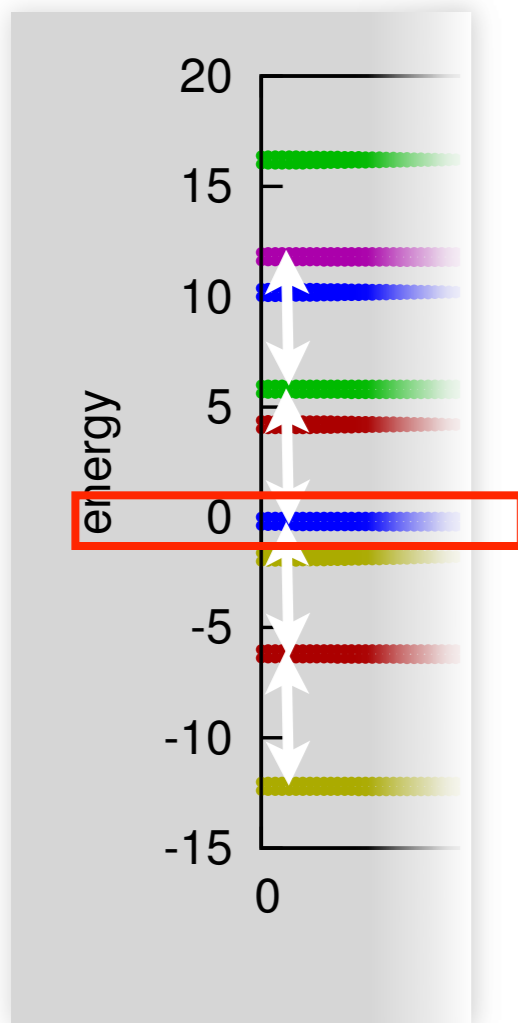
$$(|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle)/\sqrt{2}$$

$$(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)/\sqrt{2}$$

$$E \approx U$$

$$E_T = 0$$

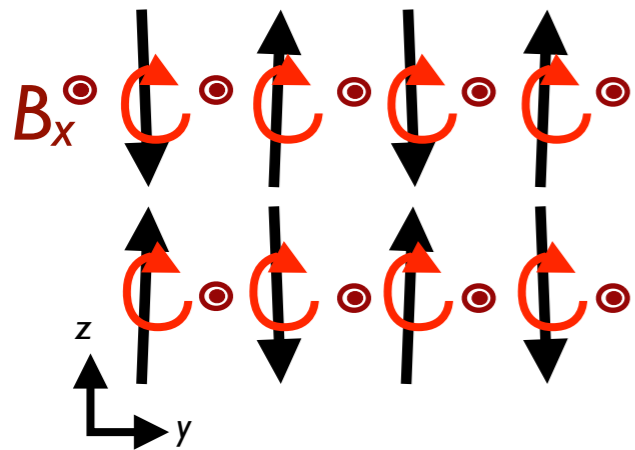
$$E_S \approx -4t_*^2/U$$



# # DMFT ...

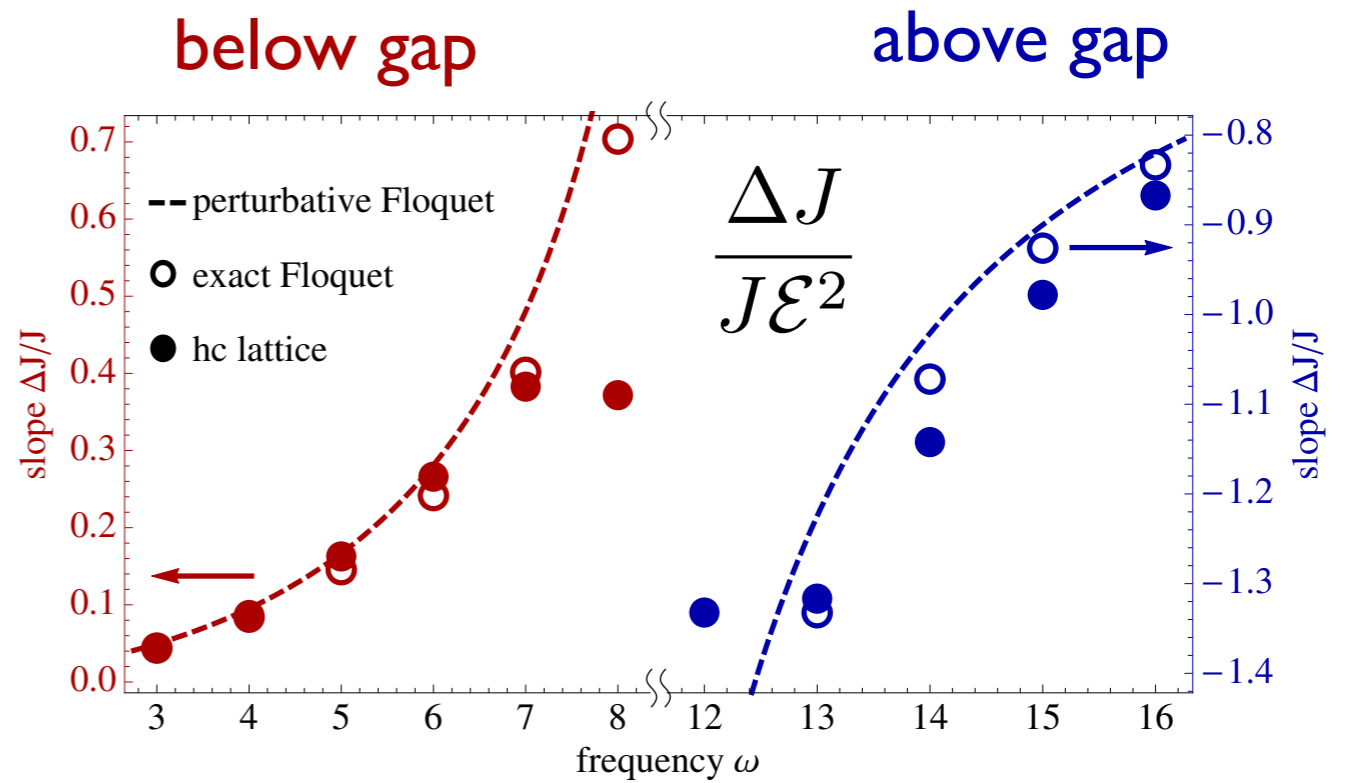
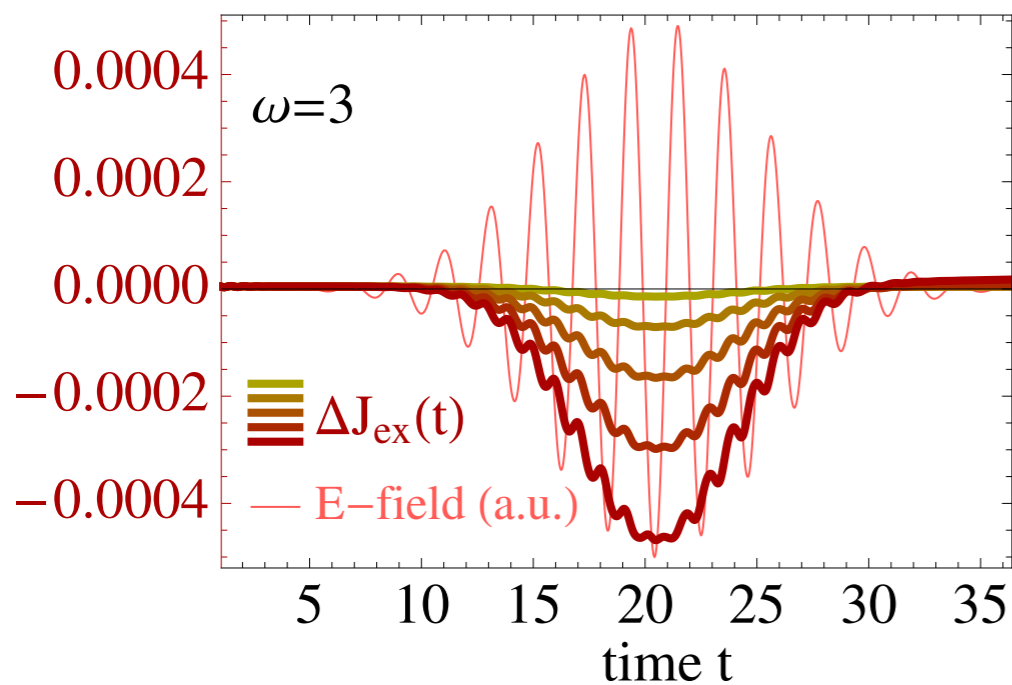
DMFT: AFM in transverse B field + electric field  $E(t) = E_0 \cos(\omega t) \exp\left[-\frac{(t-t_0)^2}{2t_0^2}\right]$

(hypercubic lattice,  $U=10$ )



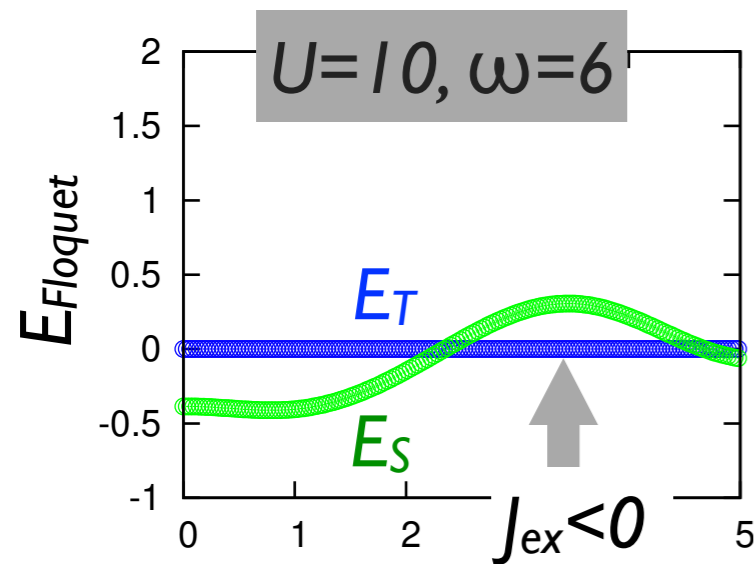
$\Rightarrow$  precession during action of laser:

$$\frac{dS_y}{dt} = S_z(t) B_x \frac{\Delta J_{\text{ex}}}{J_{\text{ex}}}$$



$\Delta J$  follows laser intensity

# # Strong driving: reversing sign of $J_{ex}$



$\Rightarrow$  transition to ferromagnetic state under driving ?

Angular momentum conserved,  
need at least spin-orbit coupling

$\Rightarrow$  evolution of AFM with FM spin-Hamiltonian?

$$\mathcal{U}_{\text{AFM}}(t) = \exp \left( -itJ_{\text{ex}} \sum_{\langle ij \rangle} \mathbf{S}_i \mathbf{S}_j \right)$$

$$\mathcal{U}_{\text{FM}}(t) = \exp \left( -it(-J_{\text{ex}}) \sum_{\langle ij \rangle} \mathbf{S}_i \mathbf{S}_j \right) = \mathcal{U}_{\text{AFM}}(t)^{-1}$$

$\Rightarrow$  „propagation backwards in time“

# # Strong driving: reversing sign of $J_{\text{ex}}$

10-site Hubbard chain  $U=20$

exact time-propagation

initial state = perfect Neel State

$$|\Psi(0)\rangle = c_{0\uparrow}^\dagger c_{1\downarrow}^\dagger \cdots c_{9\uparrow}^\dagger c_{10\downarrow}^\dagger |0\rangle$$

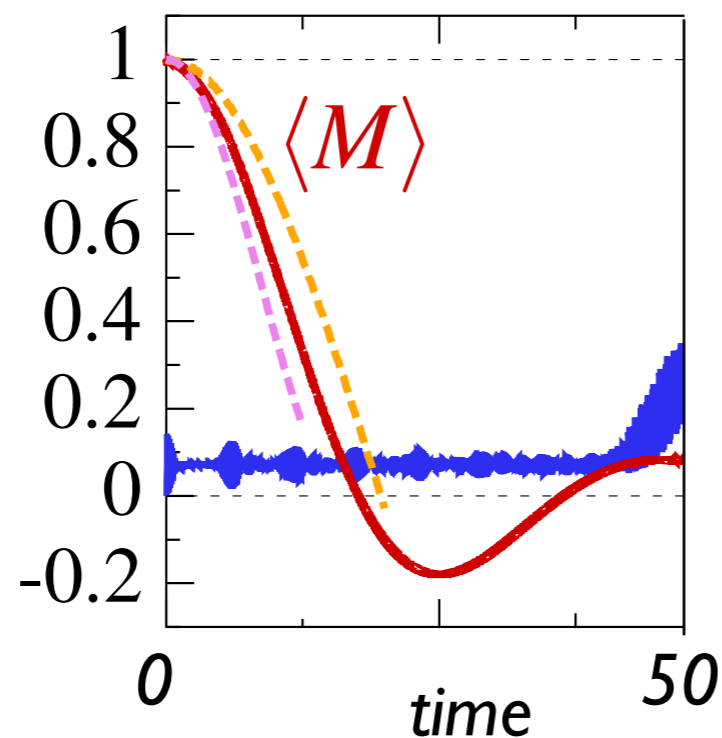
$$= |\uparrow\rangle|\downarrow\rangle|\uparrow\rangle|\downarrow\rangle|\uparrow\rangle|\downarrow\rangle|\uparrow\rangle|\downarrow\rangle|\uparrow\rangle|\downarrow\rangle$$

Free time-evolution

$$H_{\text{Hubbard}} \approx J_{\text{ex}} \sum_{\langle ij \rangle} \mathbf{S}_i \mathbf{S}_j = J_{\text{ex}} \sum_{\langle ij \rangle} S_i^z S_j^z + \frac{1}{2} (S_i^+ S_j^- + S_i^- S_j^+)$$

quantum spin-flip terms

decay of AFM-order



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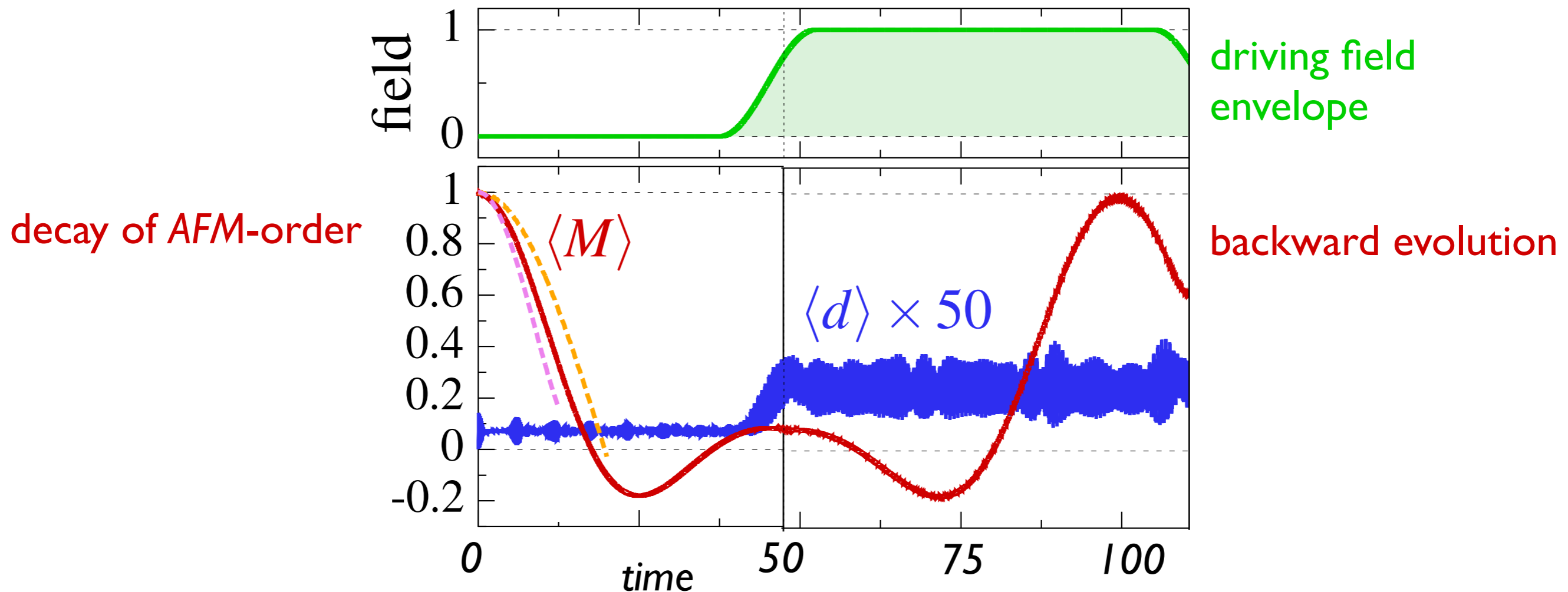
$$|\Psi(0)\rangle = c_{0\uparrow}^\dagger c_{1\downarrow}^\dagger \cdots c_{9\uparrow}^\dagger c_{10\downarrow}^\dagger |0\rangle$$

$$= |\uparrow\rangle|\downarrow\rangle|\uparrow\rangle|\downarrow\rangle|\uparrow\rangle|\downarrow\rangle|\uparrow\rangle|\downarrow\rangle|\uparrow\rangle|\downarrow\rangle$$

Free time-evolution

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quantum spin-flip terms





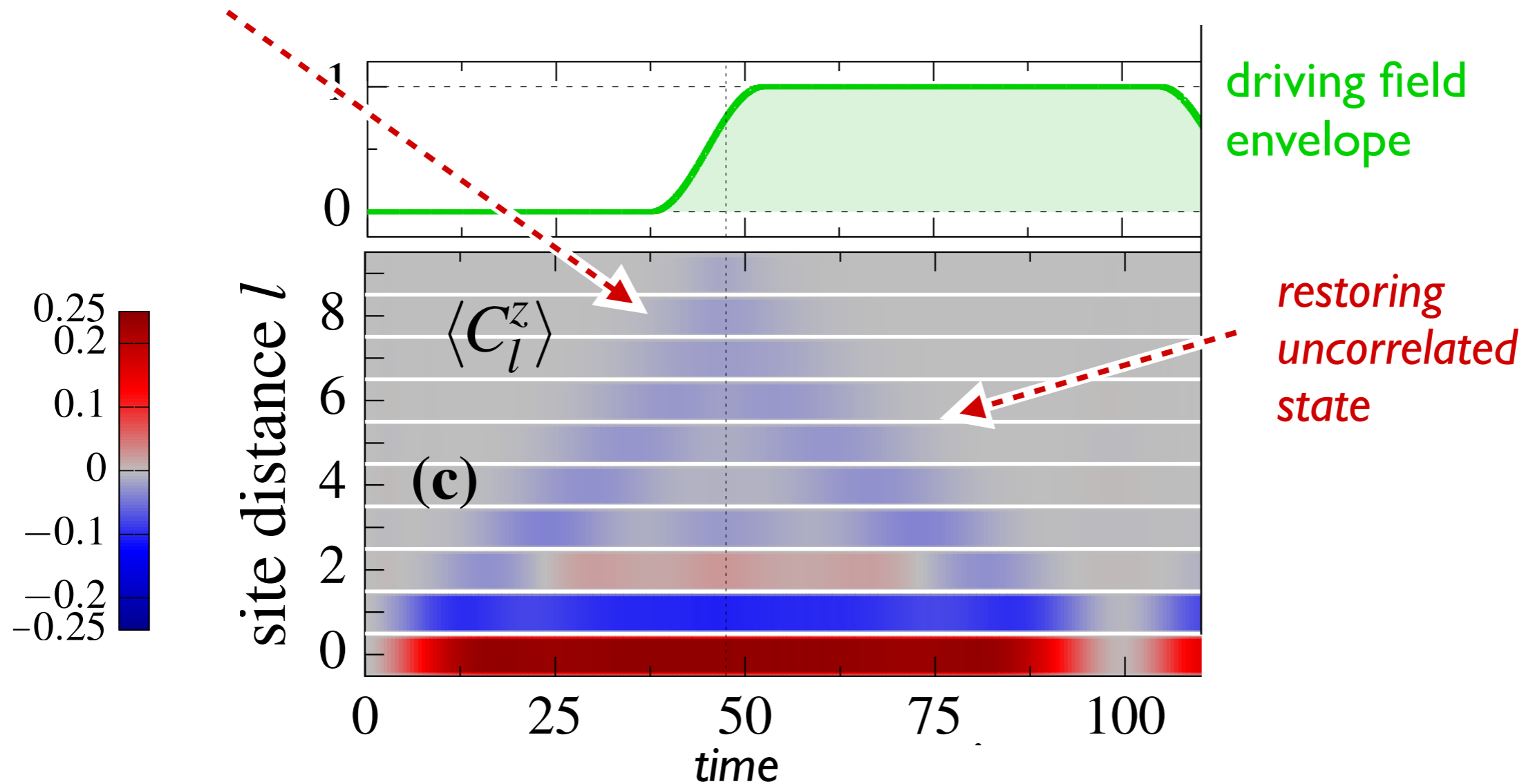
# # Strong driving: reversing sign of $J_{ex}$

10-site Hubbard chain  $U=20$   
exact time-propagation  
initial state = perfect Neel State

$$|\Psi(0)\rangle = c_{0\uparrow}^\dagger c_{1\downarrow}^\dagger \cdots c_{9\uparrow}^\dagger c_{10\downarrow}^\dagger |0\rangle$$
$$= |\uparrow\rangle |\downarrow\rangle |\uparrow\rangle |\downarrow\rangle |\uparrow\rangle |\downarrow\rangle |\uparrow\rangle |\downarrow\rangle |\uparrow\rangle |\downarrow\rangle$$

buildup of short-range correlations

$$C(l) = \langle S_0^z S_l^z \rangle - \langle S_0^z \rangle \langle S_l^z \rangle$$



# # Conclusion

The exchange interaction can be controlled with electric field of the laser

- Photo-excited states with modified exchange:  
Effect on exchange interaction ( $\sim 2\%$ ) similar to  
chemical doping

Mentink & Eckstein, arXiv 1401.5308

- exchange in laser-driven state

- anything is possible ( $J_{\text{ex}} \rightarrow -J_{\text{ex}}$ )
- driven superexchange:  $\Delta J/J \sim 0.1\%$

Mentink, Balzer, Eckstein, to be submitted