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# NUMERICAL ATTEMPTS TO OBSERVE DECONFINED CRITICALITY



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# Collaborators



## QMC Group of CMSI

Kyoto: **Harada**

Hyogo: **Suzuki**

Fudan: **Lou**

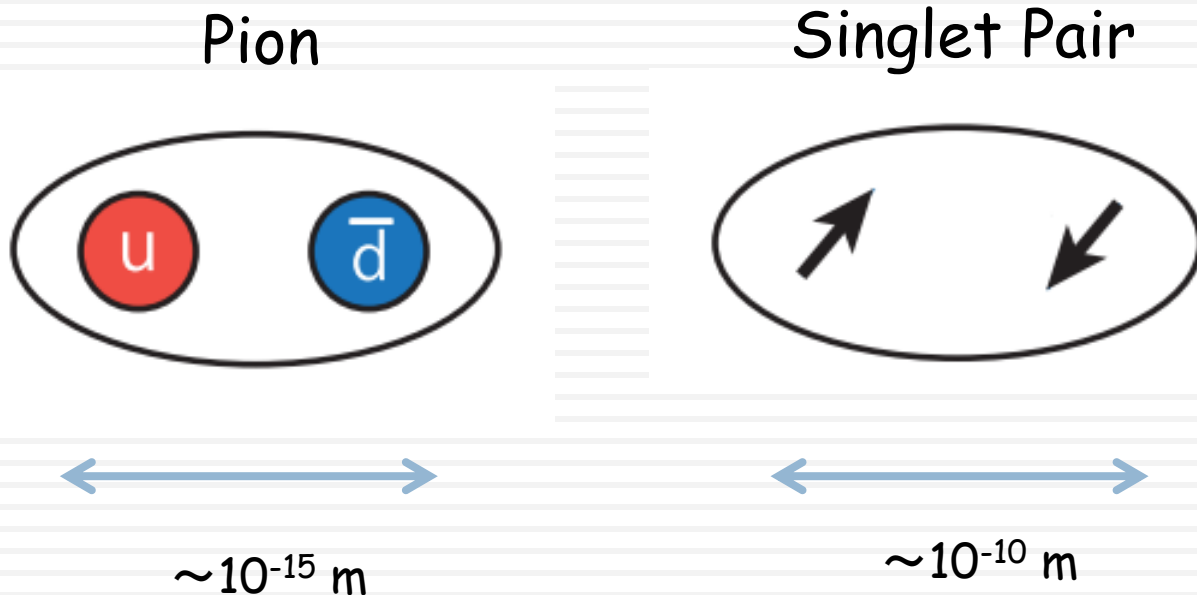
ISSP (Kashiwa): **Okubo, Masaki, Watanabe, Igarashi**

ISSP (Kobe): **Todo, Sakashita**

Fujitsu: **Shitara**

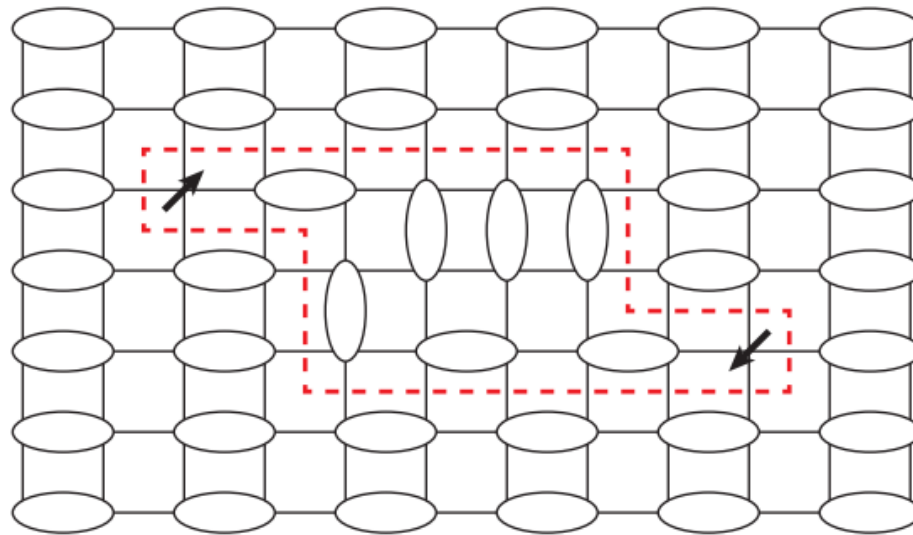
Special thanks to **Matsuo (RIST)**

# Confinement



Governed by a similar mechanism,  
in spite of big difference in scale.

# Confinement of Spinons



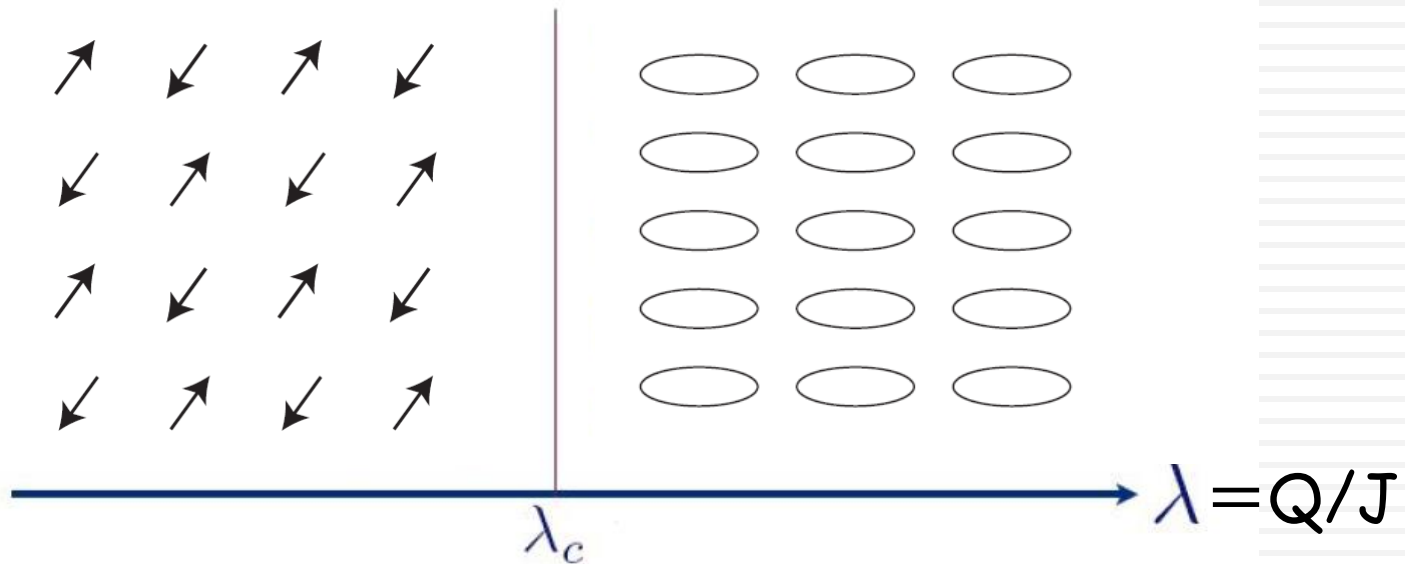
○ = ↗ ↘ - ↙ ↖

Breaking a singlet and creating two spinons cause the damage in the background texture proportional to the distance.

⇒ confinement potential

# Magnetic/Non-Magnetic Transition

Symmetry-breaking may occur **spontaneously**.



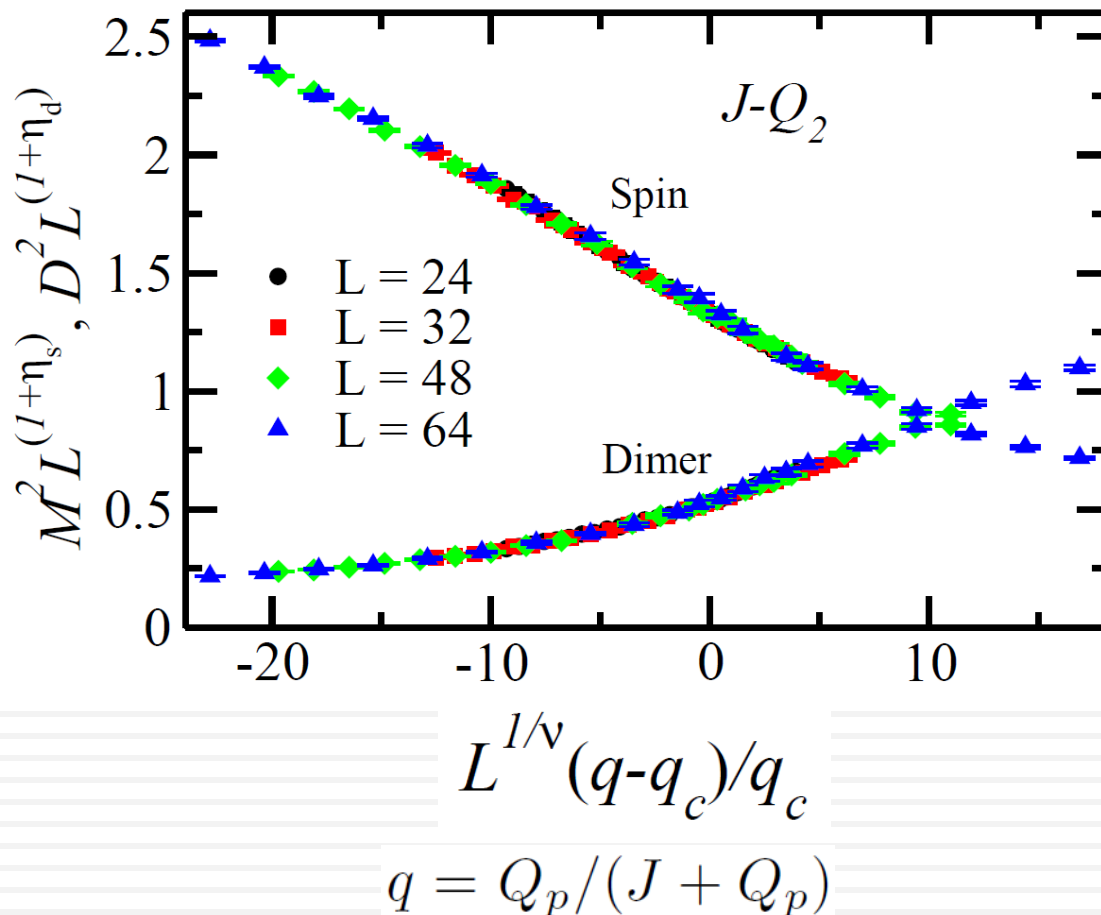
$$H = J \sum_{(ij)} S_i \cdot S_j - Q \sum_{p=(i,j,k,l)} \left( \frac{1}{4} - S_i \cdot S_j \right) \left( \frac{1}{4} - S_k \cdot S_l \right)$$

(Sandvik, 2007)

--- What universality class?

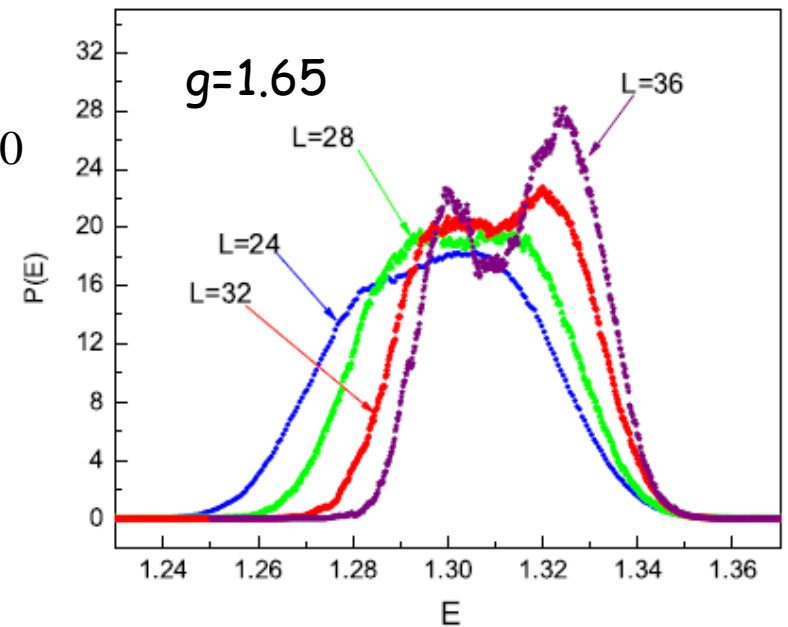
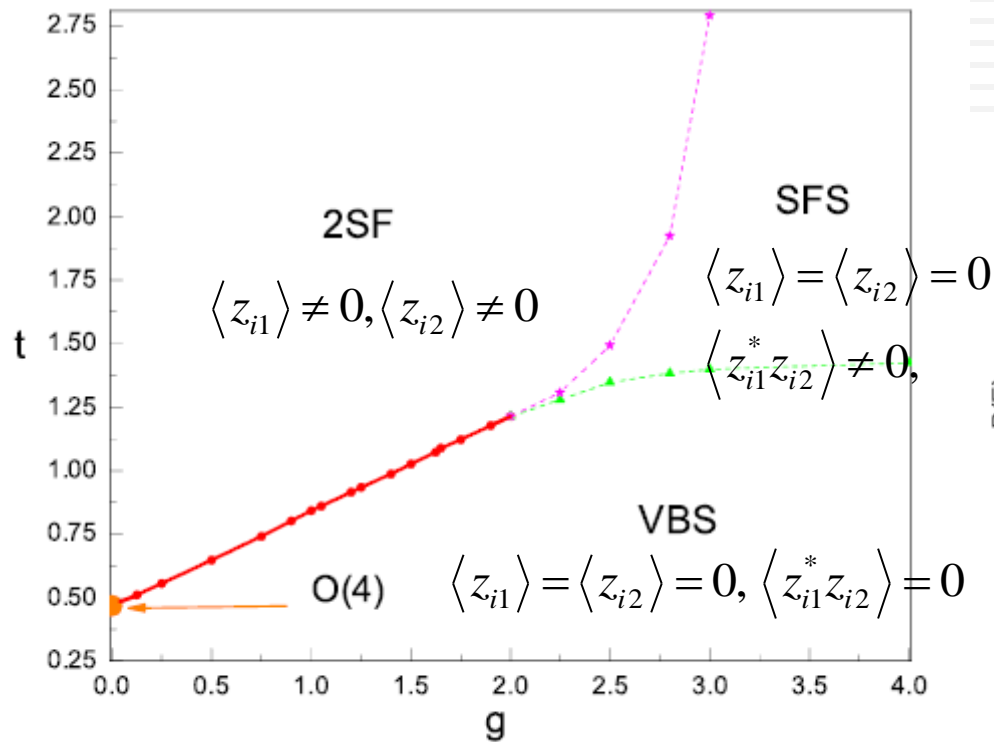
# Deconfined Critical Phenomena?

## SU(2) J-Q Model



Scaling plot works beautifully.

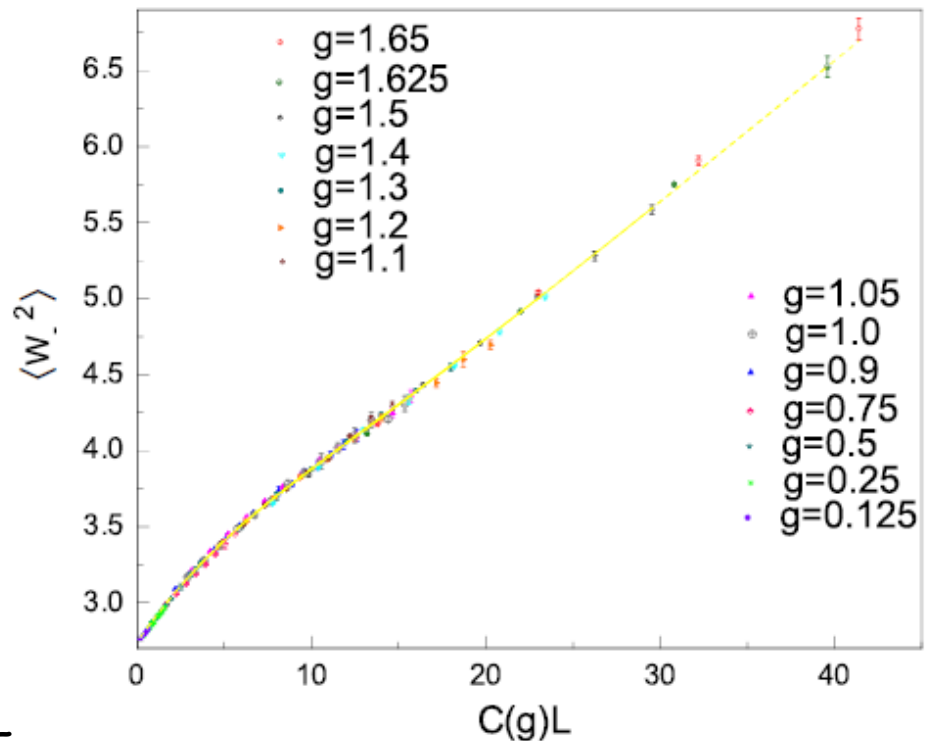
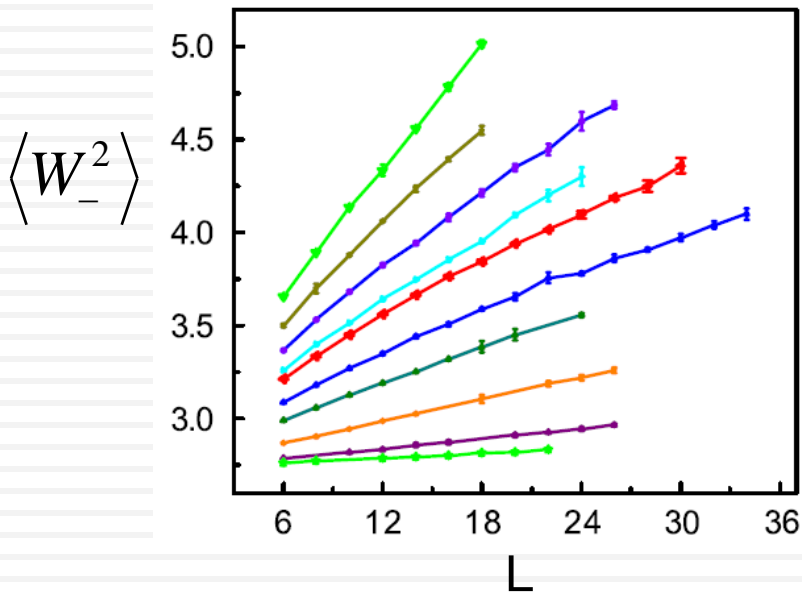
# SU(2) Symmetric NCCP<sup>1</sup> Model



$$S = -t \sum_{\langle ij \rangle} \sum_{\alpha=1,2} \left( z_{i\alpha}^* z_{j\alpha} e^{iA_{ij}} + \text{c.c.} \right) + \frac{1}{8g} \sum_{\square} (\nabla \times A)^2 \quad \left( \sum_{\alpha=1,2} |z_{j\alpha}|^2 = 1 \right)$$

# Week First-Order Transition?

Kuklov, Matsumoto, et al, PRL 101, 050405 (2008)



The coupling constant  $t$  is adjusted so that  $w > 0$  with the probability 0.75.

- ✓ Maybe too small to observe a turn around.
- ✓ Just one particular model.

$$\langle W_-^2 \rangle = f(C(g)L)$$

$$C(g) \equiv \frac{e^{bg} - 1}{e^{bg_0} - 1} \quad (b = 2.28, \quad g_0 = 1.3)$$



# SU(3) and SU(4) J-Q2 Models

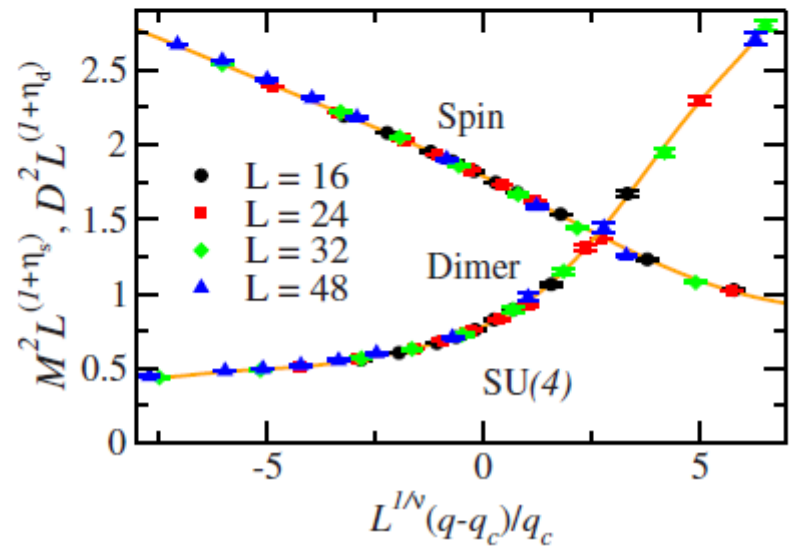
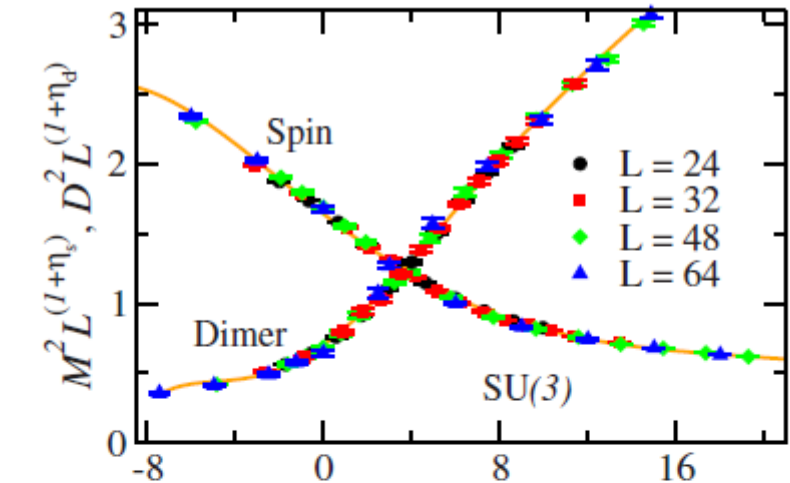
J. Lou, A. Sandvik, N.K (2009)

## SU(3) J-Q2

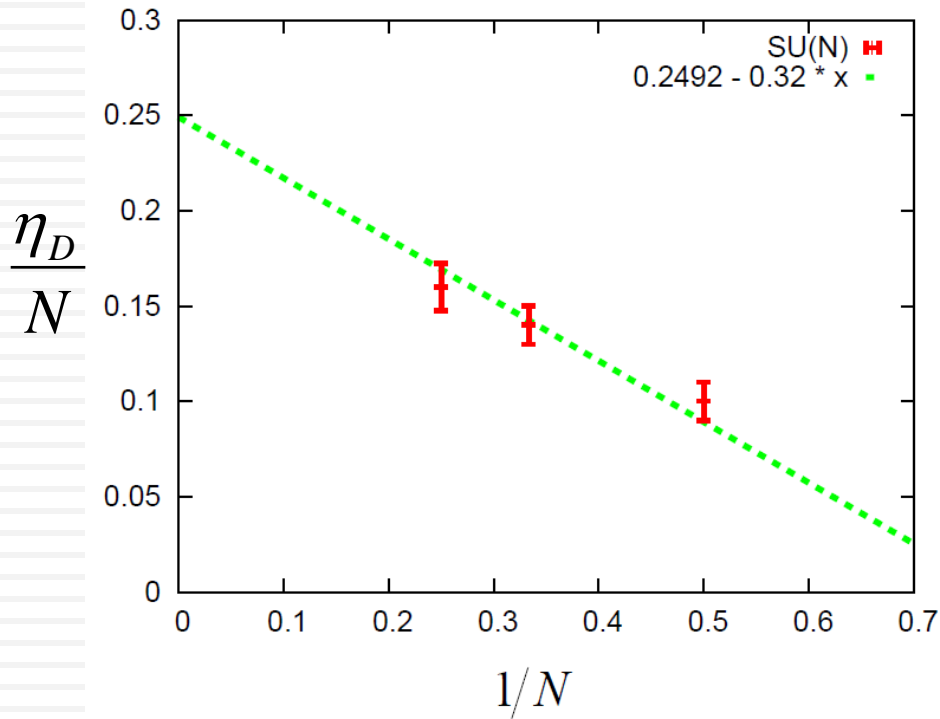
$$\eta_s = 0.38(3), \quad \nu = 0.65(3)$$

## SU(4) J-Q2

$$\eta_s = 0.42(5), \quad \nu = 0.70(2)$$



# Monopole Scaling Dimension up to $O(N^{-1})$

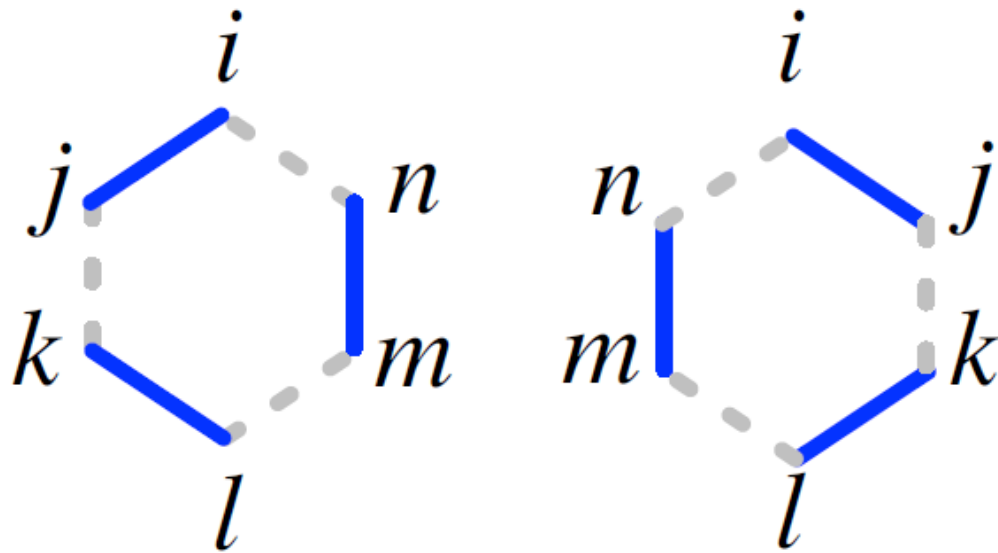


$$\frac{\eta_D}{N} = \frac{2x_\psi - 1}{N}$$

$$= 0.2492 - 0.32 \frac{1}{N} + O\left(\frac{1}{N^2}\right)$$

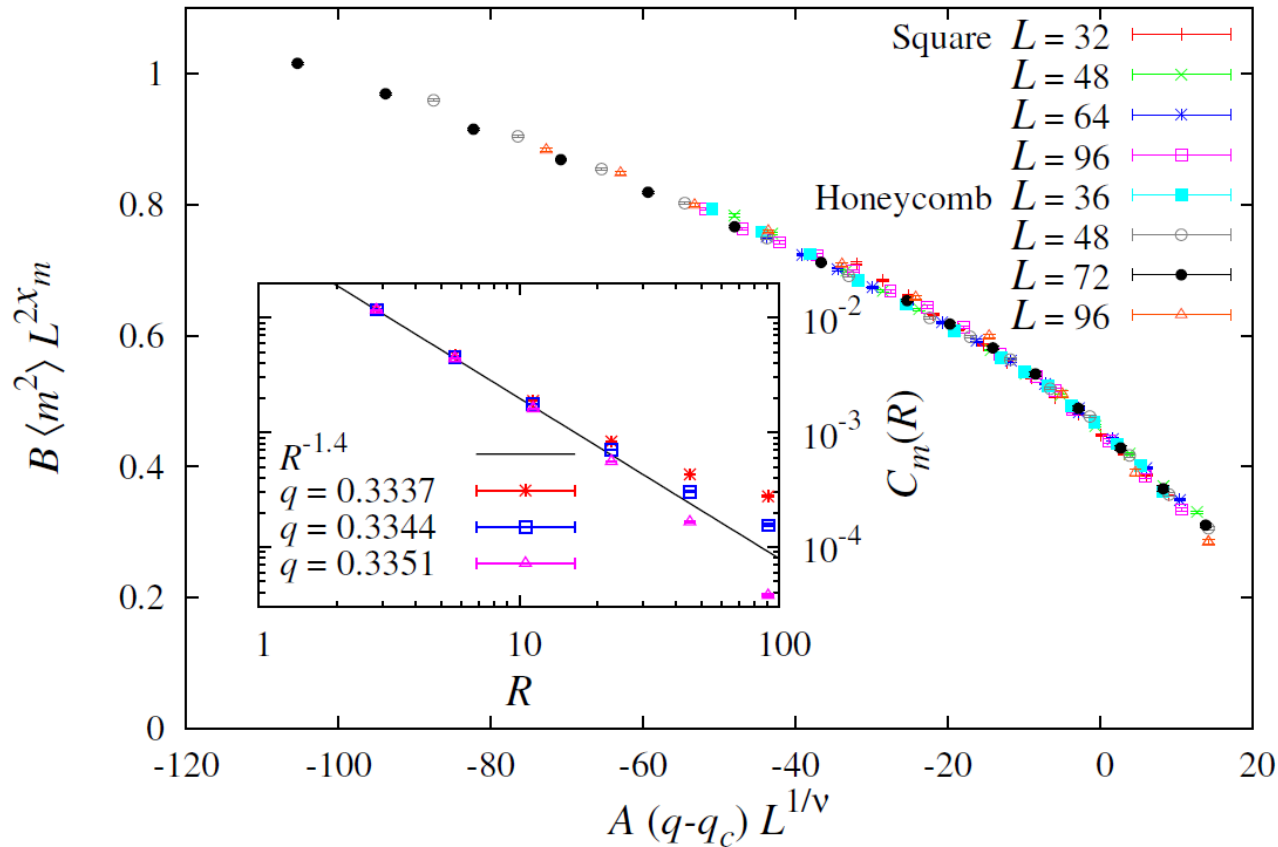
See Ribhu Kaul's  
talk for more discussions

# Honeycomb Lattice



$$H = -J \sum_{(ij)} P_{ij} - Q \sum_{(ijklmn)} (P_{ij} P_{kl} P_{mn} + P_{jk} P_{lm} P_{ni})$$

# Magnetization



SU(3) JQ Model

$$\beta J = L$$

$$2x_m = 1.40$$

$$y = 1.89$$

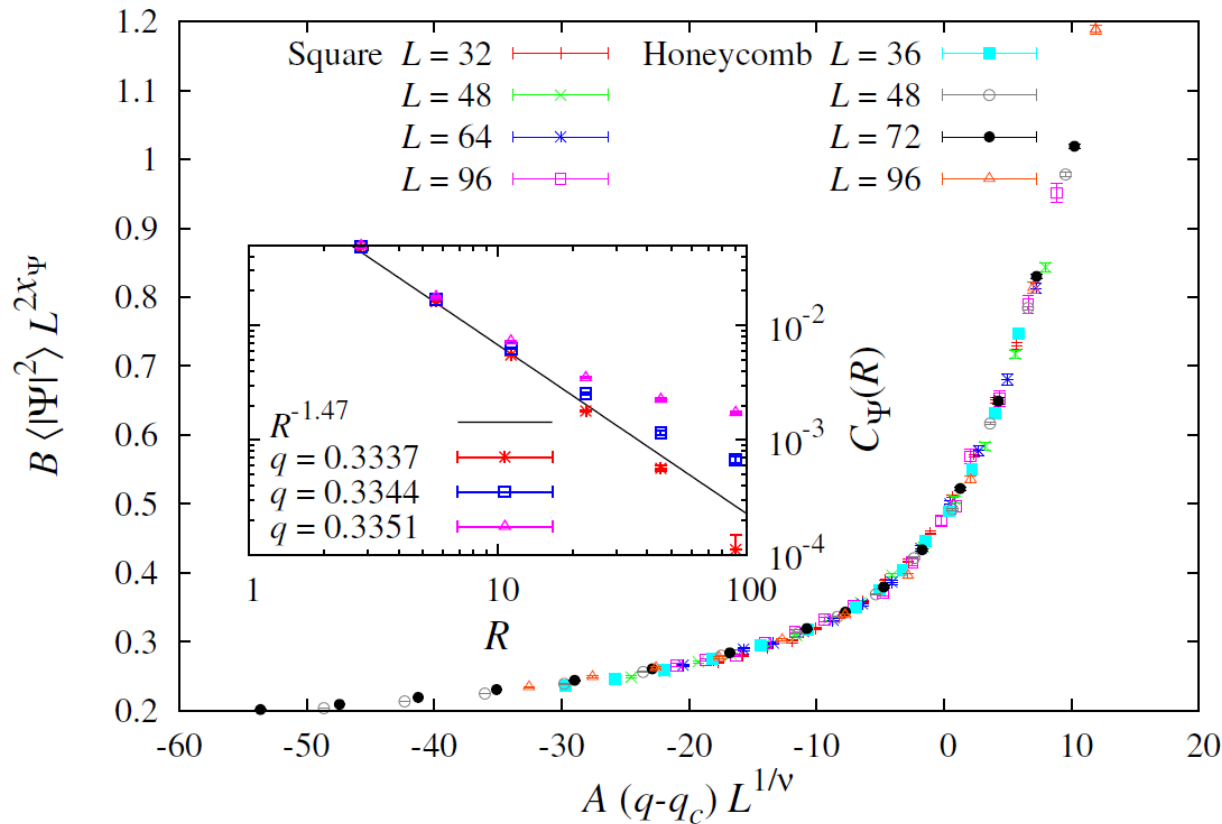
$$q_c = 0.3354 \text{ (sq)}$$

$$= 0.2036 \text{ (hc)}$$

Same exponents  
are assumed for  
both lattices

Harada et al PRB 88 220408 (2013)

# New Data (Dimerization)



SU(3) JQ Model

$$\beta J = L$$

$$2x_\Psi = 1.47$$

$$y = 1.73$$

$$q_c = 0.3339 \text{ (sq)}$$

$$= 0.2029 \text{ (hc)}$$

Same exponents  
are assumed for  
both lattices

Harada et al PRB 88 220408 (2013)

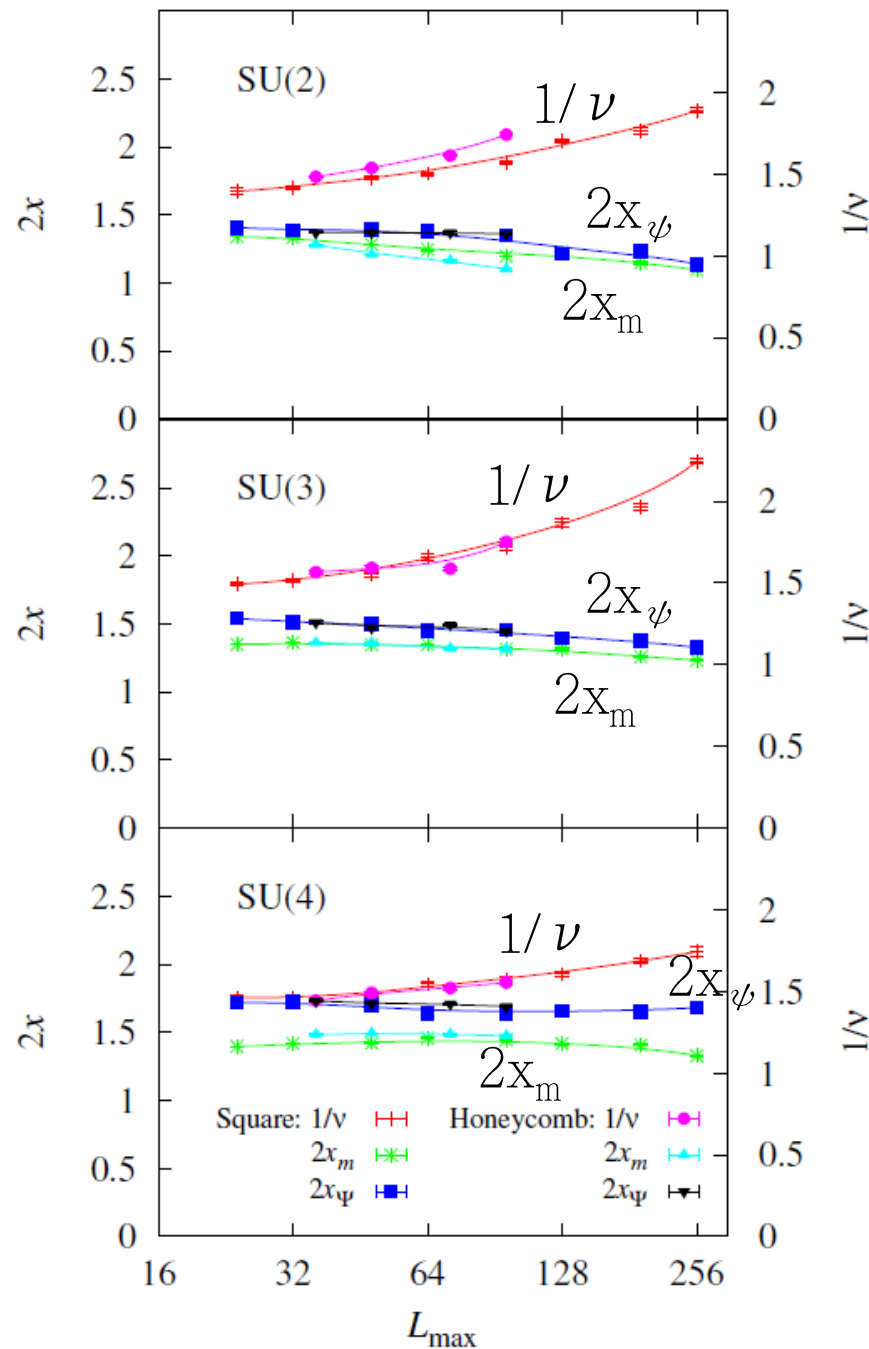
# Strong Finite-Size Corrections

FSS analysis using only triplets of systems ( $L/2, 2L/3, L$ )

Estimates still drifting at  $L=256$ .

$y=1/\nu$  may reach  $y=d$ ,  
 $x$  is still a long way to  $x=0$

Harada et al PRB 88 220408 (2013)



# Scaling Dimension of $\psi^2$

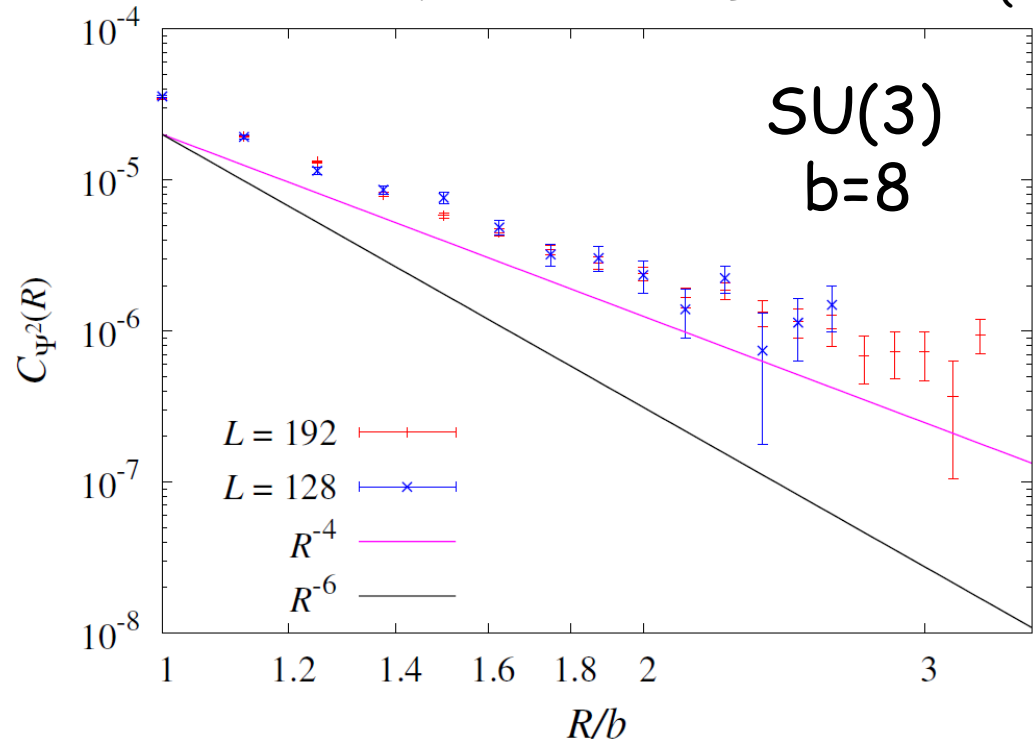
$$2x \approx 4$$

$$\Rightarrow y \approx 1 > 0$$

$\psi^2$  is relevant  
at SU(3) DCP  
(even if it exists)

2D system with  
strong spatial  
anisotropy does  
NOT have DCP

Harada et al PRB 88 220408 (2013)



$$C_{\Psi^2}(R_{ij}) \equiv \left\langle (\overline{\Psi}_j^*)^2 (\overline{\Psi}_i)^2 \right\rangle$$

$$\overline{\Psi}_i \equiv \frac{1}{b^2} \sum_{j \in b \times b} \Psi_j = (\text{block average around } i)$$

# Conclusion

◆ The transition is well described as a critical phenomena (at least within a limited size-range)


- reasonable scaling plots
- universality (Q2 and Q3, square and honeycomb)
- agreement with  $1/N$  expansion

◆ Strong corrections to scaling

- $y = 1/\nu$  is still increasing (may eventually reach the 1st order transition value  $y=d$ )
- $x = (1+\eta)/2$  is weakly decreasing (still a long way to  $x=0$ )

--- We still don't know whether it is 1st order or 2nd order.





END