Topological classification of semi-metals and nodal superconductors

Andreas P. Schnyder

MPI-FKF Stuttgart

MAX-PLANCK-GESELLSCHAFT

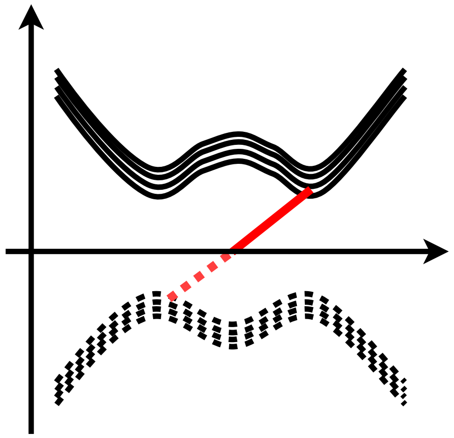
in collaboration with:

Raquel Queiroz, Johannes Hofmann
Philip Brydon, Carsten Timm
Shinsei Ryu, Shunji Matsuura, Po-Yao Chang
Ching-Kai Chiu

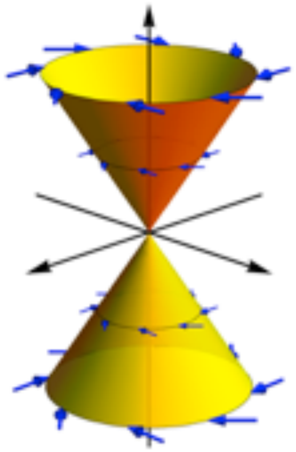
ISSP University of Tokyo, June 2014

Outline

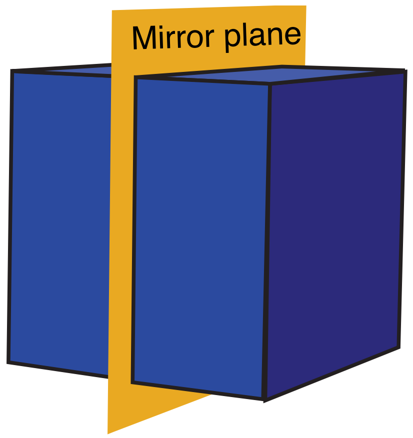
1. Introduction
   - What is a topological superconductor?
   - Chiral p-wave superconductor



2. Topological nodal superconductors
   - Superconductors without inversion symmetry
   - Spin polarization of topological surface states



3. Topological crystalline materials
   - Mirror symmetry
   - Classification schemes



4. Conclusions & Outlook
   - Candidate materials

Topological superconductors (full gap)

Superconductor = Cooper pairs (boson) + Bogoliubov quasiparticles (fermions)

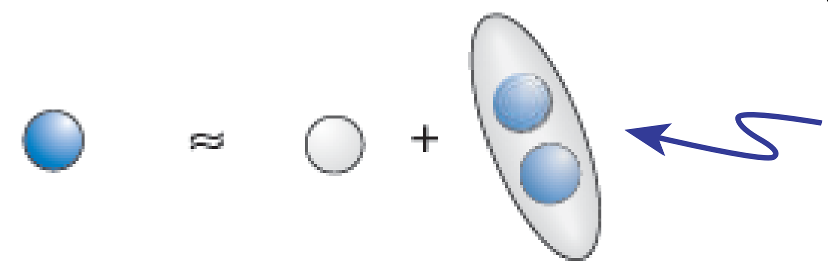
BCS mean field theory: $c^\dagger c\, c^\dagger c \quad \Rightarrow \quad \langle c^\dagger c^\dagger \rangle c\, c = \Delta^* c\, c$

$$H = \frac{1}{2} \sum_{\boldsymbol{k}} \begin{pmatrix} c^\dagger & c \end{pmatrix} H_{\mathrm{BdG}} \begin{pmatrix} c \\ c^\dagger \end{pmatrix}$$

Bogoliubov-de Gennes Hamiltonian

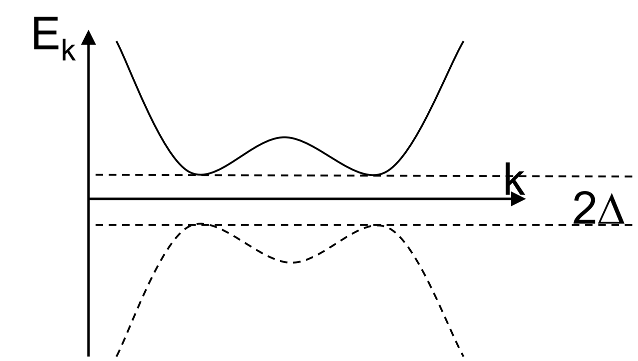
$$H_{\mathrm{BdG}} = \begin{pmatrix} h_0 & \Delta \\ \Delta^\dagger & -h_0^T \end{pmatrix}$$

Built-in particle-hole symmetry: $\Gamma^\dagger_{+E} = \Gamma_{-E}$



Cooper pair

gap in spectrum ⟶ can define top. invariant $n = \frac{i}{2\pi} \int_{\substack{\text{states} \\ \text{with } E<0}} \mathcal{F}\, d\boldsymbol{k}$



Topological equivalence:

BdG band structures are equivalent if they can be continuously deformed into one another without closing the energy gap and without breaking the symmetries of the SC.

Symmetries to consider:

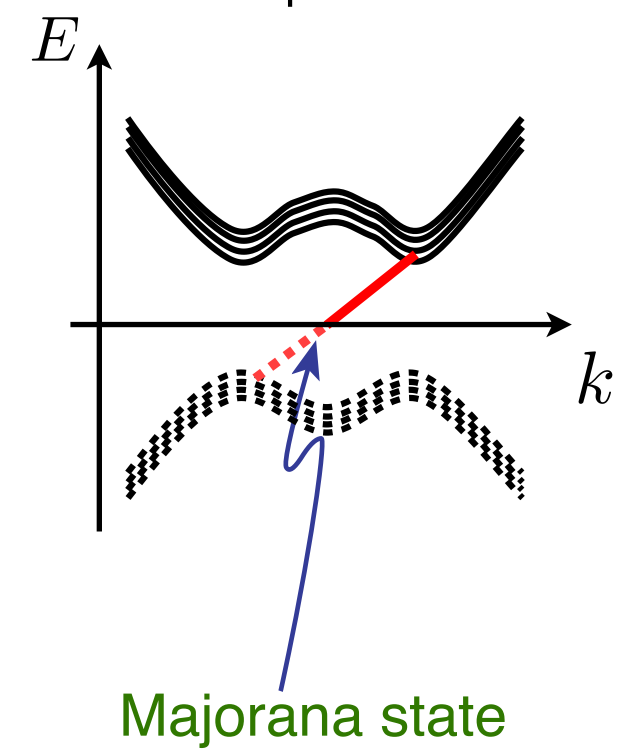
particle-hole symmetry, time-reversal, reflection symmetry, etc.

Chiral p-wave superconductor (full gap)

Lattice BdG model: **[Read & Green 00]**

$$\mathcal{H}_{\mathrm{BdG}} = \left(2t\left[\cos k_x + \cos k_y\right] - \mu\right)\tau_z + \Delta_0\left(\tau_x \sin k_x + \tau_y \sin k_y\right) = \boldsymbol{m}(\boldsymbol{k})\cdot\boldsymbol{\tau}$$

edge band structure of chiral p-wave SC



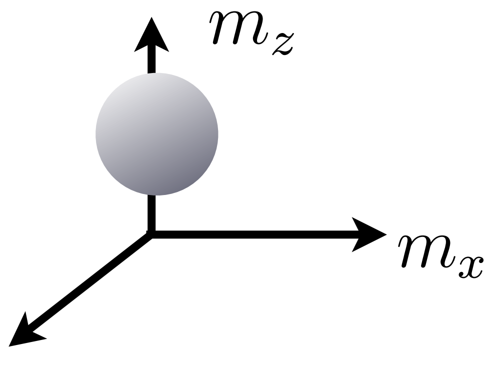
$$E = \pm\left|\boldsymbol{m}(\boldsymbol{k})\right|$$

Spectrum flattening: $\hat{\boldsymbol{m}}(\boldsymbol{k}) = \dfrac{\boldsymbol{m}(\boldsymbol{k})}{|\boldsymbol{m}(\boldsymbol{k})|}$

trivial phase

$|\mu| > 4t$

$n = 0$



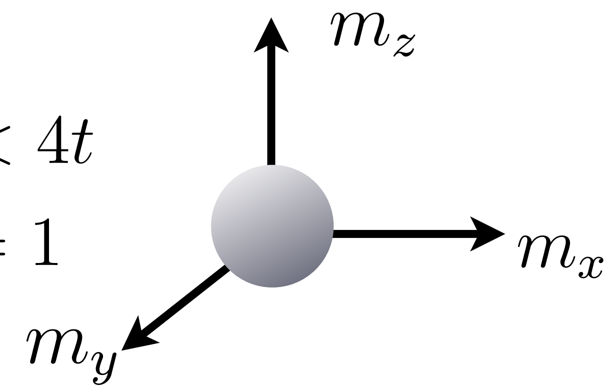
no edge state

“strong pairing”

non-trivial phase

$|\mu| < 4t$

$n = 1$



chiral Majorana edge state

“weak pairing”

classified by Chern number:

$$n = \frac{1}{8\pi}\int_{\mathrm{BZ}} d^2\boldsymbol{k}\ \epsilon^{\mu\nu}\boldsymbol{m}\cdot\left[\partial_{k_\mu}\boldsymbol{m}\times\partial_{k_\nu}\boldsymbol{m}\right]$$

Mapping $\hat{\boldsymbol{m}}(\boldsymbol{k})$: Brillouin zone $\longmapsto$ $\hat{\boldsymbol{m}}(\boldsymbol{k}) \in S^2$ $\quad \pi_2(S^2) = \mathbb{Z}$

Sr_2RuO_4 (n=2)

Summary: Topological superconductor (fully gapped)

Topological superconductors are:

- fully gapped unconventional superconductors that support stable gapless edge states (or surface states)
- surface states are robust to perturbations (e.g. insensitive to disorder) that respect the fundamental symmetries of the system
- the stability of the surface states is guaranteed by the bulk gap and by the bulk topological invariant (bulk-boundary correspondence)

Summary: Topological superconductor (fully gapped)

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Classification of topological superconductors (and insulators):

Altland-Zirnbauer Random Matrix Classes

Symmetry				dim		
Class	T	P	S	1	2	3
A	0	0	0	0	$\mathbb{Z}$	0
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	1	0	0	0	0	0
BDI	1	1	1	$\mathbb{Z}$	0	0
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$
C	0	-1	0	0	$\mathbb{Z}$	0
CI	1	-1	1	0	0	$\mathbb{Z}$

$\mathbb{Z}$: integer classification
$\mathbb{Z}_2$: binary classification
0 : no topological SC

chiral p-wave superconductor

non-centrosymmetric SC

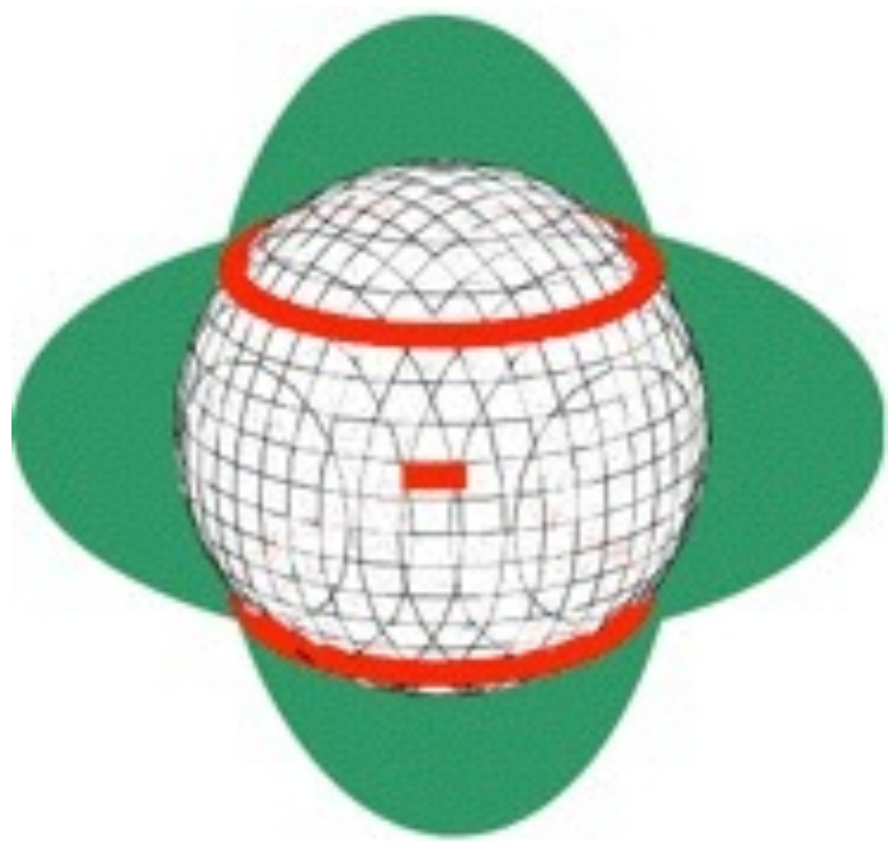
chiral d-wave superconductor

time-reversal invariant topological singlet superconductor

Schnyder, Ryu, Furusaki, Ludwig, PRB (2008)

A. Kitaev, AIP (2009)

Topological *nodal* superconductors



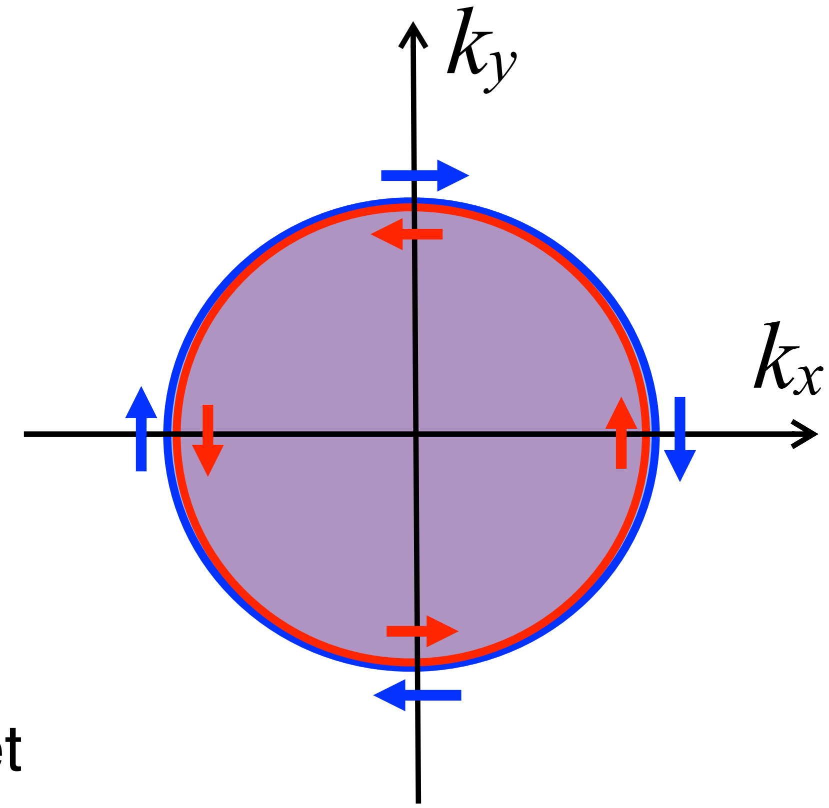
Non-centrosymmetric superconductors

$CePt_3Si$, $CeRhSi_3$, $CeIrSi_3$, Li_2Pt_3B, LaPtBi, etc. Interfaces: $LaAlO_3/SrTiO_3$

(i) Lack of center of inversion causes anti-symmetric SO coupling.

Normal state: $\mathcal{H} = \sum_{\boldsymbol{k}\mu\nu} c^\dagger_{\boldsymbol{k}\mu} \left(\varepsilon_{\boldsymbol{k}}\sigma_0 + \alpha \boldsymbol{g}_{\boldsymbol{k}} \cdot \boldsymbol{\sigma}\right)_{\mu\nu} c_{\boldsymbol{k}\nu} = \sum_{\boldsymbol{k}s} \xi_{\boldsymbol{k}s}\, b^\dagger_{\boldsymbol{k}s} b_{\boldsymbol{k}s}$

Spin basis: $\mu = \uparrow, \downarrow$ Helicity basis: $s = \pm$



Spin-split energy spectrum:

$$\boxed{\xi^\pm_{\boldsymbol{k}} = \varepsilon_{\boldsymbol{k}} \pm |\boldsymbol{g}_{\boldsymbol{k}}|}$$

(ii) Lack of center of inversion allows for admixture of singlet and triplet pairing components

$$\Delta(\boldsymbol{k}) = f(\boldsymbol{k}) \left(\Delta_s \sigma_0 + \Delta_t \boldsymbol{d}_{\boldsymbol{k}} \cdot \boldsymbol{\sigma}\right) i\sigma_y$$

$\boldsymbol{d}_{\boldsymbol{k}}$ is constrained by SO interaction: $\boldsymbol{g}_{\boldsymbol{k}} \parallel \boldsymbol{d}_{\boldsymbol{k}}$

Gaps on the two Fermi surfaces: $\boxed{\Delta^\pm_{\boldsymbol{k}} = \Delta_s \pm \Delta_t\, |\boldsymbol{d}_{\boldsymbol{k}}|}$

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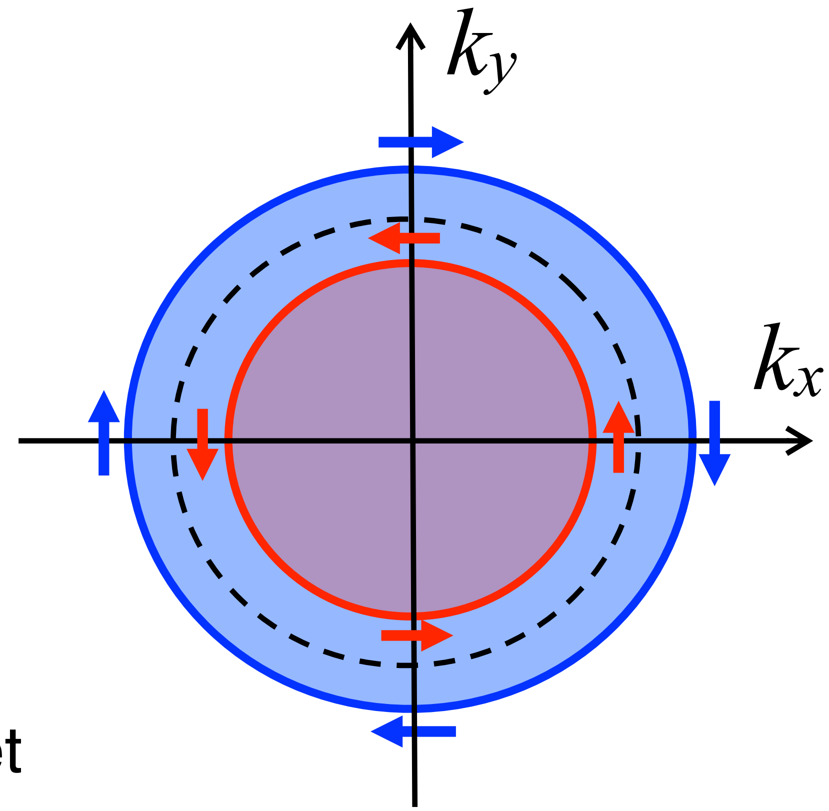

Spin-split energy spectrum:

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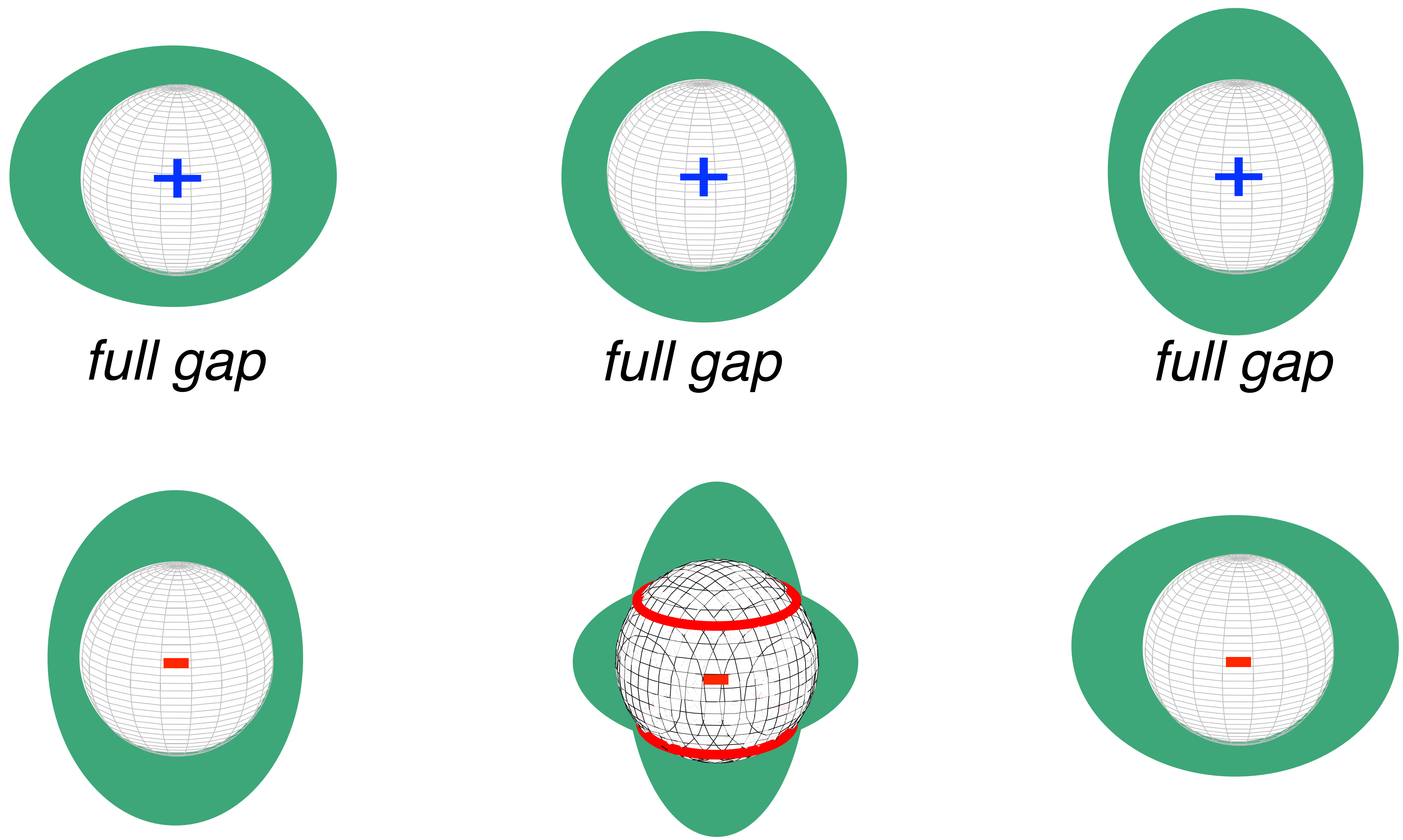
Gaps on the two Fermi surfaces: $\boxed{\Delta^{\pm}_{\boldsymbol{k}} = \Delta_s \pm \Delta_t |\boldsymbol{d_k}|}$

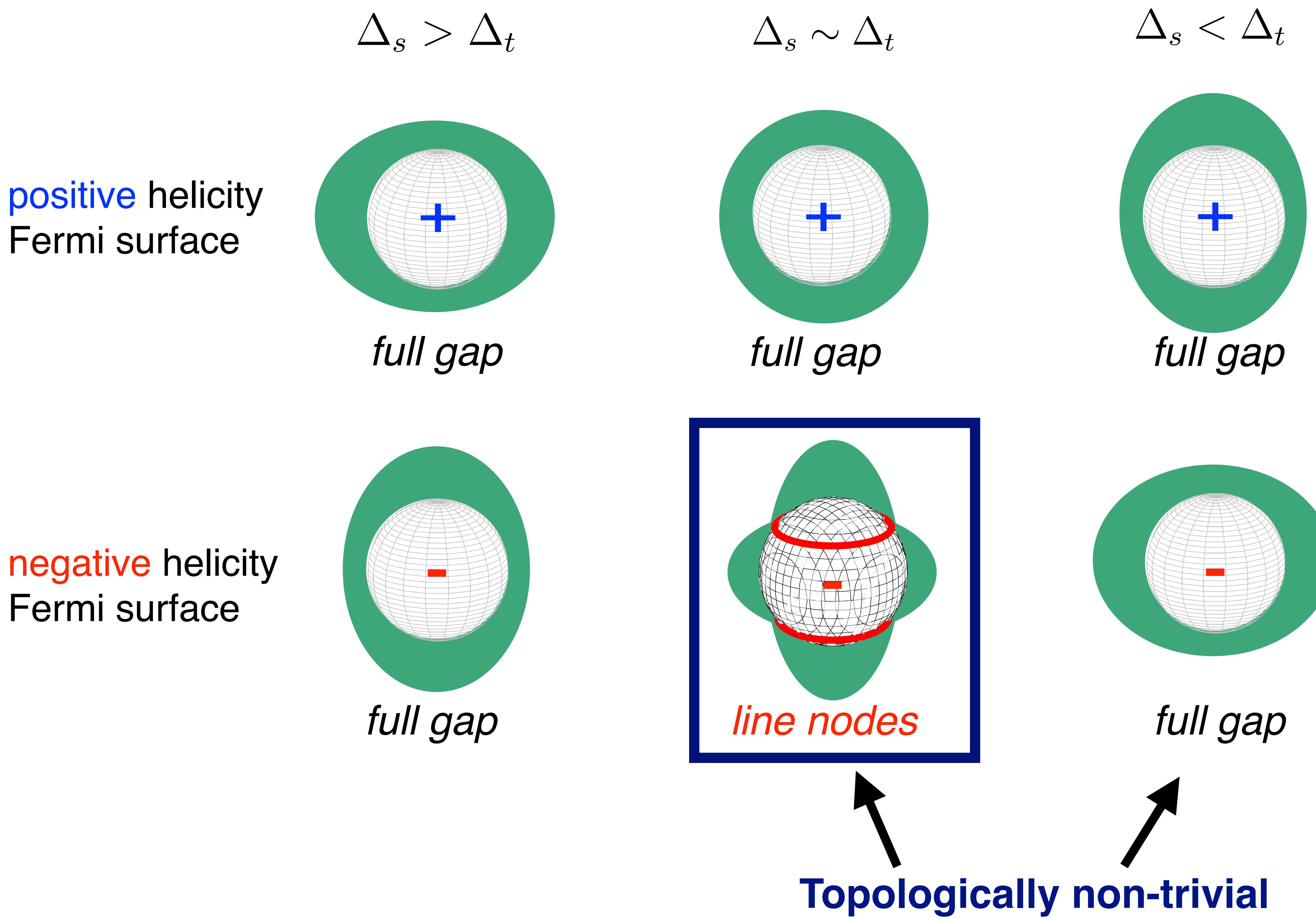
Non-centrosymmetric SCs: Structure of pairing state

	$\Delta_s > \Delta_t$	$\Delta_s \sim \Delta_t$	$\Delta_s < \Delta_t$
positive helicity Fermi surface	*full gap*	*full gap*	*full gap*
negative helicity Fermi surface	*full gap*	*line nodes*	*full gap*



Topologically non-trivial

Non-centrosymmetric SCs: Structure of pairing state



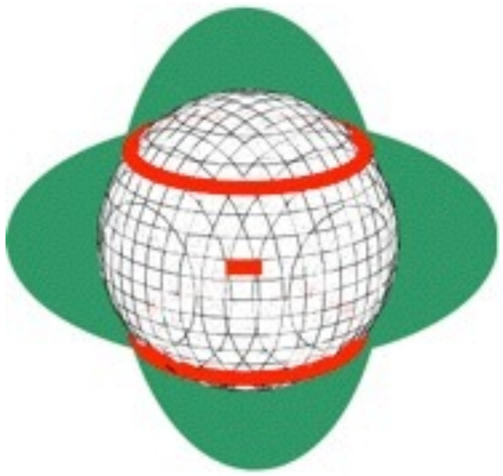
Nodal non-centrosymmetric superconductor

Non-centro SC: $\mathcal{H}_{\mathrm{BdG}}(\mathbf{k}) = \begin{pmatrix} \varepsilon_{\mathbf{k}}\sigma_0 + \lambda\,\mathbf{g}_{\mathbf{k}}\cdot\vec{\sigma} & [\Delta_s\sigma_0 + \Delta_t\mathbf{d}_{\mathbf{k}}\cdot\vec{\sigma}](i\sigma_y) \\ (-i\sigma_y)[\Delta_s\sigma_0 + \Delta_t\mathbf{d}_{\mathbf{k}}\cdot\vec{\sigma}] & -\varepsilon_{\mathbf{k}}\sigma_0 - \lambda\,\mathbf{g}_{\mathbf{k}}\cdot\vec{\sigma}^* \end{pmatrix}$

▶ Symmetries:

$$\left.\begin{array}{lll} \text{particle-hole symmetry:} & \Xi\,\mathcal{H}_{\mathrm{BdG}}(\boldsymbol{k})\,\Xi^{-1} = -\mathcal{H}_{\mathrm{BdG}}(-\boldsymbol{k}) & \Xi^2 = +1 \\ \text{time-reversal symmetry:} & \Theta\mathcal{H}_{\mathrm{BdG}}(\boldsymbol{k})\Theta^{-1} = +\mathcal{H}_{\mathrm{BdG}}(-\boldsymbol{k}) & \Theta^2 = -1 \\ \text{chiral symmetry (TRS+PHS):} & S\mathcal{H}_{\mathrm{BdG}}(\boldsymbol{k}) + \mathcal{H}_{\mathrm{BdG}}(\boldsymbol{k})S = 0 & S = \Xi\Theta \end{array}\right\} \text{DIII}$$

Gaps on the two Fermi surfaces: $\boxed{\Delta^{\pm}_{\boldsymbol{k}} = \Delta_s \pm \Delta_t\,|\boldsymbol{d}_{\boldsymbol{k}}|}$

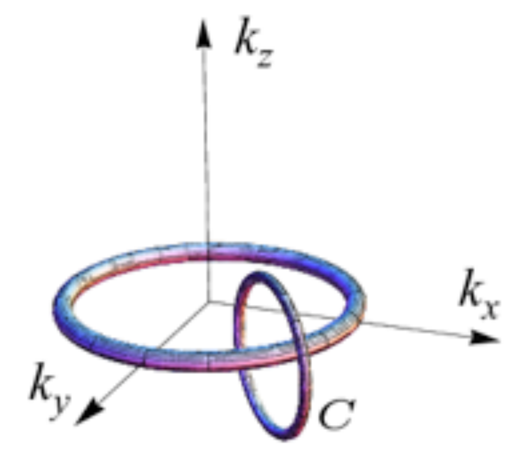


$\Delta_s \sim \Delta_t \quad\Longrightarrow$ *nodal lines* on negative helicity Fermi surface

▶ **Problem**: Global topological number *ill-defined* (**no gap!**)

Solution: (assume translational symmetry)

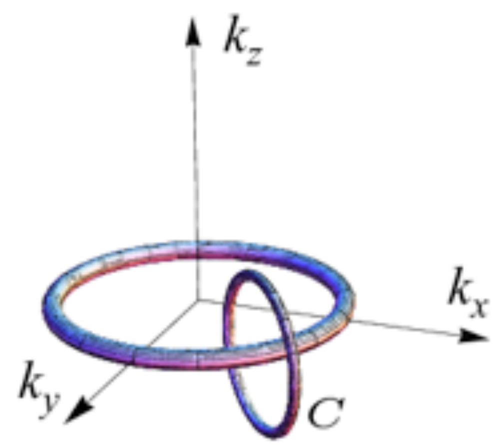
⟶ Define **momentum-dependent** topological number $\quad \nu_{\mathcal{C}} = \dfrac{1}{2\pi}\oint_{\mathcal{C}} \mathcal{F}(\mathbf{k})\,dk_l$



▶ Topological characteristics depend on the symmetries of BdG Hamiltonian restricted to contour $\mathcal{C}$

Nodal non-centrosymmetric superconductor

- Topological characteristics depend on the symmetries of BdG Hamiltonian restricted to contour $\mathcal{C}$.



Schnyder, Brydon, Timm PRB (2012)

$$\mathbf{d}_{\mathbf{k}} = (\sin k_x + \sin k_y, \sin k_x + \sin k_y, \sin k_z)^{\mathrm{T}}$$

$$\Delta_s \sim \Delta_t$$

Classification

Class	T	P	S	1	2	3
	Symmetry			dim		
A	0	0	0	0	$\mathbb{Z}$	0
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	1	0	0	0	0	0
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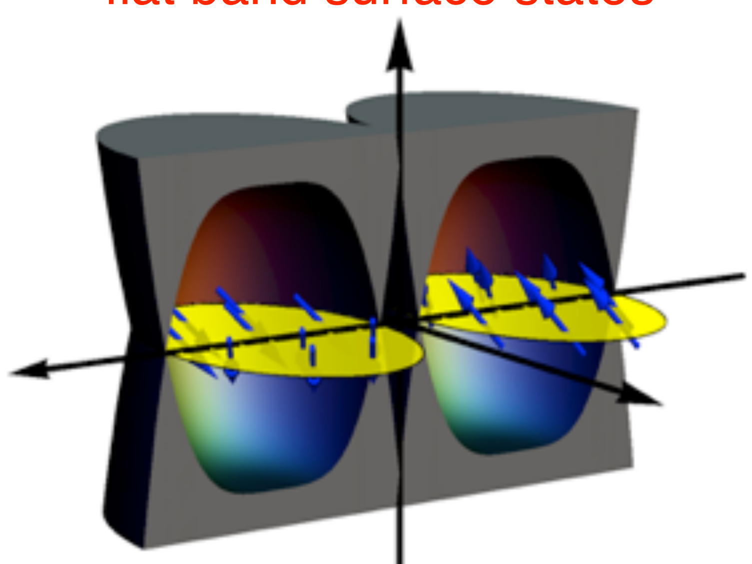
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$
C	0	-1	0	0	$\mathbb{Z}$	0
CI	1	-1	1	0	0	$\mathbb{Z}$

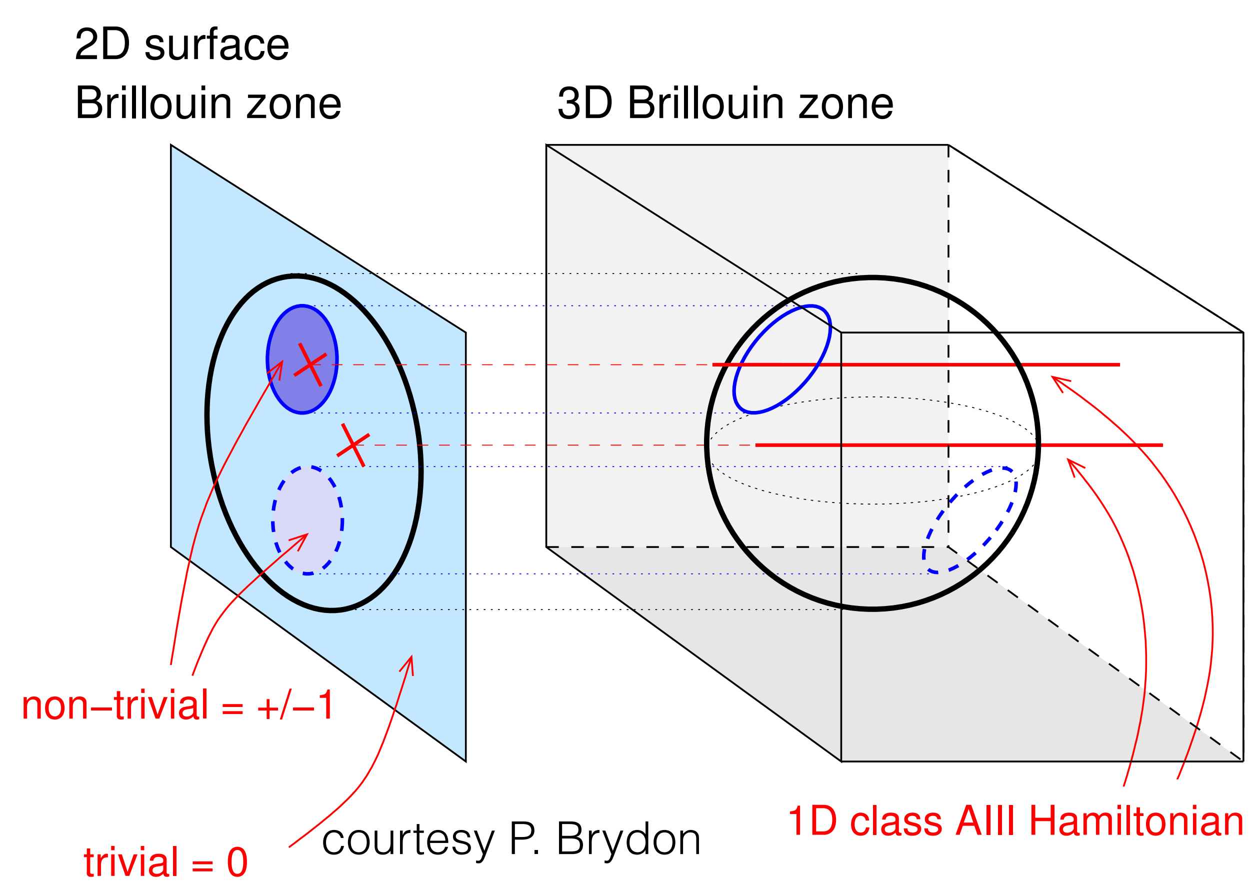
(i) 1D contour *is not* centrosymmetric: TRS ✗ PHS ✗ TRS+PHS (chiral sym S) ✓

AIII: 1D Winding number:

$$W_{\mathcal{C}} = \frac{1}{2\pi} \oint_{\mathcal{C}} dk_l \, \partial_{k_l} \left[\arg(\xi^-_{\mathbf{k}} + i\Delta^-_{\mathbf{k}}) \right]$$

flat band surface states



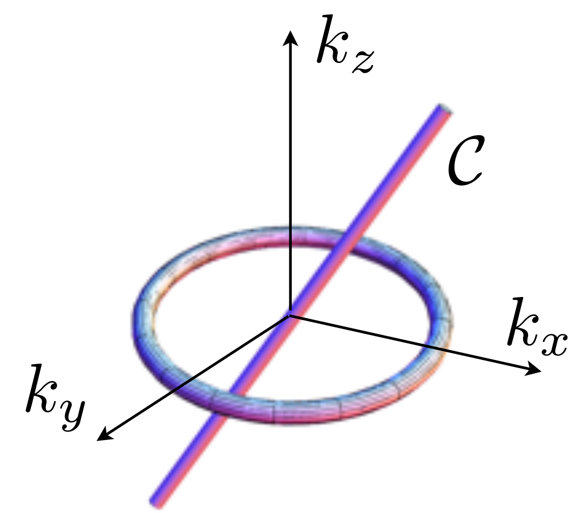


Nodal non-centrosymmetric superconductor

▶ Topological characteristics depend on the symmetries of BdG Hamiltonian restricted to contour $\mathcal{C}$.

$$\mathbf{d}_{\mathbf{k}} = (\sin k_x, \sin k_y, \sin k_z)^{\mathrm{T}}$$

$$\Delta_s \sim \Delta_t$$



Classification

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	Symmetry			dim		
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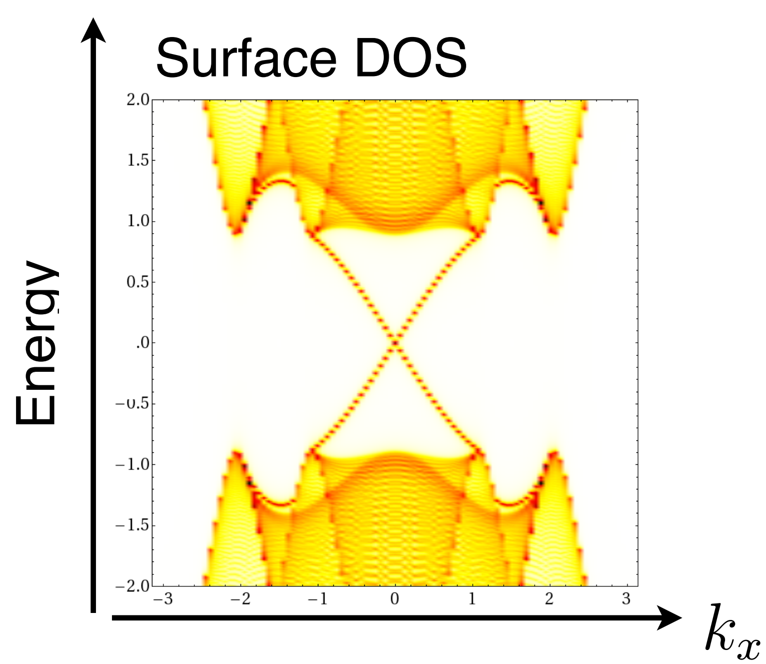
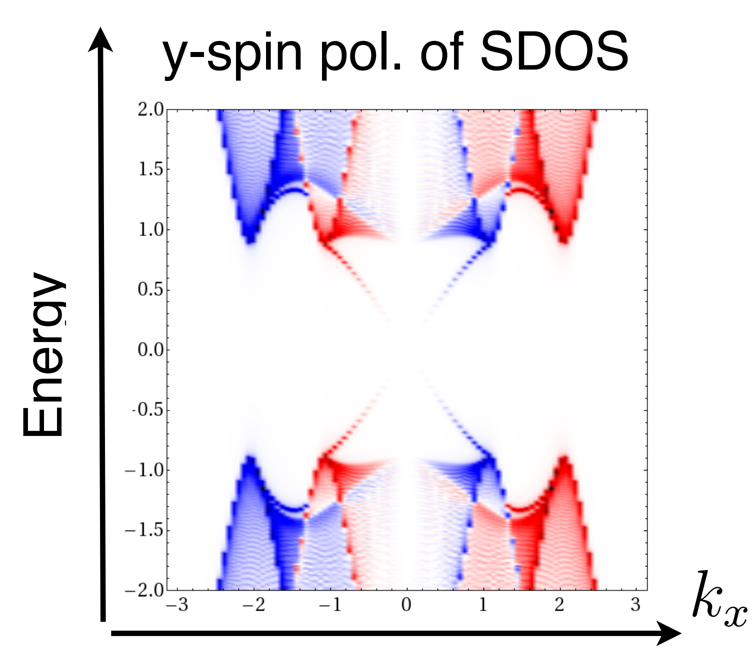
(ii) 1D contour *is* centrosymmetric: TRS ✓ PHS ✓ TRS+PHS (chiral sym S) ✓

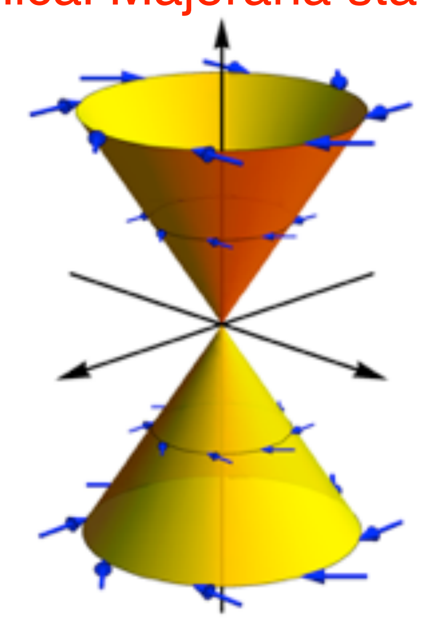
DIII: 1D Z2 number:

$$N^{1\mathrm{D}}_{\mathcal{C}} = \prod_{a=1}^{2} \frac{\mathrm{Pf}\left[\omega(\Lambda_a)\right]}{\sqrt{\det\left[\omega(\Lambda_a)\right]}} = \prod_{a=1}^{2} \frac{\mathrm{Pf}\left[q^T(\Lambda_a)\right]}{\sqrt{\det\left[q(\Lambda_a)\right]}} = \pm 1$$





helical Majorana state

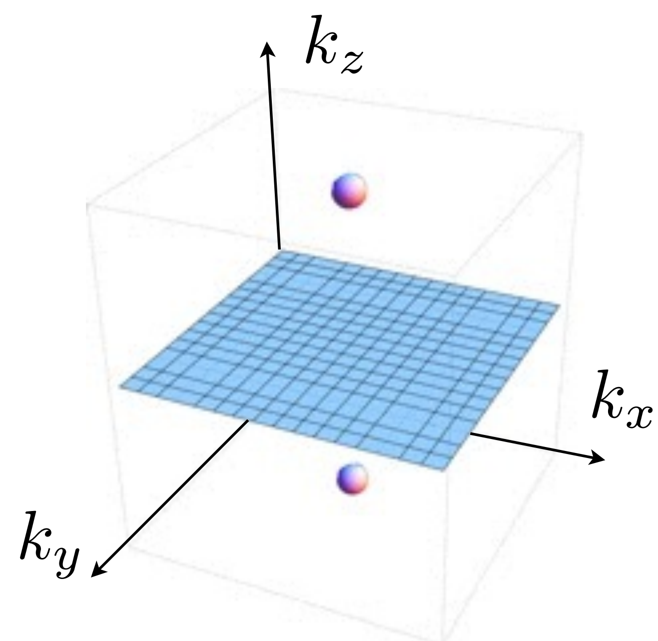


Nodal non-centrosymmetric superconductor

- Topological characteristics depend on the symmetries of BdG Hamiltonian restricted to contour $\mathcal{C}$.

$$\mathbf{d}_{\mathbf{k}} = (\sin k_y, -\sin k_x, 0)^{\mathrm{T}}$$

$$\Delta_t \neq 0, \quad \Delta_s = 0$$



Classification

Class	T	P	S	dim 1	dim 2	dim 3
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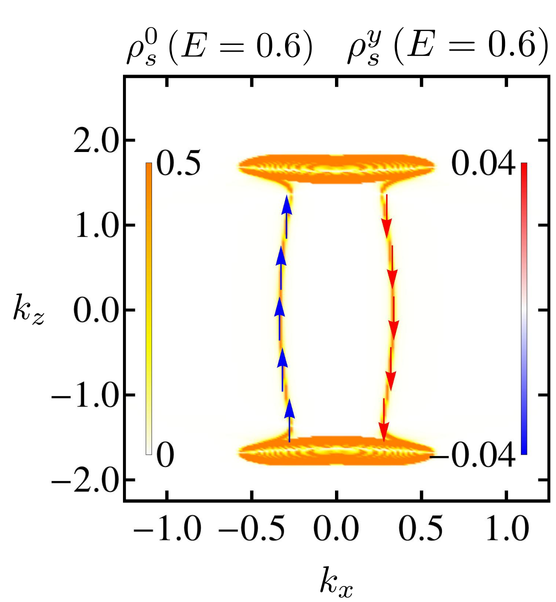
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$
C	0	-1	0	0	$\mathbb{Z}$	0
CI	1	-1	1	0	0	$\mathbb{Z}$

(iii) 2D contour *is* centrosymmetric: TRS PHS TRS+PHS (chiral sym S)

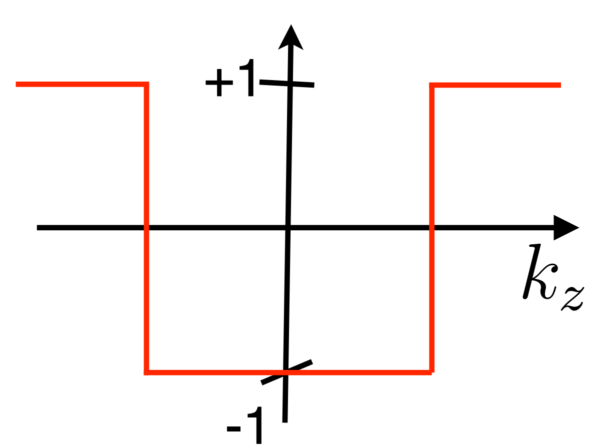
DIII: 2D Z_2 number:

$$N_E^{1\mathrm{D}} = \prod_{a=1}^{4} \frac{\mathrm{Pf}\left[\omega(\Lambda_a)\right]}{\sqrt{\det\left[\omega(\Lambda_a)\right]}} = \prod_{a=1}^{4} \frac{\mathrm{Pf}\left[q^T(\Lambda_a)\right]}{\sqrt{\det\left[q(\Lambda_a)\right]}} = \pm 1$$

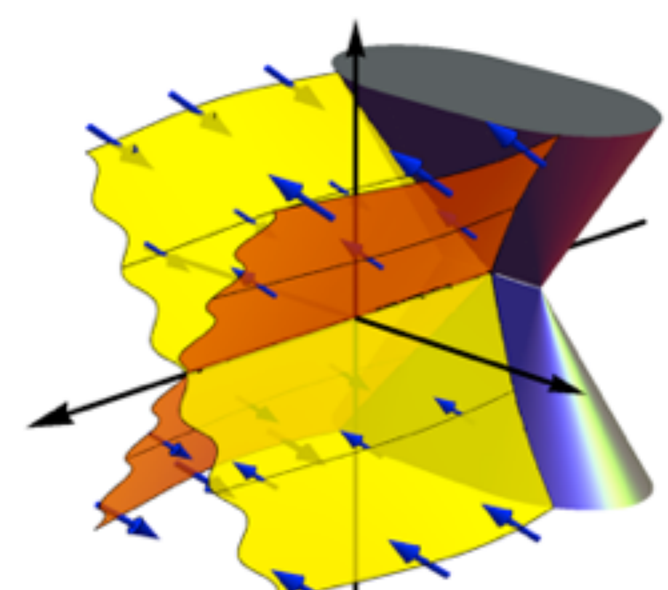
Surface density of states



2D Z_2 number



arc surface state



Topological classification of gapless materials

Classification for Fermi surfaces off high-symmetry points

Two-dimensional systems

Class	T	P	S	line	point
A	0	0	0	$\mathbb{Z}$	0
AIII	0	0	1	0	$\boxed{\mathbb{Z}}$
AI	+1	0	0	$\mathbb{Z}$	0
BDI	+1	+1	1	$\mathbb{Z}_2$	$\mathbb{Z}$
D	0	+1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
DIII	-1	+1	1	0	$\mathbb{Z}_2$
AII	-1	0	0	$\mathbb{Z}$	0
CII	-1	-1	1	0	$\mathbb{Z}$
C	0	-1	0	0	0
CI	+1	-1	1	0	0

d-wave SC

Three-dimensional systems

Class	surface	line	point
A	$\mathbb{Z}$	0	$\boxed{\mathbb{Z}}$
AIII	0	$\boxed{\mathbb{Z}}$	0
AI	$\mathbb{Z}$	0	0
BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
DIII	0	$\boxed{\mathbb{Z}_2}$	$\boxed{\mathbb{Z}_2}$
AII	$\mathbb{Z}$	0	$\mathbb{Z}_2$
CII	0	$\mathbb{Z}$	0
C	0	0	$\mathbb{Z}$
CI	0	0	0

^{3}He A phase / Weyl semimetal

TRI SC w/ S^z-spin conserv.

Non-centro SC

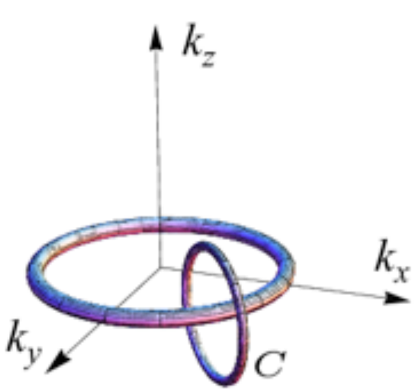
(NB: $\mathbb{Z}_2$ invariant only protects surface states, but not bulk nodes!)

$\mathbb{Z}$: integer classification
$\mathbb{Z}_2$: binary classification
0 : no top. stable nodes

- Topological invariants:

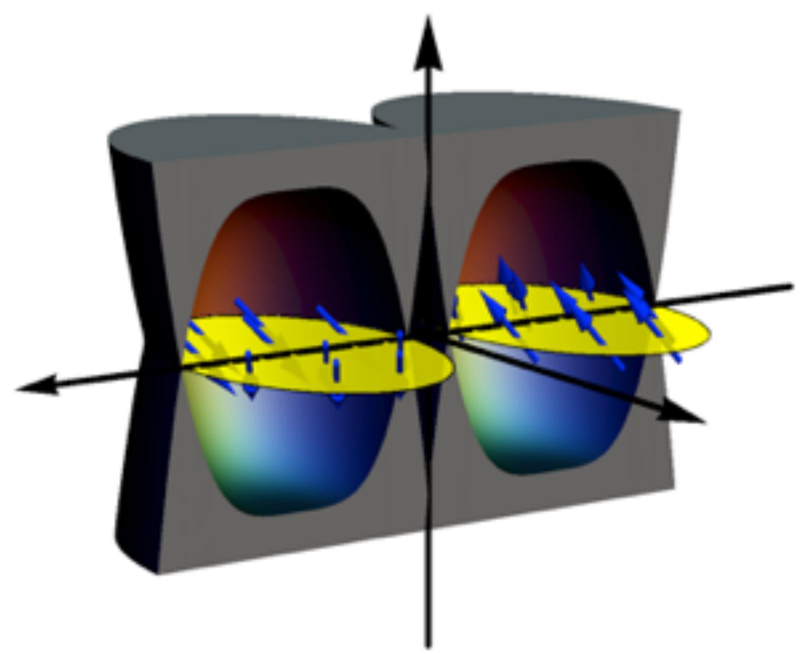
$$n_{\mathcal{C}_{2n+2}} = \frac{1}{(n+1)!} \oint_{\mathcal{C}_{2n+2}} \mathrm{tr} \left(\frac{i\mathcal{F}}{2\pi} \right)^{n+1}$$

$$\nu_{\mathcal{C}_{2n+1}} = \frac{(-1)^n n!}{(2n+1)!} \left(\frac{i}{2\pi} \right)^{n+1} \oint_{\mathcal{C}_{2n+1}} \epsilon^{\alpha_1 \alpha_2 \cdots} \mathrm{tr} \left[q^{-1} \partial_{\alpha_1} q \cdot q^{-1} \partial_{\alpha_2} q \cdots \right] d^{2n+1} k$$



Matsuura, Chang, Schnyder, Ryu NJP (2013) **Zhao, Wang, PRL (2013)**

Consequences of the surface-state spin polarization



Robustness of flat-band surface states against disorder

- Interband scattering: surface states are spin polarized

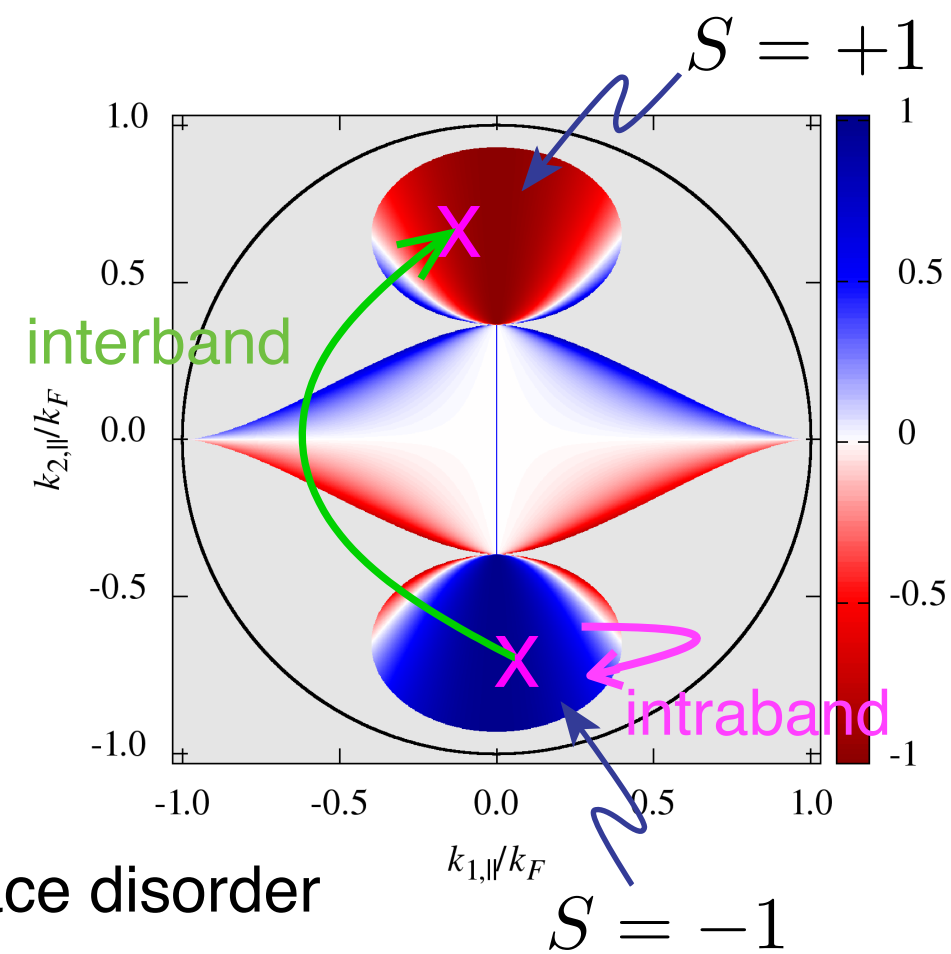
— scattering requires change in spin polarization

$$\left\langle \Psi^{+}_{-k} \middle| V^{0}_{2k} \middle| \Psi^{-}_{k} \right\rangle = 0 \qquad \left\langle \Psi^{+}_{-k'} \middle| V^{0}_{k+k'} \middle| \Psi^{-}_{k'} \right\rangle \text{ small}$$

- Intraband scattering: surface states have definite chirality

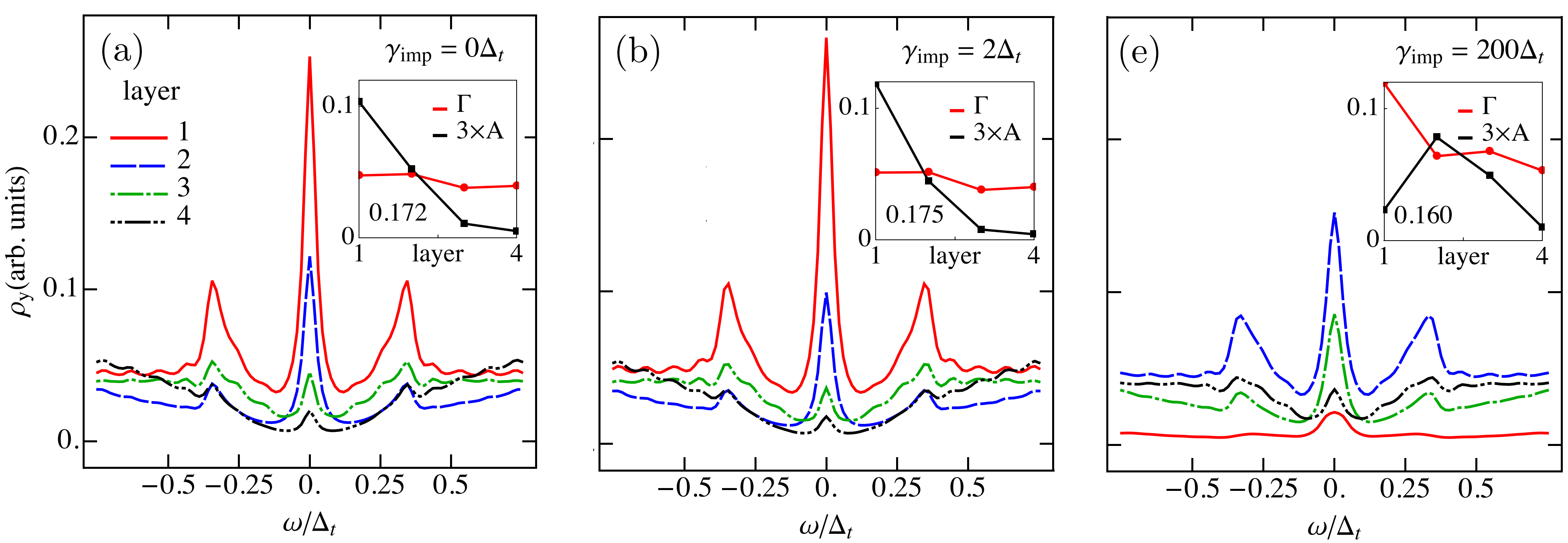
$$\Psi^{+}_{k} = \begin{pmatrix} \chi_k \\ 0 \end{pmatrix} \qquad \Psi^{-}_{k} = \begin{pmatrix} 0 \\ i\sigma_y \chi_{-k} \end{pmatrix}$$

$$\left\langle \Psi^{\pm}_{k} \middle| V^{0}_{k'-k} \middle| \Psi^{\pm}_{k'} \right\rangle = 0 \quad \text{for all } k \text{ and } k'$$



$\Longrightarrow$ some degree of protection against non-magnetic surface disorder

— for strong surface disorder: reappearance of flat-band states in 2nd layer



Hofmann, Queiroz, Schnyder, PRB (2013) **Queiroz & Schnyder, PRB (2014)**

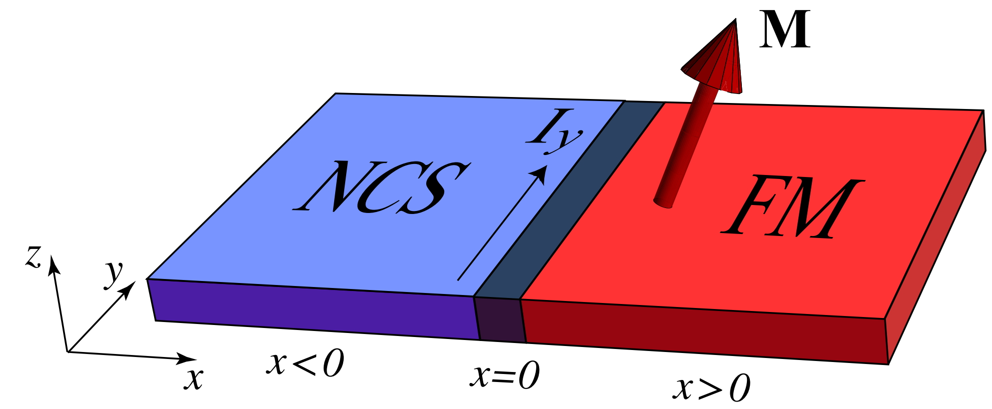
Spontaneous interface currents due to flat-band states

- ferromagnetic moment couples to spin polarization of flat-band states
  - perturbative shift of energies: $\delta E \propto \mathbf{M} \cdot \mathbf{S}^{\mu}$

    $\Longrightarrow$ both flat bands acquire dispersion

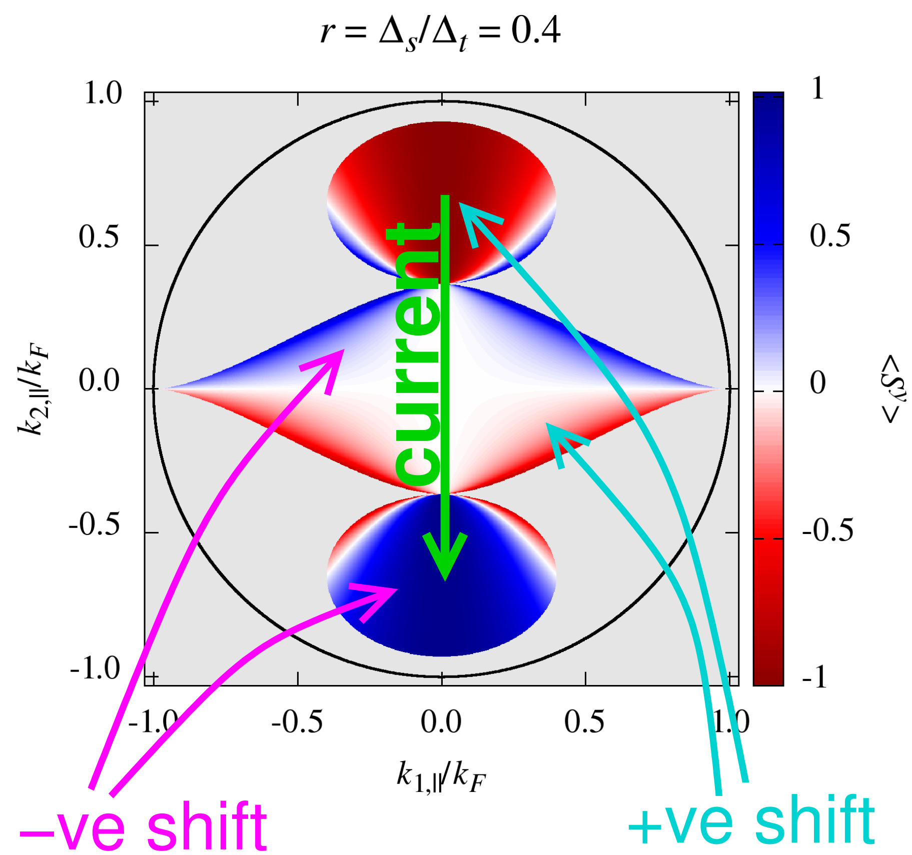
    $\Longrightarrow$ spontaneous chiral interface current

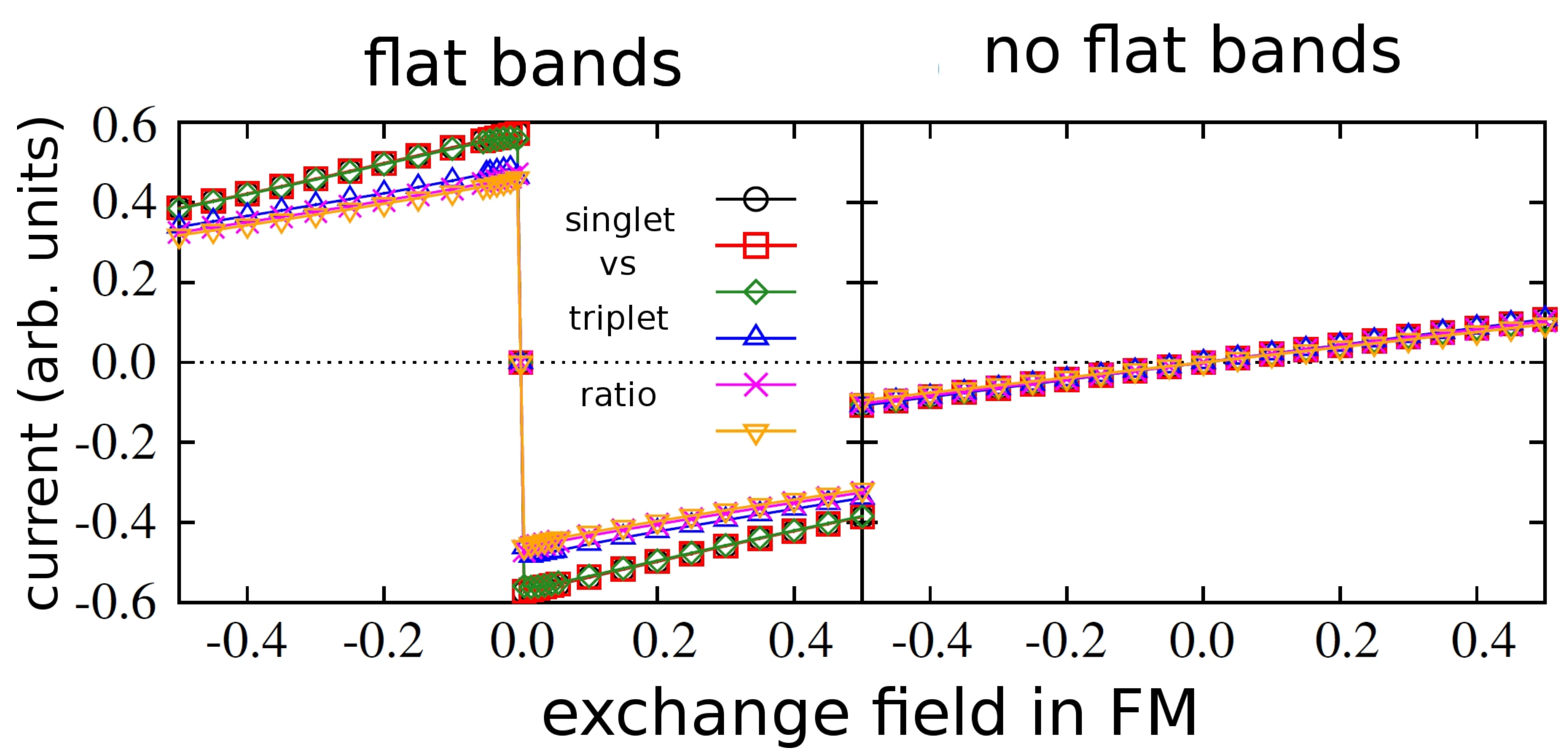
    $\Longrightarrow$ break down of linear response (singular dependence on exchange field)



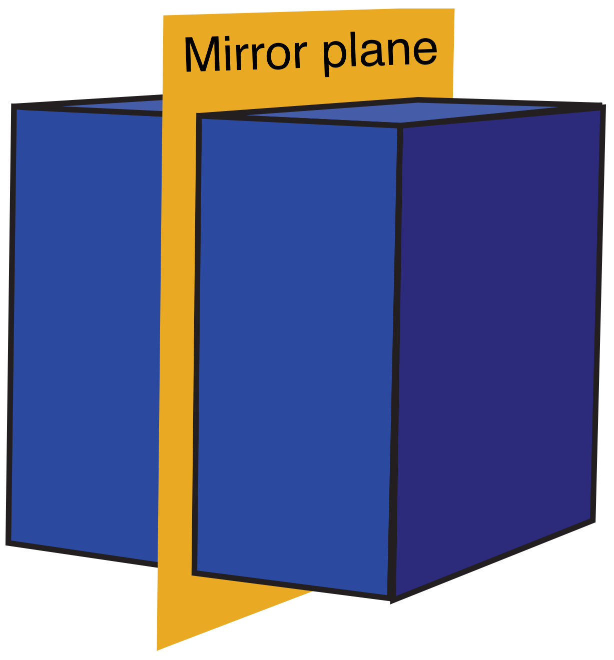
Schnyder, Brydon, Timm, PRL 111, 077001 (2013)
Brydon, Timm, Schnyder, NJP 15, 045019 (2013)

red: $\langle S^y \rangle > 0$ ↑ blue: $\langle S^y \rangle < 0$ ↓





Topological crystalline materials



Topological crystalline materials

Topological insulator/superconductor protected by global symmetry & crystal symmetry

- 32 crystallographic point groups (230 space groups)
- Surface states can exist if:

  (i) there are points/lines in the surface Brillouin zone that are invariant under the symmetry

  (ii) symmetry group has a two-dimensional irreducible representation

▶ Consider, e.g., mirror symmetry R: $x \to -x$

$$R^{-1}\mathcal{H}(-k_x, k_y, k_z)R = \mathcal{H}(k_x, k_y, k_z)$$

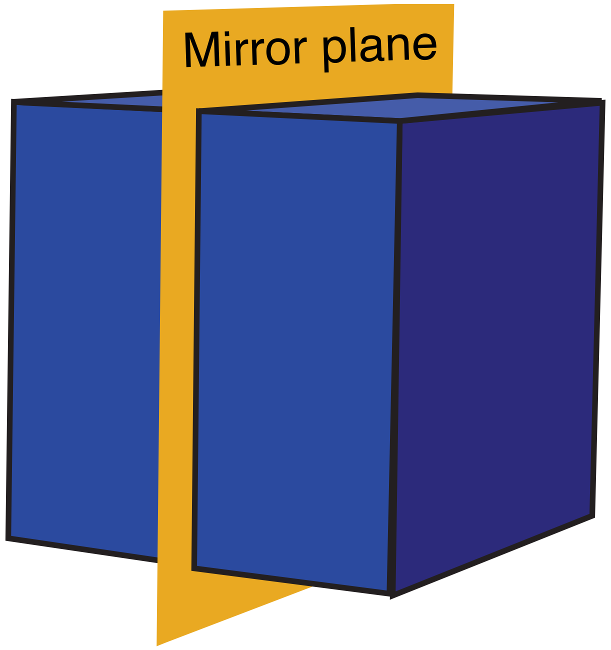
—for spin-1/2 systems: $R = s_x$

$$\Longrightarrow \mathcal{H}(0, k_y, k_z)R - R\mathcal{H}(0, k_y, k_z) = 0$$

— define Chern number for *each eigenspace of* R:

(i.e., $s_x = \pm 1$)

mirror Chern number: $$\boxed{n_{\mathcal{M}} = \frac{1}{4\pi}\int_{\text{2D BZ}} (\mathcal{F}_+ - \mathcal{F}_-)\, d^2\mathbf{k}}$$



Topological crystalline materials

▶ SnTe is a topological crystalline insulator:

Teo, Fu, Kane, 2008, Fu 2011

Tanaka, Ando, et al., 2012, 2013

$$\mathcal{H} = \varepsilon(\mathbf{k})\sigma_z + v(\sin k_x s_y - \sin k_y s_x) \otimes \sigma_x + v_z \sin k_z \sigma_y$$

$(s_i:\ \text{spin};\quad \sigma_i:\ \text{orbitals})$

Time-reversal symmetry (class AII): $(is_y)\mathcal{H}^*(\mathbf{k})(is_y)^{-1} = +\mathcal{H}(-\mathbf{k}) \qquad \Theta = is_y\mathcal{K}$

Reflection symmetry: $R = s_x,\ R^{-1}\mathcal{H}(-k_x, k_y, k_z)R = \mathcal{H}(k_x, k_y, k_z)$

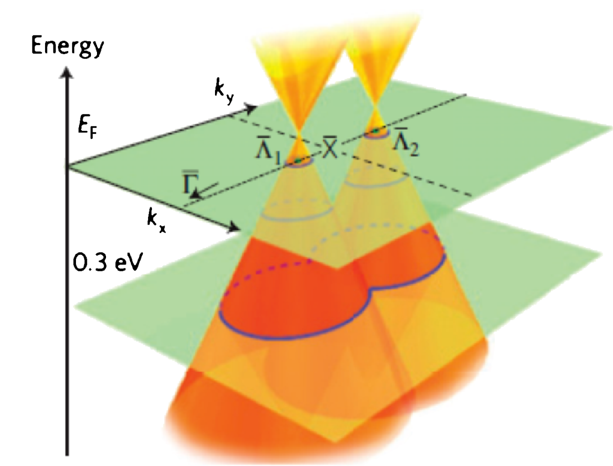
$$\Longrightarrow \Theta R = -R\Theta$$

– project $\mathcal{H}$ onto *eigenspaces of* R: $(\text{ i.e., } s_x = \pm 1)$

$$\mathcal{H}_\pm = \varepsilon_{\mathbf{k}}\sigma_z \mp v \sin k_y \sigma_x + v_z \sin k_z \sigma_y = \mathbf{m}_\pm(\mathbf{k}) \cdot \boldsymbol{\sigma}$$

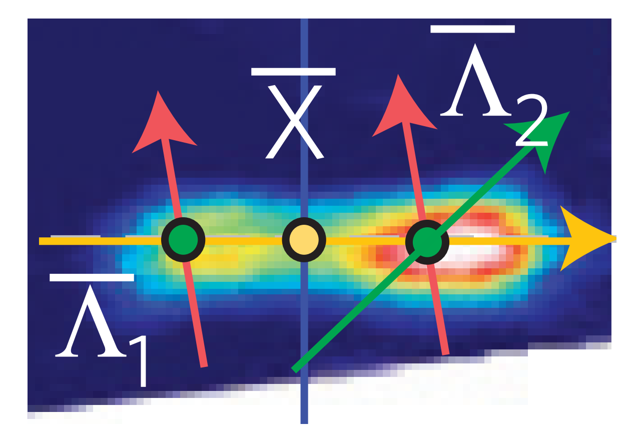
$$\Longrightarrow \text{class D: } \sigma_x \mathcal{H}^*_\pm(\mathbf{k})\sigma_x = -\mathcal{H}(-\mathbf{k})$$

$$\Longrightarrow n_\pm = \frac{1}{8\pi}\int_{\text{2D BZ}} d^2\mathbf{k}\, \epsilon^{\mu\nu}\hat{\mathbf{m}}_\pm \cdot \left[\partial_{k_\mu}\hat{\mathbf{m}}_\pm \times \partial_{k_\nu}\hat{\mathbf{m}}_\pm\right]$$



mirror Chern number: $n_{\mathcal{M}} = (n_+ - n_-)$

ARPES on SnTe



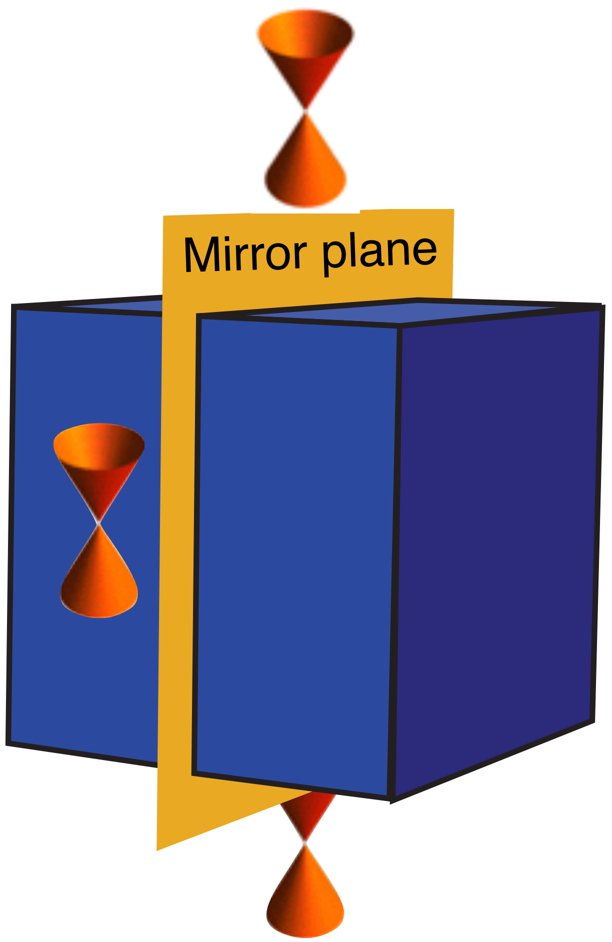
$n_{\mathcal{M}} = \#$ Dirac cones surface states

Classification topological crystalline materials

Mirror symmetry: $R^{-1}\mathcal{H}(-k_x, k_y, k_z)R = \mathcal{H}(k_x, k_y, k_z)$

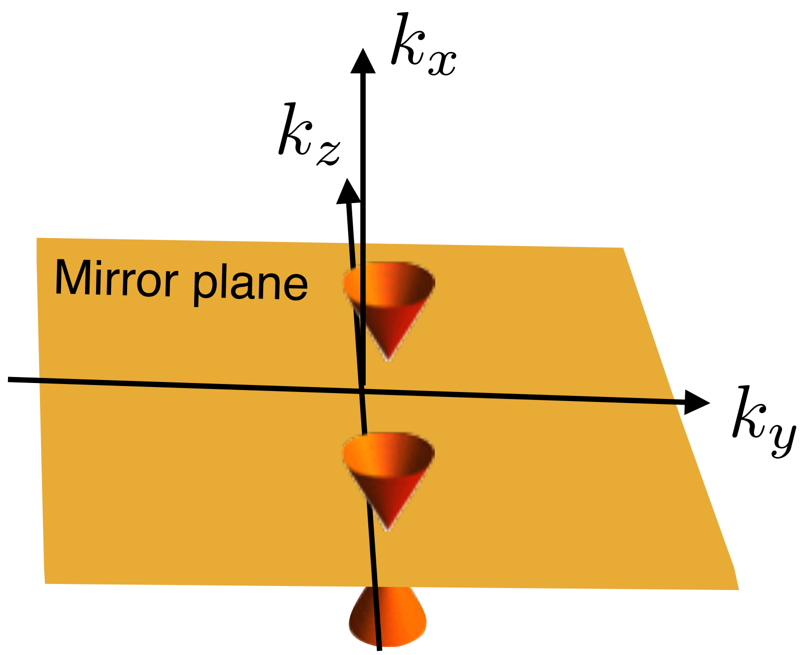
▶ Classification of *fully gapped* mirror symmetric topological materials depends on:

- non-spatial symmetries: TRS, PHS, and chiral SLS
- whether R commutes or anti-commutes with TRS, PHS, SLS
- spatial dimension of the system d



▶ Classification of *gapless* mirror symmetric topological materials depends on:

- global symmetries: TRS, PHS, and chiral SLS
- whether R (anti-)commutes with TRS, PHS, SLS
- co-dimension $p = d - d_{\rm FS}$ of Fermi surface
- how Fermi surface transforms under mirror and non-spatial symmetries



Classification of *fully gapped* topological crystalline materials

Classification in terms of mirror symmetries

$R_- \;:\; R$ anti-commutes with T (C or S)

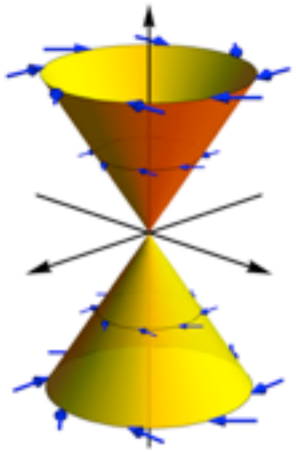
$R_+ \;:\; R$ commutes with T (C or S)

Reflection	top. insul. and top. SC	d=1	d=2	d=3	d=4	d=5	d=6	d=7	d=8
R	A	$M\mathbb{Z}$	0	$M\mathbb{Z}$	0	$M\mathbb{Z}$	0	$M\mathbb{Z}$	0
R_+	AIII	0	$M\mathbb{Z}$	0	$M\mathbb{Z}$	0	$M\mathbb{Z}$	0	$M\mathbb{Z}$
R_-	AIII	$M\mathbb{Z}\oplus\mathbb{Z}$	0	$M\mathbb{Z}\oplus\mathbb{Z}$	0	$M\mathbb{Z}\oplus\mathbb{Z}$	0	$M\mathbb{Z}\oplus\mathbb{Z}$	0
R_+,R_{++}	AI	$M\mathbb{Z}$	0	0	0	$2M\mathbb{Z}$	0	$M\mathbb{Z}_2$	$M\mathbb{Z}_2$
	BDI	$M\mathbb{Z}_2$	$M\mathbb{Z}$	0	0	0	$2M\mathbb{Z}$	0	$M\mathbb{Z}_2$
	D	$M\mathbb{Z}_2$	$M\mathbb{Z}_2$	$M\mathbb{Z}$	0	0	0	$2M\mathbb{Z}$	0
	DIII	0	$M\mathbb{Z}_2$	$M\mathbb{Z}_2$	$M\mathbb{Z}$	0	0	0	$2M\mathbb{Z}$
	AII	$2M\mathbb{Z}$	0	$M\mathbb{Z}_2$	$M\mathbb{Z}_2$	$M\mathbb{Z}$	0	0	0
	CII	0	$2M\mathbb{Z}$	0	$M\mathbb{Z}_2$	$M\mathbb{Z}_2$	$M\mathbb{Z}$	0	0
	C	0	0	$2M\mathbb{Z}$	0	$M\mathbb{Z}_2$	$M\mathbb{Z}_2$	$M\mathbb{Z}$	0
	CI	0	0	0	$2M\mathbb{Z}$	0	$M\mathbb{Z}_2$	$M\mathbb{Z}_2$	$M\mathbb{Z}$
R_-,R_{--}	AI	0	0	$2M\mathbb{Z}$	0	$T\mathbb{Z}_2$	$\mathbb{Z}_2$	$M\mathbb{Z}$	0
	BDI	0	0	0	$2M\mathbb{Z}$	0	$T\mathbb{Z}_2$	$\mathbb{Z}_2$	$M\mathbb{Z}$
	D	$M\mathbb{Z}$	0	0	0	$2M\mathbb{Z}$	0	$T\mathbb{Z}_2$	$\mathbb{Z}_2$
	DIII	$\mathbb{Z}_2$	$M\mathbb{Z}$	0	0	0	$2M\mathbb{Z}$	0	$T\mathbb{Z}_2$
	AII	$T\mathbb{Z}_2$	$\mathbb{Z}_2$	$\boxed{M\mathbb{Z}}$	0	0	0	$2M\mathbb{Z}$	0
	CII	0	$T\mathbb{Z}_2$	$\mathbb{Z}_2$	$M\mathbb{Z}$	0	0	0	$2M\mathbb{Z}$
	C	$2M\mathbb{Z}$	0	$T\mathbb{Z}_2$	$\mathbb{Z}_2$	$M\mathbb{Z}$	0	0	0
	CI	0	$2M\mathbb{Z}$	0	$T\mathbb{Z}_2$	$\mathbb{Z}_2$	$M\mathbb{Z}$	0	0
R_{-+}	BDI, CII	$2\mathbb{Z}$	0	$2M\mathbb{Z}$	0	$2\mathbb{Z}$	0	$2M\mathbb{Z}_2$	0
R_{+-}	DIII, CI	$2M\mathbb{Z}$	0	$2\mathbb{Z}$	0	$2M\mathbb{Z}$	0	$2\mathbb{Z}$	0
R_{+-}	BDI	$M\mathbb{Z}\oplus\mathbb{Z}$	0	0	0	$2M\mathbb{Z}\oplus 2\mathbb{Z}$	0	$M\mathbb{Z}_2\oplus\mathbb{Z}_2$	$M\mathbb{Z}_2\oplus\mathbb{Z}_2$
R_{-+}	DIII	$M\mathbb{Z}_2\oplus\mathbb{Z}_2$	$M\mathbb{Z}_2\oplus\mathbb{Z}_2$	$M\mathbb{Z}\oplus\mathbb{Z}$	0	0	0	$2M\mathbb{Z}\oplus 2\mathbb{Z}$	0
R_{+-}	CII	$2M\mathbb{Z}\oplus 2\mathbb{Z}$	0	$M\mathbb{Z}_2\oplus\mathbb{Z}_2$	$M\mathbb{Z}_2\oplus\mathbb{Z}_2$	$M\mathbb{Z}\oplus\mathbb{Z}$	0	0	0
R_{-+}	CI	0	0	$2M\mathbb{Z}\oplus 2\mathbb{Z}$	0	$M\mathbb{Z}_2\oplus\mathbb{Z}_2$	$M\mathbb{Z}_2\oplus\mathbb{Z}_2$	$M\mathbb{Z}\oplus\mathbb{Z}$	0

SnTe

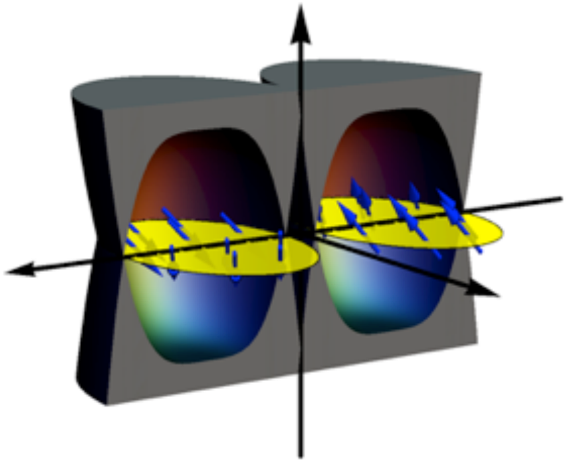
Chiu & Schnyder 2014; Chiu, Yao, Ryu, PRB 2013; Morimoto & Furusaki PRB 2013; Shiozaki & Sato 2014

Conclusions & Outlook



- Topological properties of fully gapped and nodal superconductors (and insulators and semi-metals)

- Classification in terms of global non-spatial symmetries
- Classification in terms of mirror symmetries



- Experimental fingerprints of surface states

- tunneling spectroscopy: zero-bias peak
- Fourier-transformed STS: absence of backscattering
- NCS-FM junction: spontaneous interface currents

- Candidate materials for topological superconductivity

- Non-centrosymmetric SCs: $CePt_3Si$, Li_2Pt_3B, BiPd, LuPtB, LaPtB, etc. (class DIII)

- Locally non-centrosymmetric SCs: SrPtAs (class A ?)

- Centrosymmetric SCs: Sr_2RuO_4 (class D), $Cu_xBi_2Se_3$?, $Cu_x(PbSe)_5(Bi_2Se_3)_6$ (class DIII), LuPtBi, YPtBi (class CII), $UrPt_3$, URu_2Si_2, $CeCoIn_5$

- Interfaces: $LaAlO_3/SrTiO_3$? (class DIII), InSb + Nb (class D)