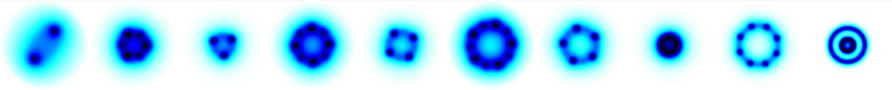

Topological defects and their experimental signature in $s + is$ superconductors

Domain walls, vortices and $\mathbb{C}P^2$ skyrmions

Julien Garaud
with J. Carlström, M. Speight and E. Babaev

Department of Physics, University of Massachusetts
Amherst, MA 01003, USA
and
Department of Theoretical Physics, Royal Institute of Technology
Stockholm, SE-10691

New Horizon of Strongly Correlated Physics
ISSP Tokyo – June, 26-th, 2014

based on J. Garaud, J. Carlström, M. Speight and E. Babaev

- *Geometrically stabilized domain walls and assisted Kibble-Zurek mechanism*, in progress (2014),
- *Domain walls and their experimental signatures in $s + is$ superconductors*, Phys. Rev. Lett. **112** *017003* (2014), arXiv:1308.3220 [cond-mat].
- *Chiral $\mathbb{C}P^2$ skyrmions in three-band superconductors*, Phys. Rev. B **87** *014507* (2013), arXiv:1211.4342 [cond-mat].
- *Topological solitons in three-band superconductors with broken time reversal symmetry*, Phys. Rev. Lett. **107** *197001*(2011), arXiv:1107.0995 [cond-mat].
- *Length scales, collective modes, and type-1.5 regimes in three-band superconductors*, Phys. Rev. B **84** *134518*(2011), arXiv:1107.4279 [cond-mat].

Motivations

Recently there has been discussions $s + is$ state in pnictides

- In hole doped $Ba_{1-x}K_xFe_2As_2$
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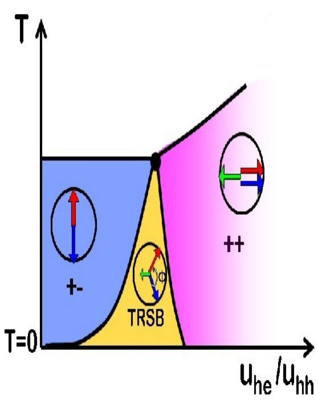

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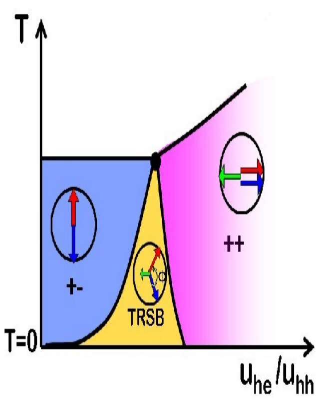

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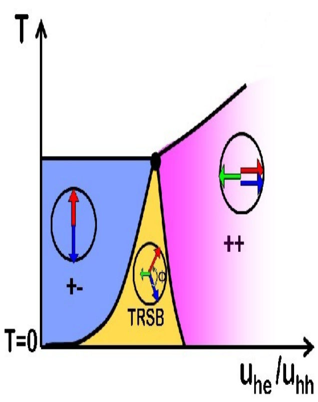

$\Rightarrow$ within Ginzburg-Landau model

(Minimal) Multicomponent Ginzburg-Landau model

Three complex fields $\psi_a = |\psi_a| \exp \mathbf{i}\varphi_a$ are the SC condensate

$$
\mathcal{F}_{\text{3CGL}} = \frac{1}{2}(\nabla \times \mathbf{A})^2 + \frac{1}{2}\sum_{a=1}^{3} |(\nabla + \mathrm{i}e\mathbf{A})\psi_a|^2 + (2\alpha_a + \beta_a|\psi_a|^2)|\psi_a|^2
$$

$$
- \sum_{b>a}^{3} \eta_{ab}|\psi_a||\psi_b| \cos\varphi_{ab} \qquad \text{where} \qquad \varphi_{ab} \equiv \varphi_b - \varphi_a \,.
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Microscopic derivation of GL model for three band Maiti, Chubukov PRB '13

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- Josephson interband interaction. Couple all ψ_a's.
  It breaks the global $\mathrm{U}(1)^3$ symmetry of the potential down to $\mathrm{U}(1)$.

Microscopic derivation of GL model for three band Maiti, Chubukov PRB '13

Phase frustration in three component systems

Each Josephson term anti-locks the phases ($\varphi_{ab}=\pi$) for $\eta_{ab}<0$:

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- if $\alpha_a = -1$, $\beta_a = 1$ and $\eta_{ab} = -1$, one cannot have all phases differences $\varphi_{ab} = \pi$.
- Then the ground state phases are ($\varphi_1 \equiv 0$)

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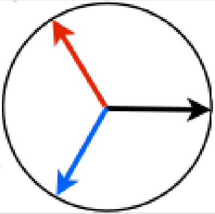

Discrete degeneracy of the ground state. It is not *c.c.* invariant
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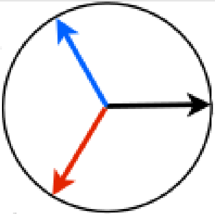

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BTRS transition as a function of the Temperature

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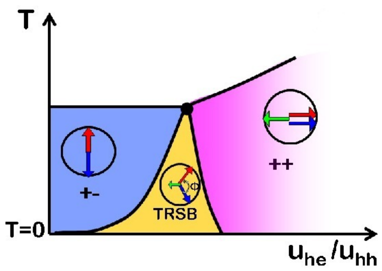

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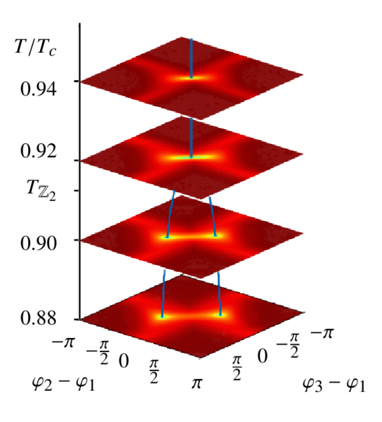

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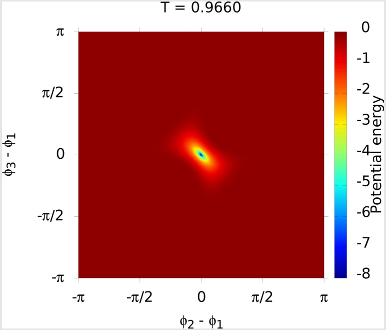

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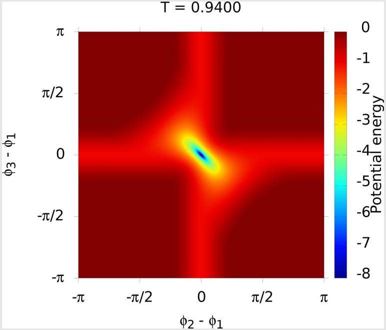

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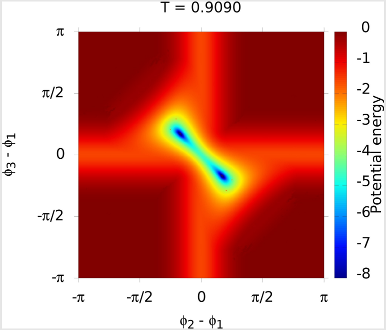

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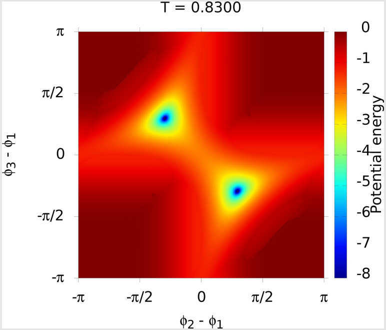
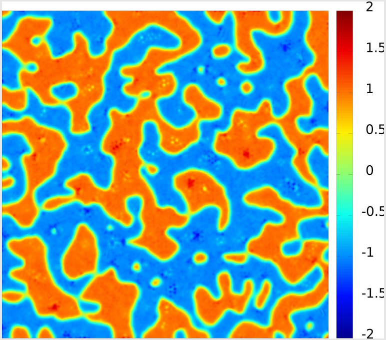


Kibble-Zurek mechanism

Topological defects are formed during phase transitions

- Domain walls are the defects associated with the breakdown of a discrete symmetry ($\mathbb{Z}_2$). Here Red and Blue are the GS



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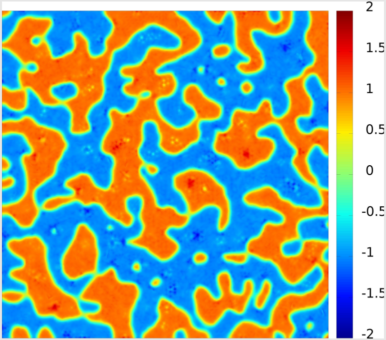

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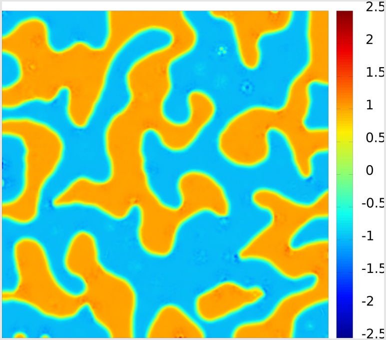

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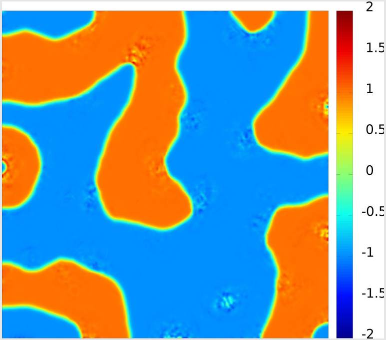

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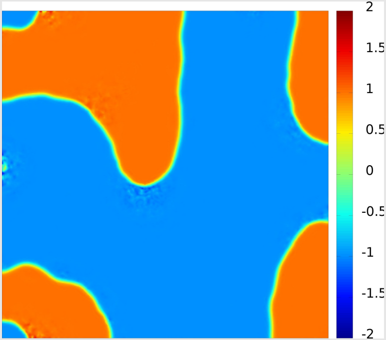

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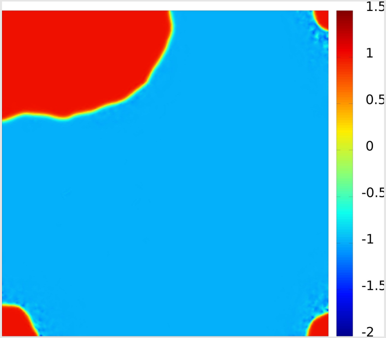

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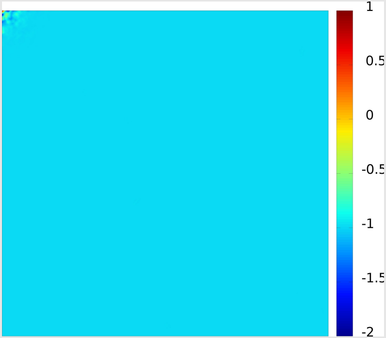

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Geometric stabilization JG, Babaev '14

So domain walls are dynamically unstable...
How can I control the stability of a domain-wall?

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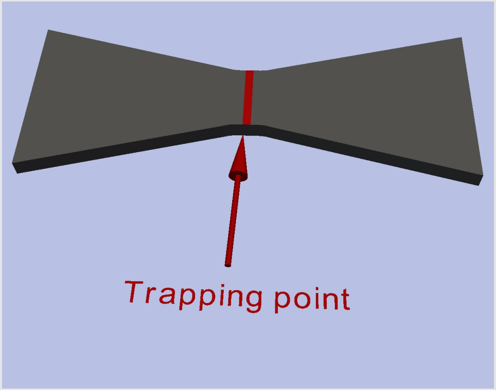
Using non-convex geometries allows stabilization of domain walls



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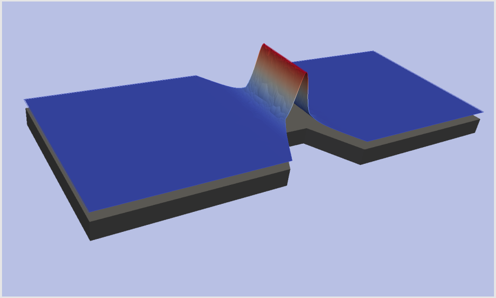
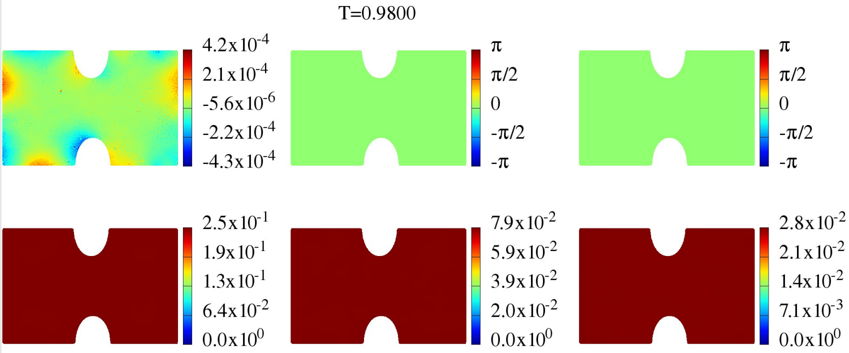


Now apply this idea to the three-component GL with BTRS

In zero field – Geometric stabilization of DW (1/2)

In zero applied field, DW can be stabilized in non-convex samples



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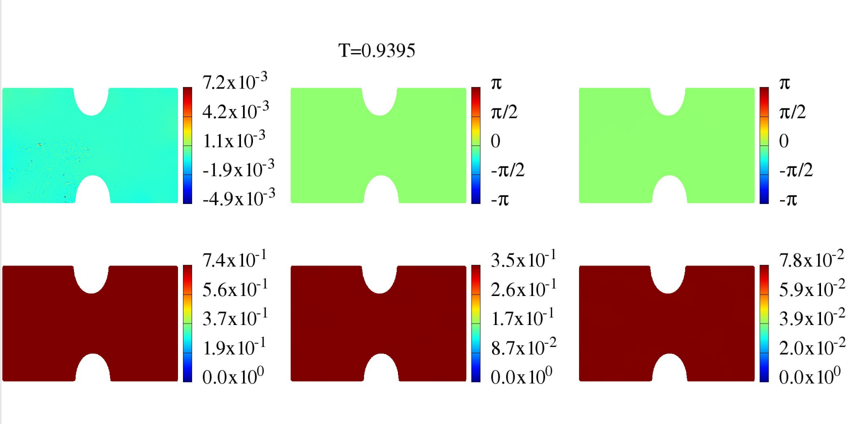

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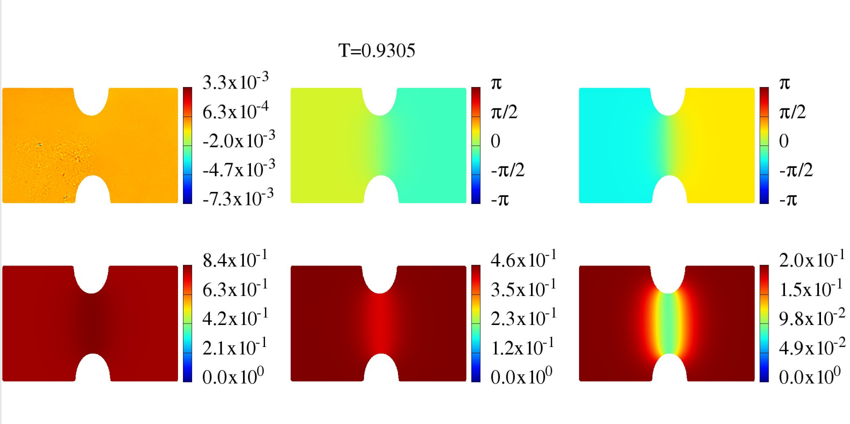

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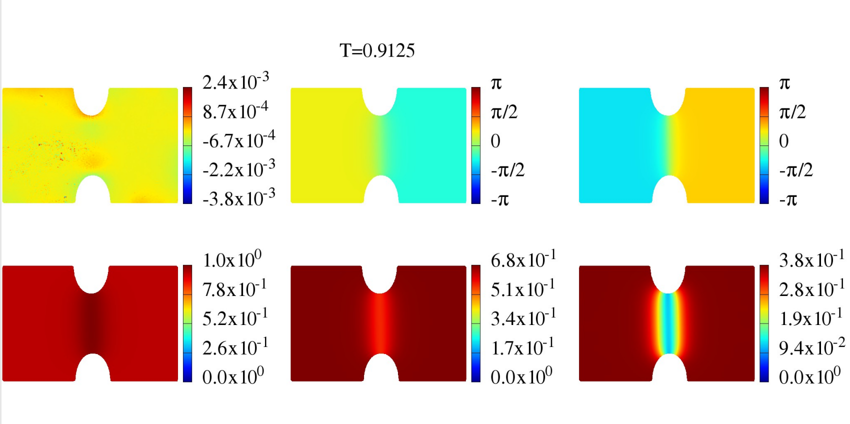

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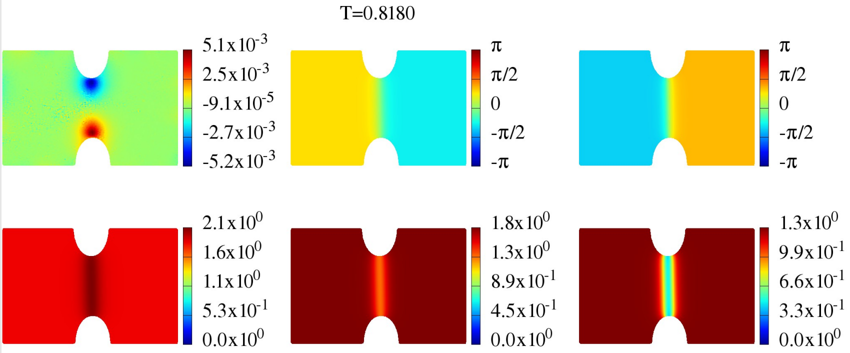

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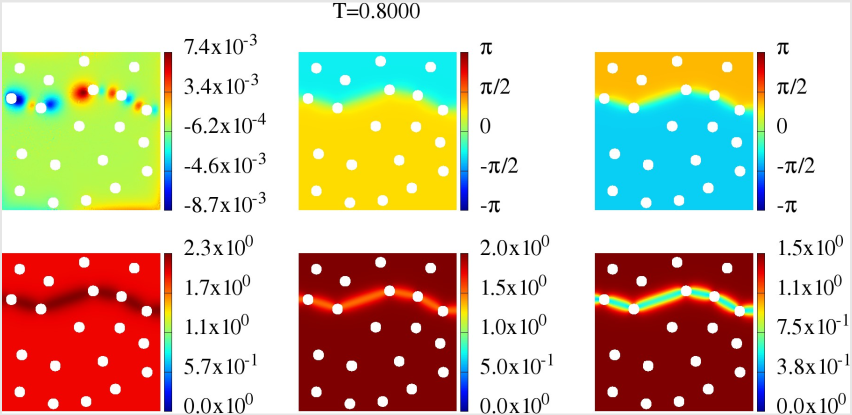
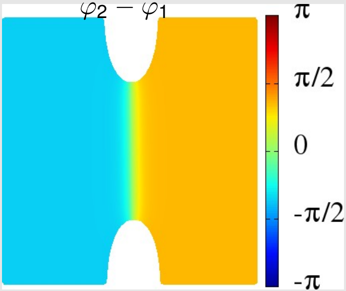


Note that stabilization also occurs with pinning centers

In zero field – Geometric stabilization of DW (2/2)

Properties of geometrically stabilized domain-walls

- Phase differences vary at the DW



In zero field – Geometric stabilization of DW (2/2)

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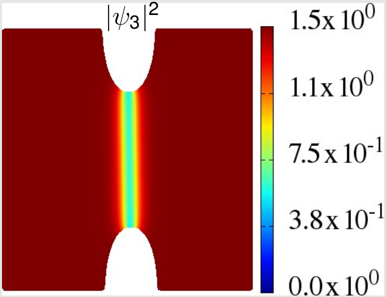

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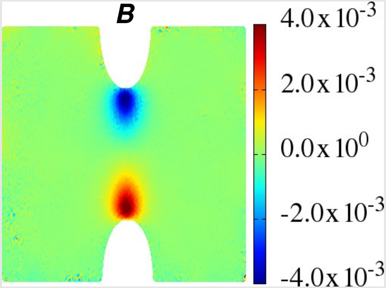
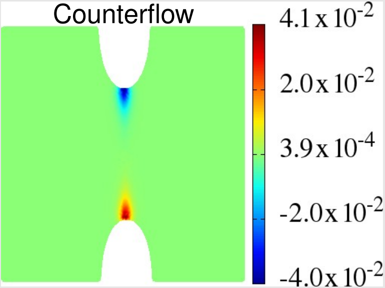


In zero field – Geometric stabilization of DW (2/2)

Properties of geometrically stabilized domain-walls

- Phase differences vary at the DW
- to accommodate these, densities are suppressed
- the ends of DW have small dipoles of the magnetic field
- this originate (partially screened) counterflows, mixing gradients of phase differences and densities. with $\Psi^\dagger = (\psi_1^*, \psi_2^*, \psi_3^*)$



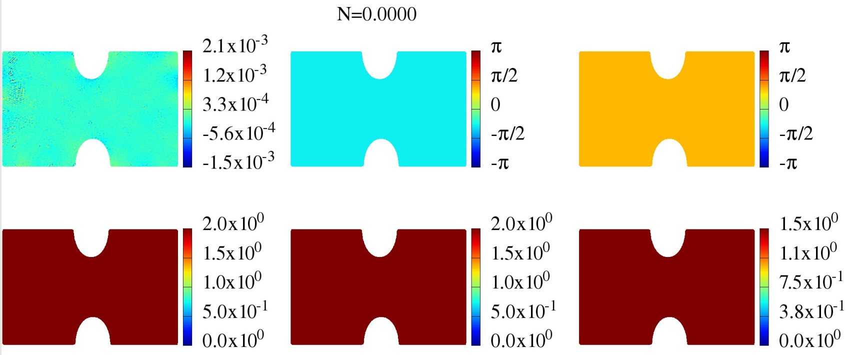
$$-eB_z = \epsilon_{ij}\partial_i \left(\frac{J_j}{e|\Psi|^2} \right) + \frac{i\epsilon_{ij}}{|\Psi|^4} \left[|\Psi|^2 \partial_i \Psi^\dagger \partial_j \Psi + \Psi^\dagger \partial_i \Psi \partial_j \Psi^\dagger \Psi \right],$$

JG, Carlström, Babaev, Speight, PRB '13

Magnetization processes (1/4)

To see the influence of the stabilized DW, first check

the reference magnetization process without a DW



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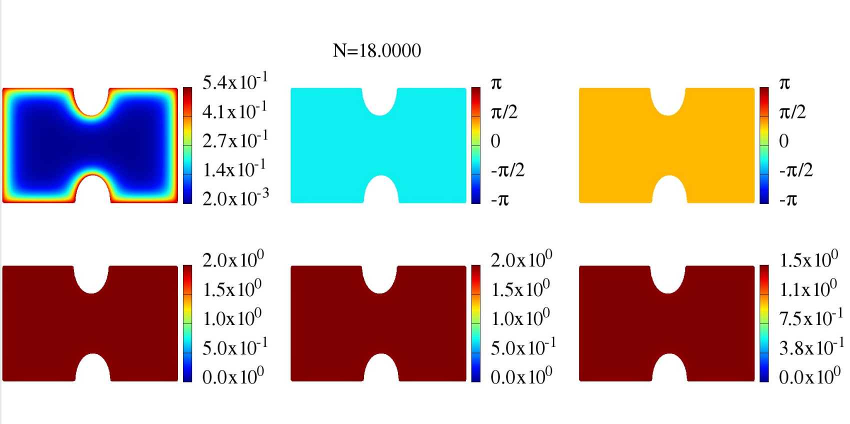

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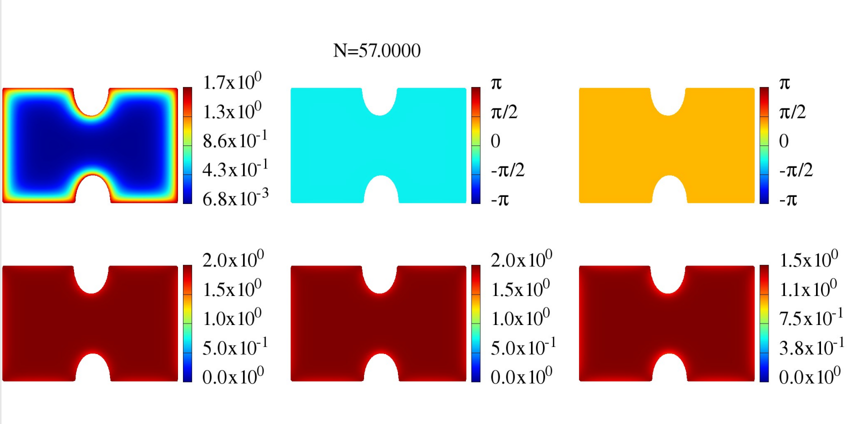

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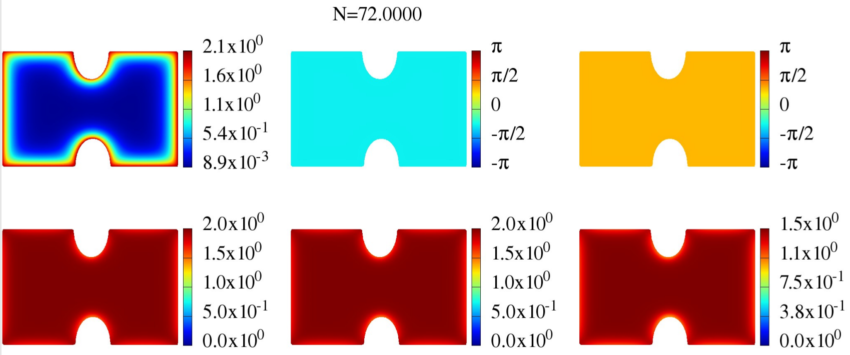

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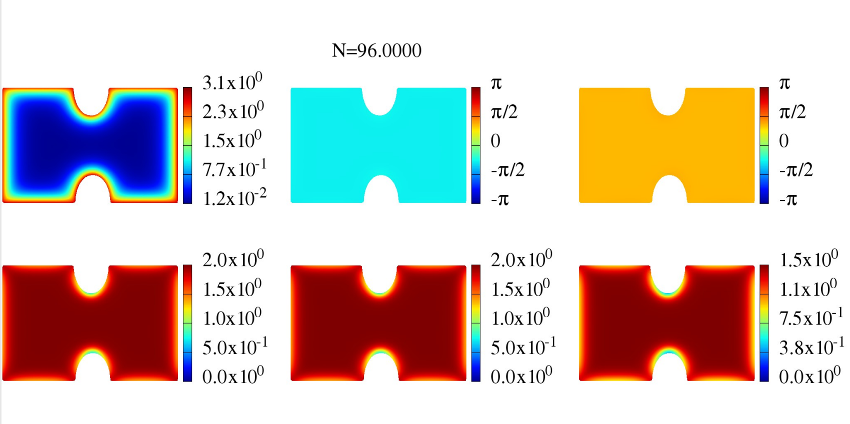

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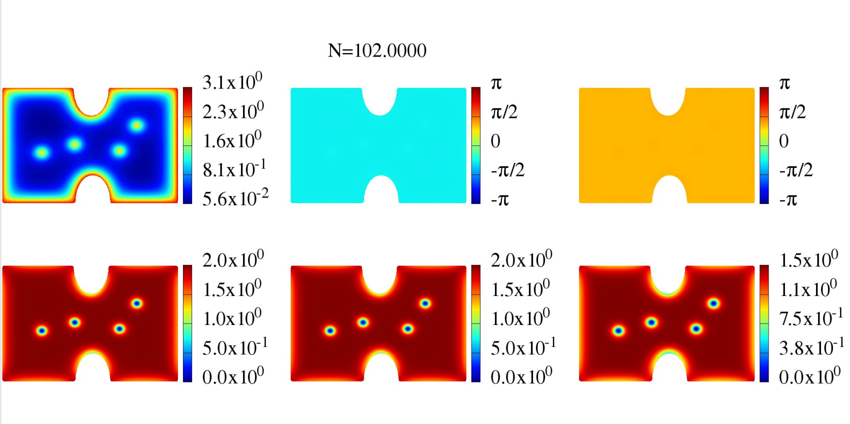

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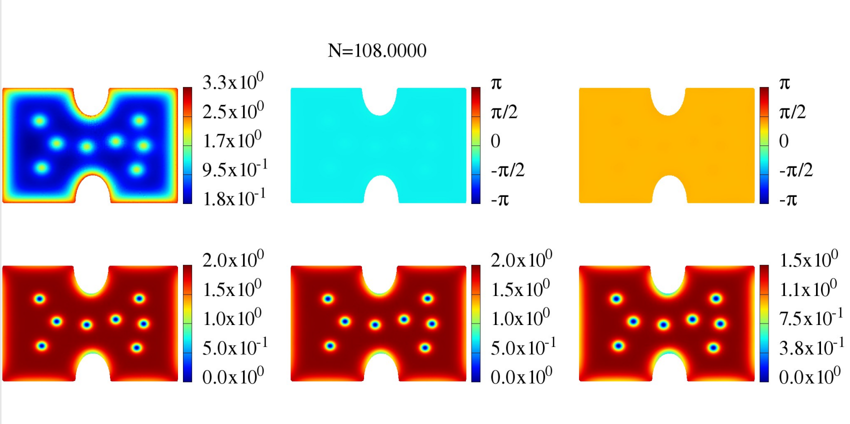

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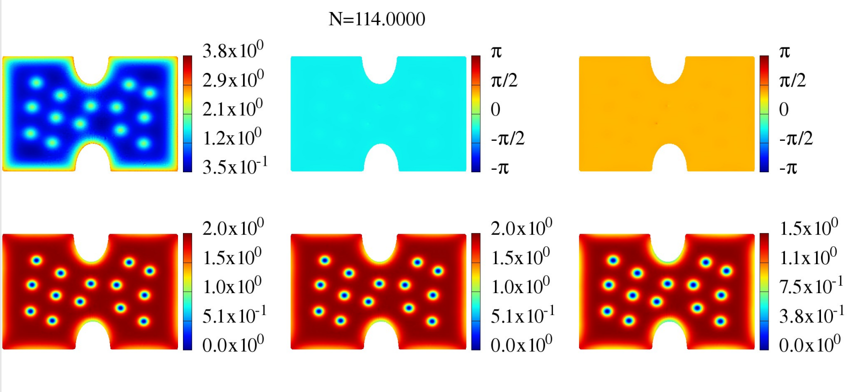

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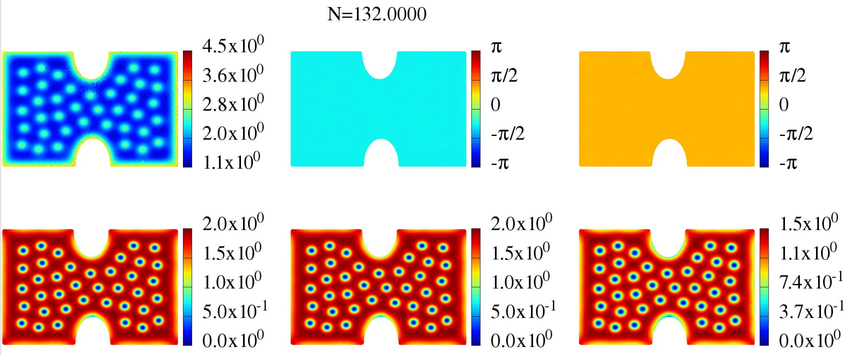

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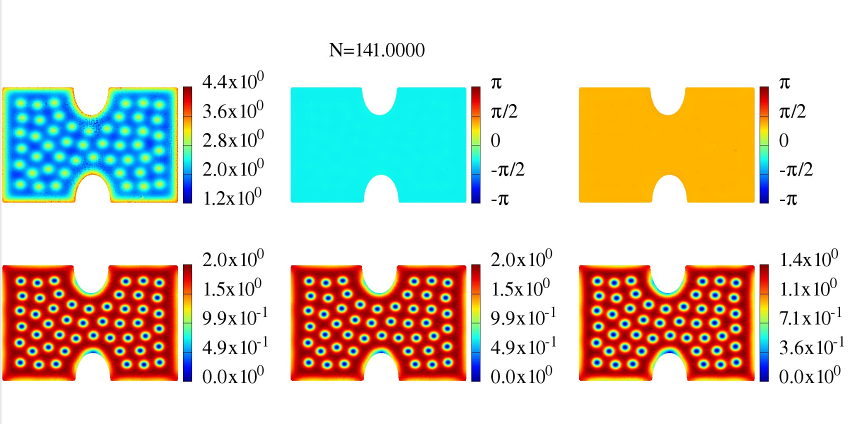

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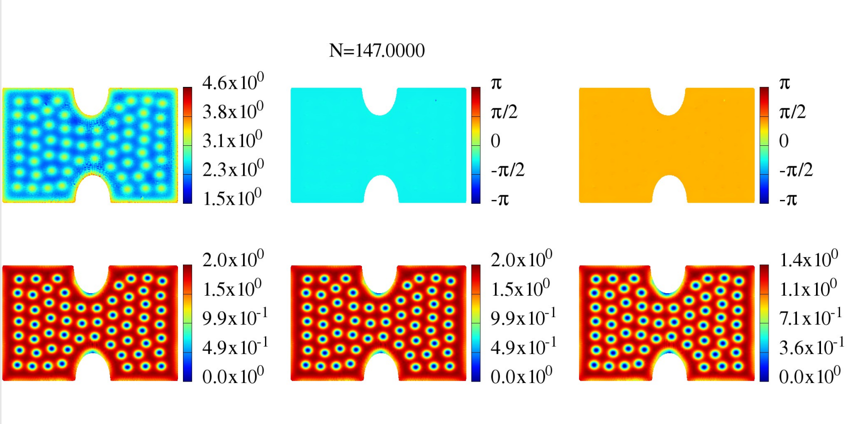
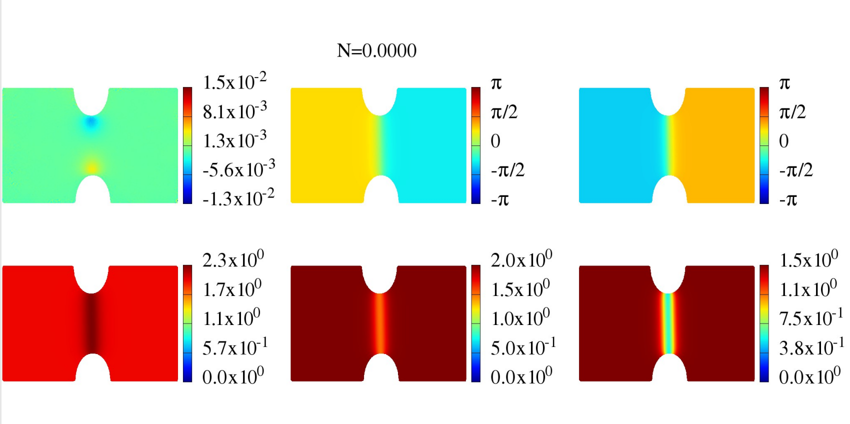


Magnetization processes (2/4)

Now, see the influence of the stabilized DW, see

magnetization process with a geometrically stabilized DW



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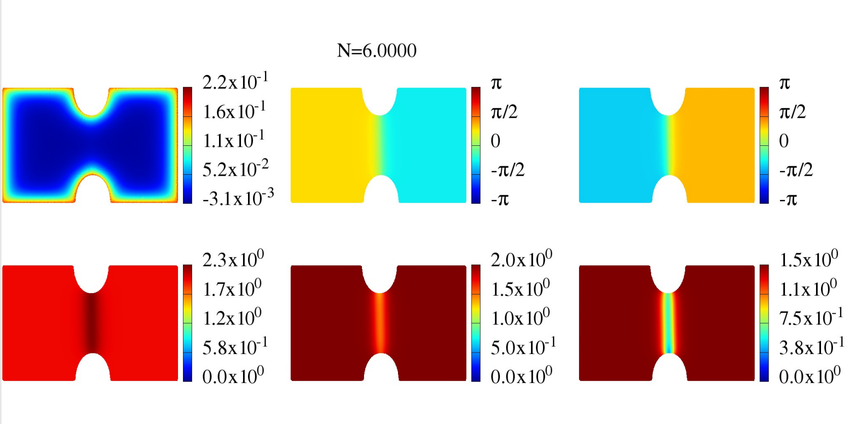

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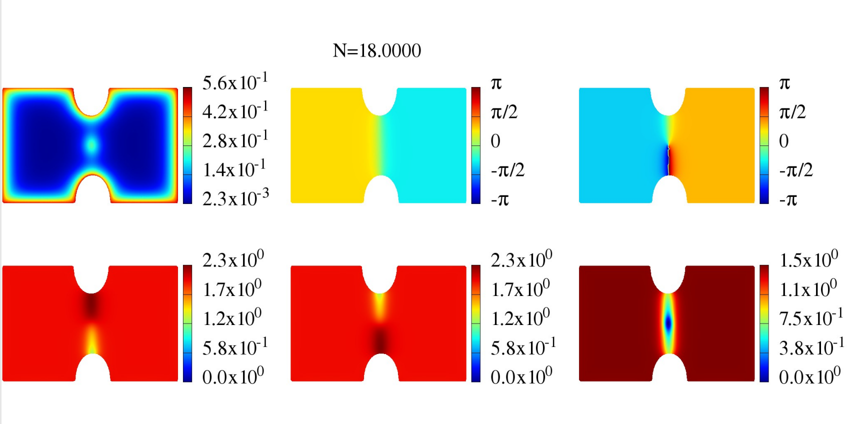

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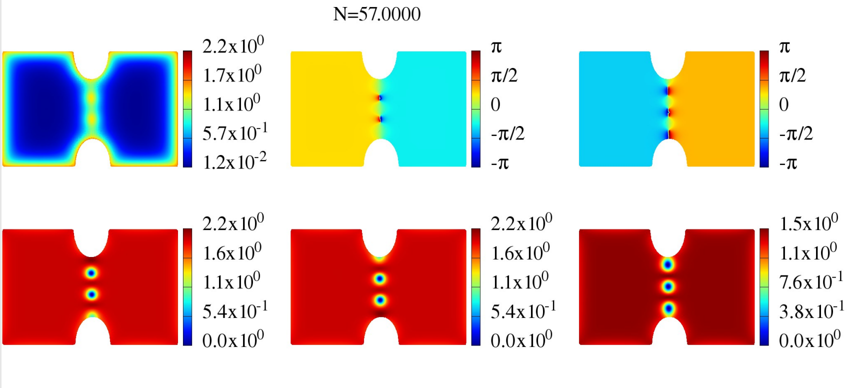

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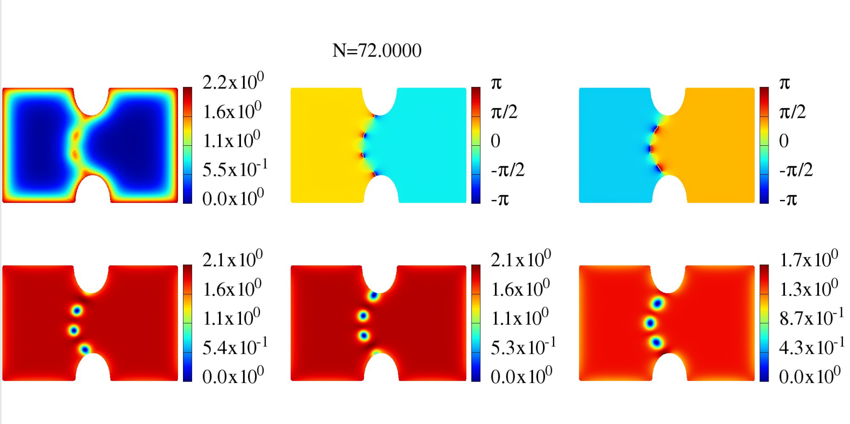

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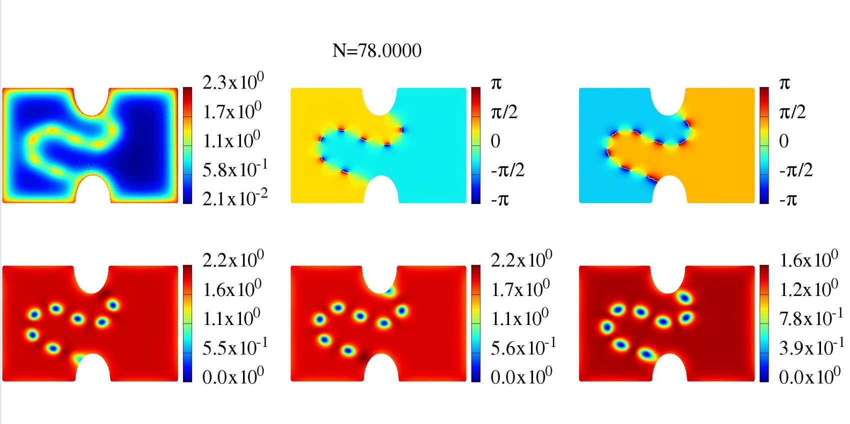

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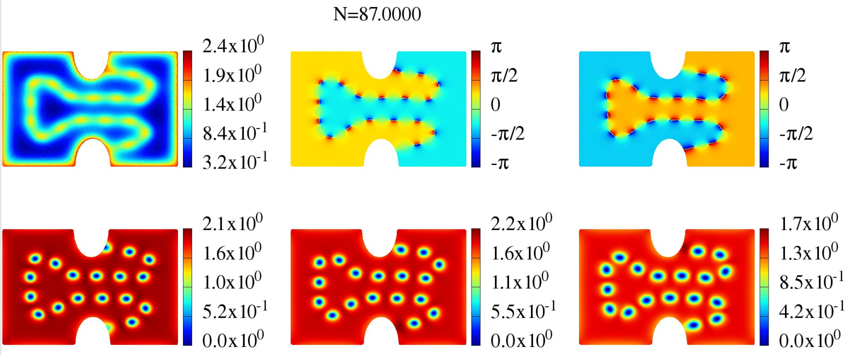

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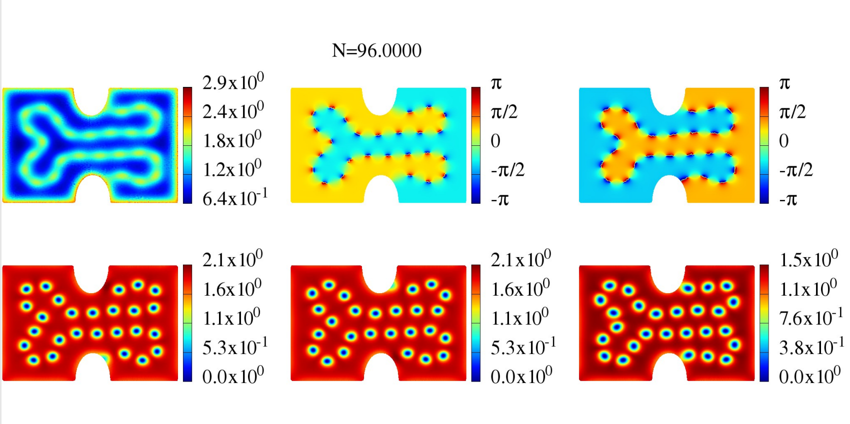

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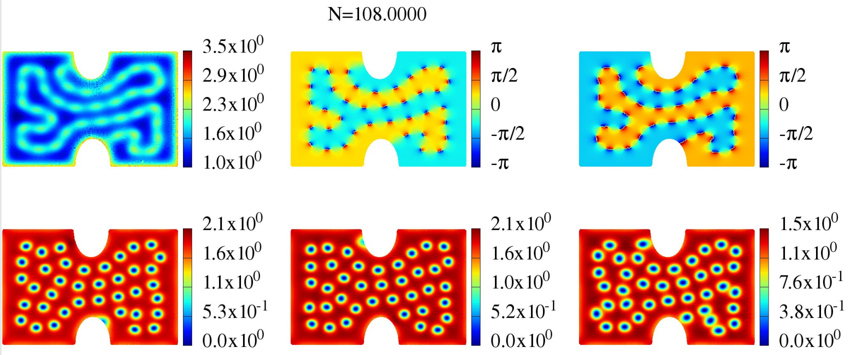

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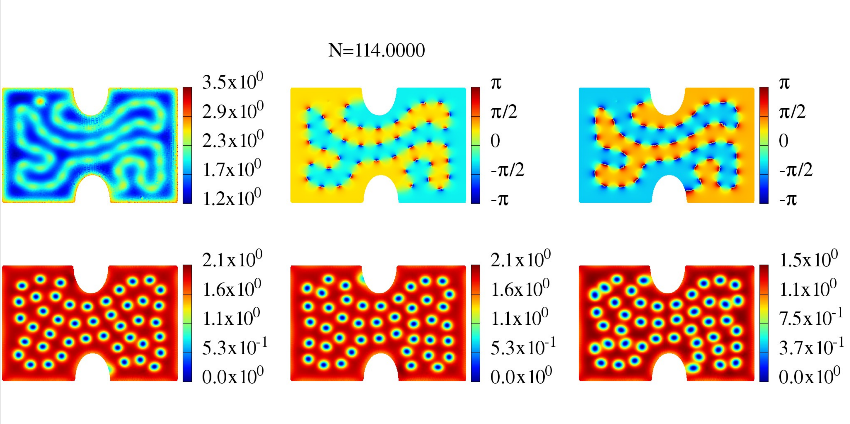

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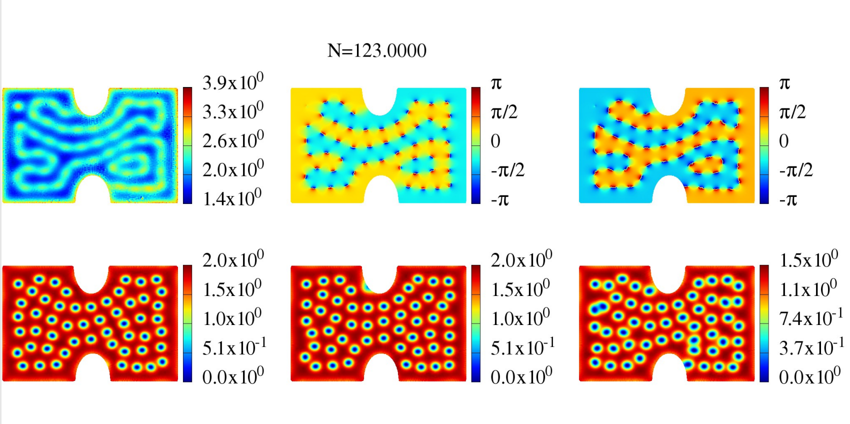

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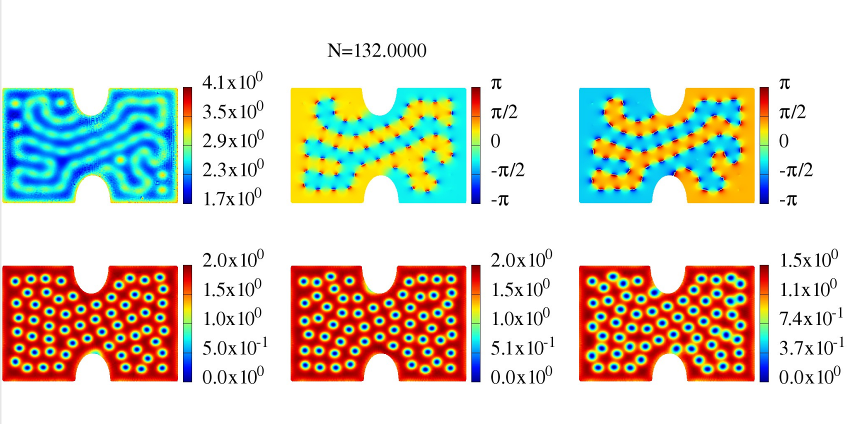
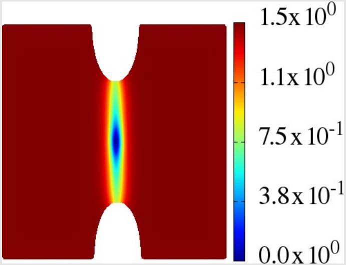


Magnetization processes (3/4)

Properties of magnetization process from a domain-walls

- densities are depleted at the DW, vortex here cost less
- it is fractional

$|\psi_3|^2$



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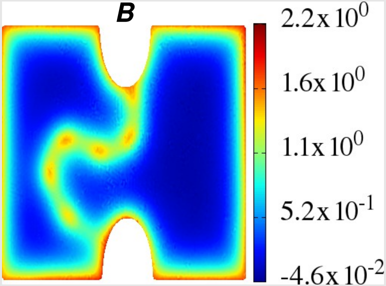

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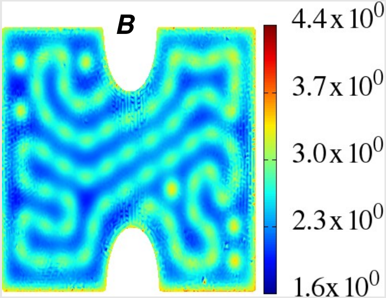
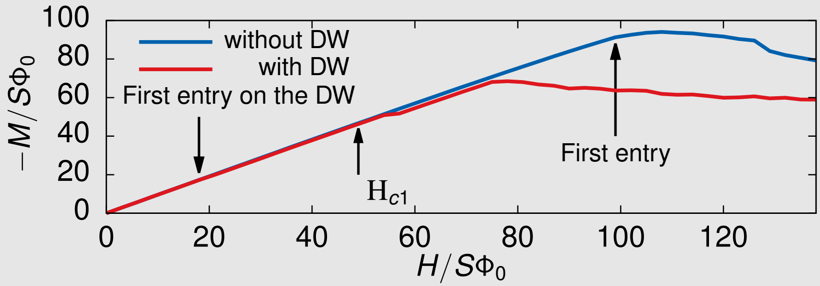
- it is beneficial to elongate the DW and enter vortices, it cost less than Bean-Livingston barrier
- eventually integer vortices enter



Magnetization processes (4/4)

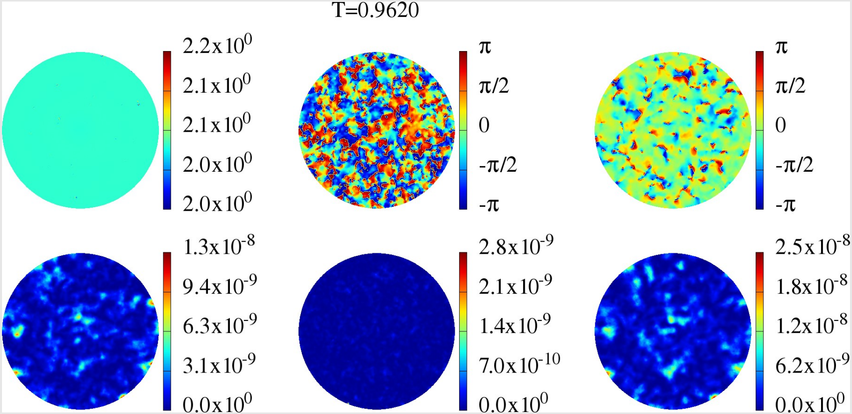
Magnetization curve



- Blue curve is the reference without DW (same sample)
- Red curve is the one with a stable DW
- $\Rightarrow$ very different magnetization curves.

Field cooled experiments JG Babaev '14

In applied field B_0, DW are stabilized by already existing vortices



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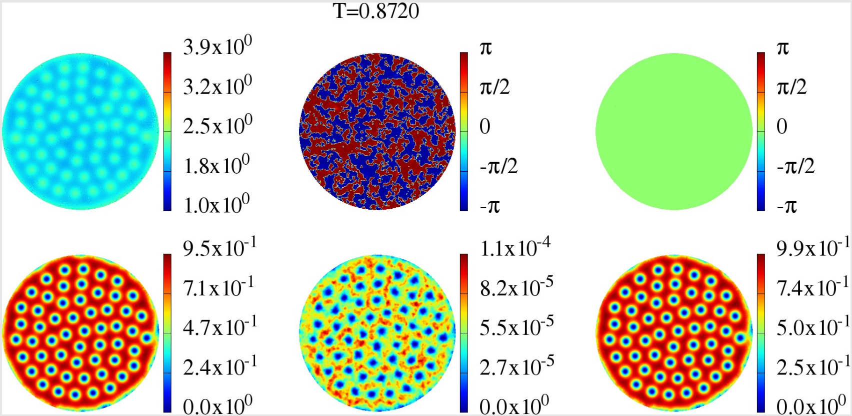

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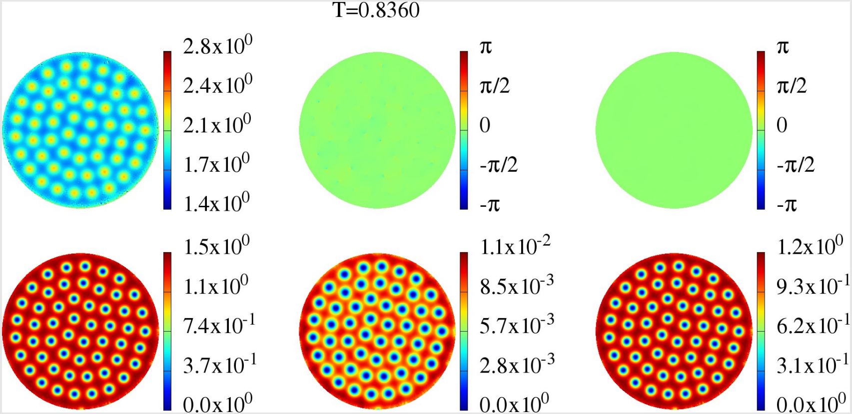

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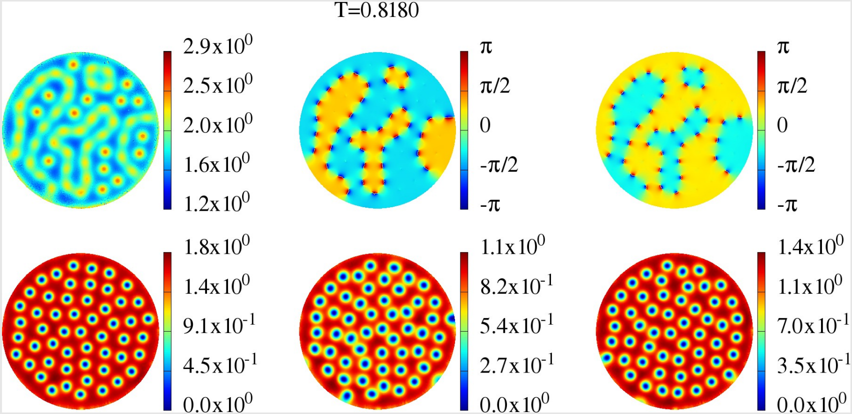

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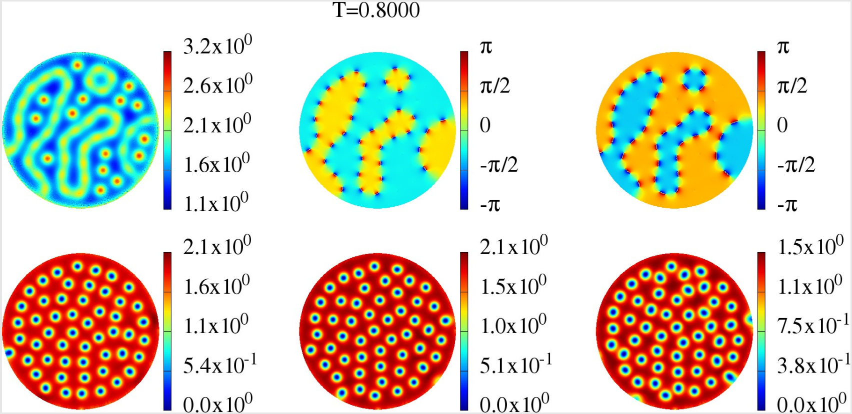

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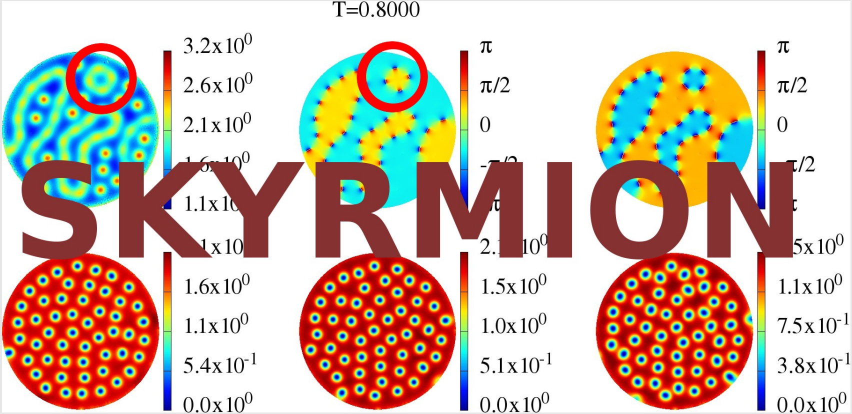

Closed DW stabilized by vortices are $\mathbb{C}P^2$ Skyrmions (discussed later)

$\mathbb{C}P^2$ Skyrmions

JG, Carlström, Babaev, Speight, PRB '13/ PRL '11

Reasons for stability

- Closed domain-wall are unstable to collapse because of their own line tension.

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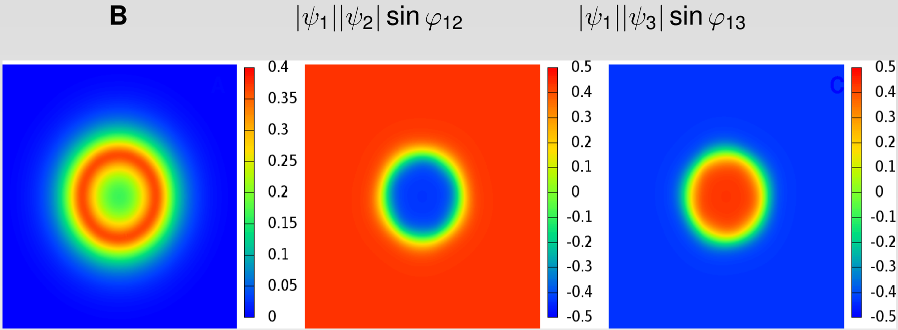
Composite vortex/domain-wall solitons are in fact $\mathbb{C}P^2$ Skyrmion

- If vortex interaction is repulsive enough (repulsion between fractional vortices), captured vortices can stabilize the domain wall.

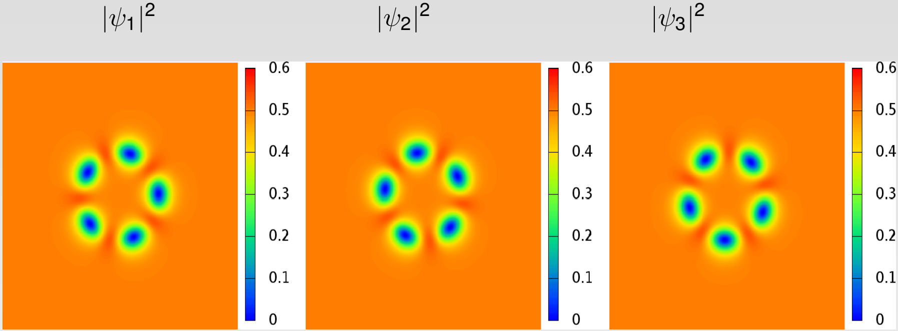
Skyrmion's structure ($\eta_{ab} = -3$; $\alpha_a, \beta_a = 1$; $N = 5$)



$\mathbb{C}P^2$ skyrmion's features

- Ringlike Magnetic field, spread along the domain wall
- Phase difference $\sin \varphi_{12}$ interpolate between the two inequivalent ground states $-2\pi/3$ and $2\pi/3$ $\Rightarrow$ Domain-wall

Skyrmion's structure ($\eta_{ab} = -3$; $\alpha_a, \beta_a = 1$; $N = 5$)



$\mathbb{C}P^2$ skyrmion's features

- Singularity for each component do not superimpose
  $\Rightarrow$ fractionalized vortices
  even with Josephson interaction

Skyrmions topology – conserved quantities

$\mathbb{C}P^2$ Topological invariant associated with the mapping $\Psi : \mathbb{R}^2 \to \mathbb{C}P^2$

$$\mathcal{Q}(\Psi) = \int_{\mathbb{R}^2} \frac{i\epsilon_{ij}}{2\pi|\Psi|^4} \left[|\Psi|^2 \partial_i \Psi^\dagger \partial_j \Psi + \Psi^\dagger \partial_i \Psi \partial_j \Psi^\dagger \Psi\right] \mathrm{d}^2 x \ \in \mathbb{N} ,$$

where $\Psi^\dagger = (\psi_1^*, \psi_2^*, \psi_3^*)$ denotes the vector of the 3 complex fields.
JG, Carlström, Babaev, Speight, PRB '13

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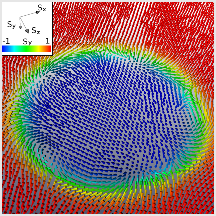
JG, Carlström, Babaev, Speight, PRB '13

Texture can be defined, by projecting on spin-1 Pauli matrices :

Additionally, to the topological charge $\mathcal{Q}$, The pseudo-spin texture, is defined as

$$\mathbf{S} \equiv (S_x, S_y, S_z) = \frac{\Psi^\dagger \boldsymbol{\sigma} \Psi}{\Psi^\dagger \Psi} \,,$$

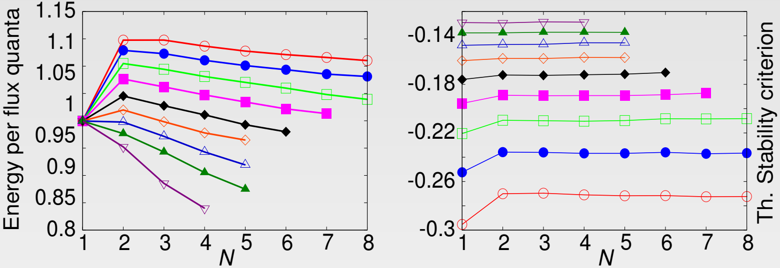
provides a good visualization of the Skyrmion



Few interesting properties

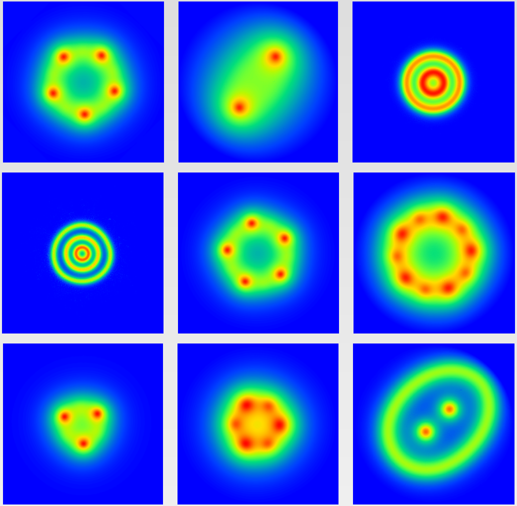
Adding $\sum_{b>a}^{3} \gamma_{ab}|\psi_a|^2|\psi_b|^2$ makes Skyrmions preferred over vortices

- Skyrmions are at least meta-stable (also thermodynamically stable)
- better to have more vortices on the DW

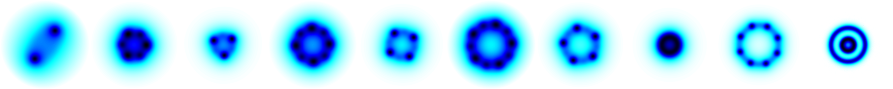


Here, different curves for γ_{ab}. Skyrmions can be preferred over vortices

Skyrmions have very exotic profile of *B*



Summary (1/2)

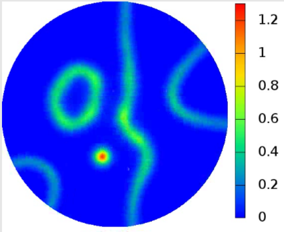


$\mathbb{C}P^2$ skyrmions could be an observable signature of TRSB states

- very exotic profile of magnetic field should be detected in scanning SQUID, scan Hall or magnetic force microscopy experiments

Skyrmions few percent more energetic than vortices (if $\gamma_{ab} = 0$)

- They are at least metastable
- can be excited by thermal fluctuations
- or created in field cooled experiments
- Other plausible formation mechanism, relaxing an initially dense vortex cluster [Movie]



Summary (2/2)

Stable domain wall as observable signatures of TRSB states

- very remarkable magnetic response

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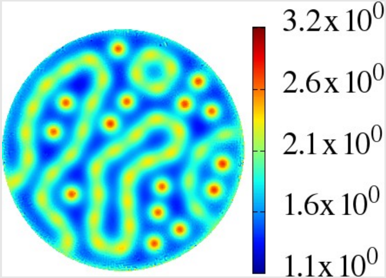

Summary (2/2)

Stable domain wall as observable signatures of TRSB states

- very remarkable magnetic response
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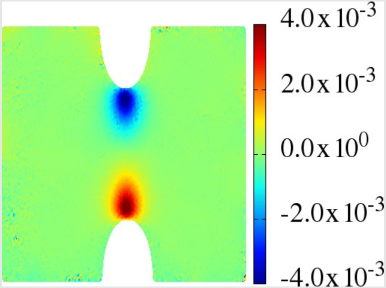
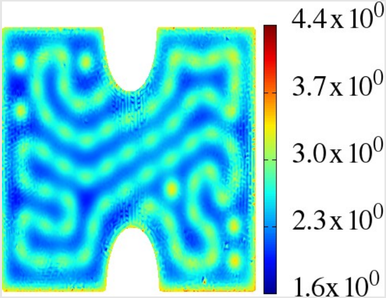
- or in zero field geometric stabilization



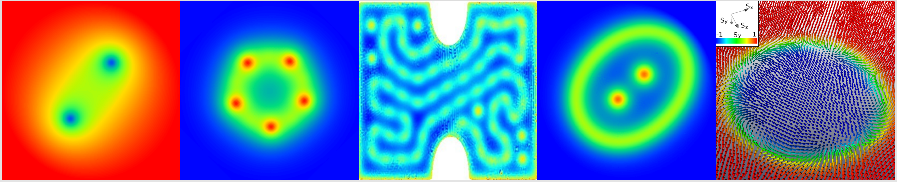
Summary (2/2)

Stable domain wall as observable signatures of TRSB states

- very remarkable magnetic response
- either in field cooled
- or in zero field geometric stabilization
- and the associated magnetization process which should be different depending on the cooling



Thank you for your attention!



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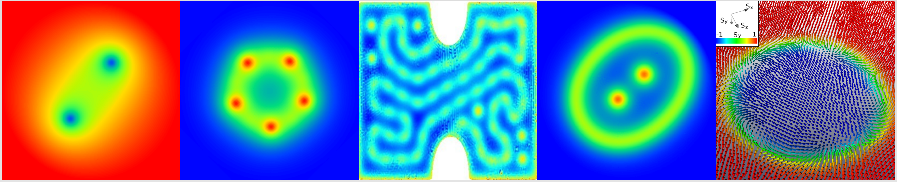

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