

FFLO-like state in oxide interface superconductors

Daniel F. Agterberg, U. Wisconsin - Milwaukee

J. Garaud, E. Babaev (KTH, U. Mass-Amherst)

M. Sigrist (ETH-Zurich), H. Tsunetsugu (ISSP)

R.P. Kaur, S. Mukherjee, Z. Zheng (UWM)

NSF DMR-0906633 and NSF DMR-1335215

- 1- Relevant Materials
- 2- Superconductivity with Rashba spin-orbit
- 3- FFLO-like state in magnetic fields
- 4- Abrikosov, Fractional, Skyrmion vortices (arXiv:1403.6655)

2D Superconductors

Ohtomo, Hwang, *Nature* 427, 423 (2004):
2D electron gas at LaAlO_3 and SrTiO_3 interface

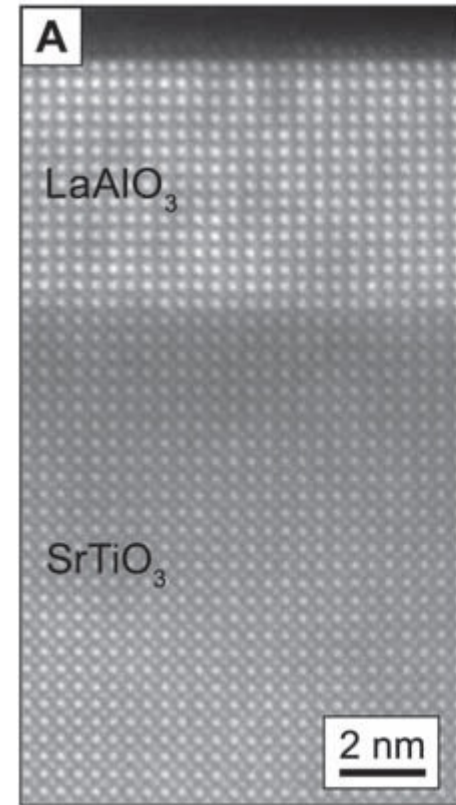
Superconductivity in the 2D electron gas
Reyren et al, *Science* 317, 1196 (2007):

Many 2D superconducting materials:

MoS_2 : *Science* 338, 1193 (2012)

Pb on GaAs : *PRL* 111, 057005 (2013)

KTaO_3 : *Nat. Nano* 6, 408 (2011)

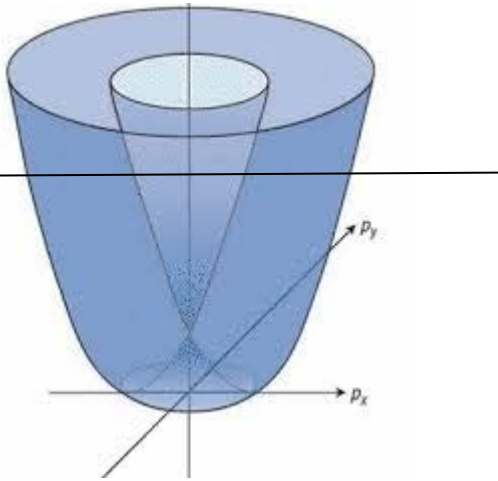


All these materials lack spatial parity symmetry and allow for a Rashba spin-orbit interaction

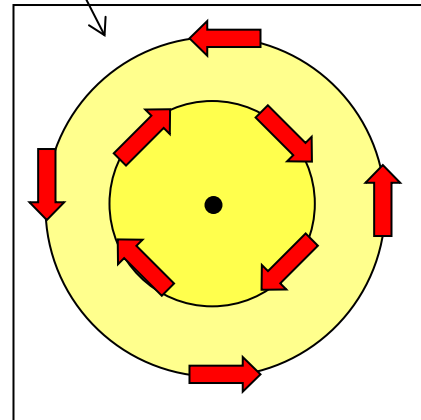
Microscopic Model

$$H = \sum_{k,s} \xi_k c_{ks}^t c_{ks} + \frac{1}{2} \sum_{k,k',q,s,s'} V c_{k+qs}^t c_{-k+qs}^t c_{-k'+qs} c_{k'+qs}$$

$$H_{spin} = \sum_{k,s,s'} (\mu_B \vec{h} + \vec{g}_k) \cdot \vec{\sigma}_{s,s'} c_{ks}^t c_{ks'}$$



$$\vec{g}_k = \alpha(\hat{x}k_y - \hat{y}k_x)$$



$$\delta N = \frac{N_1 - N_2}{N_1 + N_2} (\cong 0.05)$$

Free Energy: Broken Parity

GLW free energy: constrained by symmetry:

$$f = -\alpha |\psi|^2 + \beta |\psi|^4 + \frac{\hbar^2}{2m} (\nabla \psi)(\nabla \psi)^*$$

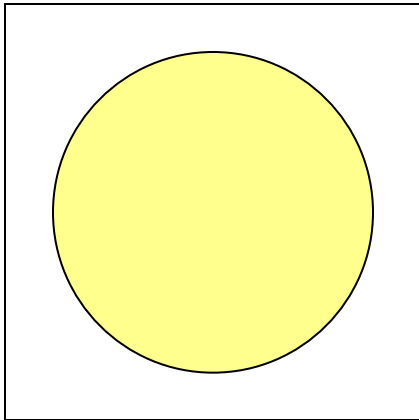
Broken parity symmetry allows a new term (Lifshitz invariant)

$$\varepsilon \hat{z} \cdot \vec{B} \times [\psi (\nabla \psi)^* + \psi^* (\nabla \psi)] = \varepsilon \hat{z} \cdot \vec{B} \times \vec{j}_{s,0}$$

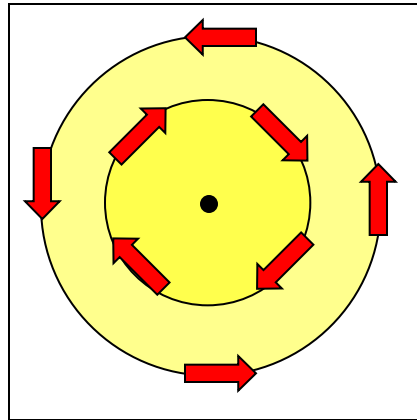
$$\Rightarrow \psi = \psi_0 e^{i2\vec{q} \cdot \vec{r}} \quad \vec{q} = -m\varepsilon \hat{z} \times \vec{B}$$

- finite momentum pairing solution in a uniform magnetic field when $|\Psi|$ is uniform (this state carries no current)
- finite momentum pairing guaranteed with in-plane field

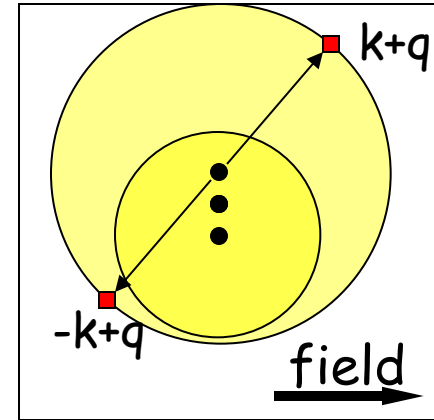
Finite Momentum Pairing



No Rashba



With Rashba



With Rashba and Zeeman Field

$$\xi_{\pm}(k) = \varepsilon(k) \pm | \alpha \vec{g}(k) + \mu_B \vec{B} |$$

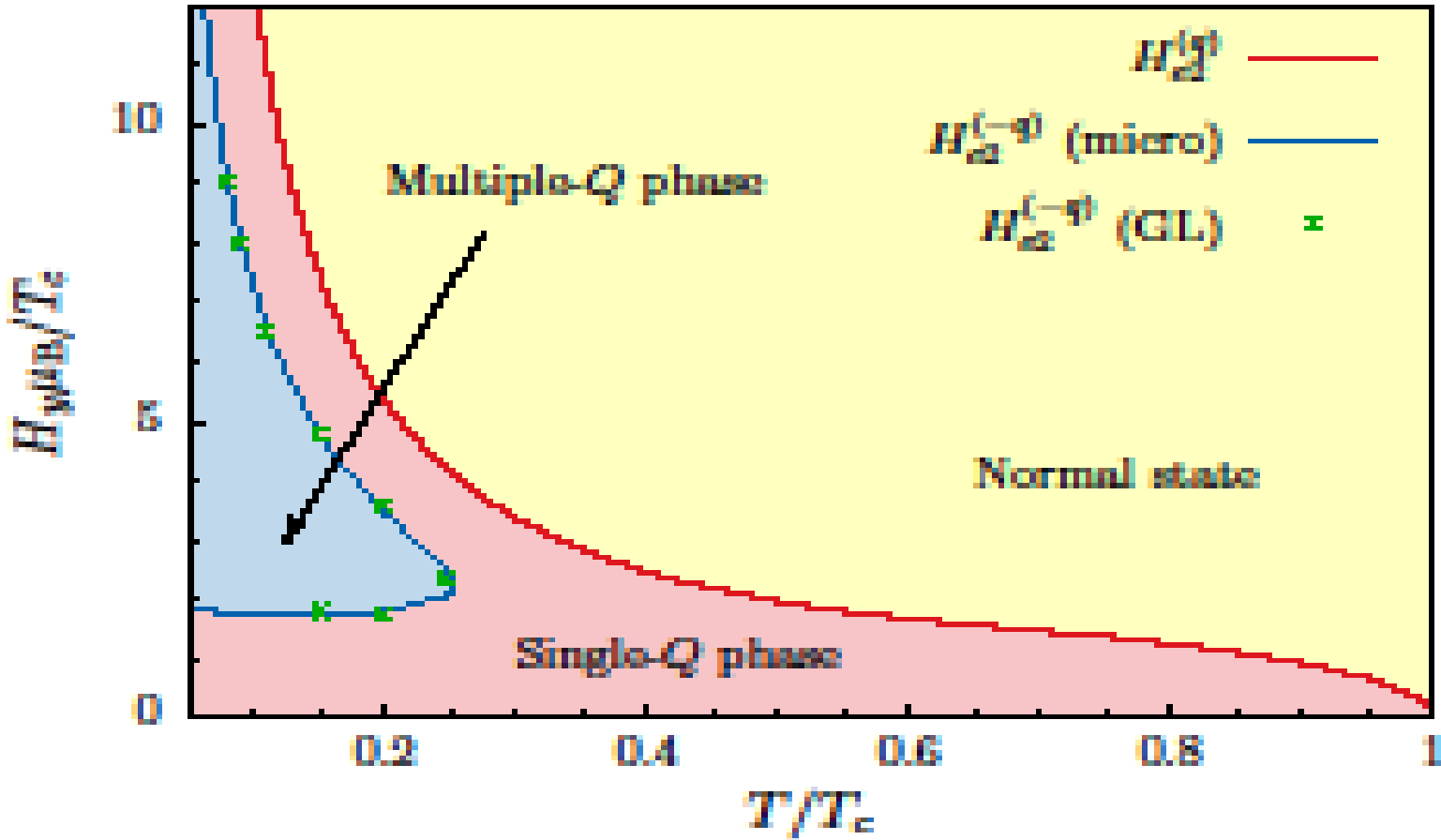
$$\xi_{\pm}(k) \approx \varepsilon(k) \pm \alpha \pm \mu_B \vec{B} \cdot \hat{g}(k)$$

$$\mu_B \vec{B} \cdot \hat{g}(k) = \mu_B B_x k_y / k_F$$

$$\Delta(\vec{r}) = \Delta_0 e^{i2\vec{q} \cdot \vec{r}}$$

The two bands prefer opposite q vectors.

Phase Diagram



Single-Q phase: $\psi(\vec{r}) = \psi_0 e^{i2\vec{q}\cdot\vec{r}}$

Multiple-Q phase: $\psi(\vec{r}) = \psi_q e^{i2qr} + \psi_{-q} e^{-2iqr}$

Theory of ψ_q and ψ_{-q}

$$\psi = \psi_q e^{iqx} + \psi_{-q} e^{-iqx}$$

$$f = \alpha_+ |\psi_q|^2 + \alpha_- |\psi_{-q}|^2 + \beta_+ |\psi_q|^4 + \beta_- |\psi_{-q}|^4 \\ + \beta_m |\psi_q|^2 |\psi_{-q}|^2 + \kappa_+ |\nabla \psi_q|^2 + \kappa_- |\nabla \psi_{-q}|^2$$

$U(1) \times U(1)$ symmetry!

General feature: $(\psi_{qx})^n (\psi_{qx}^*)^m (\psi_{-qx})^p (\psi_{-qx}^*)^k$

$$\mathbf{n} - \mathbf{m} + \mathbf{p} - \mathbf{k} = 0 \quad \text{Gauge invariance} \quad \mathbf{n} = \mathbf{m}$$

$$\mathbf{n} - \mathbf{m} - \mathbf{p} + \mathbf{k} = 0 \quad \text{Translational invariance} \quad \mathbf{p} = \mathbf{k}$$

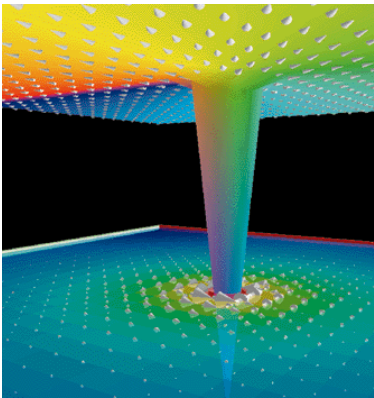
Fractional Vortices

$$(n,m) \quad \psi_q(r,\phi) = |\psi_q(r)| e^{in\phi} \quad \psi_{-q}(r,\phi) = |\psi_q(r)| e^{im\phi}$$

Consider (1,0) vortex:

$$\vec{j} = i\hbar m [\psi_q (\nabla \psi_q)^* - \psi_q^* (\nabla \psi_q)] - \frac{2me}{c} (|\psi_q|^2 + |\psi_{-q}|^2) \vec{A}$$

$$\oint \vec{A} \cdot d\vec{l} = \frac{|\psi_1|^2}{|\psi_1|^2 + |\psi_2|^2} \Phi_0$$



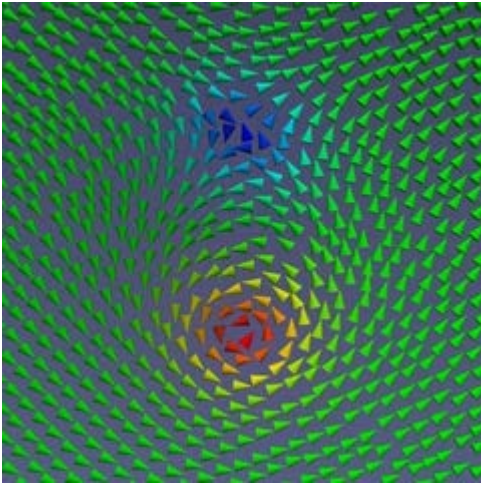
Fractional Flux vortices have line energies that diverge as $\log R$

(1,1) vortex has usual flux Φ_0 and finite line energy

Two kinds of (1,1) vortices: Abrikosov vortices and Skyrmion vortices

Abrikosov/Skyrmion Vortices

$$\Psi = \begin{pmatrix} \psi_q \\ \psi_{-q} \end{pmatrix} \quad \hat{n} = \frac{\Psi^t \vec{\partial} \Psi}{\Psi^t \Psi} \quad S_2 \rightarrow S_2 \text{ Map}$$



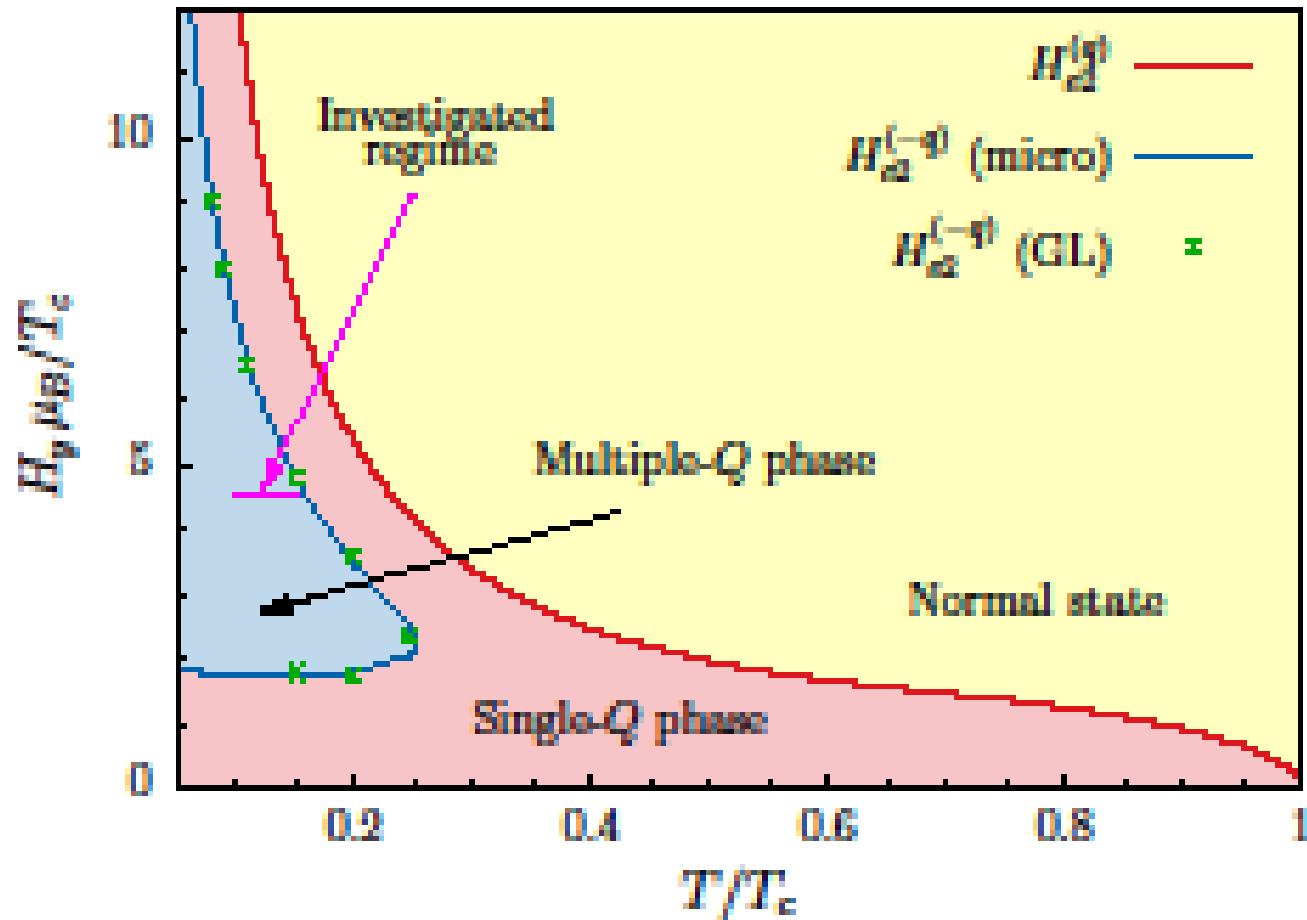
$$Q = \frac{1}{4\pi} \int_{R^2} (\hat{n} \cdot \partial_x \hat{n} \times \partial_y \hat{n}) dx dy$$

Q is non-zero (integer) when the two components have different core positions (Skyrmion)

Q = 0 when the cores coincide (Abrikosov)

- Which defects are stable in field a c-axis field, fractional vortices, Abrikosov vortices, or Skyrmion vortices?

c-axis Field

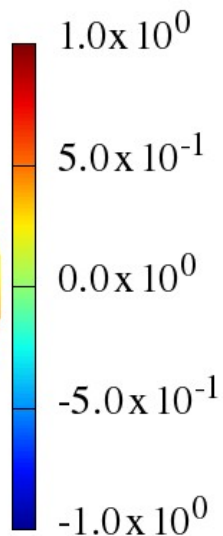
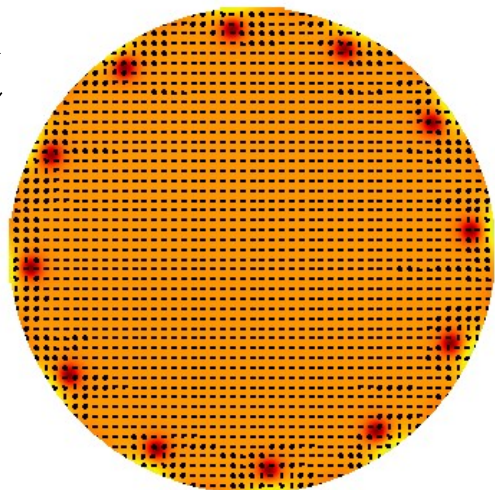


For parameters investigated GL theory is valid since $1/q \ll \xi_0$

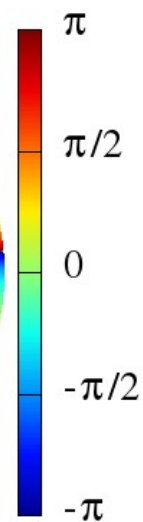
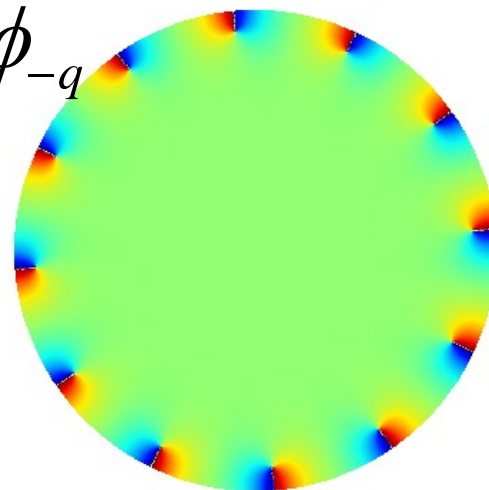
Results

T=0.1100

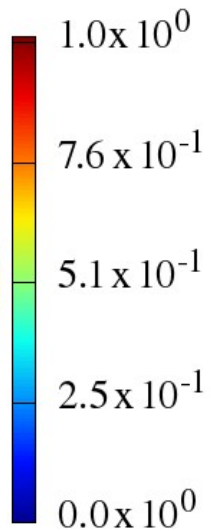
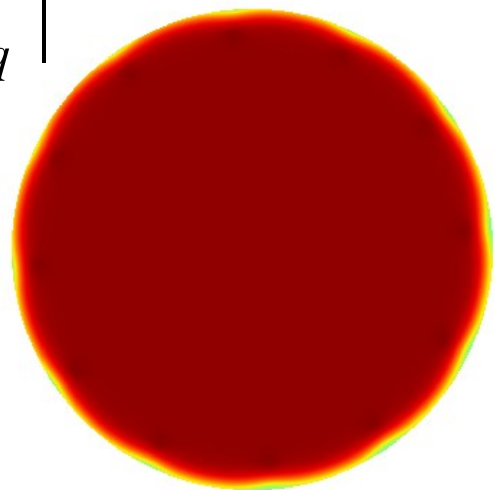
\hat{n}



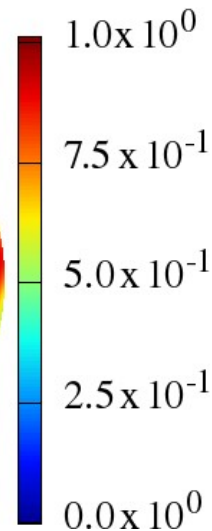
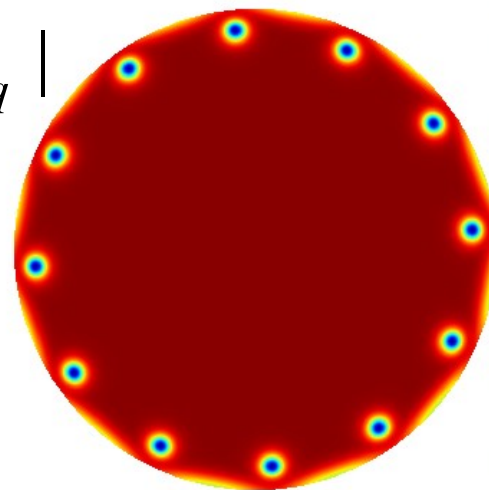
$\phi_q - \phi_{-q}$



$|\psi_q|$



$|\psi_{-q}|$



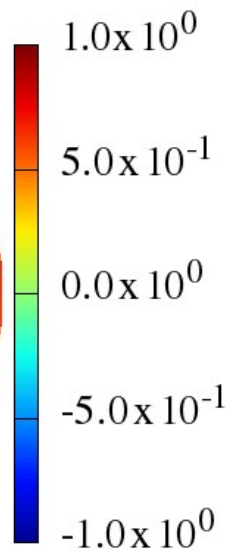
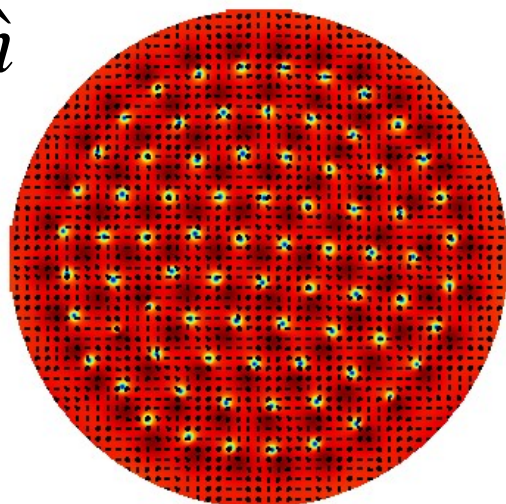
Fractional vortices near the boundary

Boundary Fractional Vortices

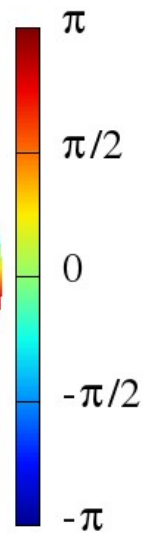
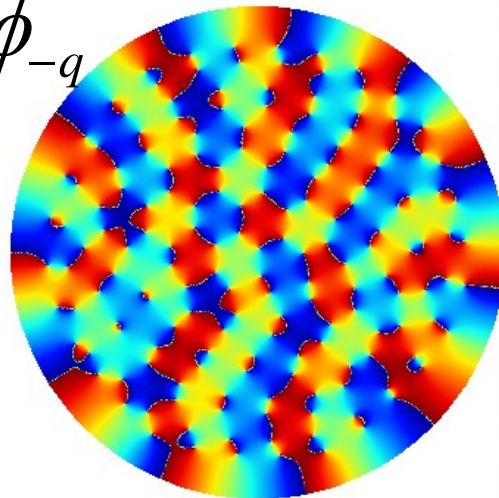
- Stable because condition that no current flows through boundary.
- Can be implemented by image fractional anti-vortex.
- Fractional vortex anti-vortex pair does not have divergent line energy
- Fractional vortices stay near boundary because line energy grows with $\log L$, $L =$ vortex separation

$T=0.1410$

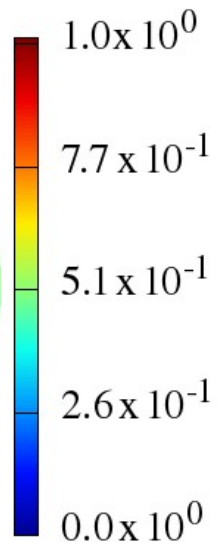
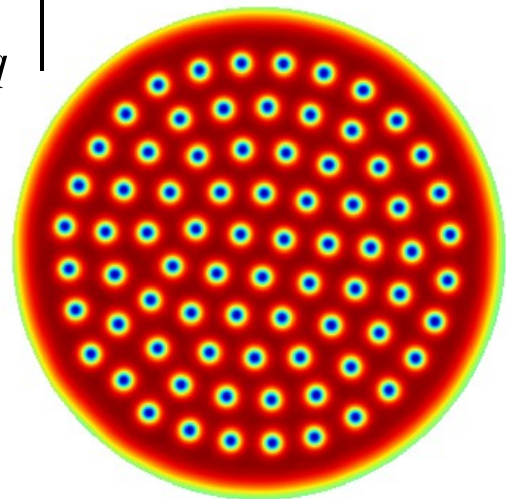
\hat{n}



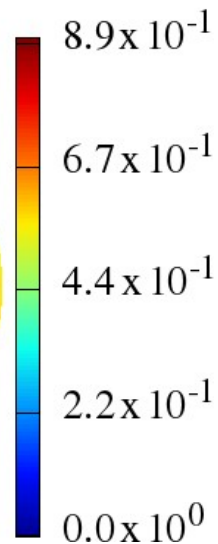
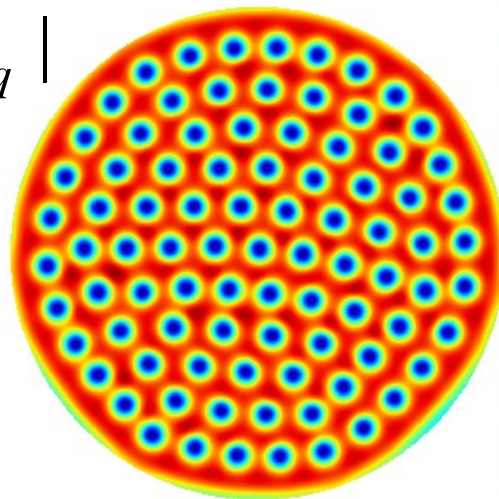
$\phi_q - \phi_{-q}$



$|\psi_q|$

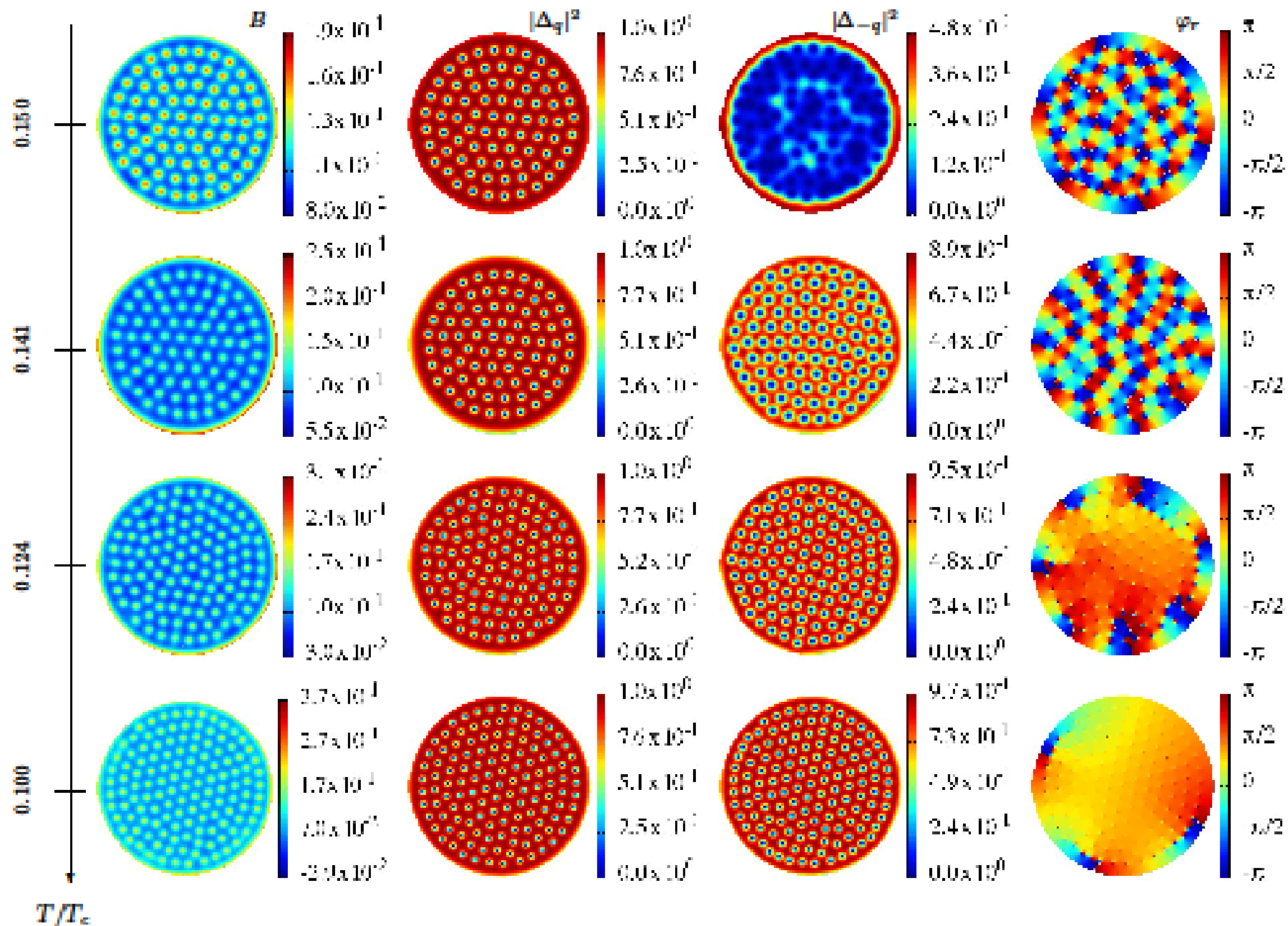


$|\psi_{-q}|$



Skymion vortex lattice at high fields

Moderate Field: Temperature Evolution



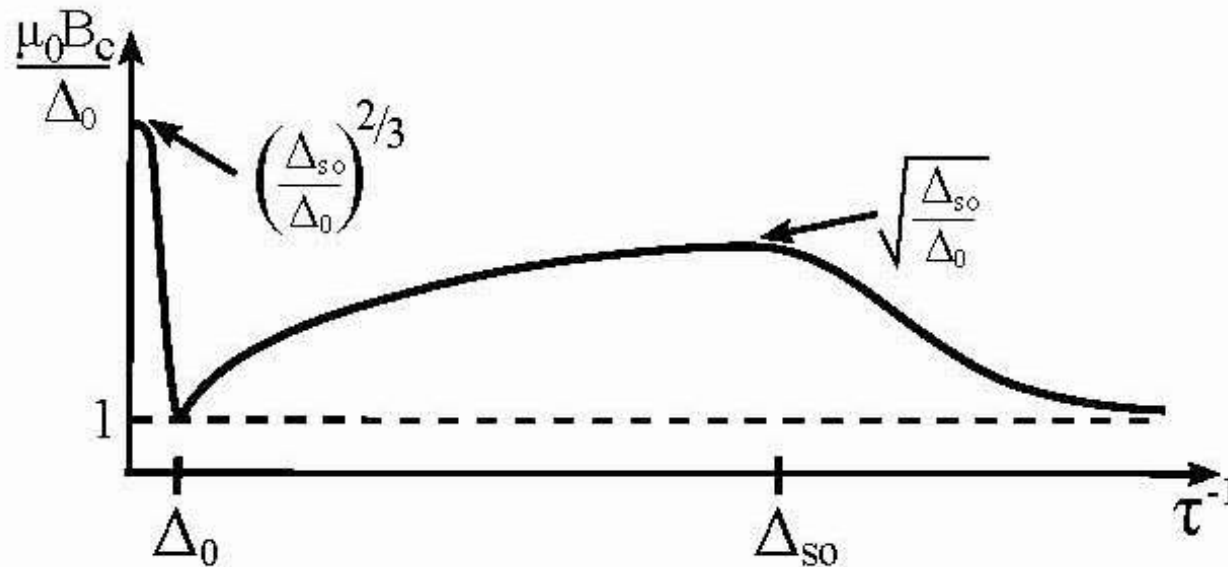
Conclusions

- 2D superconductors with Rashba spin-orbit coupling has finite momentum phases with in-plane fields.
- In clean limit, FFLO-like phase exists that supports fractional vortices, Abrikosov vortices, and Skyrmion vortices.
- Microscopic weak-coupling theory shows fractional vortices are stable near boundaries and Skyrmion vortex lattices appear when c-axis fields are applied

Results in-plane field Rashba Spin-Orbit

Barzykin and Gorkov, PRL (2002); DFA, Physica C (2003);
Kaur, DFA, Sigrist PRL (2004); DFA and Kaur PRB (2007);
Dimitrova and Feigel'man, PRB (2007); Yanase and Sigrist
JPSJ (2007); Samokhin, PRB (2008); Mineev and Samokhin
PRB (2008); Zhang et al. Nat. Comm. (2013)

Michaeli, Potter, PRL (2012)

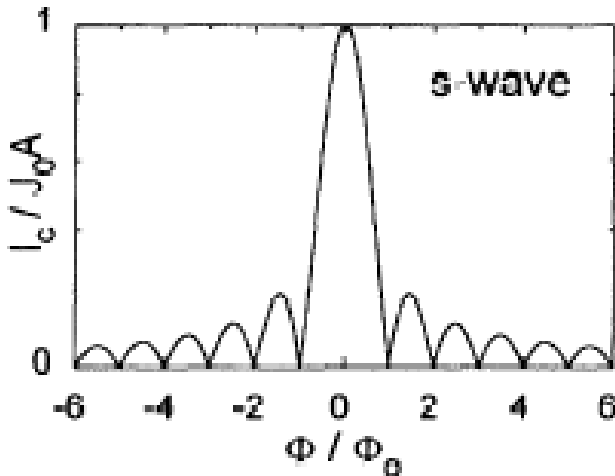
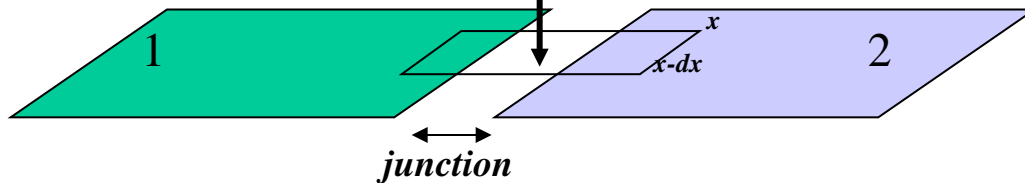


Survives to high Fields in disordered 2D Rashba

Josephson detection of helical phase

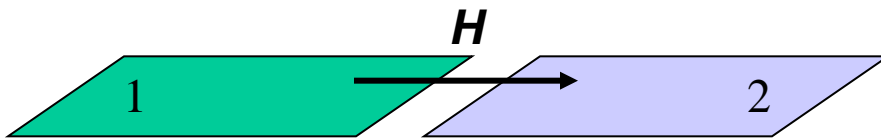
- The Helical phase can be *unambiguously* detected by a Josephson experiment

$$\psi_1 = |\psi_1| e^{i\phi_1} \quad \mathbf{H} \quad \psi_2 = |\psi_2| e^{i\phi_2}$$



Fraunhofer pattern

$$I = I_c \sin(\gamma_0) \frac{|\sin(\pi\Phi/\Phi_0)|}{|\pi\Phi/\Phi_0|}$$

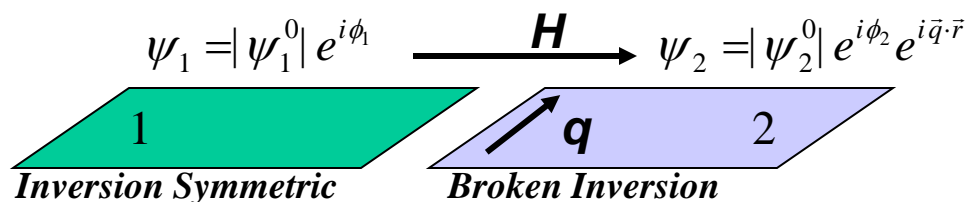


For this field in the plane, no Fraunhofer pattern

If one superconductor has single- q phase order parameter (say 2). The Josephson current will exhibit a *Fraunhofer pattern for field in the plane*.

$$I = \tilde{I}_c \frac{\sin(qL)}{qL}$$

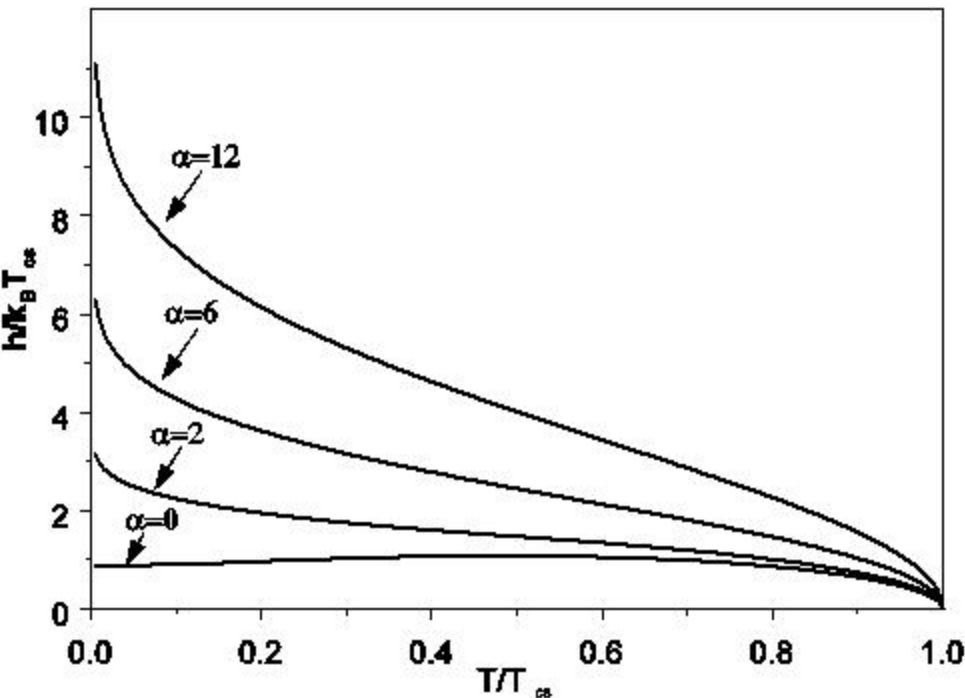
$$\vec{q} = -2m\varepsilon\hat{n} \times \vec{H}$$



For verification of helical phases by Josephson experiment, uniform $|\Psi|$ and in-plane H are required.

Enhancement of Pauli field

- Pauli field for **spin-singlet** case is strongly enhanced (also found by Bulaevski):



for $\vec{g} \cdot \vec{h} = 0$

Spin singlet pairing:

Paramagnetic limit diverge for $T=0$:

Spin triplet pairing:

No limitation if $\hat{g}_{\vec{k}} \parallel \vec{d}$

