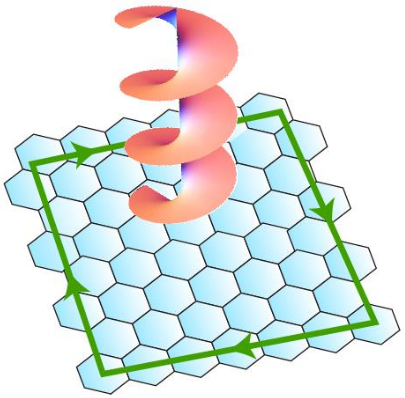
Floquet topological phase transition: Control of quantum matter by laser

using synthetic fields

$$H_{\mathrm{eff}} = H_0 + \frac{[H_{-1}, H_1]}{\Omega} + \mathcal{O}(A^4)$$

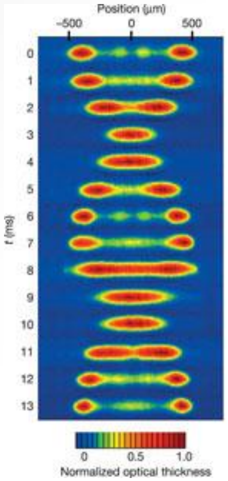
Takashi Oka (U-Tokyo, Dept. of Applied Physics)



M. Sato (Aoyama), S. Takayoshi (NIMS), T. Kitagawa (Harvard), T. Mikami (U-Tokyo), M. Sentef (Stanford, Bonn),
N. Tsuji (U-Tokyo), L. Fu (MIT), H. Aoki (U-Tokyo), E. Demler (Harvard),
J. Freericks (Georgetown), T. Devereaux (Stanford)

Quantum Control by laser

cold atom

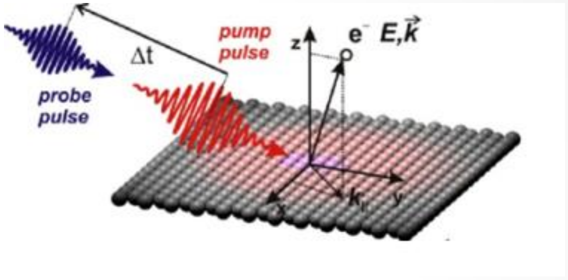


high controllability

- synthetic gauge field, optical lattice
- control of interaction by Feshbach resonance
- simulated spin models

realization of topological QHE

Quantum Control of Solid state materials



http://www.fhi-berlin.mpg.de/pc/PCres_methods.html

even higher controllability

- Floquet topolotical phase transition

  laser induced quantum Hall effect

  Oka-Aoki 2009

- Control of interaction via dynamical localization

  Tsuji-Oka-Aoki 2011

- Control of quantum magnets

  Takayoshi-Aoki-Oka 2013
  Takayoshi-Sato-Oka 2014
  Sato-Sasaki-Oka 2014

Theory (1/3): Floquet theory (time-version of Bloch theorem)

time periodic system

$$i\partial_t \psi = H(t)\psi \qquad H(t) = H(t+T) \qquad \Omega = 2\pi/T$$

"Floquet mapping"

=discrete Fourier trans.

$$\Psi(t) = e^{-i\varepsilon t} \sum_m \phi^m e^{-im\Omega t}$$

Floquet Hamiltonian (static eigenvalue problem)

$$\sum_{m=-\infty}^{\infty} \mathcal{H}^{mn} \phi_\alpha^m = \varepsilon_\alpha \phi_\alpha^n$$

ε: Floquet quasi-energy

$$(\mathcal{H})^{mn} = \frac{1}{T}\int_0^T dt H(t) e^{i(m-n)\Omega t} \boxed{+ m\delta_{mn}\Omega I}$$

comes from the $i\partial_t$ term

$$H_m = \mathcal{H}^{m0}$$

~ absorption of m "photons"

Theory (2/3): Floquet theory (time-version of Bloch theorem)

Time-periodic quantum system = Floquet theory (exact) ~ effective theory

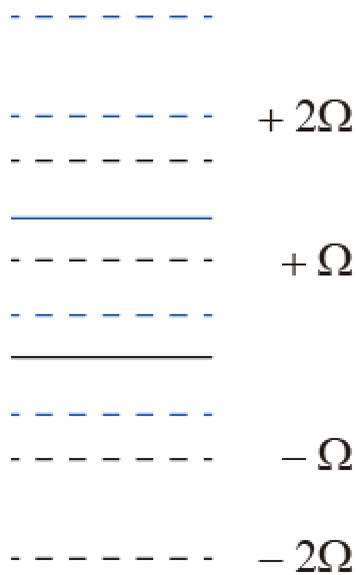
$$i\partial_t \psi = H(t)\psi \qquad \mathcal{H}\phi = \varepsilon\phi \qquad H_{\mathrm{eff}} = H_0 + \frac{[H_{-1}, H_1]}{\Omega} + \mathcal{O}(A^4)$$

$$H(t) = H(t+T)$$

"synthetic fields (term)"

Floquet theory

two states + periodic driving



Hilbert sp. size
= original system

n-photon dressed state

Theory (3/3): Floquet theory (time-version of Bloch theorem)

$1/\Omega$ expansion of the effective Hamiltonian

$H_{\rm eff}^{(0)}$	$H_{0,0}$
$H_{\rm eff}^{(1)}$	$\sum_{n\neq0}\frac{H_{0,n}H_{n,0}}{n\Omega}=\frac{1}{2}\sum_{n\neq0}\boxed{\frac{[H_{0,n},H_{n,0}]}{n\Omega}}$
$H_{\rm eff}^{(2)}$	$\sum_{m,n\neq0}\frac{H_{0,m}H_{m,n}H_{n,0}}{mn\Omega^2}-\sum_{n\neq0}\frac{H_{0,n}H_{n,0}H_{0,0}}{n^2\Omega^2}$
$H_{\rm eff}^{(3)}$	$\sum_{l,m,n\neq0}\frac{H_{0,l}H_{l,m}H_{m,n}H_{n,0}}{lmn\Omega^3}+\sum_{m\neq0}\frac{H_{0,m}H_{m,0}H_{0,0}H_{0,0}}{m^3\Omega^3}$ $-\sum_{m,n\neq0}\left(\frac{H_{0,m}H_{m,n}H_{n,0}H_{0,0}}{mn^2\Omega^3}+\frac{H_{0,m}H_{m,n}H_{n,0}H_{0,0}}{m^2n\Omega^3}+\frac{H_{0,m}H_{m,0}H_{0,n}H_{n,0}}{m^2n\Omega^3}\right)$
$H_{\rm eff}^{(4)}$	$\sum_{k,l,m,n\neq0}\frac{H_{0,k}H_{k,l}H_{l,m}H_{m,n}H_{n,0}}{klmn\Omega^4}-\sum_{m\neq0}\frac{H_{0,m}H_{m,0}H_{0,0}H_{0,0}H_{0,0}}{m^4\Omega^4}$ $-\sum_{l,m,n\neq0}\left(\frac{H_{0,l}H_{l,m}H_{m,n}H_{n,0}H_{0,0}}{l^2mn\Omega^4}+\frac{H_{0,l}H_{l,m}H_{m,n}H_{n,0}H_{0,0}}{lm^2n\Omega^4}+\frac{H_{0,l}H_{l,m}H_{m,n}H_{n,0}H_{0,0}}{lmn^2\Omega^4}\right)$ $-\sum_{l,m,n\neq0}\left(\frac{H_{0,l}H_{l,m}H_{m,0}H_{0,n}H_{n,0}}{l^2mn\Omega^4}+\frac{H_{0,l}H_{l,m}H_{m,0}H_{0,n}H_{n,0}}{lm^2n\Omega^4}+\frac{H_{0,l}H_{l,0}H_{0,m}H_{m,n}H_{n,0}}{l^2mn\Omega^4}\right)$ $+\sum_{m,n\neq0}\left(\frac{H_{0,m}H_{m,n}H_{n,0}H_{0,0}H_{0,0}}{m^3n\Omega^4}+\frac{H_{0,m}H_{m,n}H_{n,0}H_{0,0}H_{0,0}}{m^2n^2\Omega^4}+\frac{H_{0,m}H_{m,n}H_{n,0}H_{0,0}H_{0,0}}{mn^3\Omega^4}\right)$ $+\sum_{m,n\neq0}\left(\frac{H_{0,m}H_{m,0}H_{0,0}H_{0,n}H_{n,0}}{m^3n\Omega^4}+\frac{H_{0,m}H_{m,0}H_{0,n}H_{n,0}H_{0,0}}{m^3n\Omega^4}+\frac{H_{0,m}H_{m,0}H_{0,n}H_{n,0}H_{0,0}}{m^2n^2\Omega^4}\right)$

Mikami, Yasuda, Tsuji, Oka, Aoki *in prep.*

cf) Floquet-Magnus expansion has a initial time dependence
and is not the correct $1/\Omega$ expansion

Application (1/4): "Lattice systems"

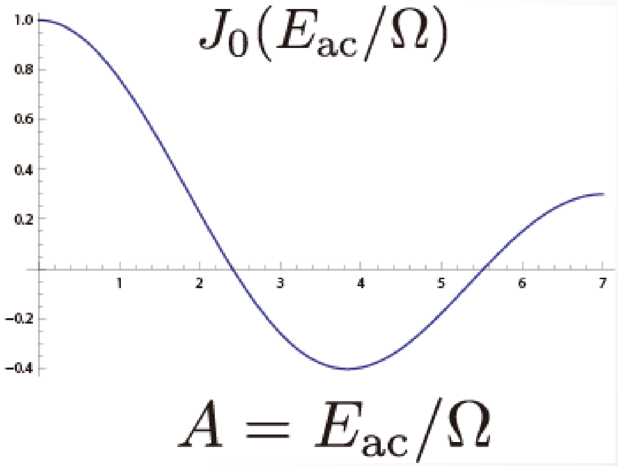
(cold atom) Eckardt-Weiss-Holthaus'05,
(solid state) Tsuji-Oka-Aoki '11

$$H_{\rm eff} = H_0 + \frac{[H_{-1}, H_1]}{\Omega} + \mathcal{O}(A^4)$$

hopping in H_0

$$t_{\rm hopp} e^{iA(t)} \xrightarrow[\frac{1}{T}\int_0^T dt]{\text{time average}} J_0(E_{\rm ac}/\Omega) t_{\rm hopp}$$

$$A(t) = \frac{E_{\rm ac}}{\Omega} \cos \Omega t$$

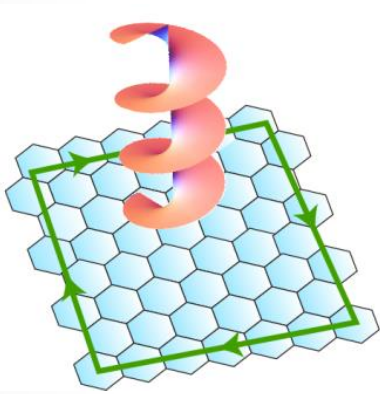


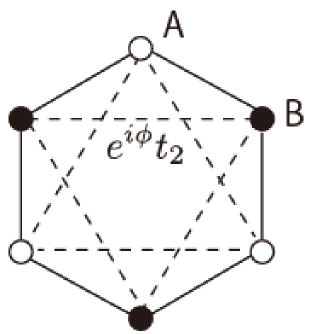
Realizable with strong pulse lasers

Application (2/4): “Dirac systems”

2D Dirac in
circularly polarized laser → parity anomaly Oka, Aoki ’09

Graphene in
circularly polarized laser → Haldane model (1988) of
quantum Hall effect without Landau levels





Chern insulator
(topological insulator)

Oka, Aoki ’09
Kitagawa, Oka, Fu, Brataas, Demler ’11

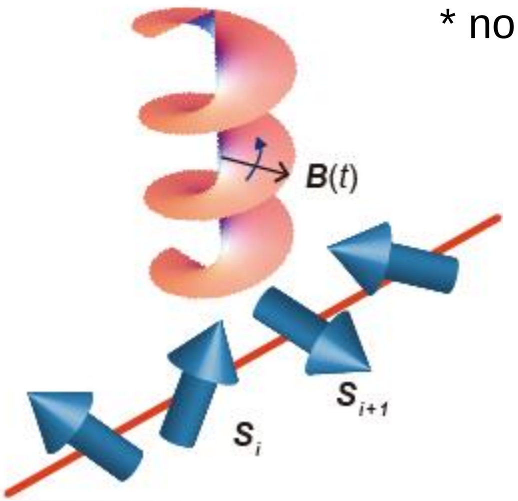
Laser induced Quantum Hall effect

Floquet topological insulator

Application (3/4): "Spin systems"

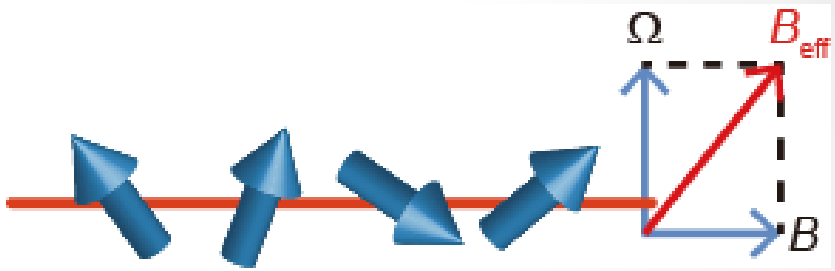
Takayoshi-Aoki-Oka 2013
Takayoshi-Sato-Oka 2014

$$\boldsymbol{B}(t) = (B\cos\Omega t, B\sin\Omega t)$$



* no E field here

spin model with slanted magnetic fields



rotating frame

Large, and dynamical synthetic (effective) magnetic field

$$B^z_{\rm eff} = \Omega$$

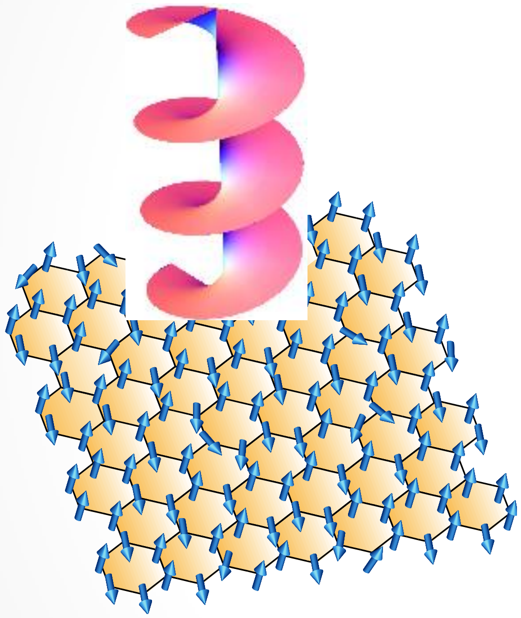
~ 50 Tesla for Ω = 1THz

"laser induced magnetization curve"

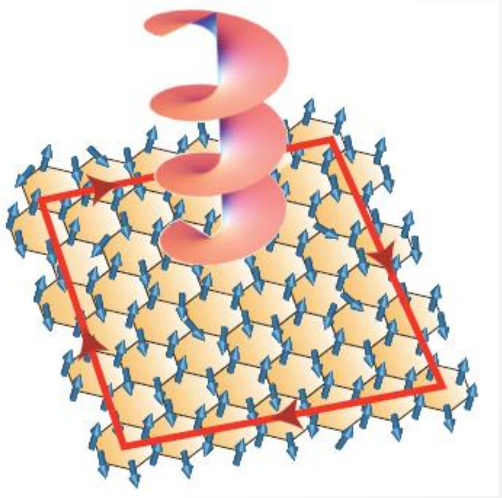
Application (4/4): “ Spin liquid”

Sato-Sasaki-Oka 2014

Kitaev model in a circularly polarized laser



Floquet Topological Spin Liquid
= Quantum spin version of
Floquet Topological Insulator



Plan of talk

1. Dirac + circularly polarized laser = parity anomaly (QHE)

Explanation of $H_{\text{eff}} = H_0 + \frac{[H_{-1}, H_1]}{\Omega} + \mathcal{O}(A^4)$

How experiments are done

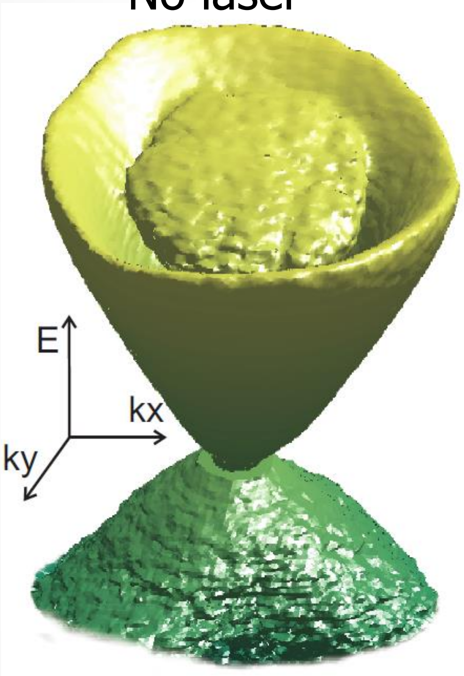
2. Kitaev spin model + multiferroic coupling + circularly polarized laser

1. Dirac + circularly polarized laser = parity anomaly (QHE)

Time resolved ARPES

surface Dirac state of a TI

No laser

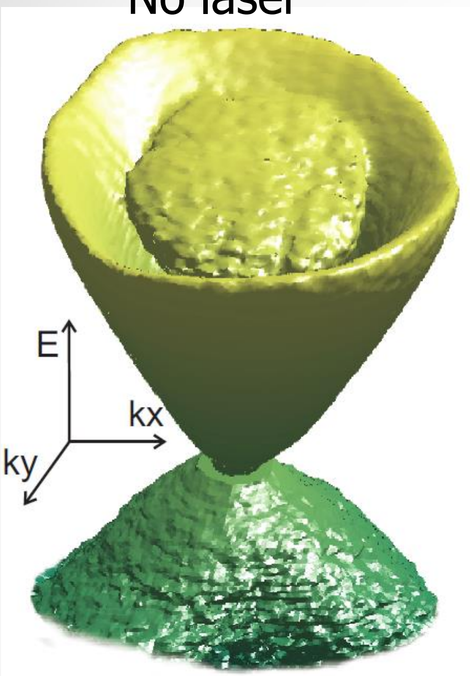


Wang et al. … N. Gedik ***Phys. Rev. Lett. 109***, 127401 (2012)

Experiment using time resolved ARPES

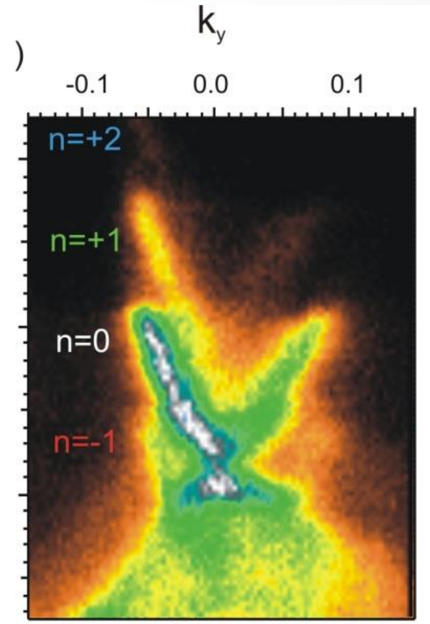
surface Dirac state of a TI

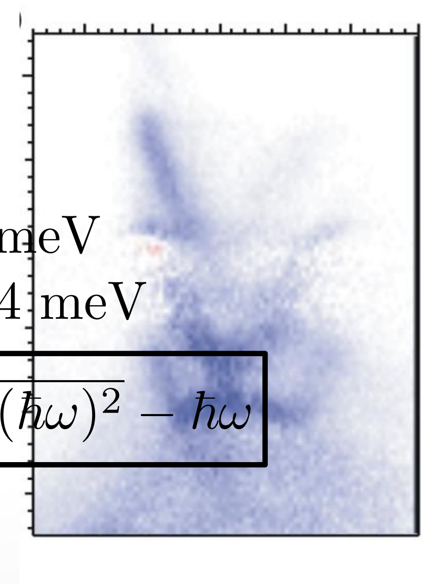
No laser

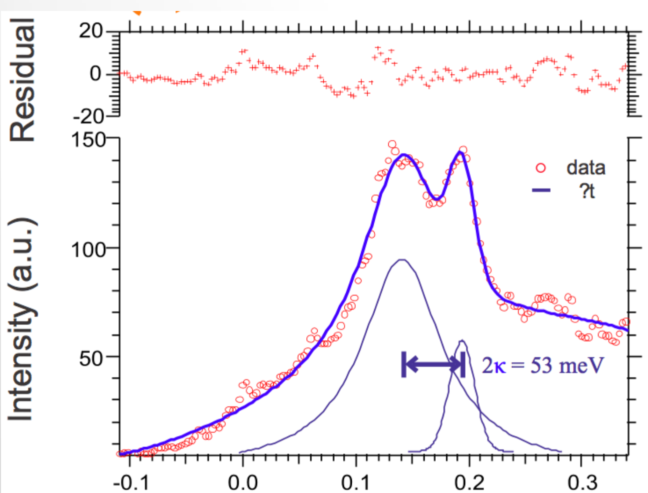


Wang et al. Gedik (MIT) Science '13

circularly
polarized laser







Exp.: $2\kappa = 53$ meV

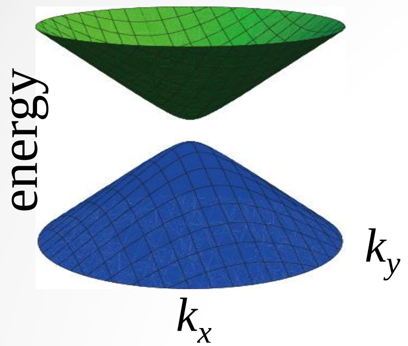
Theory: $2\kappa = 54$ meV

$$\boxed{2\kappa = \sqrt{4V^2 + (\hbar\omega)^2} - \hbar\omega}$$

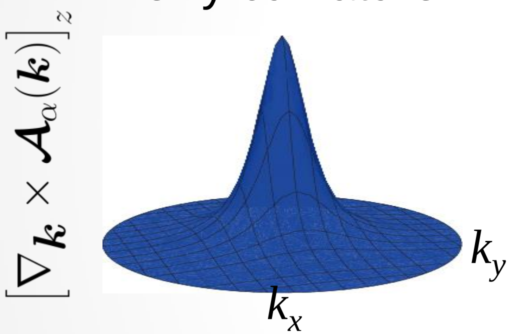
TO, Aoki '09

Chern number of a 2d Dirac system

Niemi Semenoff '83, Redlich '84, Ishikawa '84



Berry curvature



$$H = \begin{pmatrix} m & \pm k_x - i k_y \\ \pm k_x + i k_y & -m \end{pmatrix}$$

$$\sigma_{xy} = e^2 \int \frac{d\boldsymbol{k}}{(2\pi)^d} \left[\nabla_{\boldsymbol{k}} \times \boldsymbol{\mathcal{A}}_1(\boldsymbol{k})\right]_z$$

$$= \pm \frac{1}{2} \frac{e^2}{h} \frac{m}{|m|}$$

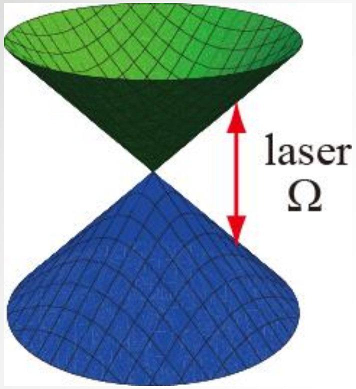
Parity anomaly in QFT

1. Dirac cone has half quantum unit

non-integer because BZ is not periodic

2. The sign depends on the chirality, and mass sign

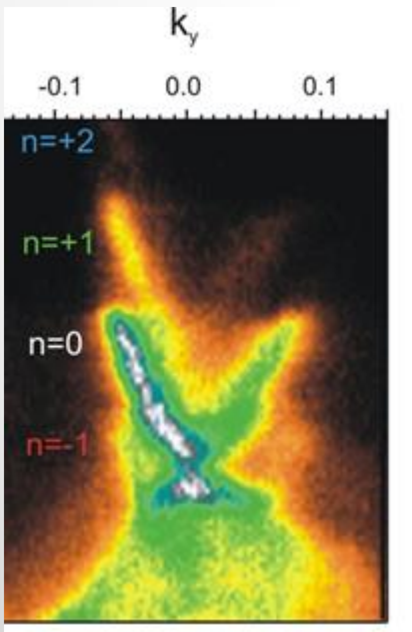
Floquet spectrum: Dirac model + circularly polarized laser



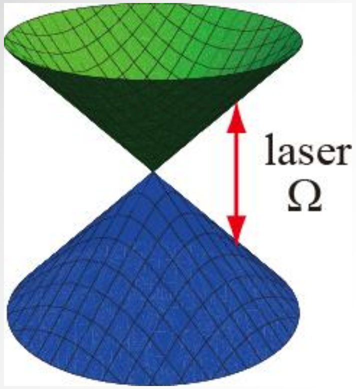
$$H_{\mathrm{Dirac}} = \begin{pmatrix} 0 & k \\ \bar{k} & 0 \end{pmatrix}$$

$k = k_x + i k_y$

??



Floquet spectrum: Dirac model + circularly polarized laser



coupling to AC field

$$\boldsymbol{k} \longrightarrow \boldsymbol{k} + \boldsymbol{A}(t)$$

$$k = k_x + ik_y$$
$$\boldsymbol{A}(t) = (F/\Omega \cos \Omega t, F/\Omega \sin \Omega t)$$
$$A = F/\Omega$$

time dependent Schrodinger equation

$$i\partial_t \psi_k = \begin{pmatrix} 0 & k + Ae^{i\Omega t} \\ \bar{k} + Ae^{-i\Omega t} & 0 \end{pmatrix} \psi_k$$

Floquet theory

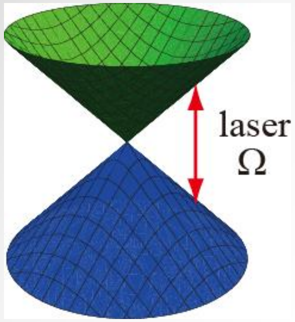
$$(\mathcal{H})^{mn} = \frac{1}{T}\int_0^T dt H(t) e^{i(m-n)\Omega t} + m\delta_{mn}\Omega I$$

$$H^{\text{Floquet}} = \begin{pmatrix} \Omega & k & 0 & A & 0 & 0 \\ \bar{k} & \Omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k & 0 & A \\ A & 0 & \bar{k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega & k \\ 0 & 0 & A & 0 & \bar{k} & -\Omega \end{pmatrix}$$

truncated at m=0,+1, -1 for display

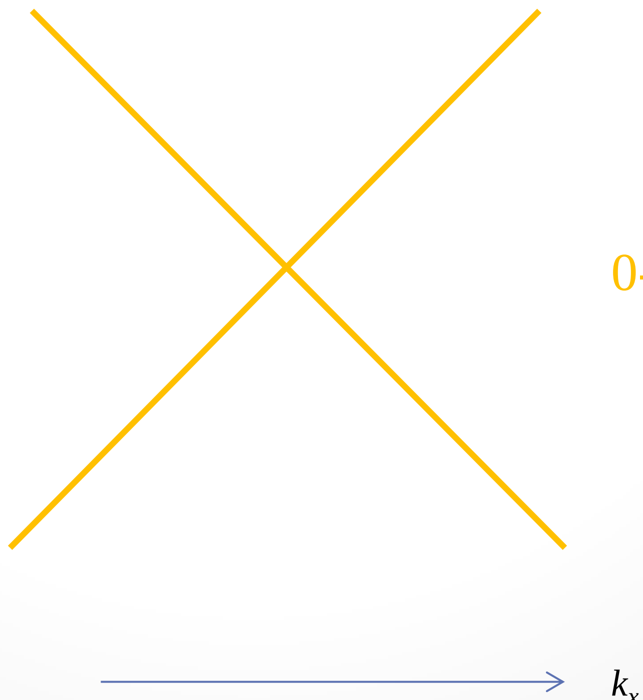
Floquet spectrum: Dirac model + circularly polarized laser

TO, Aoki 2009



$$
H^{\mathrm{Floquet}} = \begin{pmatrix} \Omega & k & 0 & A & 0 & 0 \\ \bar{k} & \Omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k & 0 & A \\ A & 0 & \bar{k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega & k \\ 0 & 0 & A & 0 & \bar{k} & -\Omega \end{pmatrix}
$$

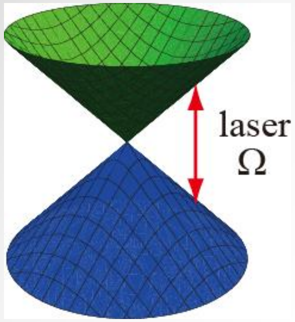
0-photon absorbed state



0-photon absorbed state

Floquet spectrum: Dirac model + circularly polarized laser

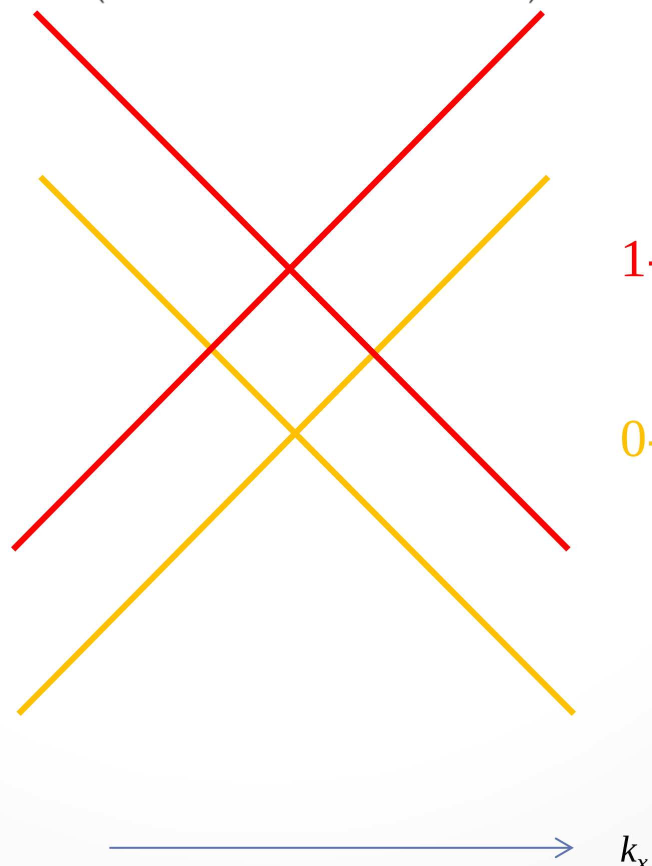
TO, Aoki 2009



$$H^{\mathrm{Floquet}} = \begin{pmatrix} \Omega & k & 0 & A & 0 & 0 \\ \bar{k} & \Omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k & 0 & A \\ A & 0 & \bar{k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega & k \\ 0 & 0 & A & 0 & \bar{k} & -\Omega \end{pmatrix}$$

1-photon absorbed state

0-photon absorbed state

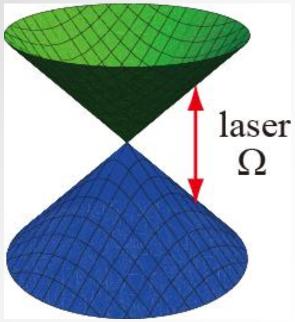


1-photon absorbed state

0-photon absorbed state

Floquet spectrum: Dirac model + circularly polarized laser

TO, Aoki 2009

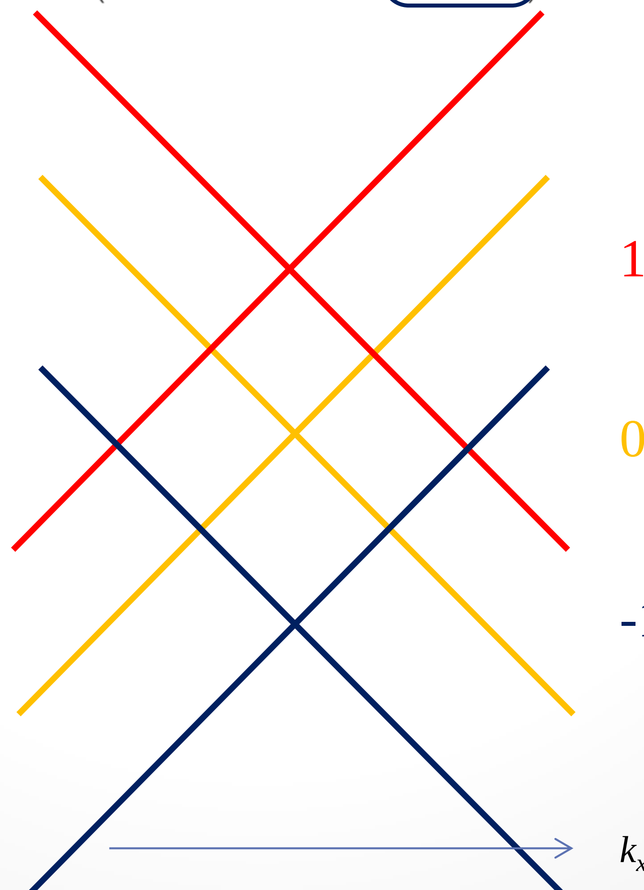


$$H^{\text{Floquet}} = \begin{pmatrix} \Omega & k & 0 & A & 0 & 0 \\ \bar{k} & \Omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k & 0 & A \\ A & 0 & \bar{k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega & k \\ 0 & 0 & A & 0 & \bar{k} & -\Omega \end{pmatrix}$$

1-photon absorbed state

0-photon absorbed state

-1-photon absorbed state



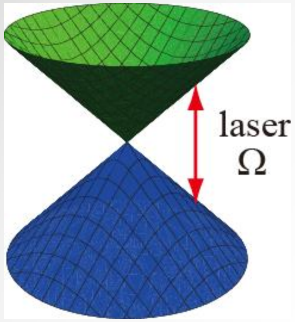
1-photon absorbed state

0-photon absorbed state

-1-photon absorbed state

Floquet spectrum: Dirac model + circularly polarized laser

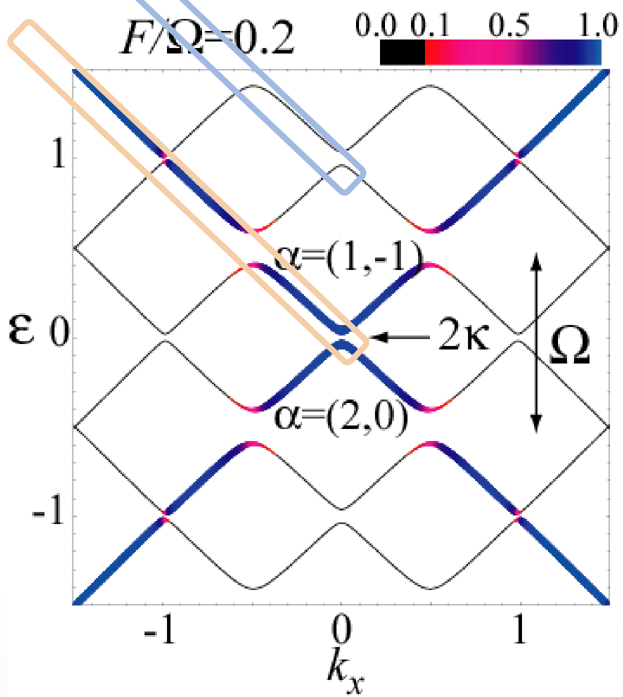
TO, Aoki 2009

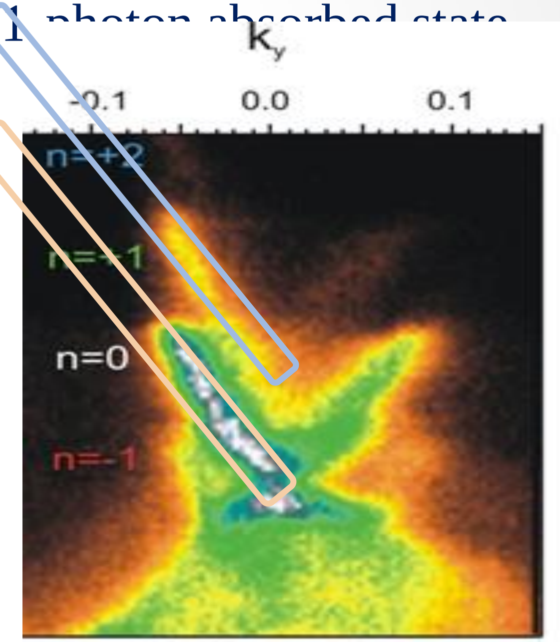


$$H^{\mathrm{Floquet}} = \begin{pmatrix} \Omega & k & 0 & A & 0 & 0 \\ \bar{k} & \Omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k & 0 & A \\ A & 0 & \bar{k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega & k \\ 0 & 0 & A & 0 & \bar{k} & -\Omega \end{pmatrix}$$

1-photon absorbed state

0-photon absorbed state

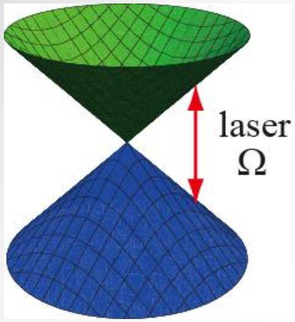




-1-photon absorbed state

Floquet spectrum: Dirac model + circularly polarized laser

TO, Aoki 2009

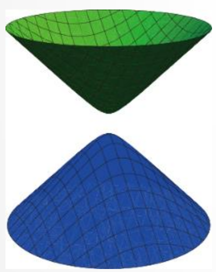


$$H^{\text{Floquet}} = \begin{pmatrix} \Omega & k & 0 & A & 0 & 0 \\ \bar{k} & \Omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k & 0 & A \\ A & 0 & \bar{k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega & k \\ 0 & 0 & A & 0 & \bar{k} & -\Omega \end{pmatrix}$$

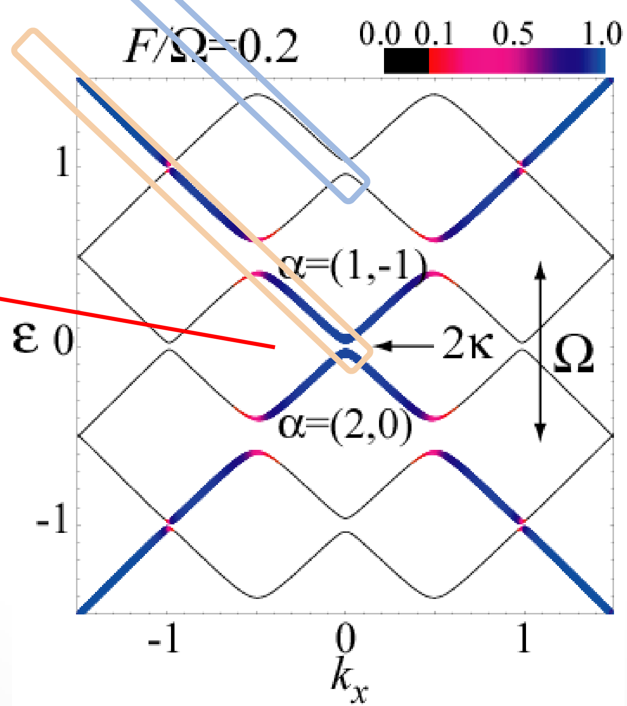
1-photon absorbed state

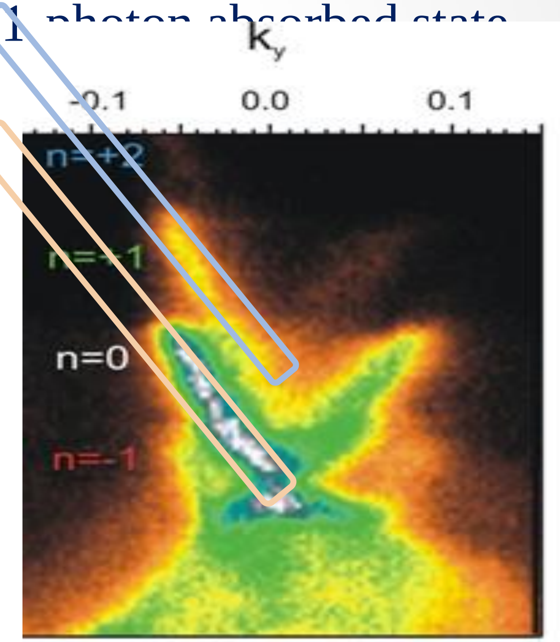
0-photon absorbed state

near Dirac point



2κ





-1-photon absorbed state

Dirac gap

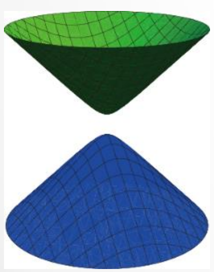
$$\kappa = \frac{\sqrt{4A^2+\Omega^2}-\Omega}{2} \sim A^2/\Omega$$

Theory II: Synthetic fields (terms) from Floquet

$$H^{\text{Floquet}} = \begin{pmatrix} \Omega & k & 0 & A & 0 & 0 \\ \bar{k} & \Omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k & 0 & A \\ A & 0 & \bar{k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega & k \\ 0 & 0 & A & 0 & \bar{k} & -\Omega \end{pmatrix}$$

2nd order perturbation

near Dirac point



2κ

$$H_{\text{eff}} = H_0 + \boxed{\frac{[\overset{\sim A\sigma_-}{\boxed{H_{-1}}}, \overset{\sim A\sigma_+}{\boxed{H_1}}]}{\Omega}} + \mathcal{O}(A^4)$$

synthetic term

Dynamical gap

$$\sim v(k_x\sigma_y - \tau_z k_y \sigma_x) \boxed{\pm \tau_z \frac{v^2 A^2}{\Omega}\sigma_z}$$

$$A = F/\Omega$$

$$\kappa = \frac{\sqrt{4A^2 + \Omega^2} - \Omega}{2} \sim A^2/\Omega$$

Realization of the Haldane model of QHE without Landau levels

Kitagawa, TO, Fu, Brataas, Demler '11

$$H_{\mathrm{eff}} = H_0 + \boxed{\frac{[\boxed{H_{-1}}, \boxed{H_1}]}{\Omega}} + \mathcal{O}(A^4)$$

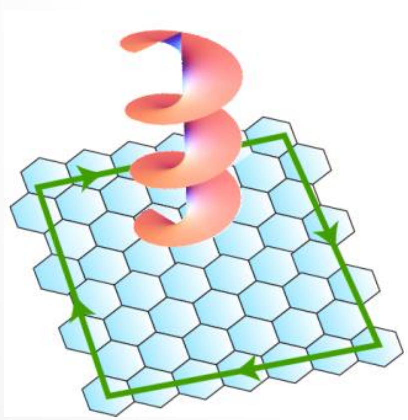
$A = F/\Omega$

applied to honeycomb lattice

n. hopping + n. hopping = n.n. hopping with phase $\pi/2$

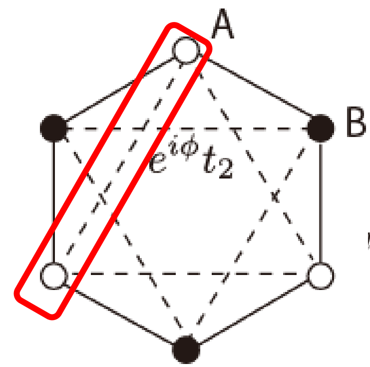
honeycomb + circularly polarized light

$$\boldsymbol{A}(t) = (A\cos\Omega t, A\sin\Omega t)$$

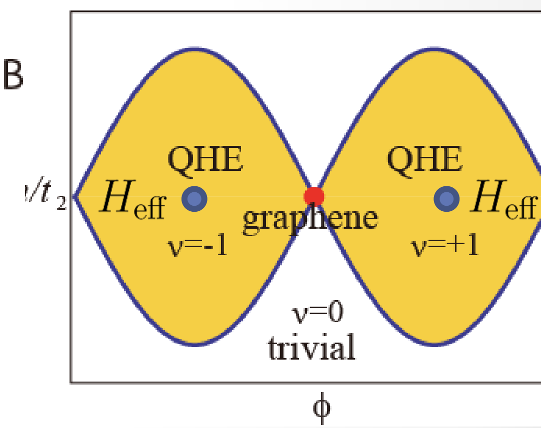


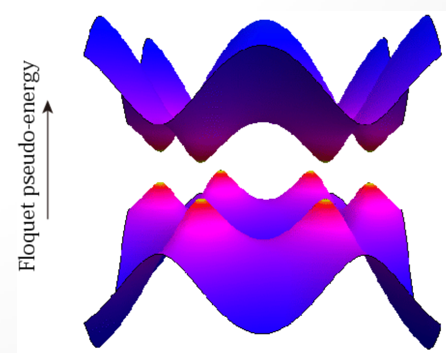
large Ω

Haldane's Model of QHE without LL (1988)



local magnetic field ϕ
AB-level offset m





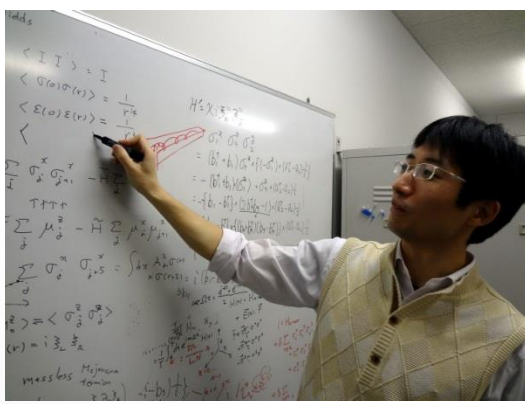
Plan of talk

1. Dirac + circularly polarized laser = parity anomaly (QHE)

Explanation of $H_{\text{eff}} = H_0 + \dfrac{[H_{-1}, H_1]}{\Omega} + \mathcal{O}(A^4)$

How experiments are done

2. Kitaev spin model + multiferroic coupling + circularly polarized laser

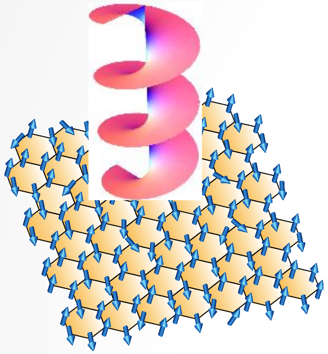


Masahiro Sato (Aoyama Gakuin)
佐藤正寛 (青学)

Floquet Majorana edge state and Nonabelian anyons in a Driven Kitaev model

M. Sato, Y. Sasaki and T. Oka arXiv 2014

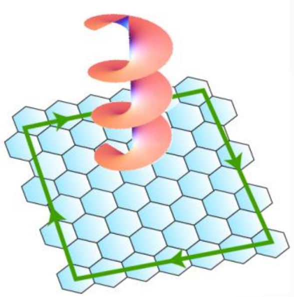
Kitaev's Spin liquid model
+ circularly polarized light



Fermionization

Real (not complex) Fermion

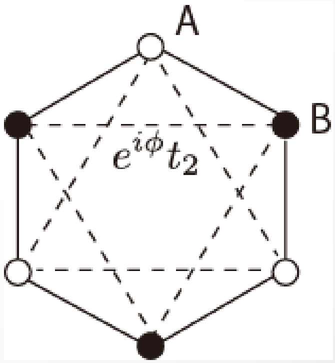
Honeycomb lattice
+ circularly polarized light

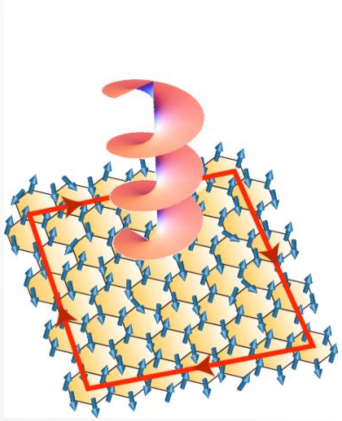


weak field expansion

$$H_{\text{eff}} = H_0 + \frac{[H_{-1}, H_1]}{\Omega} + \mathcal{O}(A^4)$$

Haldane model

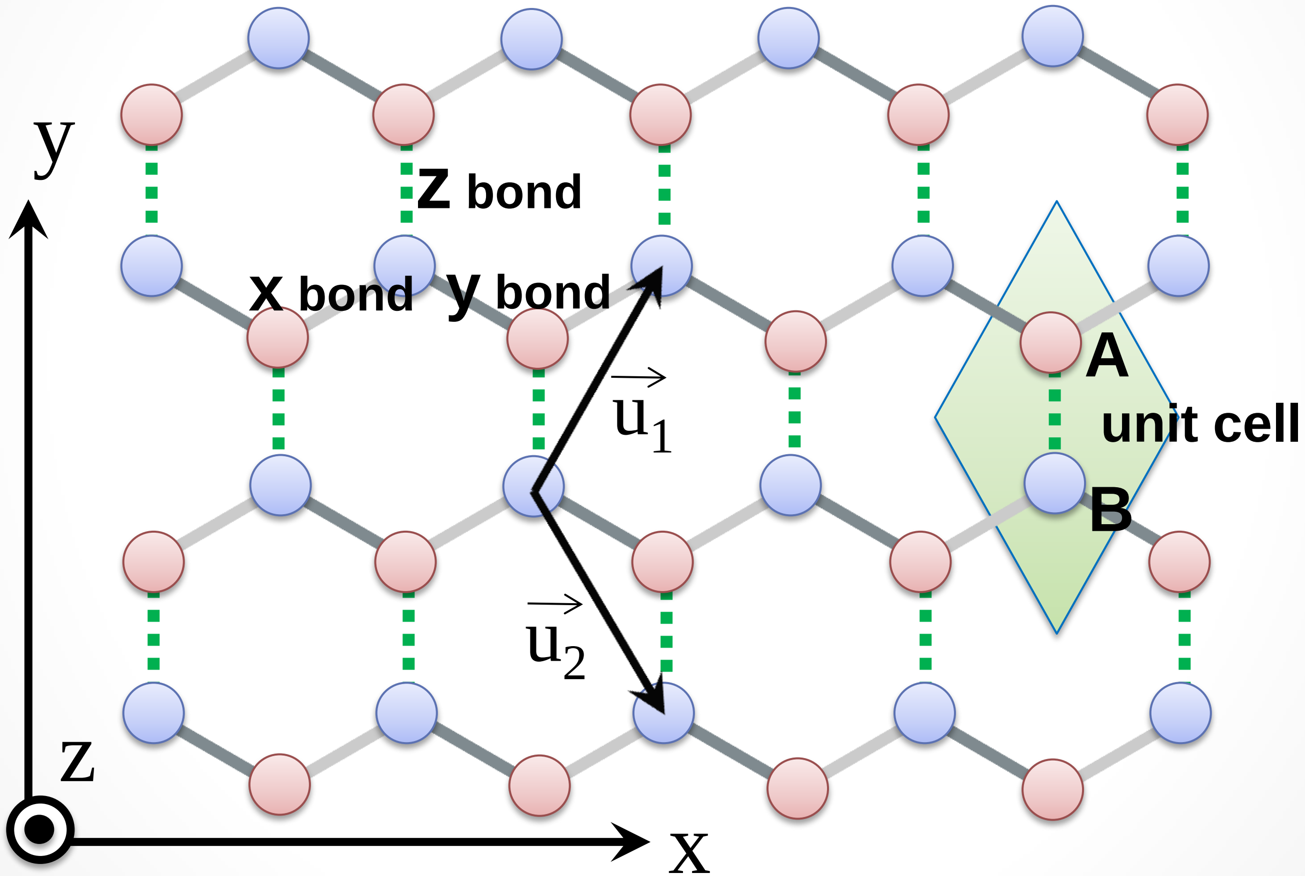




Gapped topological spin liquid
with Majorana edge mode
and nonabelian anyon excitation

Kitaev honeycomb model (an anisotropic spin model)

$$\hat{\mathcal{H}}_{\text{Kitaev}} = \sum_{\alpha=x,y,z} J_\alpha \sum_{\langle \vec{r},\vec{r}'\rangle_\alpha} \sigma^\alpha_{\vec{r}} \sigma^\alpha_{\vec{r}'}$$



Kitaev honeycomb model (an anisotropic spin model)

$$\hat{\mathcal{H}}_{\text{Kitaev}} = \sum_{\alpha=x,y,z} J_\alpha \sum_{\langle \vec{r},\vec{r}' \rangle_\alpha} \sigma^\alpha_{\vec{r}} \sigma^\alpha_{\vec{r}'}$$

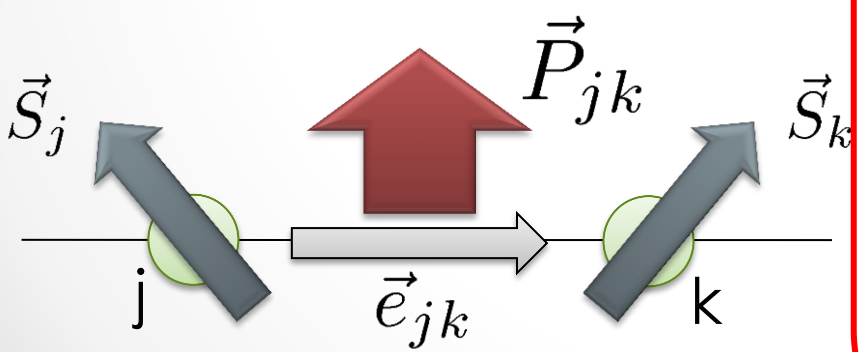
We assume magneto-electric coupling exists

Anti-symmetric magneto striction type

Katsura, Nagaosa, Balatsky (2005),
Tanabe, Moriya, Sugano (1965), etc.

$$\vec{P}_{jk} \propto \vec{e}_{jk} \times (\vec{S}_j \times \vec{S}_k)$$

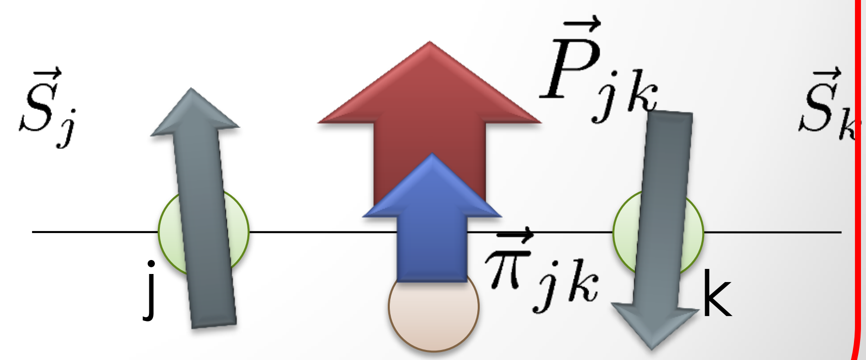
Electric polarization **Vector spin chirality**



$$\otimes\, \vec{\mathcal{V}}_{jk} = \vec{S}_j \times \vec{S}_k$$

Symmetric magneto striction type

$$\vec{P}_{jk} \propto \vec{\pi}_{jk} \vec{S}_j \cdot \vec{S}_k$$



Kitaev honeycomb model (an anisotropic spin model)

$$\hat{\mathcal{H}}_{\mathrm{Kitaev}} = \sum_{\alpha=x,y,z} J_\alpha \sum_{\langle \vec{r},\vec{r}' \rangle_\alpha} \sigma^\alpha_{\vec{r}} \sigma^\alpha_{\vec{r}'}$$

Application of circularly polarized laser

$$\hat{\mathcal{H}}(t) = \hat{\mathcal{H}}_{\mathrm{Kitaev}} + \hat{\mathcal{H}}_E(t)$$

$$\hat{\mathcal{H}}_{\mathrm{E}}(t) = -\vec{E}(t) \cdot \vec{P}_{\mathrm{tot}}$$

$$\vec{E}(t) \quad = \quad E(\mp \cos(\Omega t + \delta), \sin(\Omega t), 0)$$

We ignore the Zeeman term

$$\mathcal{H}_{\mathrm{Zeeman}}(t) = \vec{B}(t) \cdot \vec{S}$$

Effective Hamiltonian for Kitaev model in a laser

$$\hat{\mathcal{H}}_{\text{eff}} = \hat{\mathcal{H}}_0 - \frac{1}{\Omega}[\hat{\mathcal{H}}_{+1}, \hat{\mathcal{H}}_{-1}] + \mathcal{O}(\Omega^{-2})$$

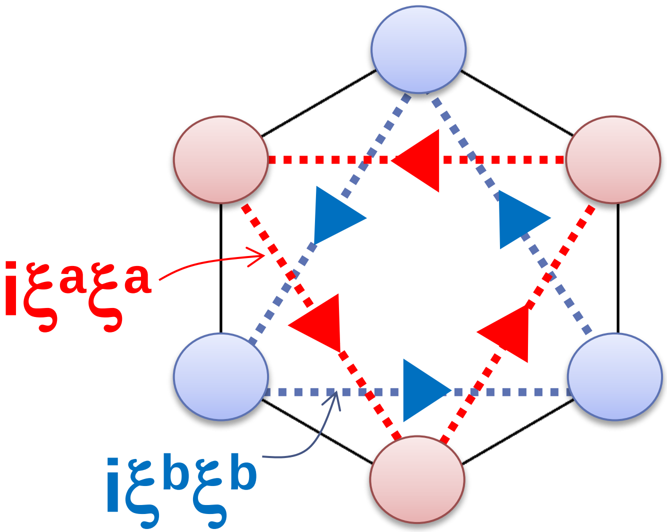
after Fermionization

$$\begin{aligned}\hat{\mathcal{H}}_{\Omega} &= \pm\frac{1}{\Omega}E^2\sum_{\vec{r}} iG_{12}\left[\xi^b_{\vec{r}}\xi^b_{\vec{r}+\vec{u}_1+\vec{u}_2} + \xi^a_{\vec{r}}\xi^a_{\vec{r}-\vec{u}_1-\vec{u}_2}\right] \\ &+ i\hat{I}_{\vec{r}}G_{23}\left[\xi^b_{\vec{r}}\xi^b_{\vec{r}-\vec{u}_2} + \xi^a_{\vec{r}}\xi^a_{\vec{r}+\vec{u}_2}\right] \\ &+ i\hat{I}_{\vec{r}}G_{31}\left[\xi^b_{\vec{r}}\xi^b_{\vec{r}-\vec{u}_1} + \xi^a_{\vec{r}}\xi^a_{\vec{r}+\vec{u}_1}\right]\end{aligned}$$

Effective Hamiltonian for Kitaev model in a laser

$$\hat{\mathcal{H}}_{\rm eff} \;=\; \hat{\mathcal{H}}_0 - \frac{1}{\Omega}[\hat{\mathcal{H}}_{+1}, \hat{\mathcal{H}}_{-1}] + \mathcal{O}(\Omega^{-2})$$

after Fermionization

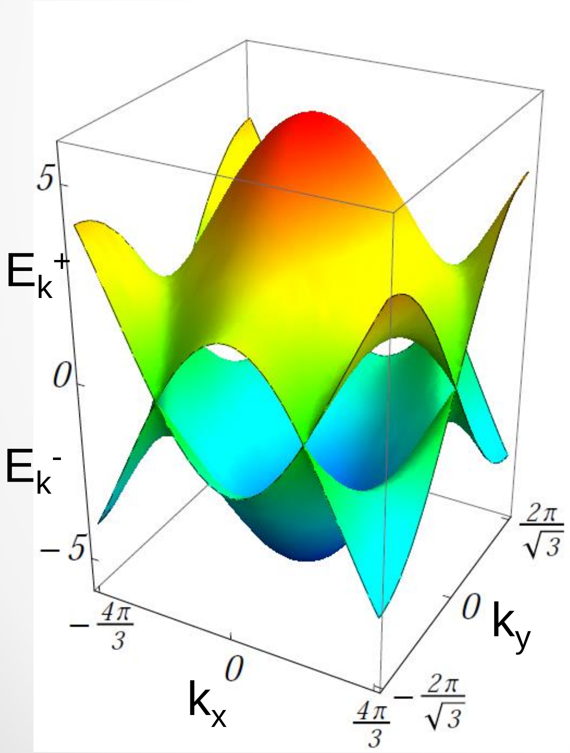


Haldane model

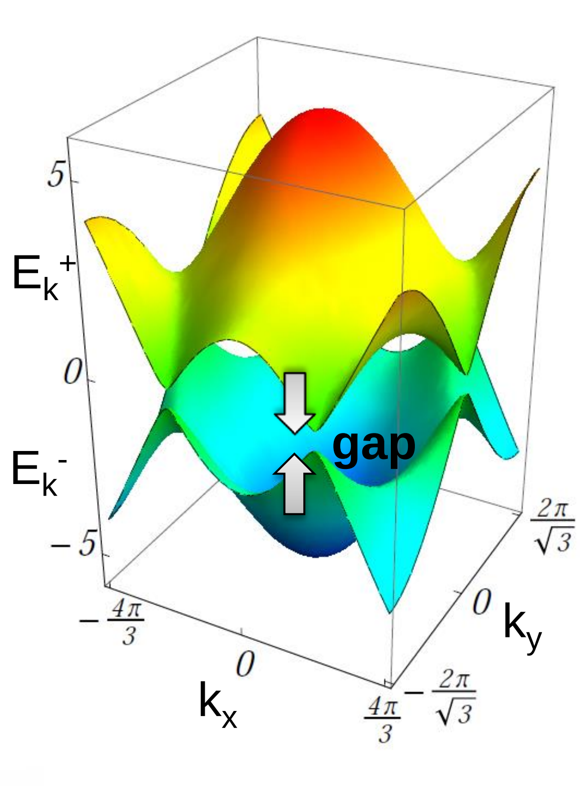
Effective Hamiltonian for Kitaev model in a laser

$$\hat{\mathcal{H}}_{\text{eff}} = \hat{\mathcal{H}}_0 - \frac{1}{\Omega}[\hat{\mathcal{H}}_{+1}, \hat{\mathcal{H}}_{-1}] + \mathcal{O}(\Omega^{-2})$$

fermion dispersion without laser

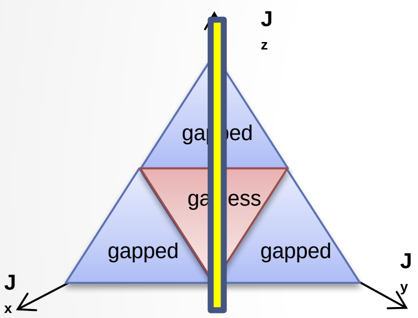


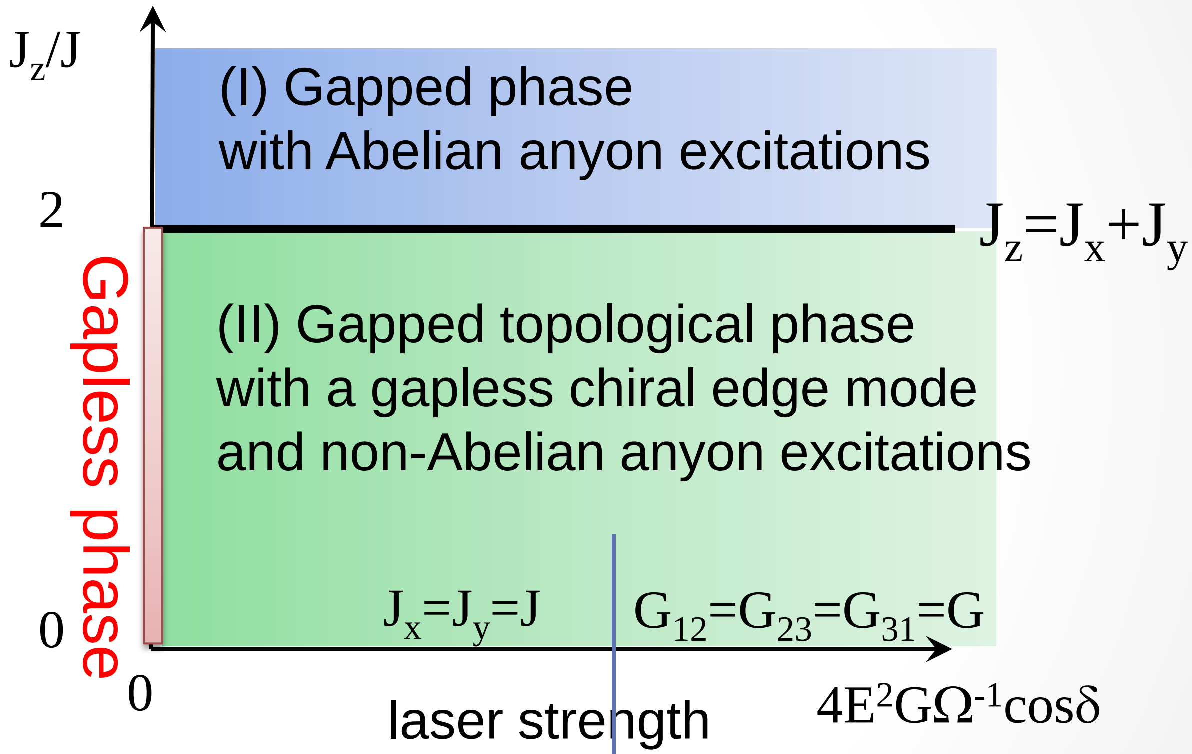
fermion dispersion with laser



Gap opens at the Dirac point

Nonequilibrium phase diagram of the effective Hamiltonian for the Kitaev model in a laser

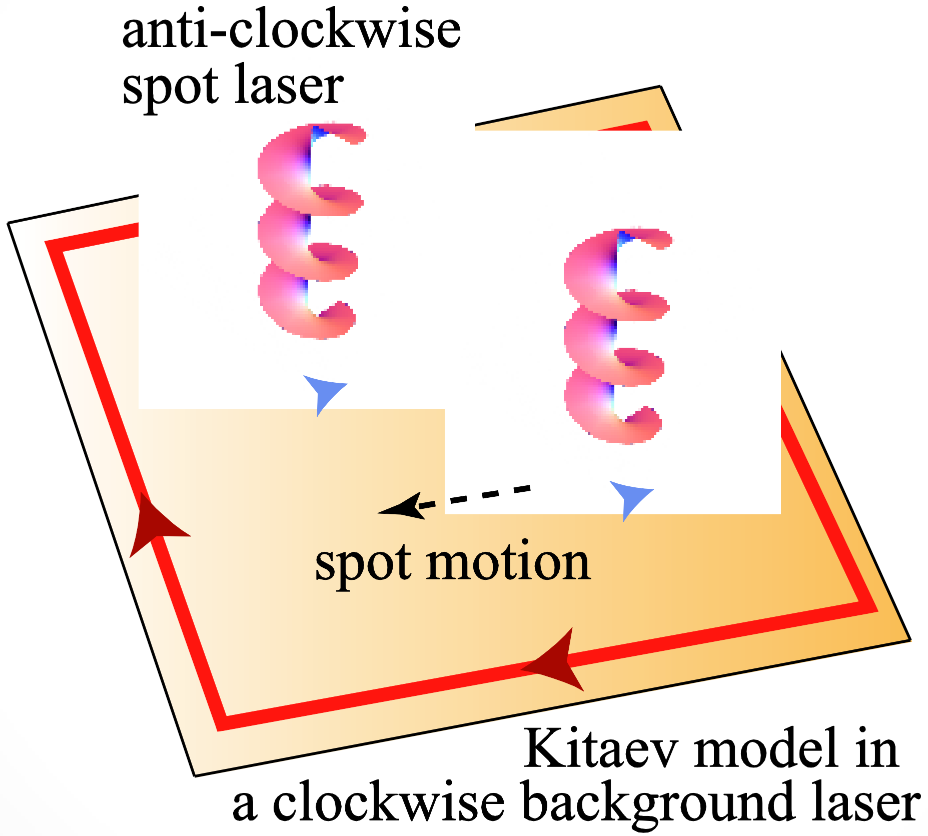




same phase can be obtained by B_z (Kitaev)

Majorana edge mode
and nonabelian anyon excitation

Manipulation of island of topological state by spot laser ??



Conclusion

1. Quantum coherent control of solid state materials by laser

2. Floquet theory is easy (just remember the following)

Time-periodic quantum system = Floquet theory (exact) ~ effective theory

$$i\partial_t \psi = H(t)\psi$$

$$H(t) = H(t+T)$$

$$\mathcal{H}\phi = \varepsilon\phi$$

$$H_{\text{eff}} = H_0 + \frac{[H_{-1}, H_1]}{\Omega} + \mathcal{O}(A^4)$$

"synthetic fields (term)"

3. Many applications

strongly correlated system, topological system, quantum spins, …