A faint, light blue background image of a complex crystal structure, likely a quadrupolar ordered phase, consisting of interconnected polyhedral units and spheres.

Quadrupolar Ordered Phases in Pr-based Superconductors $\text{PrT}_2\text{Zn}_{20}$ (T = Rh and Ir)

Takahiro Onimaru¹

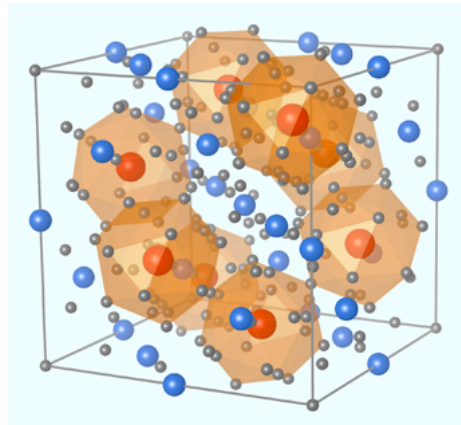
K. T. Matsumoto¹, N. Nagasawa¹, K. Wakiya¹, K. Umeo²,
S. Kittaka³, T. Sakakibara³, and T. Takabatake^{1,4}

¹*AdSM*, ²*N-BARD*, ³*IAMR*, *Hiroshima University*

⁴*ISSP, University of Tokyo*

Outline

- Multipoles of non-Kramers Pr^{3+} ion with $4f^2$ configuration
- $\text{PrT}_2\text{Zn}_{20}$ (T=Rh and Ir)
 - ❖ Crystal electric field (CEF) effect
 - Non-Kramers doublet ground state
 - ❖ Quadrupole order and superconducting transition
 - ❖ Further topic: Non-Fermi liquid behavior,
Multi-channel Kondo effect, etc.
- Summary



Multipoles of 4f electrons

- Electric Monopole, e^-

Strong spin-orbit interaction $\rightarrow J = L \pm S$

- Magnetic Dipole $\dots J_x, J_y, J_z$

- Electric Quadrupole

$$O_2^0 (= 2J_z^2 - J_x^2 - J_y^2) : \Gamma_3$$

$$O_2^2 (= J_x^2 - J_y^2)$$

$$O_{yz}, O_{zx}, O_{xy} : \Gamma_5$$

$$(O_{\alpha\beta} = J_\alpha J_\beta + J_\beta J_\alpha)$$

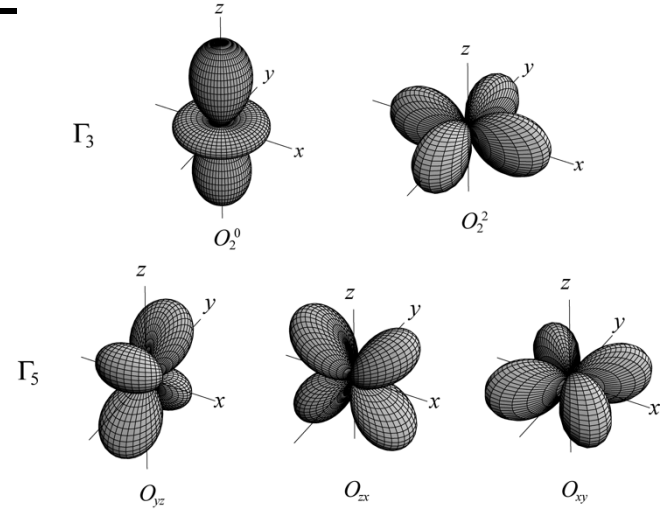
- Magnetic Octupole

$$T_x^a, T_y^a, T_z^a (\Gamma_4)$$

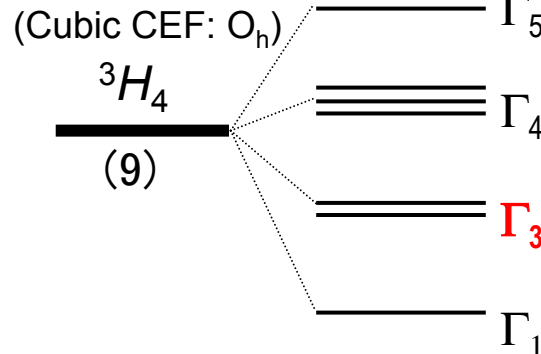
$$T_x^b, T_y^b, T_z^b (\Gamma_5)$$

$$T_{xyz} (\Gamma_2)$$

Quadruples of 4f electrons



$\text{Pr}^{3+} : 4f^2$ configuration ($J=4$)



	Dipole	Quadru -pole	Octu- pole
Γ_5	○	○	×
Γ_4	○	○	×
Γ_3	×	○	○
Γ_1	×	×	×

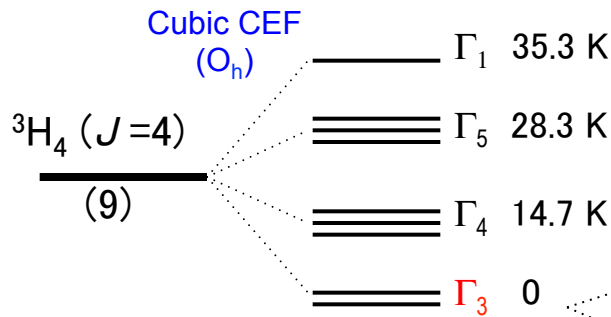
How to remove the entropy of the degenerated state?

Cubic Pr-based compounds

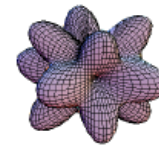
	Crystal Structure	Point group	CEF GS	Magnetic order	Multipole order	SC transition
PrSn₃	AuCu ₃	O _h	$\Gamma_1 - \Gamma_4$	AFM $T_N=8.6$ K	—	—
PrPb₃	AuCu ₃	O _h	Γ_3	—	AFQ $T_Q=0.4$ K	—
PrPtBi	MgAgAs	O _h	Γ_3	—	FQ $T_Q=1.35$ K	—
PrInAg₂	BiF ₃	O _h	Γ_3	—	—	—
PrMg₃	Fe ₃ Al	O _h	Γ_3	—	—	—
Pr₃Pd₂₀Ge₆	C ₆ Cr ₂₃	O _h , T _d	Γ_5, Γ_3	—	AFQ $T_Q=0.26$ K	—
Pr₃Pd₂₀Si₆	C ₆ Cr ₂₃	O _h , T _d	Γ_5, Γ_3	AFM $T_N=0.14$ K	AFQ $T_Q=0.21$ K	—
PrOs₄Sb₁₂	AT ₄ X ₁₂	T _h	$\Gamma_1 - \Gamma_4^{(1)}$	—	AFQ (B>2 T)	$T_c=1.85$ K
PrRu₄P₁₂	AT ₄ X ₁₂	T _h	$\Gamma_1, \Gamma_4^{(2)}$ ($T < T_{co}=63$ K)	—	AFQ $T_Q=0.11$ K	—
PrFe₄P₁₂	AT ₄ X ₁₂	T _h	$\Gamma_1 - \Gamma_4^{(1)}$	—	Scalar-type $T_M=6.5$ K	—
PrPt₄Ge₁₂	AT ₄ X ₁₂	T _h	Γ_1	—	—	$T_c=8$ K

Non-Kramers Pr³⁺ ions in crystals

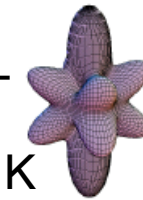
Pr³⁺ ion: 4f² configuration ($L=5, S=1, J=4$)



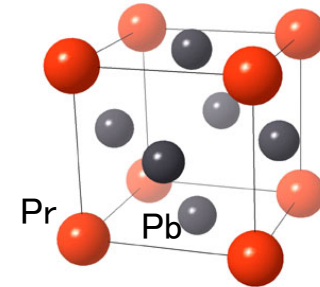
In the Γ_3 doublet, there is no magnetic dipole, but quadrupoles O_2^0 and O_2^0 , and octupole T_{xyz} .



Quadrupoles of 4f electrons



($O_2^0 < 0$)



Crystal structure of PrPb₃

PrPb₃ Simple cubic structure

Antiferro-quadrupole (AFQ) order at $T_Q=0.4$ K

E. Bucher *et al.*, J. Low. Temp. **2** (1974) 322.

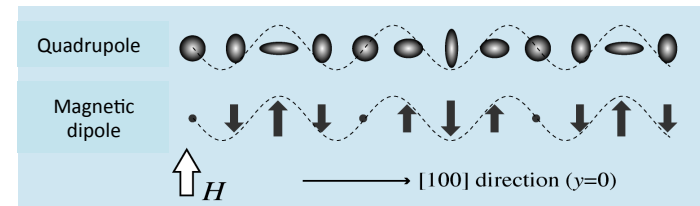
AFQ order

Incommensurate quadrupole structures

⇒ Indirect RKKY-type interaction.

Quadrupoles are partially quenched.

T. Onimaru *et al.*, Phys. Rev. Lett. **94** (2005) 197201.



Incommensurate quadrupole structure in PrPb₃. (T. Onimaru *et al.*, PRL, 2005.)

PrInAg₂, PrMg₃:

No ordering, Quadrupole Kondo effect

A. Yatskar *et al.*, Phys. Rev. Lett. **77** (1996) 3637., H. Tanida *et al.*, J. Phys. Soc. Jpn. **75** (2006) 073705.

Composite order in two-channel Kondo lattice

S. Hoshino *et al.*, J. Phys. Soc. Jpn. **82**, 044707 (2013.)

Crystal structure of $\text{PrT}_2\text{Zn}_{20}$ (T=Ru, Rh, Ir)

T. Nasch *et al.*, Z. Naturforsch. B **52** (1997) 1023.

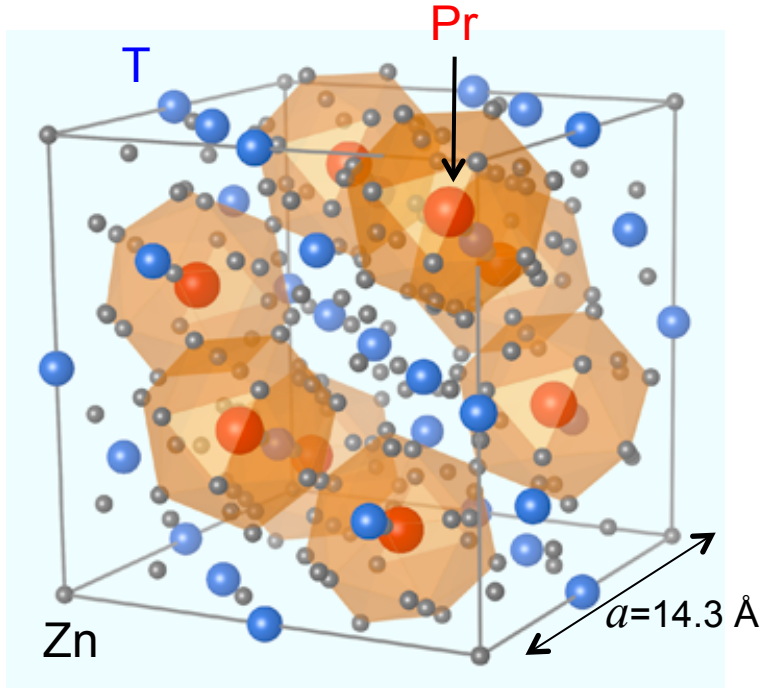
T. Onimaru *et al.*, J. Phys. Soc. Jpn. **79** (2010) 033704.

Cubic $\text{CeCr}_2\text{Al}_{20}$ -type
(Space group: $Fd\bar{3}m$)

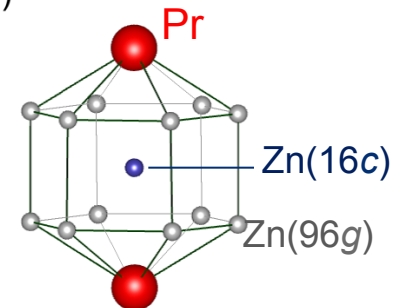
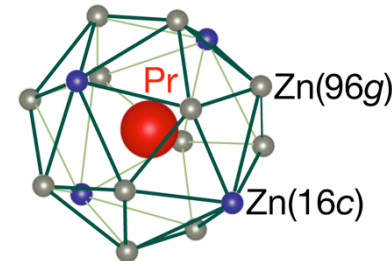


T=Ru
Rh
Ir

The Pr ions are encapsulated into highly symmetric Zn-cages.



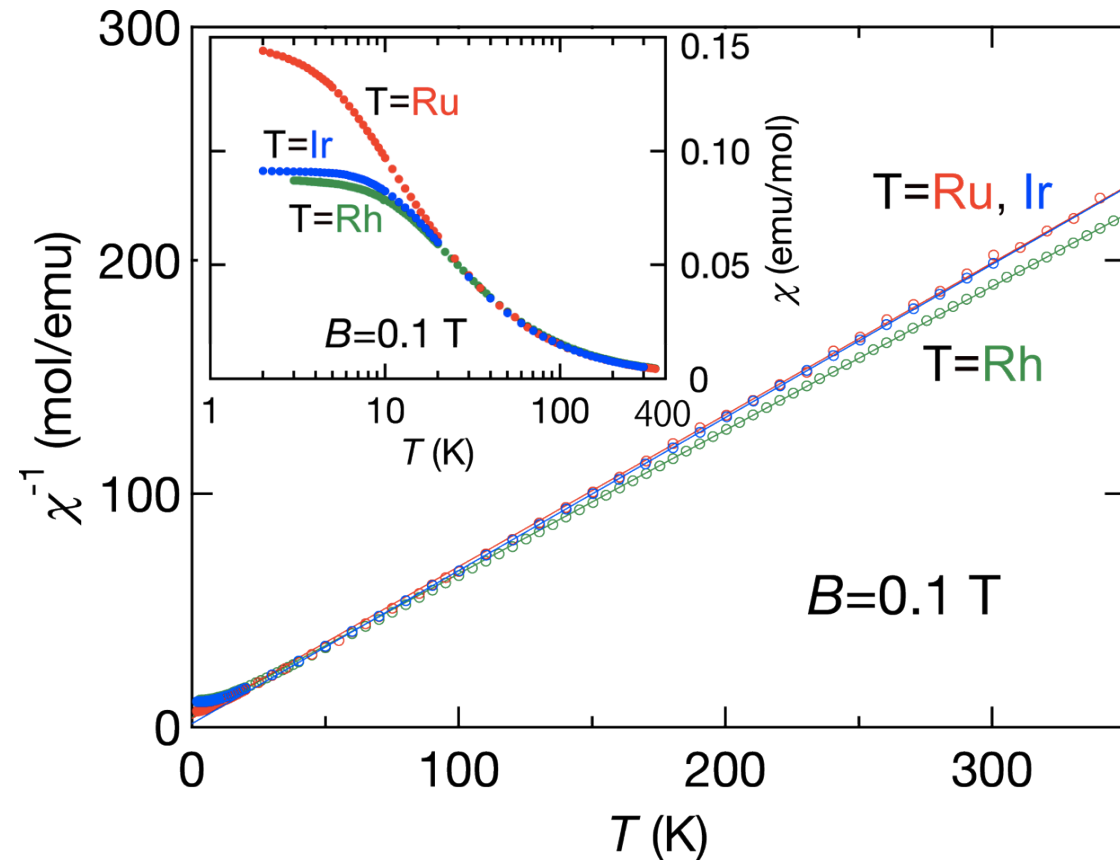
Pr site : Cubic T_d point group



Zn(16c) atoms insides
Pr and Zn(96g) cage

- **Non-Kramers Γ_3 doublet** for Pr^{3+} ion \rightarrow Quadrupole moments are active.
- **Strong hybridization** of the 4f electrons with conduction electrons
 \rightarrow Heavy fermion state and Kondo effect
- Anharmonic atomic vibrations couple with 4f and conduction electrons.

Magnetic susceptibility: $\text{PrT}_2\text{Zn}_{20}$ (T=Ru, Rh, Ir)



T. Onimaru *et al.*, J. Phys.: Condens. Matter **24** (2012) 294207.

χ obeys the Curie-Weiss law above 30 K.

⇒ Effective magnetic moment μ_{eff}

3.50(2) μ_{B} /f.u. for T=Ru

3.54(2) μ_{B} /f.u. for T=Rh

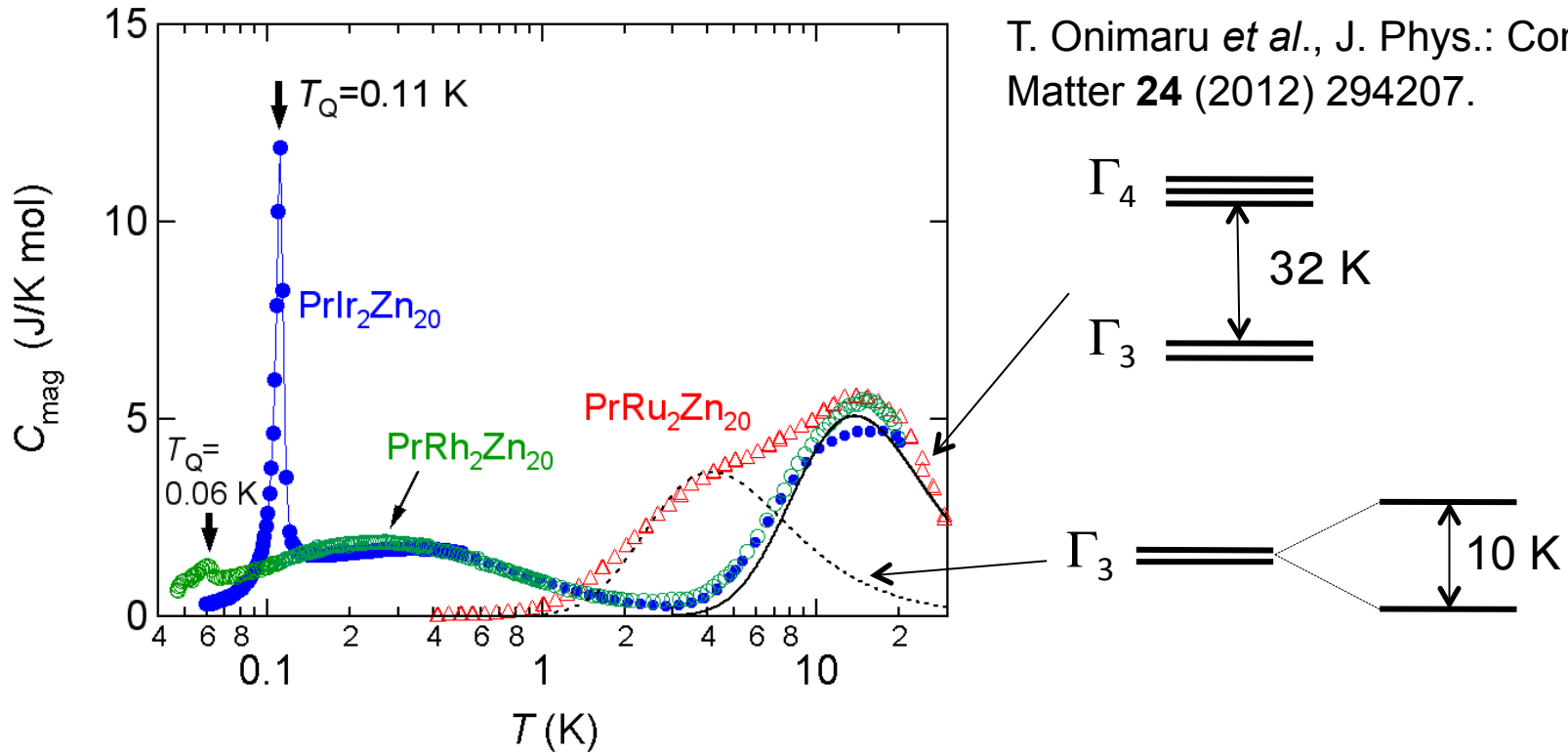
3.49(2) μ_{B} /f.u. for T=Ir

Pr ions are trivalent.

⇒ The CEF effect is weak.

- The small negative θ_{p} 's indicate weak magnetic interaction between the Pr-ions.
- Van-Vleck paramagnetic behavior.
 - ⇒ The CEF ground states is non-magnetic Γ_1 or Γ_3 .

Magnetic specific heat



- Schottky-type peaks appear at 12 K. → CEF ground state: Γ_3 doublets

$\text{PrRu}_2\text{Zn}_{20}$: Structural transition at $T_S=138$ K

- Shoulder-like anomaly → Splitting of Γ_3 doublet by the lattice modulation.

T. Onimaru *et al.*, Phys. Rev. Lett. **106** (2011) 177001.

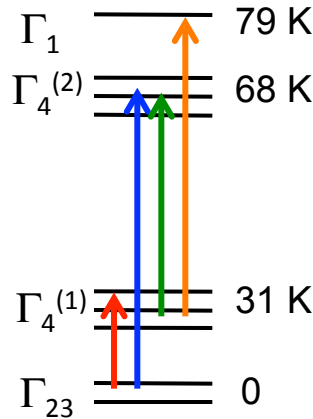
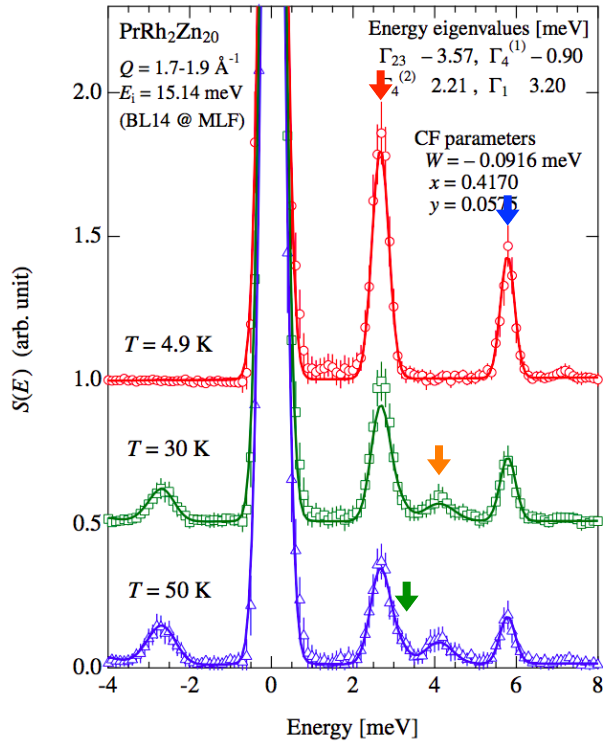
I. Ishii, *et al.*, J. Phys. Soc. Jpn. **80**, 093601 (2011).

T. Onimaru *et al.*, Phys. Rev. B **86** (2012) 184426.

Inelastic neutron scattering

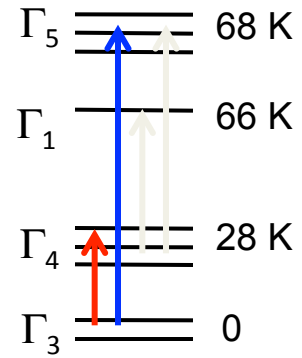
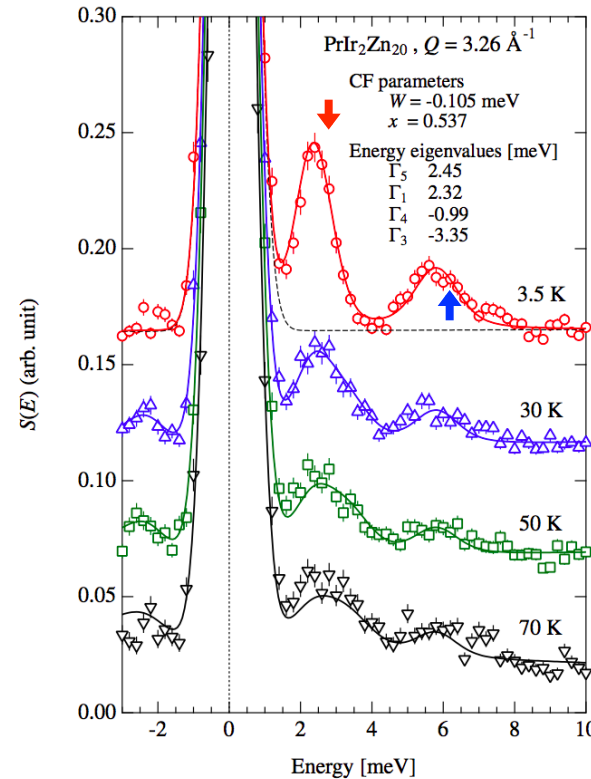
K. Iwasa *et al.*, J. Phys. Soc. Jpn. **82** (2013) 043707.

PrRh₂Zn₂₀



Cubic T

PrIr₂Zn₂₀



Cubic T_d

cf. Structural transition at 170~470 K.

- Spectra are well reproduced by the CEF level schemes with **the nonmagnetic doublet ground state.**

PrT₂X₂₀ (T: Transition metal, X: Al, Zn)

	Lattice parameter (Å)	Structural transition	CEF ground state	Quadrupole order	SC transition
PrRu ₂ Zn ₂₀	14.3467(4)	T _S =138 K	Singlet (T < T _S)	—	— (>0.04 K)
PrRh ₂ Zn ₂₀	14.2702(3)	T _S =170 ~470 K	Γ ₂₃ doublet (T) (T < T _S)	AFQ T _Q =0.06 K	T _c =0.06 K
PrOs ₂ Zn ₂₀	14.365(5)	T _S =87 K	?	— (>0.4 K)	— (>0.4 K)
PrIr ₂ Zn ₂₀	14.2729(2)	—	Γ ₃ doublet	AFQ T _Q =0.11 K	T _c =0.05 K
PrTi ₂ Al ₂₀	14.723(7)	—	Γ ₃ doublet	FQ T _Q =2 K	T _c =0.2 K (a. p.) T _c =1 K (~8 GPa)
PrV ₂ Al ₂₀	14.591(2)	—	Γ ₃ doublet	AFQ T _Q =0.6 K	—
PrNb ₂ Al ₂₀	14.7730(3)	—	Γ ₃ doublet	—	—

T. Onimaru *et al.*, JPSJ **79** (2010) 033704.

T. Onimaru *et al.*, PRL **106** (2011) 177001.

T. Onimaru *et al.*, PRB **86** (2012) 184426.

K. Wakiya *et al.*, JKPS **62** (2013) 2143.

A. Sakai and S. Nakatsuji, JPSJ **80** (2011) 063701.

R. Higashinaka *et al.*, JPSJ **80** (2011) SA048.

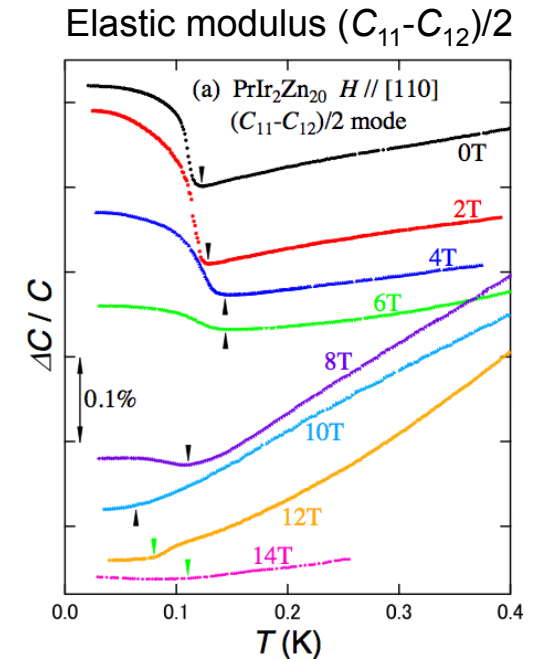
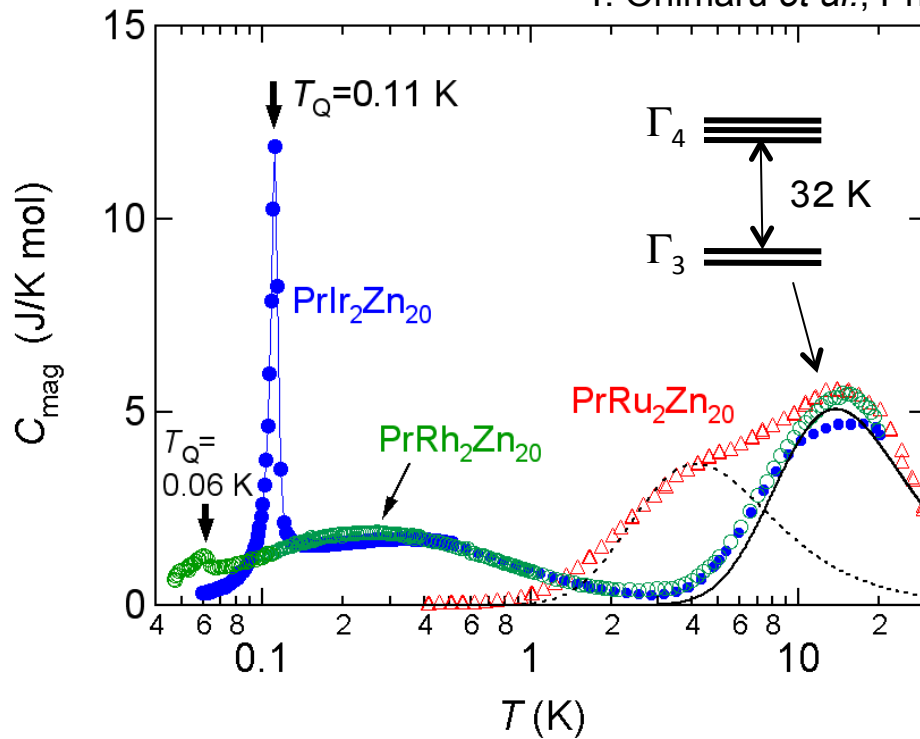
A. Sakai *et al.*, JPSJ **81** (2012) 083702.

K. Matsubayashi *et al.*, PRL **109** (2012) 187004.

Magnetic specific heat

T. Onimaru *et al.*, Phys. Rev. Lett. **106** (2011) 177001.

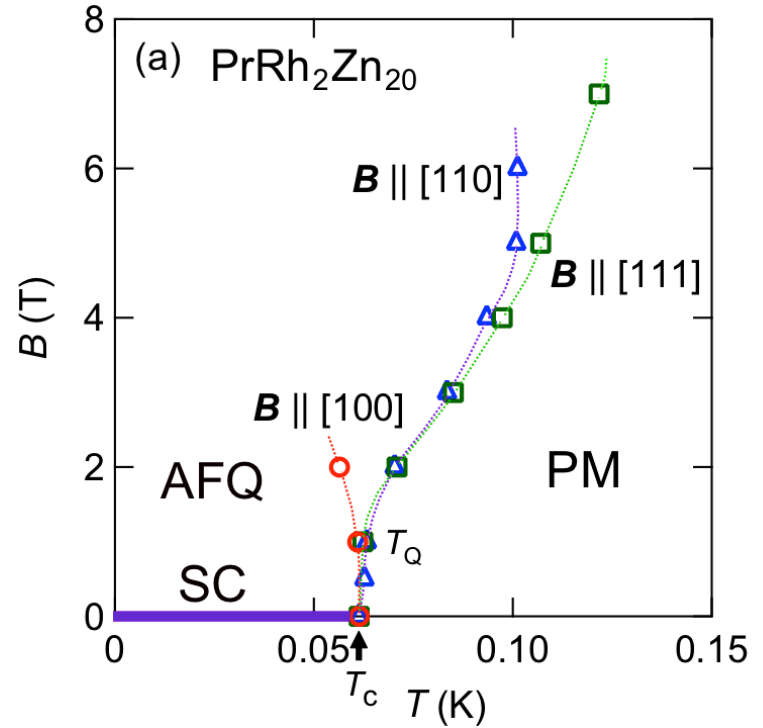
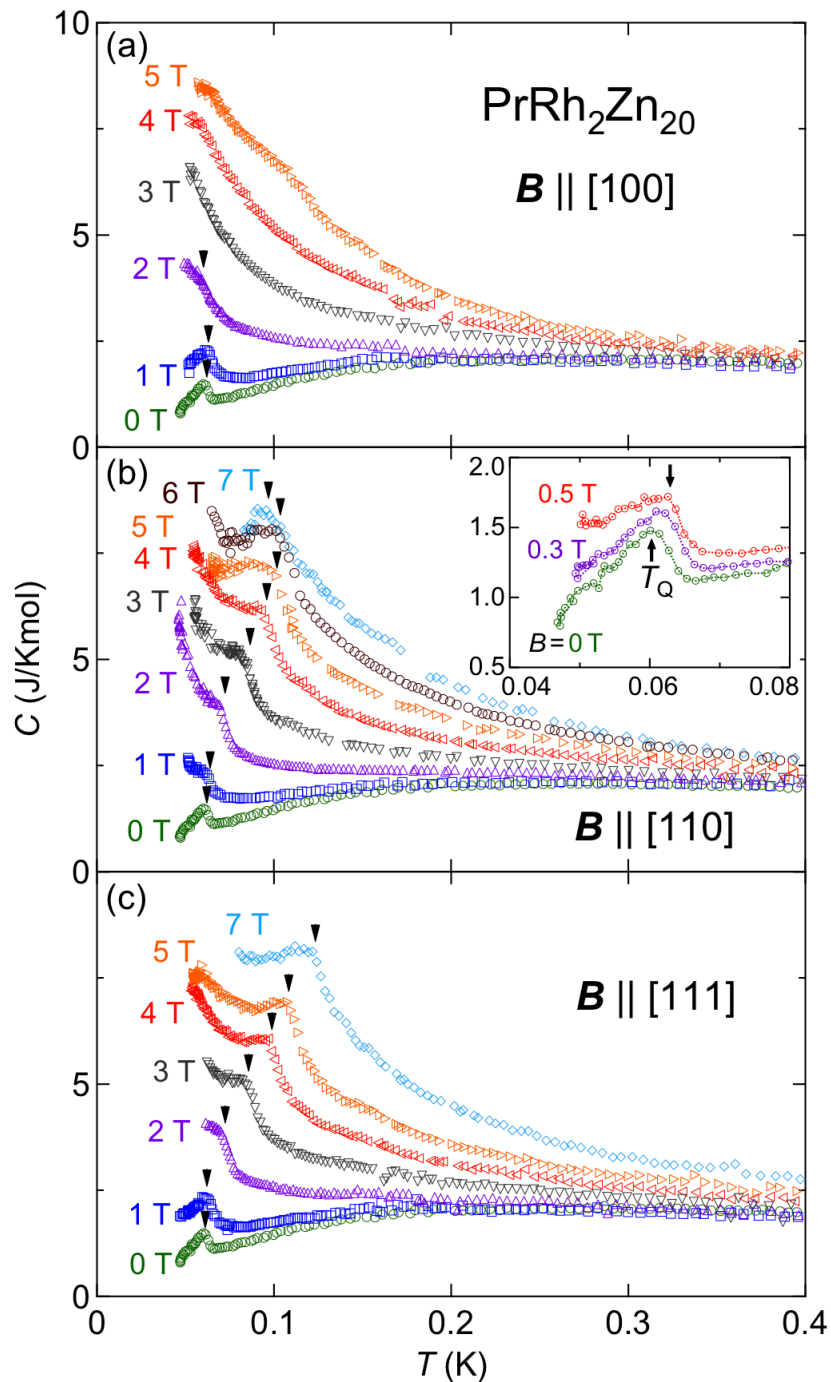
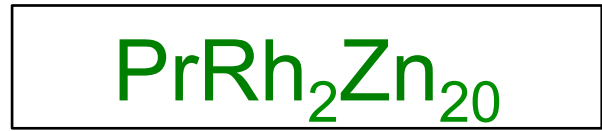
T. Onimaru *et al.*, Phys. Rev. B **86** (2012) 184426.



I. Ishii *et al.*, J. Phys. Soc. Jpn. **80** (2011) 093601.

- Sharp peaks appear at low temperatures.

$\text{PrRh}_2\text{Zn}_{20}$	Antiferroquadrupole (AFQ) ordering at $T_Q = 0.06$ K. I. Ishii <i>et al.</i> , Phys. Rev. B 87 (2013) 205106.
$\text{PrIr}_2\text{Zn}_{20}$	AFQ ordering at $T_Q = 0.11$ K. I. Ishii <i>et al.</i> , J. Phys. Soc. Jpn. 80 (2011) 093601.



- i. Anisotropic behavior of T_Q .
 $T_Q: [100] < [110] < [111]$
- ii. T_Q increases with increasing B for [110] and [111].

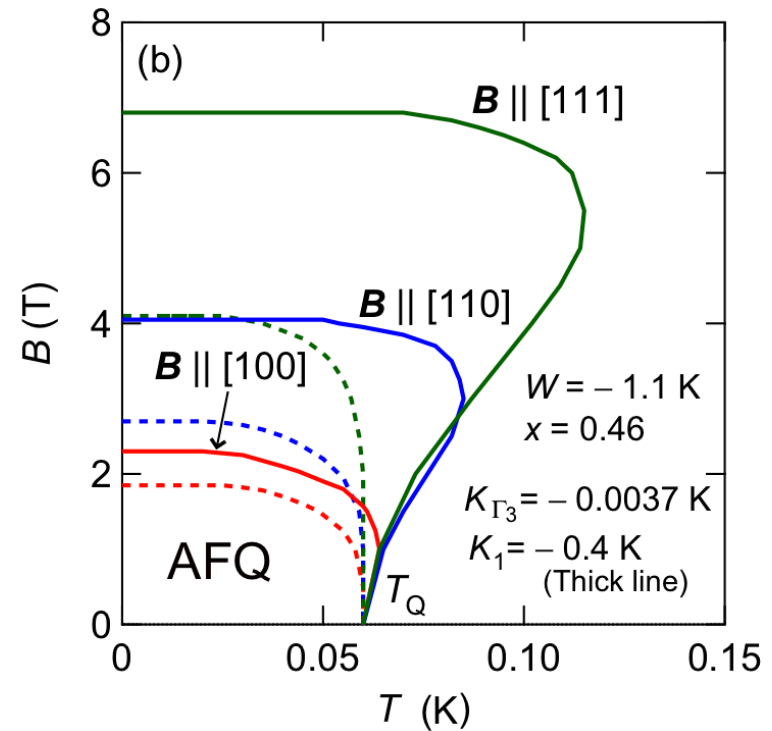
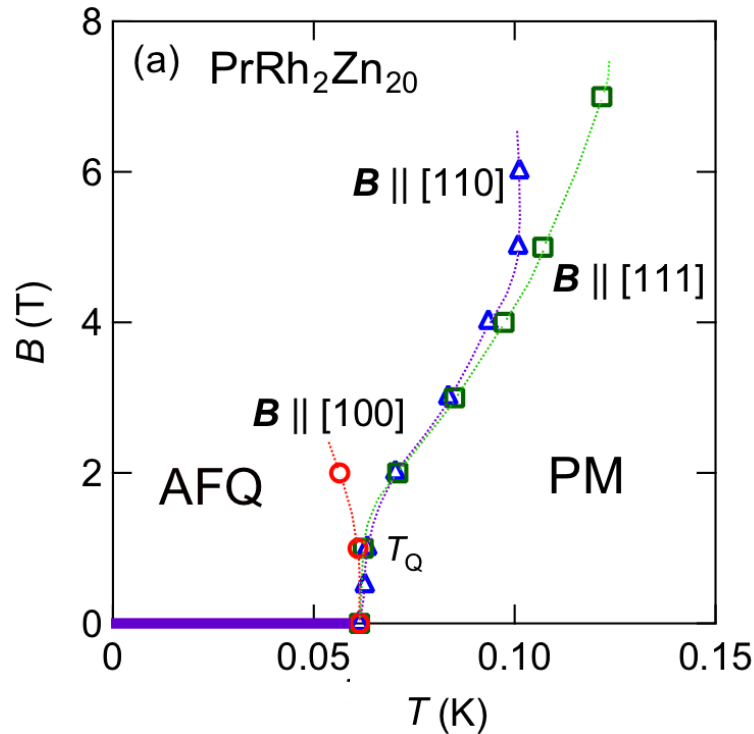
AFQ ordering occurs at $T_Q=0.06$ K.

AFQ ordered state: $\text{PrRh}_2\text{Zn}_{20}$

T. Onimaru *et al.*, Phys. Rev. B **86** (2012) 184426.

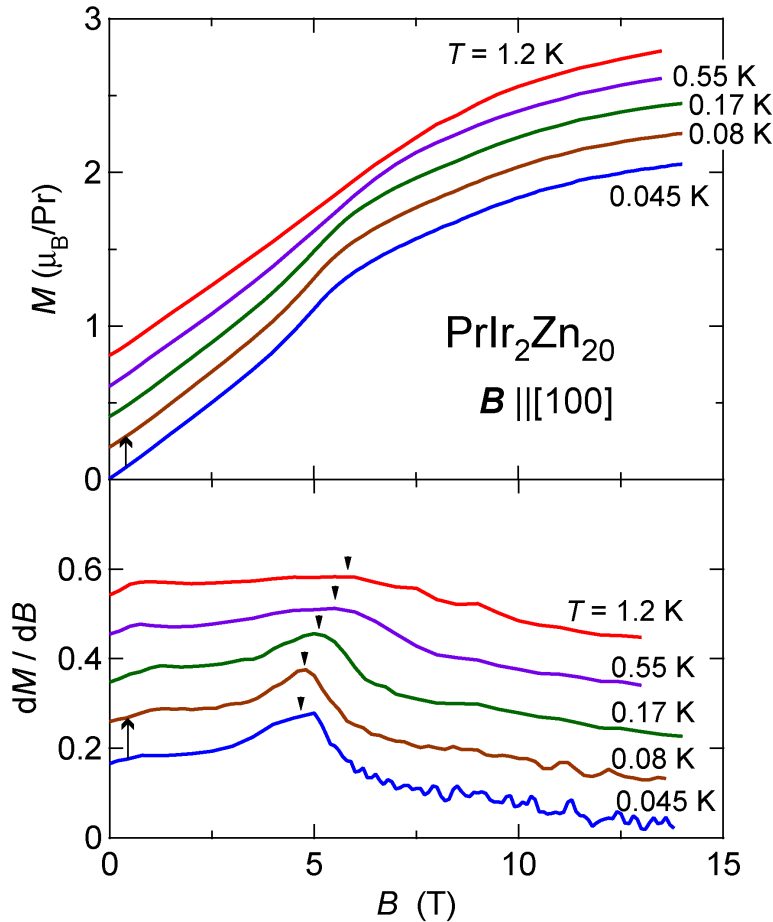
Two-sublattice mean-field model

$$H_{A(B)}^I = H_{CEF} - g_J \mu_B JH - \left(K_1 \langle J \rangle_{B(A)} + K_2 \langle J \rangle_{A(B)} \right) J - K_{\Gamma_3} \left(\langle O_2^0 \rangle_{B(A)} O_2^0 + \langle O_2^2 \rangle_{B(A)} O_2^2 \right)$$



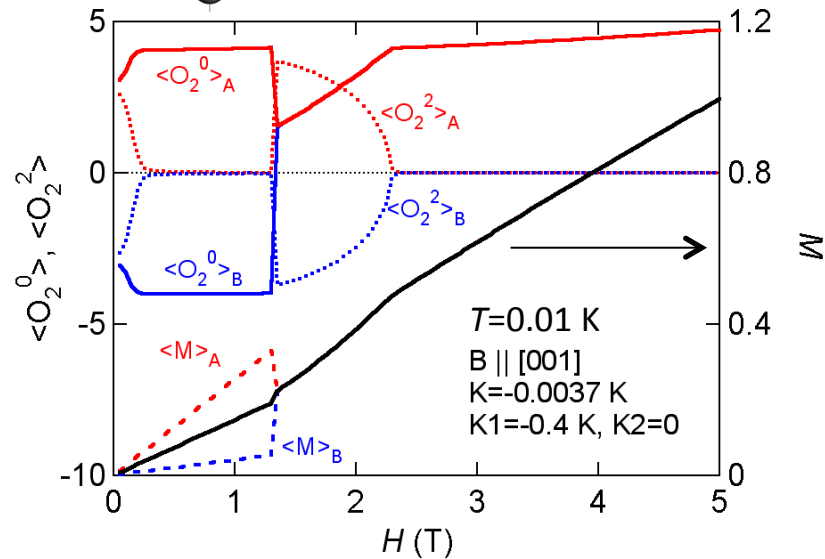
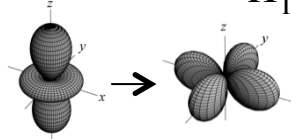
- Two sub-lattice mean-field calculation well reproduces the anisotropic B - T phase diagrams.
- Antiferromagnetic interaction between the field-induced magnetic moments of the Pr^{3+} ions stabilizes the AFQ ordered state.

Metamagnetic behavior in $B \parallel [100]$



Two-sublattice mean-field model

$$H_{A(B)}^I = H_{CEF} - g_J \mu_B J H - \left(K_1 \langle J \rangle_{B(A)} + K_2 \langle J \rangle_{A(B)} \right) J - K_{\Gamma 3} \left(\langle O_2^0 \rangle_{B(A)} O_2^0 + \langle O_2^2 \rangle_{B(A)} O_2^2 \right)$$



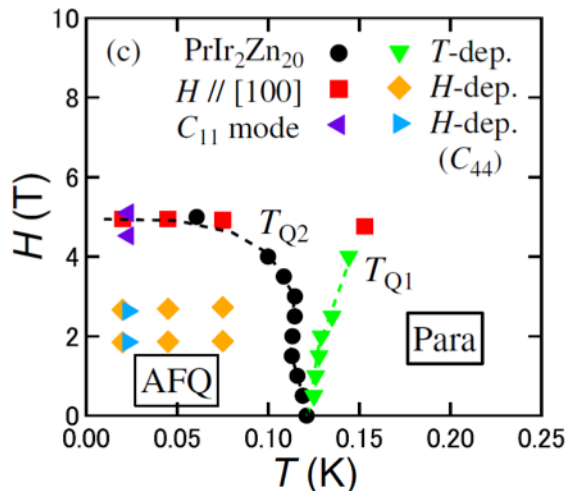
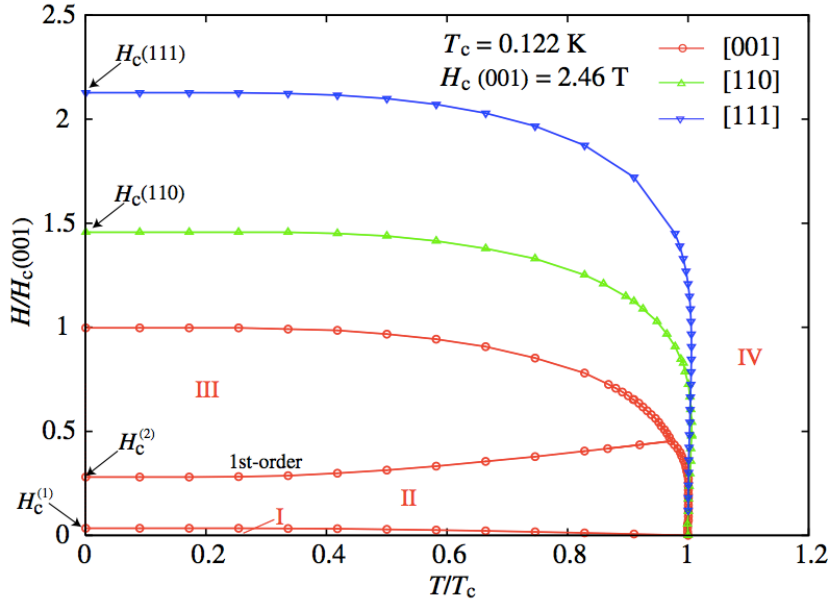
- Metamagnetic behavior was observed at around $B=5$ T.
- In mean-field model calculation, order parameter is changed in B , where metamagnetic behavior is found, resulting from level-crossing.

➔ Switching of the OP possibly changes the conduction band?

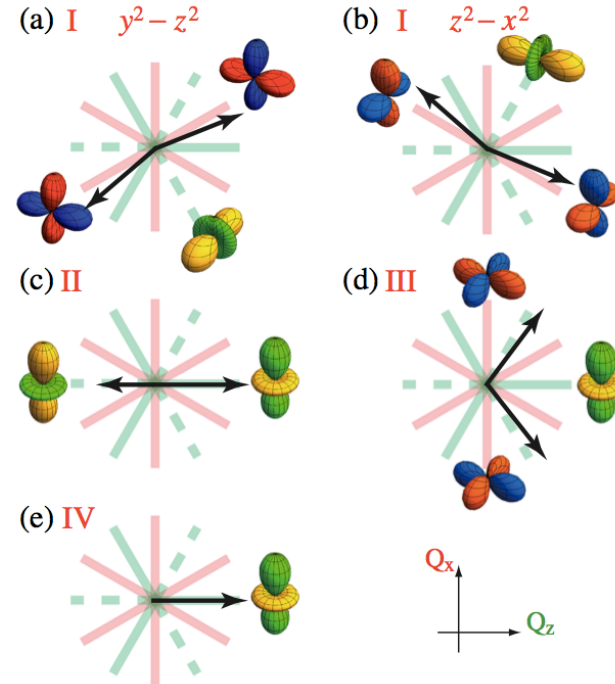
Order parameter of the AFQ phase

Mean-field calculation

K. Hattori and H. Tsunetsugu, JPSJ **83**, 034709 (2013).



I. Ishii *et al.*, JPSJ. **80** (2011) 093601.

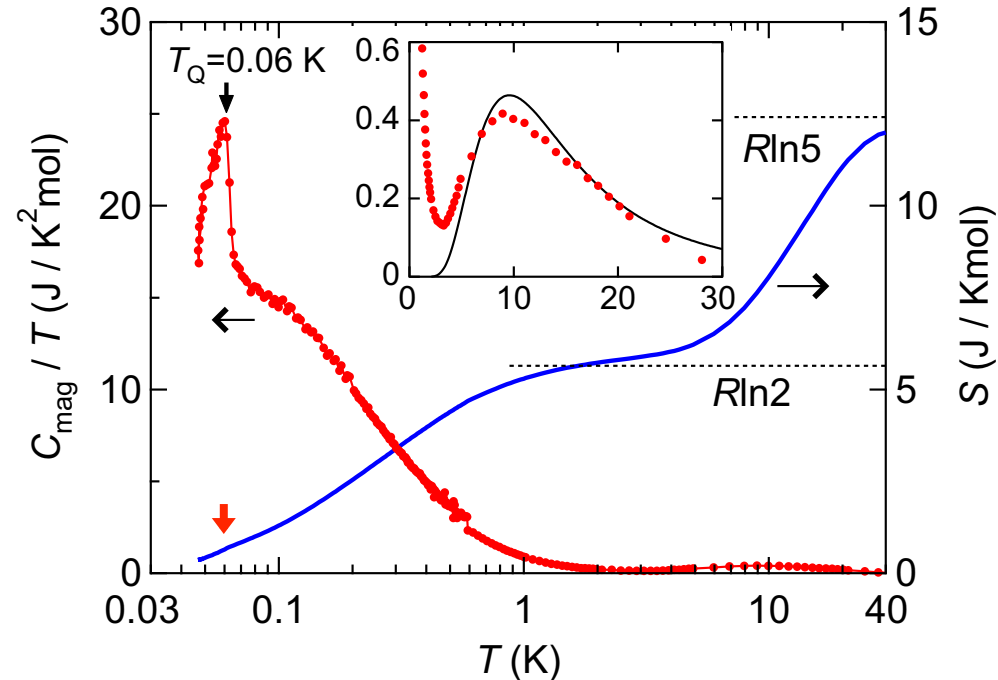
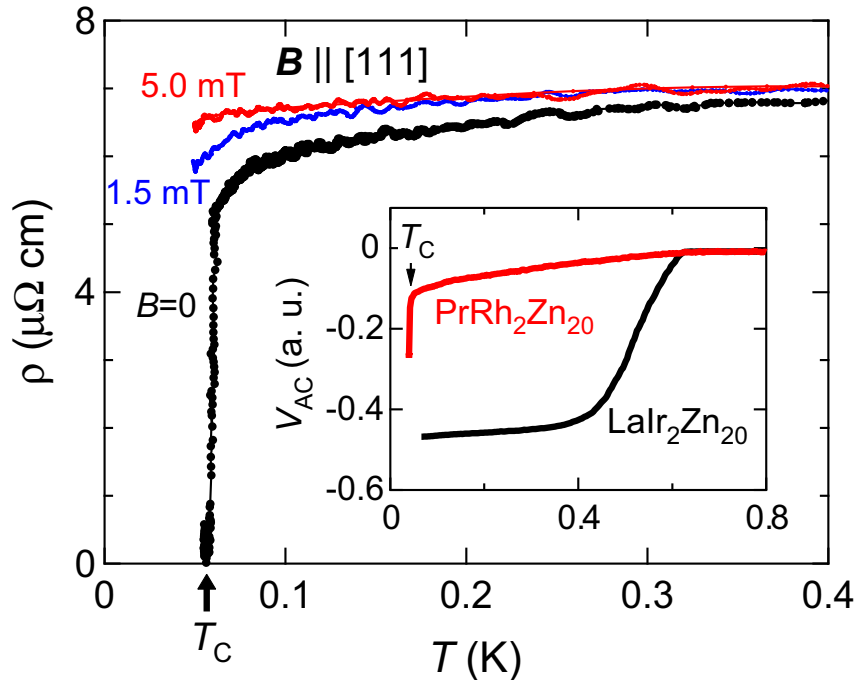


Phase I: O_2^2 AFQ + small O_2^0 FQ
 Phase II: Collinear O_2^0 AFQ
 Phase III: Canted AFQ
 Phase IV: Paramagnetic state

- Gapless spin-wave excitation at the border of the O_2^2 AFQ phases.
- Unusual singularities of χ_Q at $T=T_Q$ for $H=0$ and the critical field for [111].

Simultaneous Superconducting and AFQ transitions in $\text{PrRh}_2\text{Zn}_{20}$

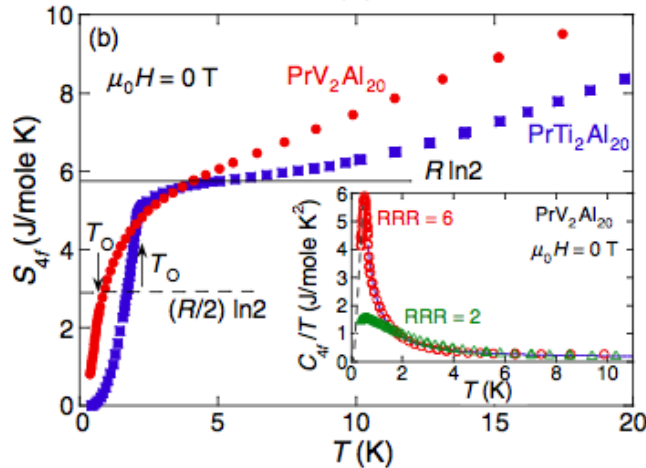
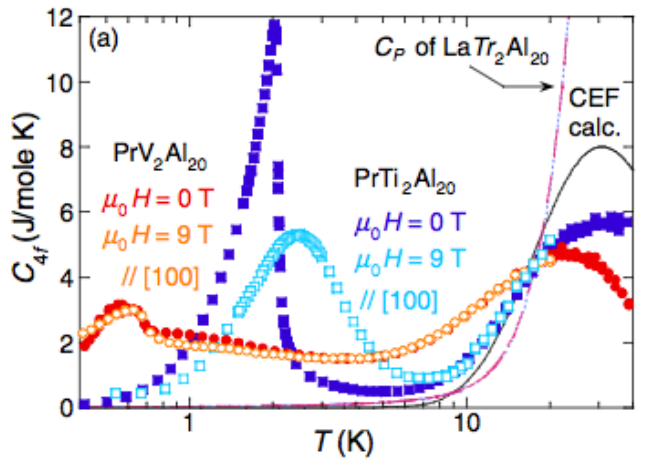
T. Onimaru *et al.*, Phys. Rev. B **86** (2012) 184426.



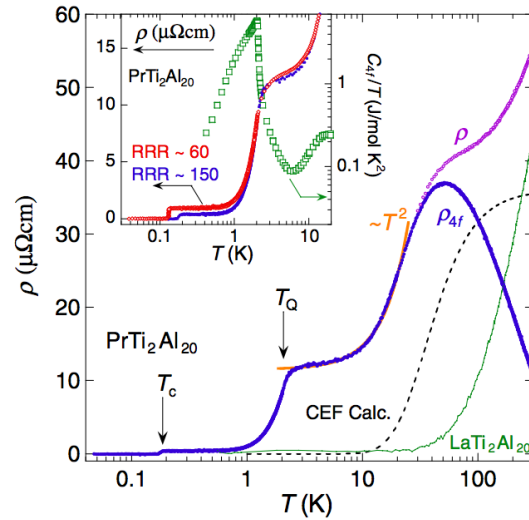
- Superconducting transition occurs at $T_c=0.06$ K.
- The magnetic entropy at T_Q is only $0.1R\ln 2$ which is much smaller than $R\ln 2$ expected for an order of the nonmagnetic doublet ground state.
 \Rightarrow The quadrupole fluctuations remain active below T_Q ?

Ferroquadrupole order and heavy-fermion superconductivity in the isostructural $\text{PrTi}_2\text{Al}_{20}$

- FQ transition at $T_Q=2$ K.
- Magnetic entropy at T_Q is $0.9R\ln 2$.

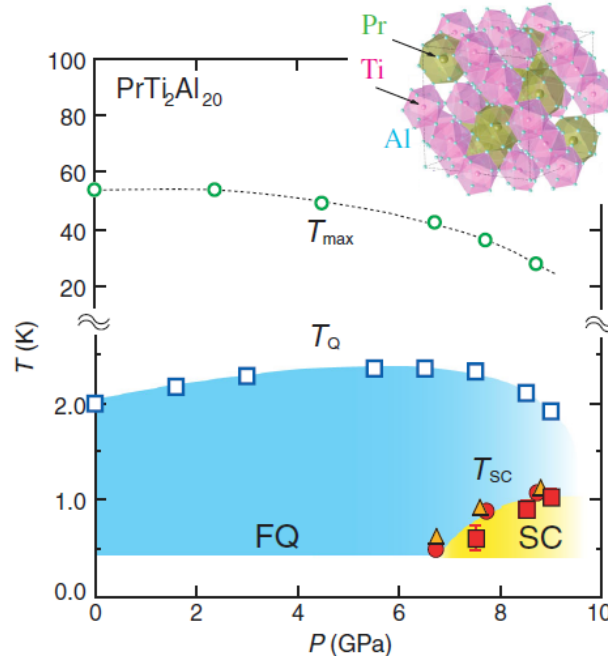


A. Sakai and S. Nakatsuji, J. Phys. Soc. Jpn. **80** (2011) 063701.



- Superconducting transition at $T_c=0.2$ K.
- $m^*=16m_0$

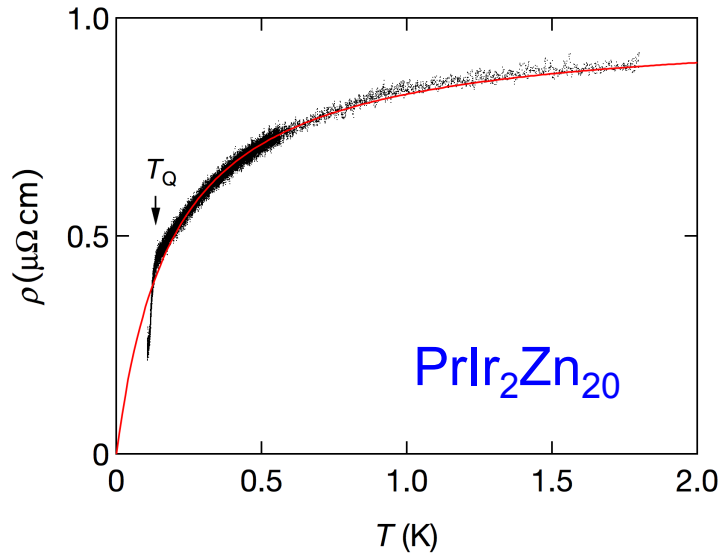
A. Sakai *et al.*,
J. Phys. Soc. Jpn. **81**
(2012) 083702.



- T_c is increased up to $T_c=1$ K at 8 GPa
- Heavy-fermion superconductor

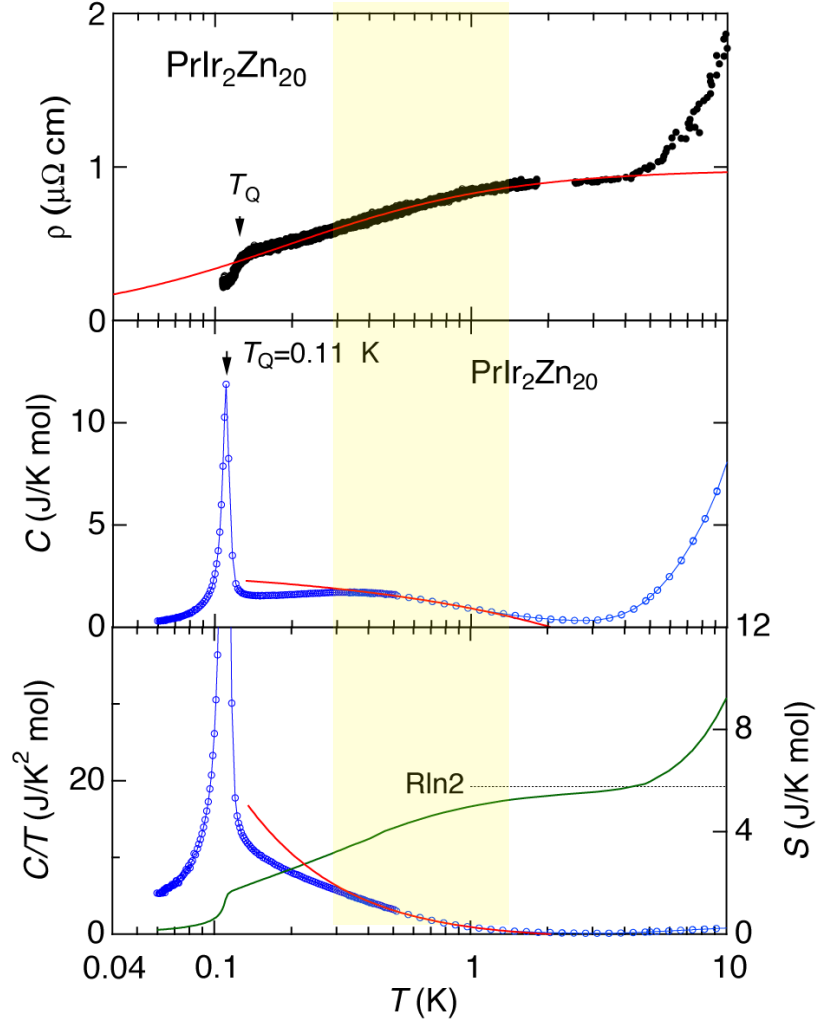
K. Matsubashi *et al.*, Phys. Rev. Lett. **109** (2012) 187004.

Anomalous temperature dependence of $\rho(T)$ and $C(T)$



$S/T \sim 0.031 \mu\text{V}/\text{K}^2$ (T. Ikeura and K. Izawa)

➔ Weak hybridization between the 4f and conduction electrons ?



❖ Entropy of the Γ_3 doublet is released by dynamic Jahn-Teller distortion in PrMg_3 .

Summary $\text{PrT}_2\text{Zn}_{20}$ (T = Ru, Rh, and Ir)

- For T=Rh and Ir, the CEF ground state are the **non-magnetic Γ_3 doublet** with quadrupolar degrees of freedom.
- **Antiferro-quadrupolar (AFQ) ordering** at T_Q was confirmed.
 - To determine the AFQ structure and order parameters, microscopic measurements are in progress.
- **Non-Fermi liquid behavior** was observed above T_Q .
 - Hybridization effect must play an important role in forming the ground state.

$\text{PrT}_2\text{X}_{20}$ (X=Al and Zn) family is appropriate for revealing characteristics of the quadrupoles such as quadrupole order, multi-channel Kondo effect, exotic superconductivity.