

Momentum differentiation enhanced by Hund's coupling: A multi-orbital cluster DMFT study

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Outline

- Effect of local U and J in correlated electron systems

	Onsite correlation	Intersite correlation
U	DMFT	Cluster DMFT
J	Multi-orbital DMFT	multi-orbital cluster DMFT

- Efficient CTQMC algorithm for multi-orbital DCA

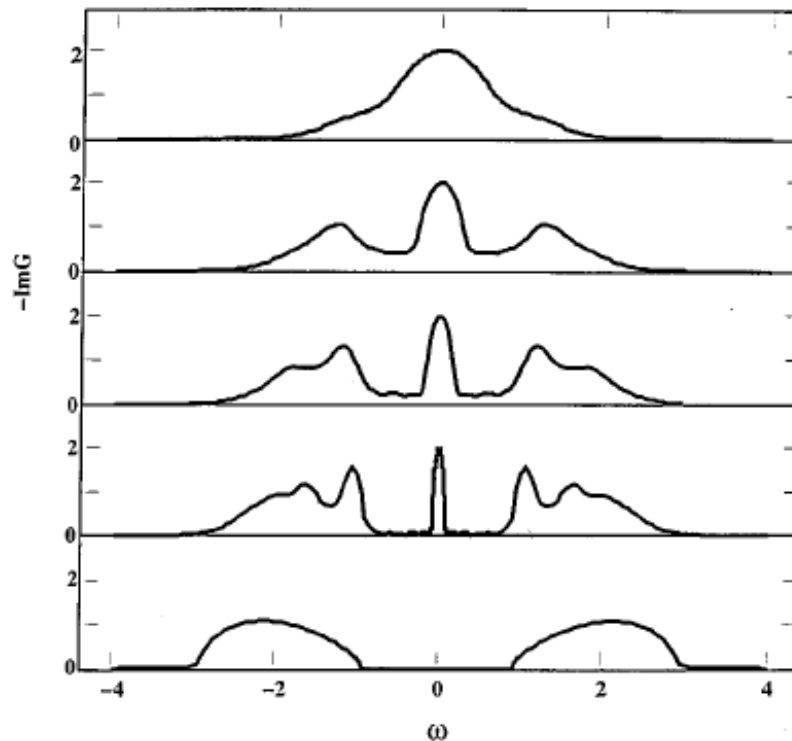
Y. Nomura, S. Sakai, and RA, Phys. Rev. B **89**, 195146 (2014)

- How local J enhances intersite correlations

DMFT study on the on-site correlation due to U

	Onsite correlation	Intersite correlation
U	DMFT	Cluster DMFT
J	Multi-orbital DMFT	multi-orbital cluster DMFT

Mott-Hubbard transition



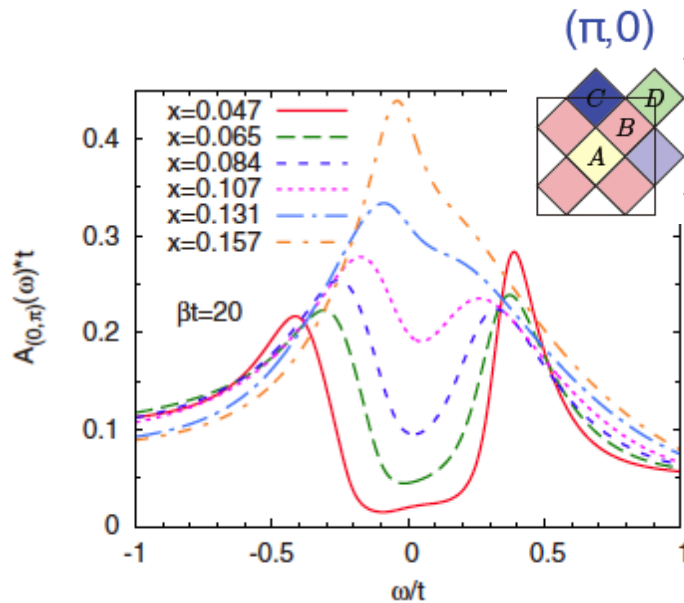
A. Georges et al., RMP 68 13 (1996)

Cluster DMFT study on the intersite correlation due to U

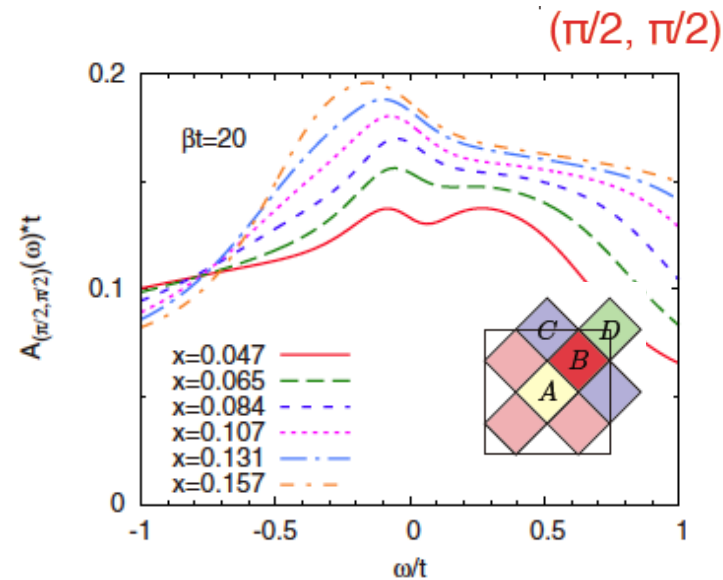
	Onsite correlation	Intersite correlation
U	DMFT	Cluster DMFT
J	Multi-orbital DMFT	multi-orbital cluster DMFT

Analytically continued spectral function $A(\omega)$: $U = 7t$, $t'/t=0.15$, $\beta t=20$

for the **antinodal** sector.



for the **nodal** sector.

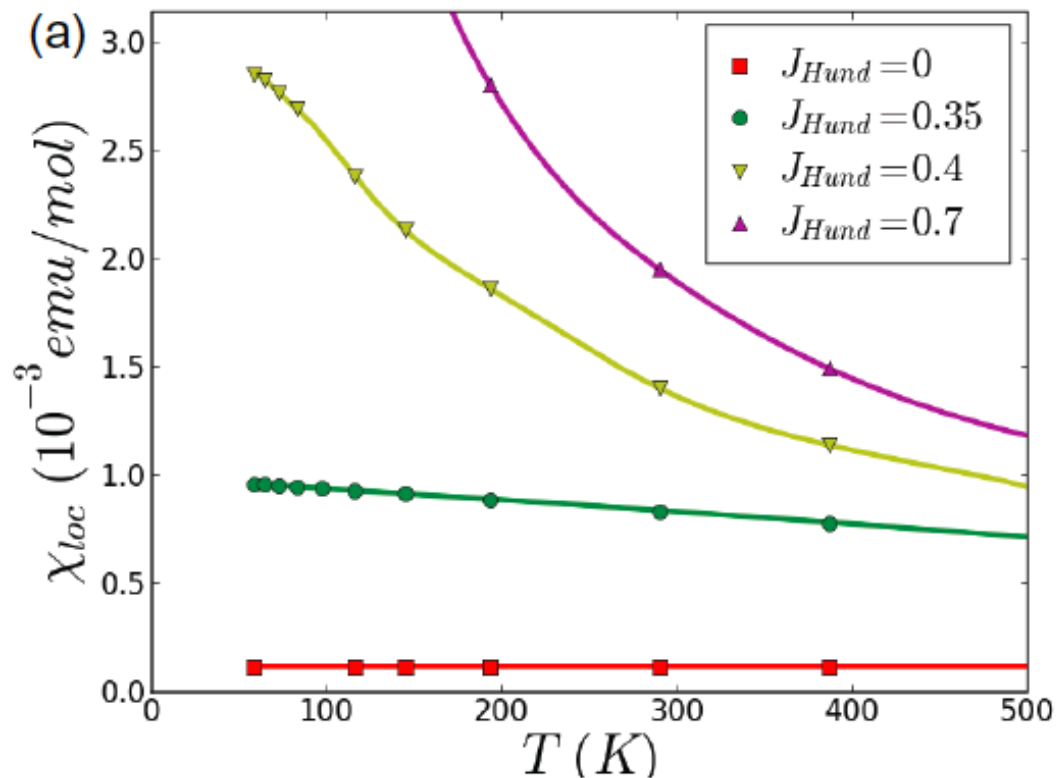


N. Lin, E. Gull, and A.J. Millis, Phys. Rev. B 82, 045104 (2010)

DMFT study on the on-site correlation due to J

	Onsite correlation	Intersite correlation
U	DMFT	Cluster DMFT
J	Multi-orbital DMFT	multi-orbital cluster DMFT

“Hund’s metal” (Strong correlation induced by J)



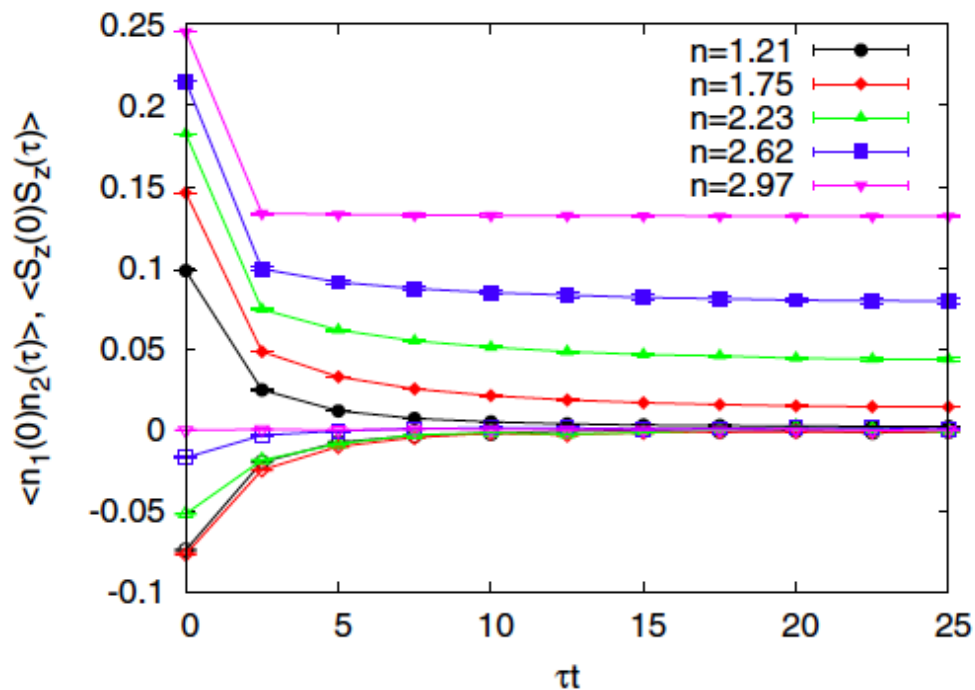
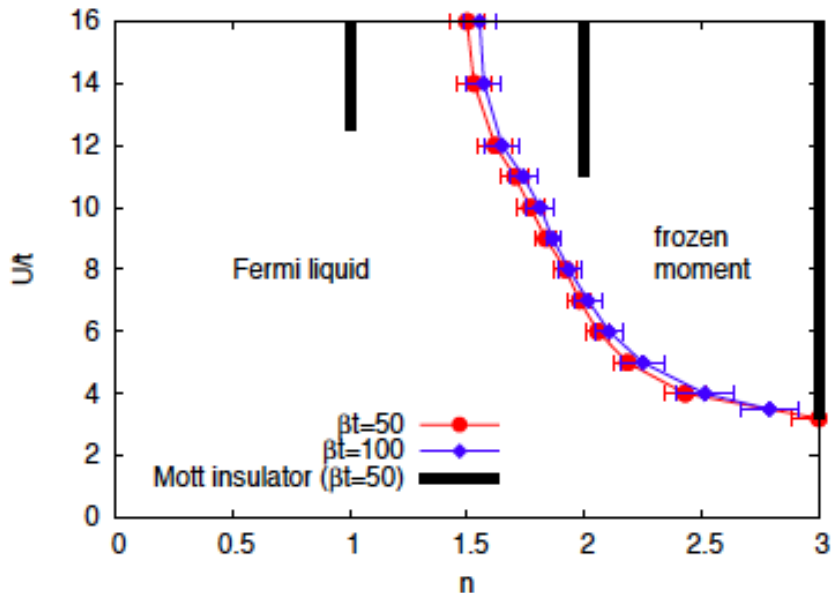
Haule and Kotliar, New J. Phys. 11, 025021 (2009)

DMFT study on the on-site correlation due to J

	Onsite correlation	Intersite correlation
U	DMFT	Cluster DMFT
J	Multi-orbital DMFT	multi-orbital cluster DMFT

“spin-freezing” transition

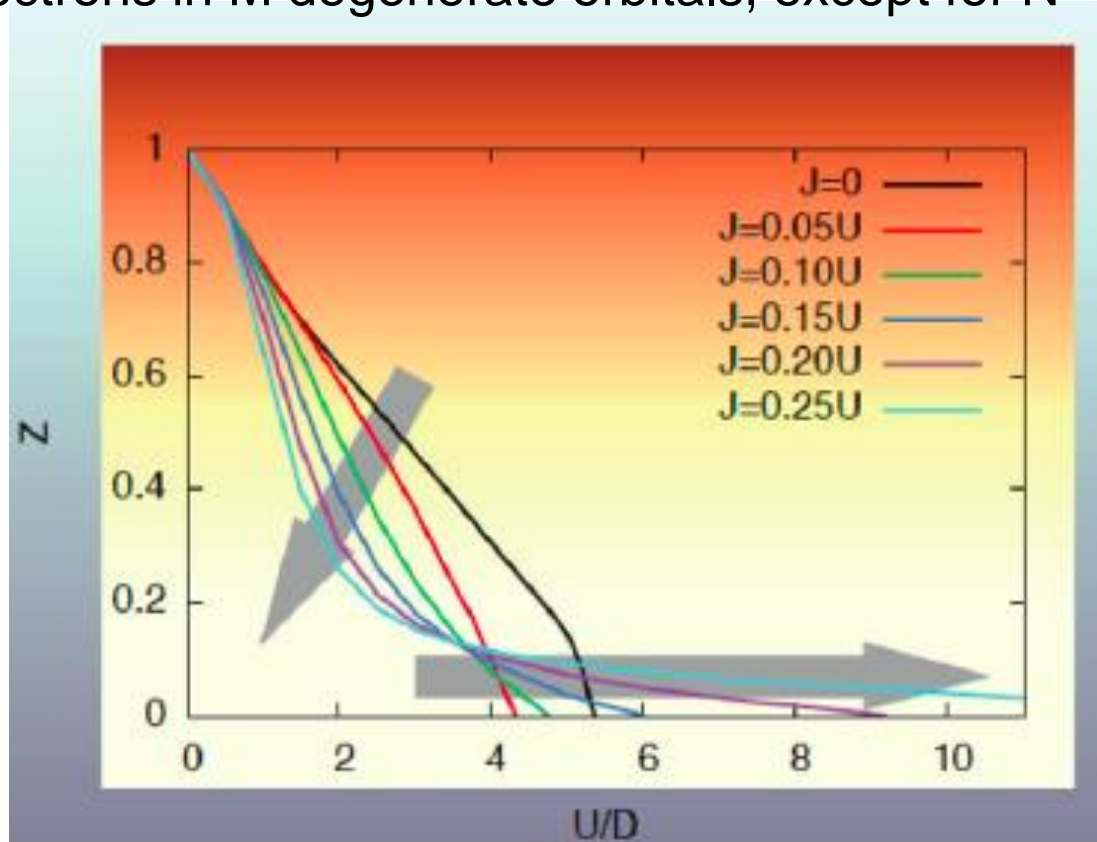
3-orbital Hubbard model



DMFT study on the on-site correlation due to J

	Onsite correlation	Intersite correlation
U	DMFT	Cluster DMFT
J	Multi-orbital DMFT	multi-orbital cluster DMFT

“Janus-Faced” effect
for N electrons in M degenerate orbitals, except for $N=1$ or $N=M$



Medici *et al.*, PRL 107, 256401 (2011)

Purpose of the present study

	Onsite correlation	Intersite correlation
U	DMFT	Cluster DMFT
J	Multi-orbital DMFT	multi-orbital cluster DMFT

First multi-orbital cluster DMFT calculation using CTQMC with including spin-flip and pair-hopping terms

Y. Nomura, S. Sakai, and RA, Phys. Rev. B **89**, 195146 (2014)

CTQMC: weak coupling expansion

- A. N. Rubtsov et al., Phys. Rev. B 72, 035122 (2005)
F. F. Assaad and T. C. Lang, Phys. Rev. B 76, 035116 (2007)
E. Gull et al., Rev. Mod. Phys. 83 349 (2011)

Diagrammatic expansion of the partition function of an impurity model
→ sampling of the series stochastically up to infinite order

$$Z = \text{Tr} \left[\underset{\substack{\uparrow \\ \text{Non-interacting part}}}{e^{-\beta H_0} T_\tau} \exp \left(- \int_0^\beta d\tau \underset{\substack{\uparrow \\ \text{Interacting part}}}{H_1(\tau)} \right) \right]$$

$$\frac{Z}{Z_0} = \sum_{k=0}^{\infty} \int_k d\tau P(q_k), \quad P(q_k) = (-1)^k \langle T_\tau H_1(\tau_k) \cdots H_1(\tau_2) H_1(\tau_1) \rangle_0$$
$$\langle A \rangle_0 \equiv \frac{1}{Z_0} \text{Tr} [e^{-\beta H_0} A]$$

We used the sub-matrix update method originally developed for the Hirsch-Fye algorithm (Nukula et al., PRB09) and the Continuous-Time Auxiliary Field algorithm (Gull et al., PRB11)

CTQMC: weak coupling expansion

E. Gorelov et al., Phys. Rev. B 80, 155132 (2009)

Action for the **multi-orbital** model

$$\begin{aligned}
 S_0 &= - \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{ij,\sigma} [\tilde{G}_{0\sigma}^{-1}(\tau - \tau')]_{ij} c_{i\sigma}^\dagger(\tau) c_{j\sigma}(\tau') \\
 S_{\text{int}} &= \int_0^\beta d\tau \sum_{s=\pm 1} \left[\sum_i \frac{U}{2} [n_{i\uparrow}(\tau) - \alpha_{1\uparrow}(s)] [n_{i\downarrow}(\tau) - \alpha_{1\downarrow}(s)] + \sum_{i<j,\sigma} \frac{U'}{2} [n_{i\sigma}(\tau) - \alpha_{1\sigma}(s)] [n_{j\bar{\sigma}}(\tau) - \alpha_{1\bar{\sigma}}(s)] \right. \\
 &+ \sum_{i<j,\sigma} \frac{U' - J_H}{2} [n_{i\sigma}(\tau) - \alpha_{1\sigma}(s)] [n_{j\sigma}(\tau) - \alpha_{1\sigma}(s)] + \sum_{i\neq j} \frac{J_H}{2} [c_{i\uparrow}^\dagger(\tau) c_{j\uparrow}(\tau) - \alpha_{2\uparrow}(s)] [c_{j\downarrow}^\dagger(\tau) c_{i\downarrow}(\tau) - \alpha_{2\downarrow}(s)] \\
 &\left. + \sum_{i\neq j} \frac{J_H}{2} [c_{i\uparrow}^\dagger(\tau) c_{j\uparrow}(\tau) - \alpha_{2\uparrow}(s)] [c_{i\downarrow}^\dagger(\tau) c_{j\downarrow}(\tau) - \alpha_{2\downarrow}(s)] \right].
 \end{aligned}$$

We introduce additional parameters

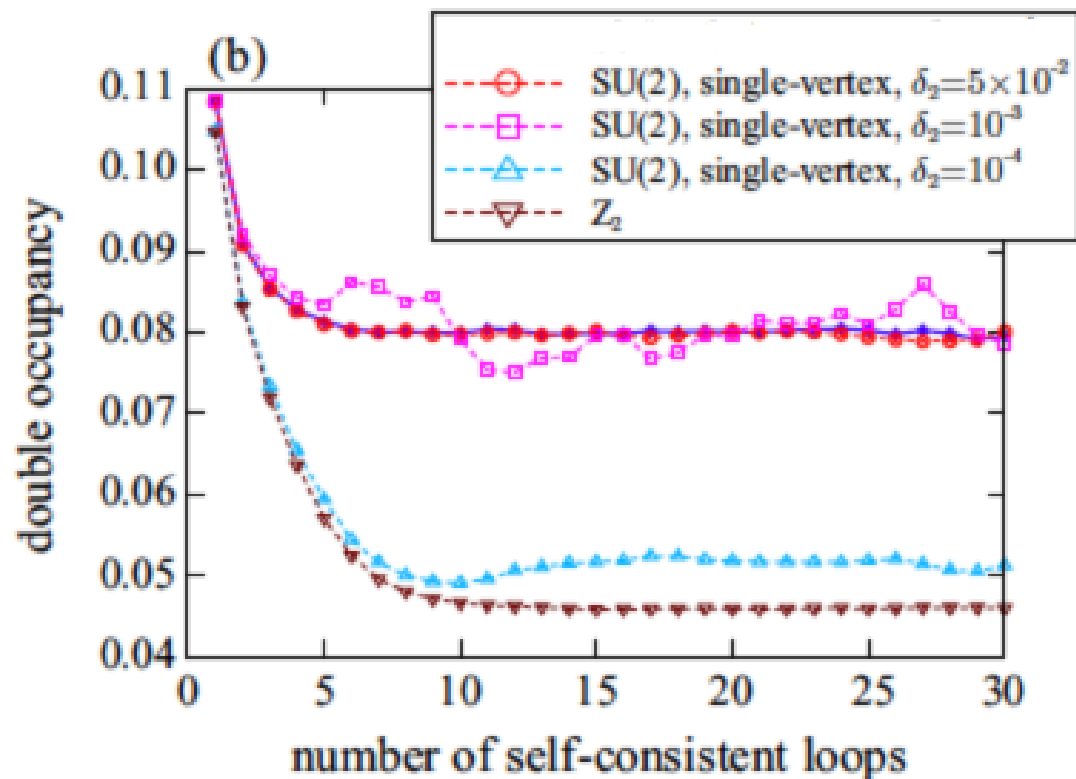
$$\begin{cases} \alpha_{1\uparrow}(s) = 1/2 + s\delta_1 \\ \alpha_{1\downarrow}(s) = 1/2 - s\delta_1 \end{cases} \quad \delta_1 = 1/2 + 0^+ \quad \rightarrow \text{To reduce sign problem}$$

$$\begin{cases} \alpha_{2\uparrow}(s) = +s\delta_2 \\ \alpha_{2\downarrow}(s) = -s\delta_2 \end{cases} \quad \delta_2: \text{small positive number} \quad \rightarrow \text{To ensure the ergodicity (if } [G_0]_{ij} = 0 \text{)} \\ \text{large } \delta_2 \rightarrow \text{sign problem}$$

Two-orbital Hubbard model

half-filling, $t=1$

$T=0.05$, $U=6$, $J=1.5$

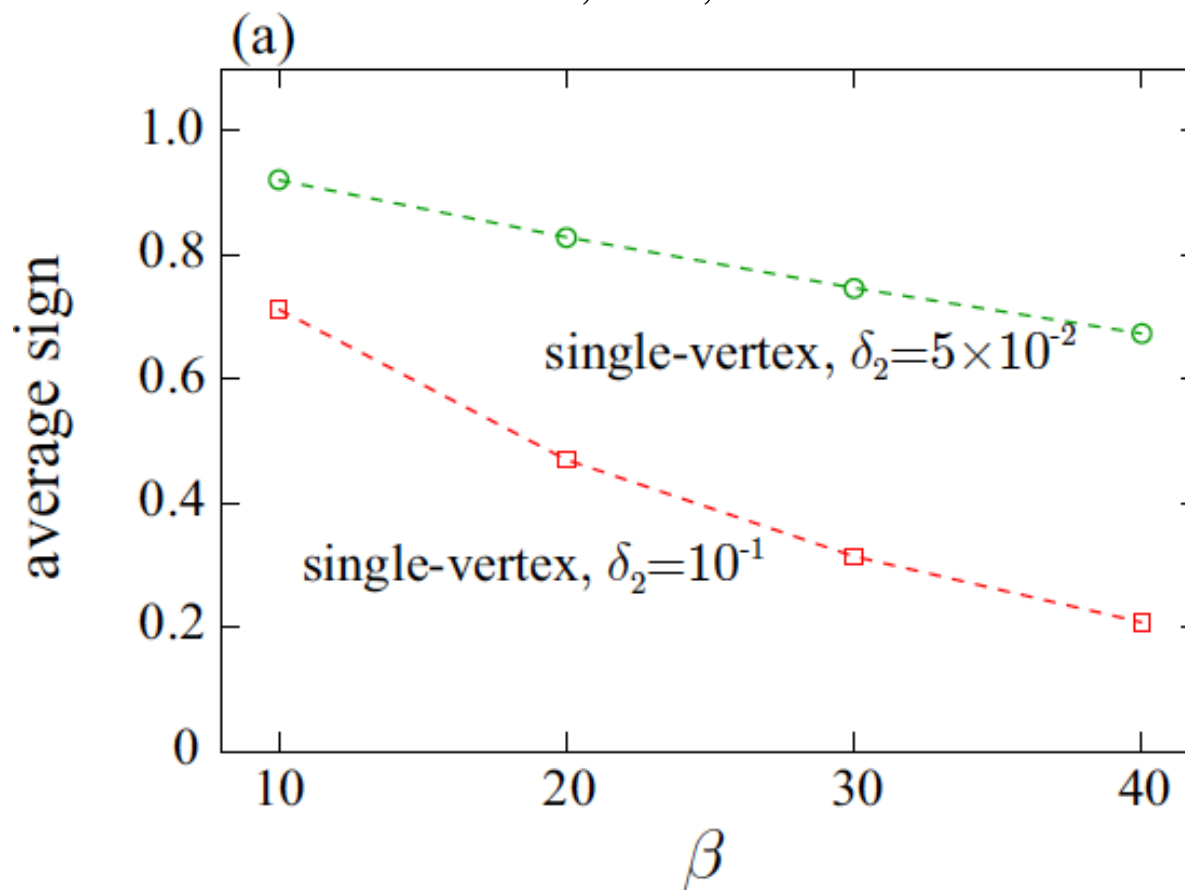


We need sufficiently large δ_2 to guarantee the ergodicity

Two-orbital Hubbard model

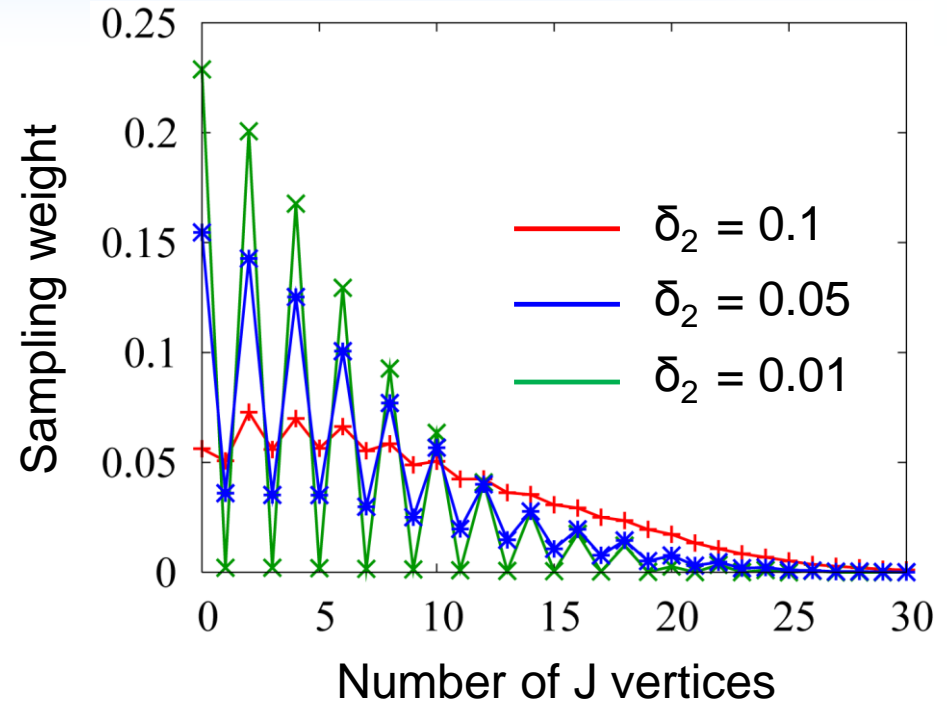
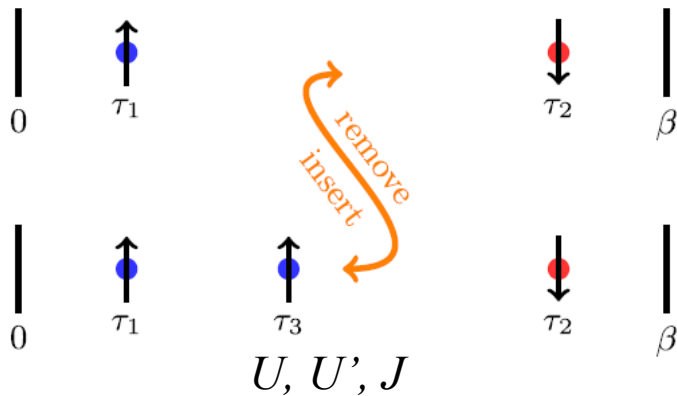
half-filling, $t=1$

$T=0.05$, $U=6$, $J=1.5$



Large $\delta_2 \rightarrow$ Severer sign problem

Efficient sampling for spin-flip & pair-hopping terms



Weight of configurations with odd number of J vertices is zero for $\delta_2 \rightarrow 0$.

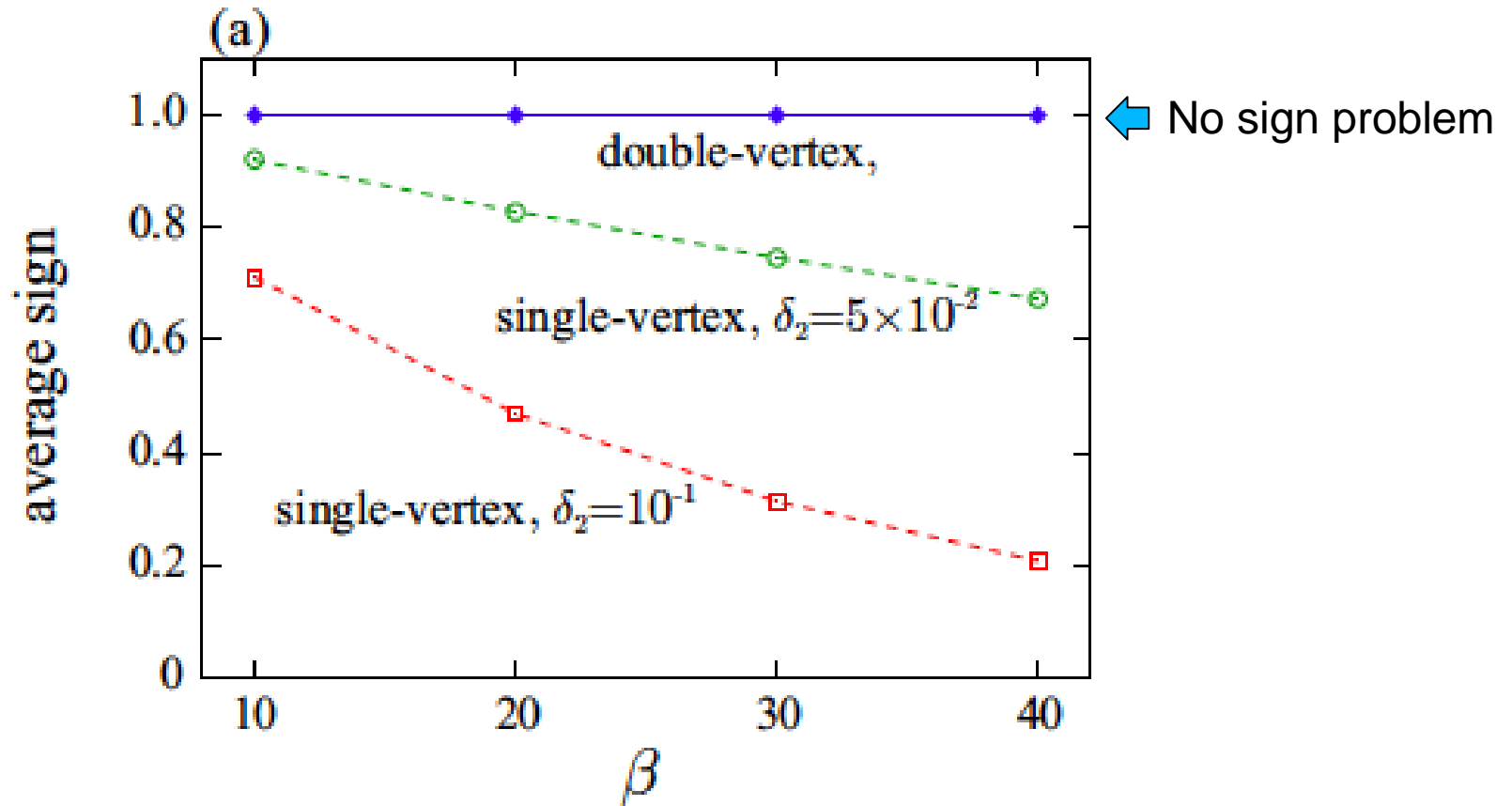
The odd-order terms are artificial but necessary, since the number of non-density J vertices can not be changed without passing through the odd-order terms.

➡ We only need to insert/remove two J vertices simultaneously (**double vertex update**)
 We do not need to introduce δ_2

Double vertex update

half-filling, $t=1$

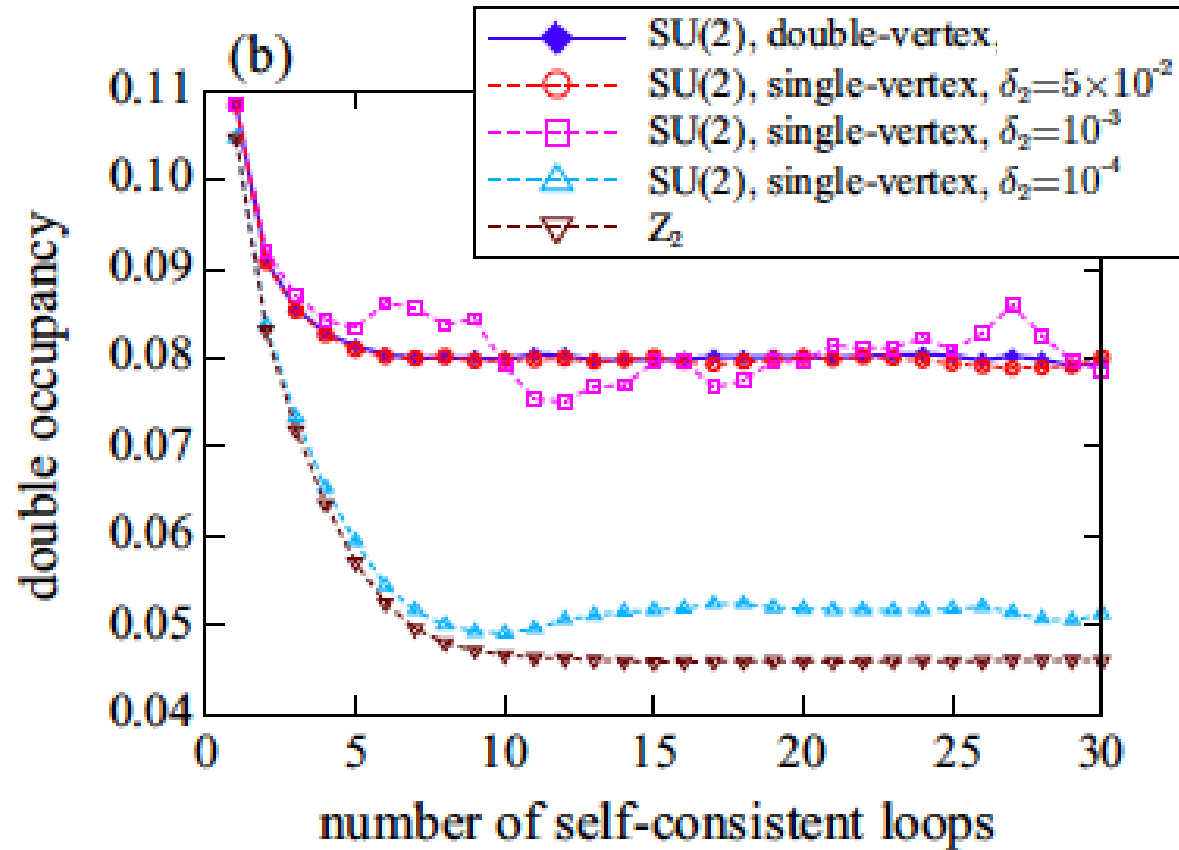
$T=0.05$, $U=6$, $J=1.5$



Double vertex update

half-filling, $t=1$

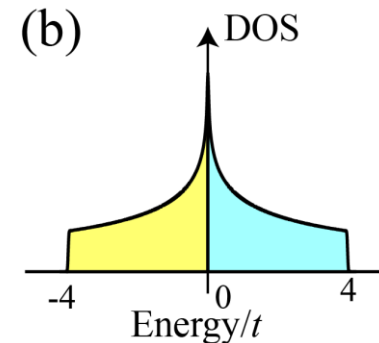
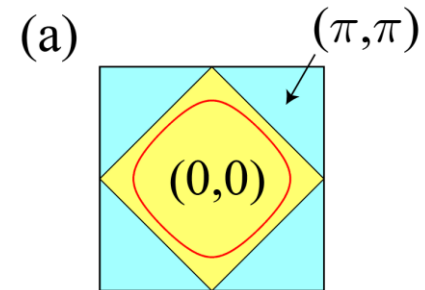
$T=0.05$, $U=6$, $J=1.5$



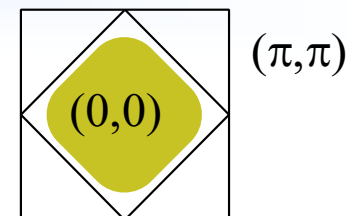
Three-orbital Hubbard model

Calculation conditions:

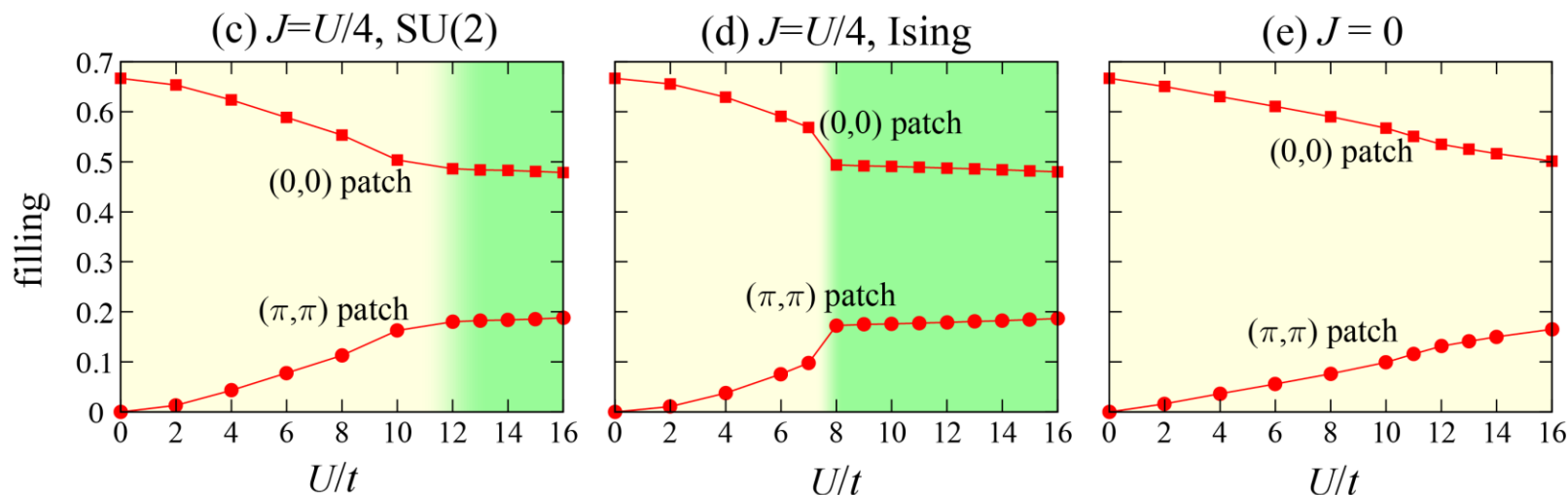
- 2D square lattice, $t = 1$, $t' = 0$
- Three-orbital Hubbard model with 2 electrons/site
- $J = U/4$
- cluster DMFT with 2 site cluster (DCA)
- Paramagnetic and para-orbital solution



Three-orbital Hubbard model: one-particle quantity (1)

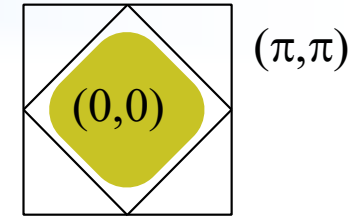


In the presence of J
 filling redistribution within BZ: $(0,0)$ patch tends to be half-filling



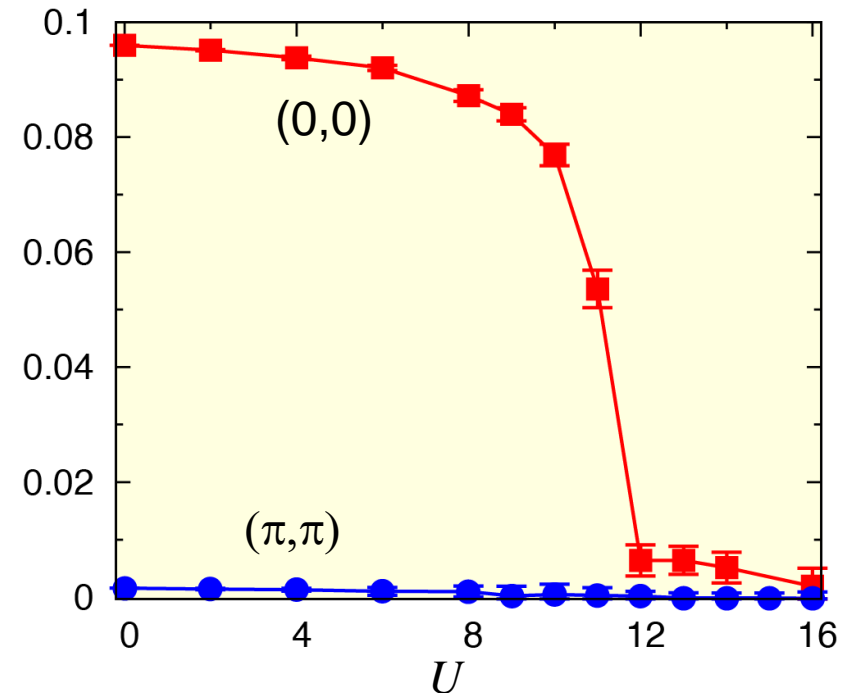
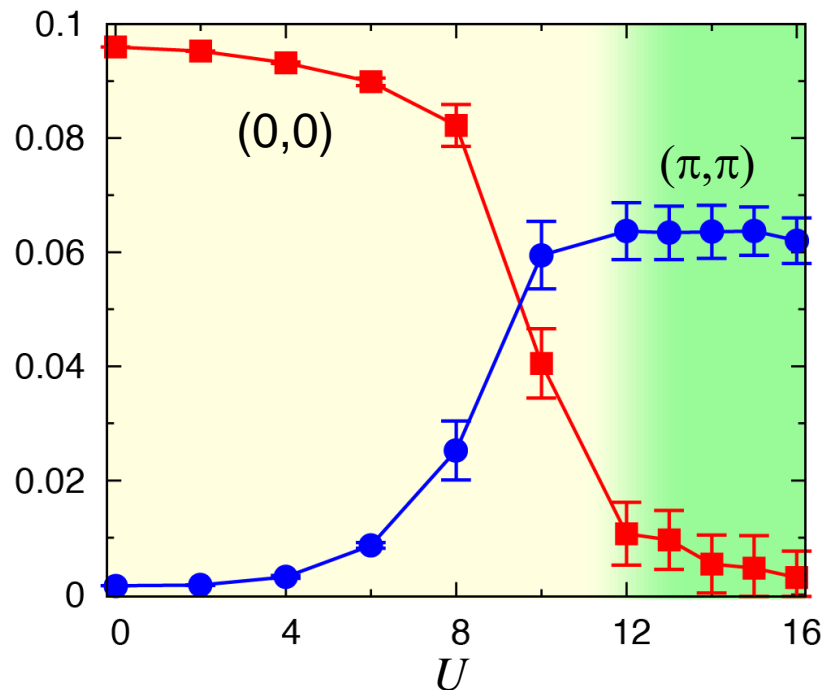
Three-orbital Hubbard model: one-particle quantity (2)

$$G(\tau = \beta/2)$$



$J=U/4$ (SU(2) symmetric)

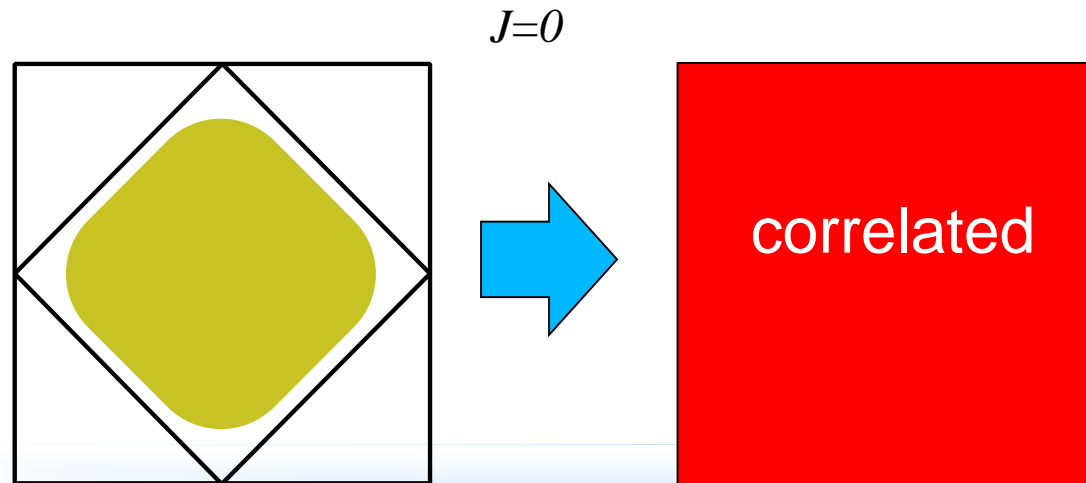
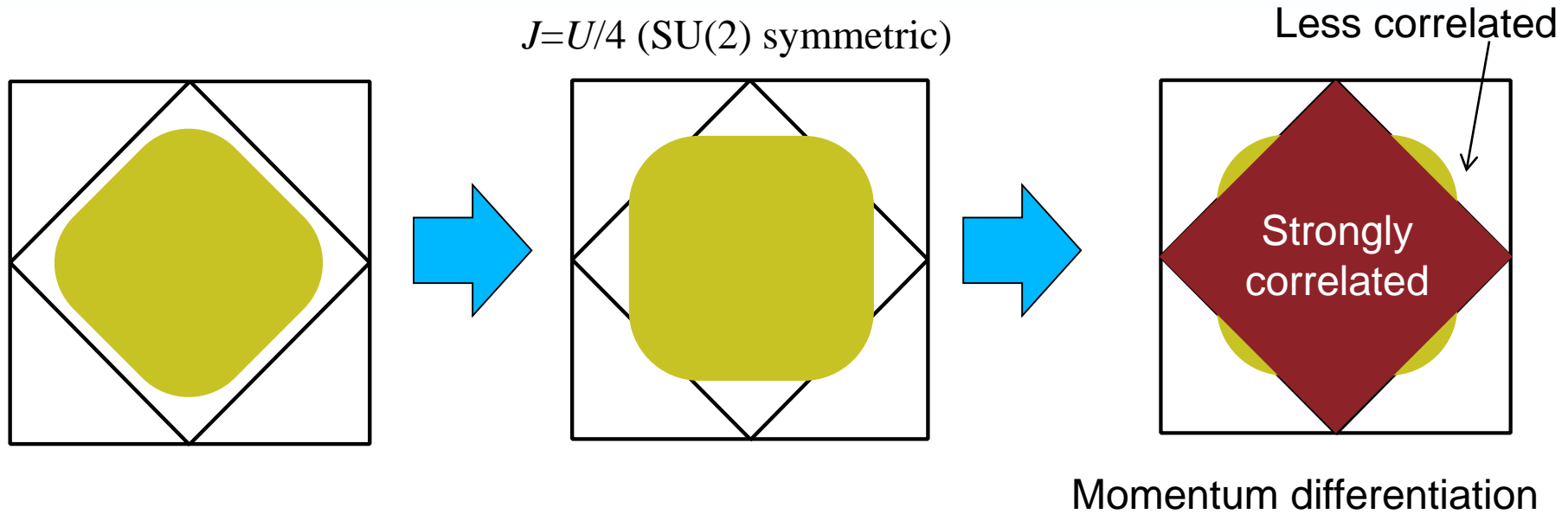
$J=0$



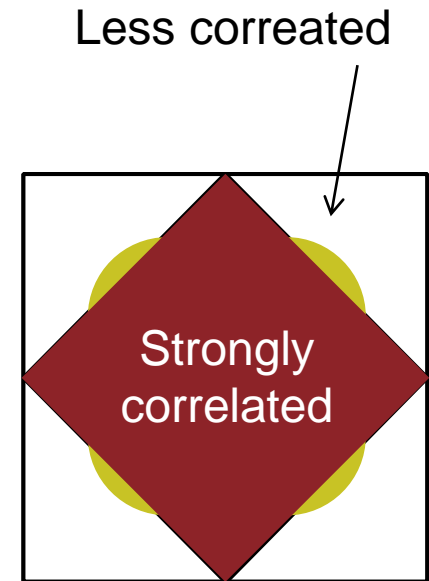
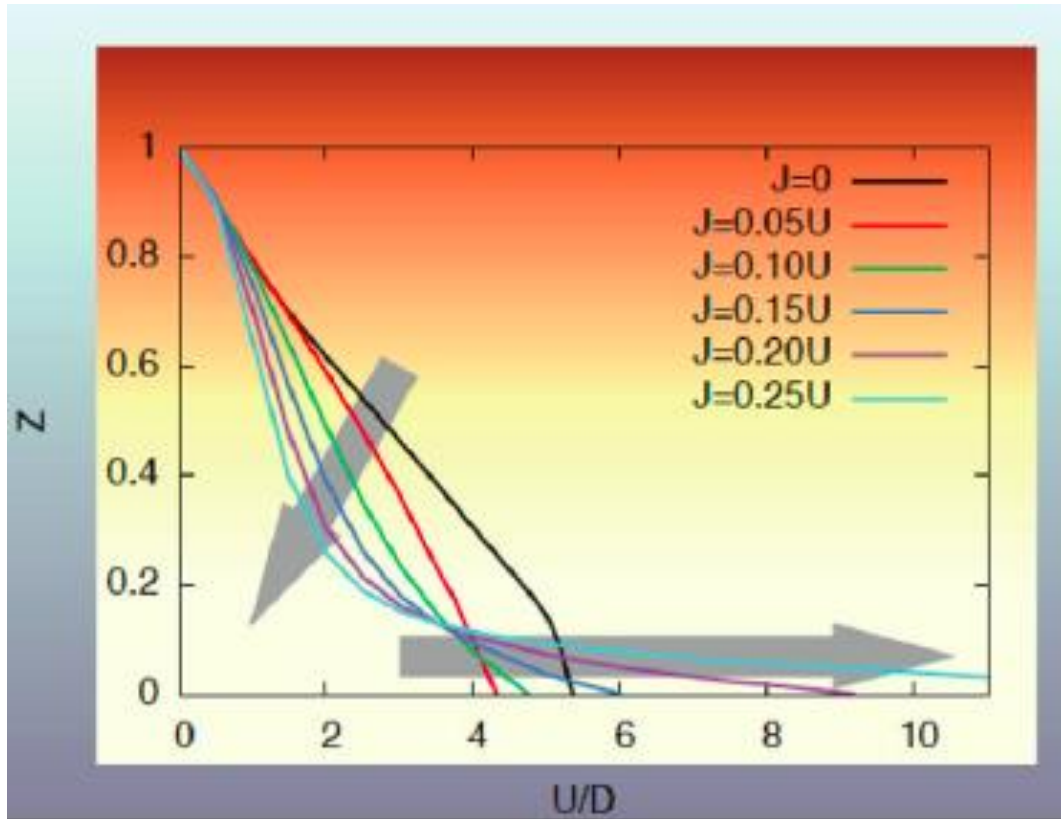
Low energy spectrum appears
in (π, π) patch

Three-orbital Hubbard model: one-particle quantity

(3)



Janus-faced effect of J

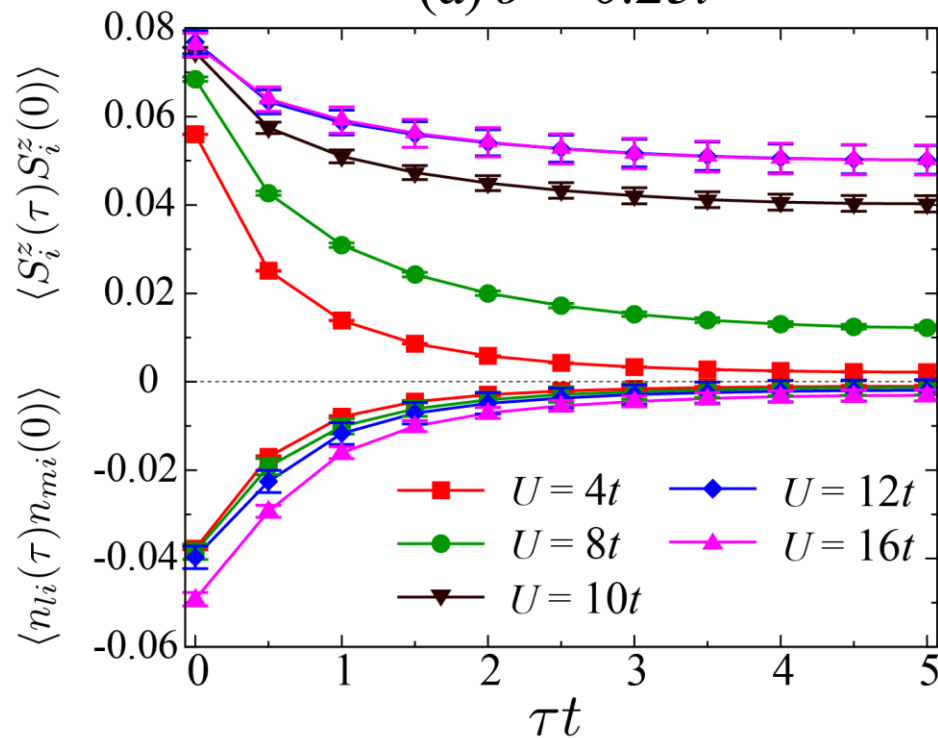


Medici *et al.*, PRL 107, 256401 (2011)

Three-orbital Hubbard model: two-particle quantity (1)

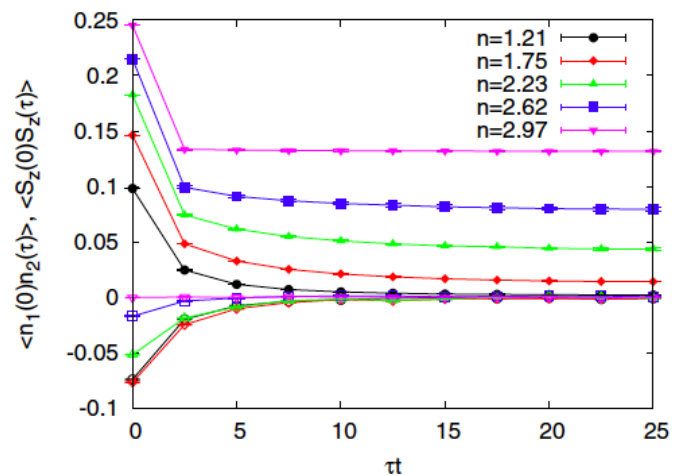
Onsite correlation

(a) $J = 0.25t$



i, j : site
 l, m : orbital

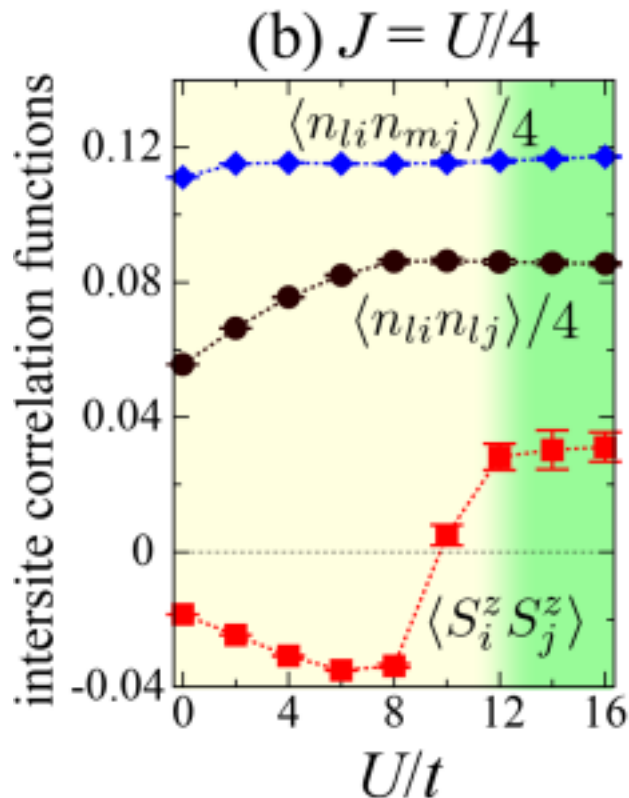
“Spin freezing” behavior for large U



Werner *et al.*, PRL 101, 166405 (2008)

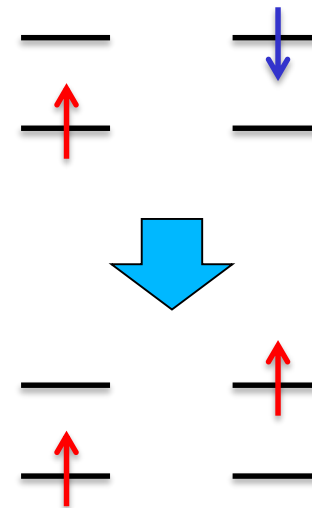
Three-orbital Hubbard model: two-particle quantity (2)

Intersite correlation



i, j : site
 l, m : orbital

Ferromagnetic ground state
 for 1D 2-orbital Hubbard model



K. Kubo, JPSJ 1982

K. Kusakabe & H. Aoki, Physica 1994

DMFT

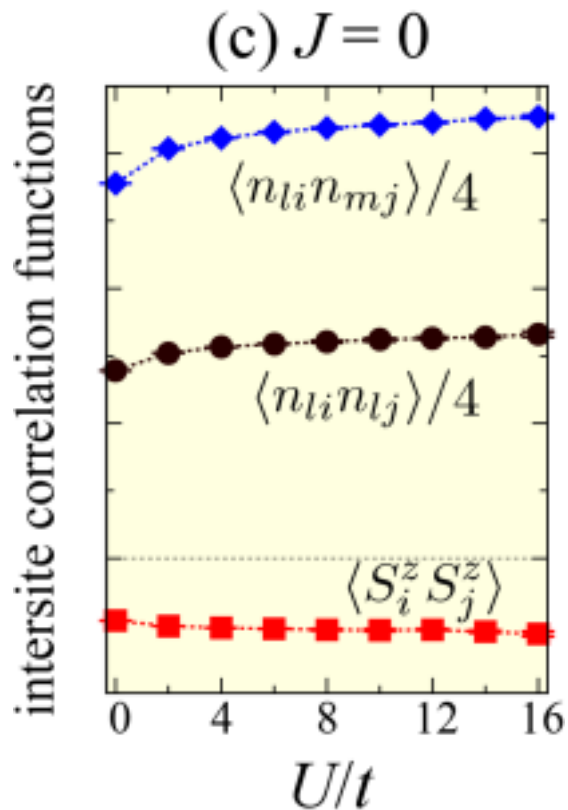
T. Momoi & K. Kubo, PRB 1998

K. Held & D. Vollhardt EPJ 1998

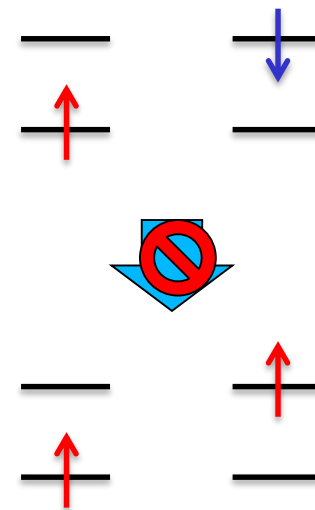
C-K. Chan et al., PRB2009

Three-orbital Hubbard model: two-particle quantity (2)

Intersite correlation



i, j : site
 l, m : orbital

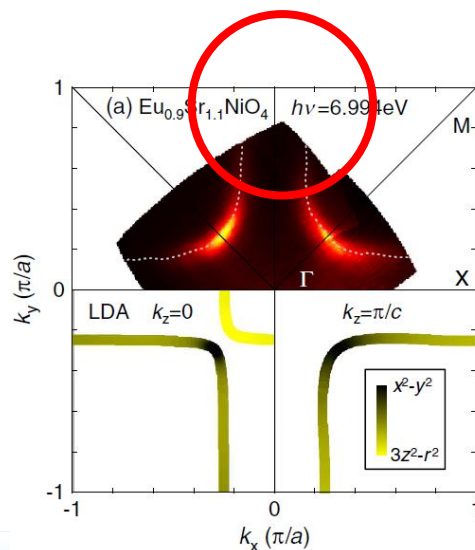


Conclusion

- Efficient CTQMC algorithm for multi-orbital DCA
- Effect of local J in correlated electron systems
 - J enhances spatial correlations
- Future problem: LDA+cDMFT calculation for multi-orbital systems

LnSrNiO_4

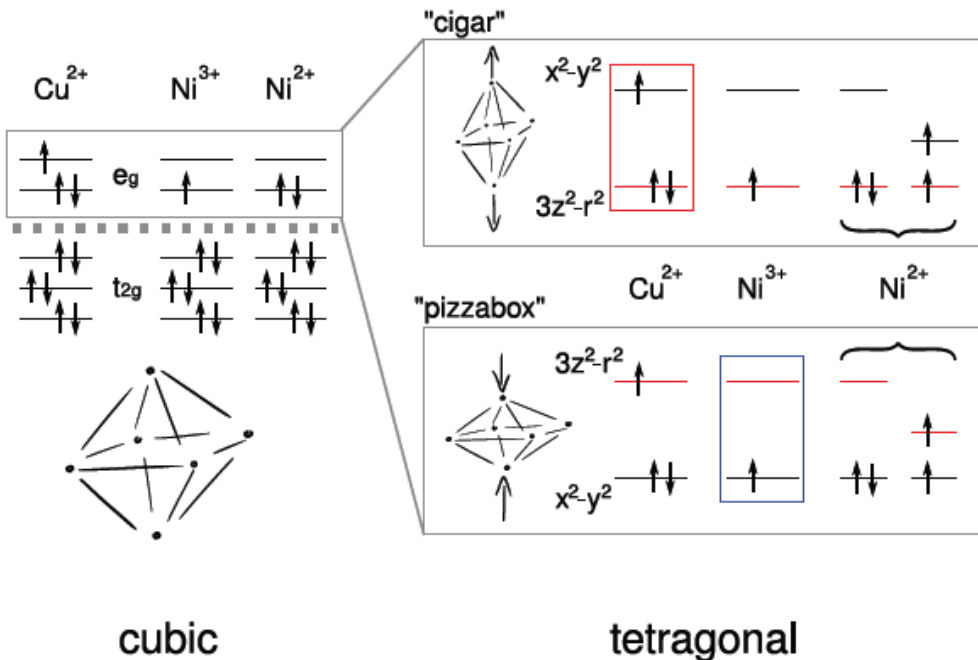
Uchida, RA *et al.*, PRL
106, 027001(2011)



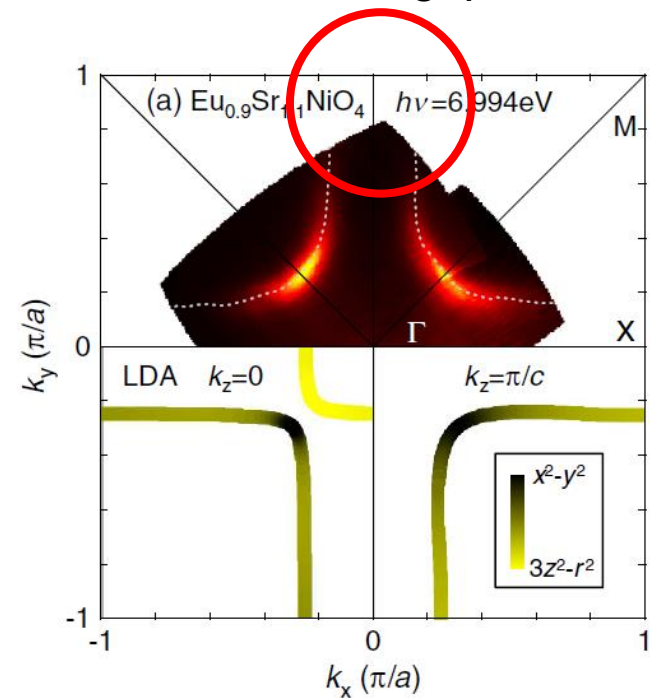
Intersite correlation due to J

LnSrNiO₄: Layered perovskite 3d⁷ system
(Same crystal structure as La₂CuO₄)

	Onsite correlation	Intersite correlation
U	DMFT	Cluster DMFT
J	Multi-orbital DMFT	multi-orbital cluster DMFT



Pseudogap ?

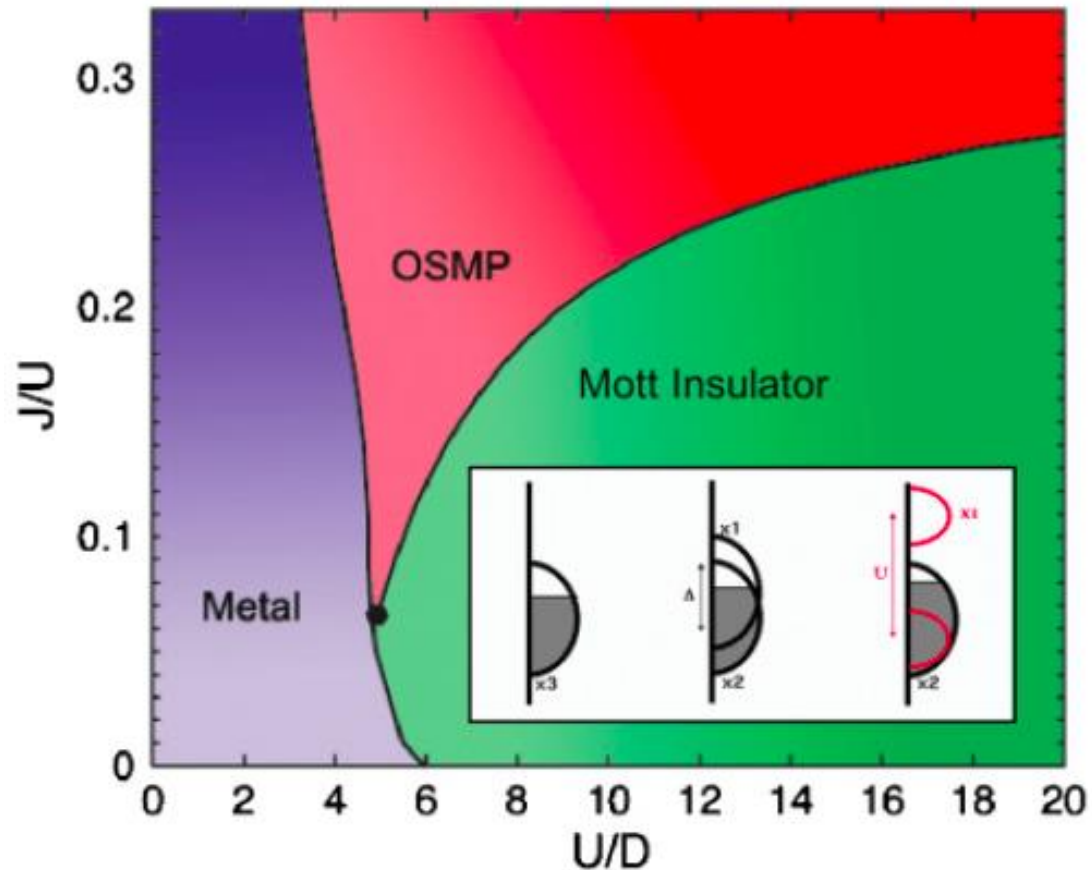


Uchida, RA *et al.*, PRL **106**, 027001(2011)

Motivation: effect of J

	Onsite correlation	Intersite correlation
U	DMFT	Cluster DMFT
J	Multi-orbital DMFT	multi-orbital cluster DMFT

“band decoupler” and orbital selective Mott transition

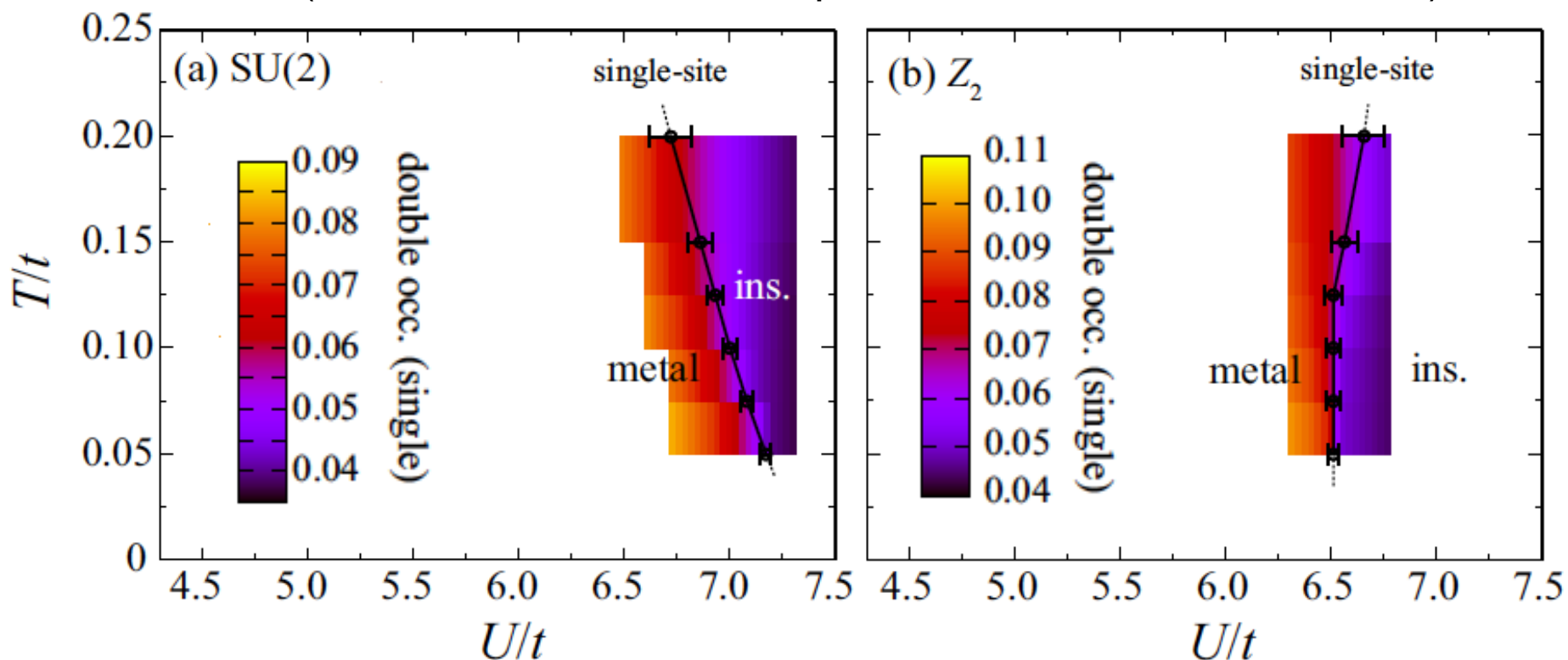


Medici *et al.*, PRL 102, 126401 (2009);
Medici, PRB 83 205112 (2011)

Two-orbital Hubbard model: Mott transition

$$U=6J$$

(DMFT self-consistent loop starts with a metallic solution)



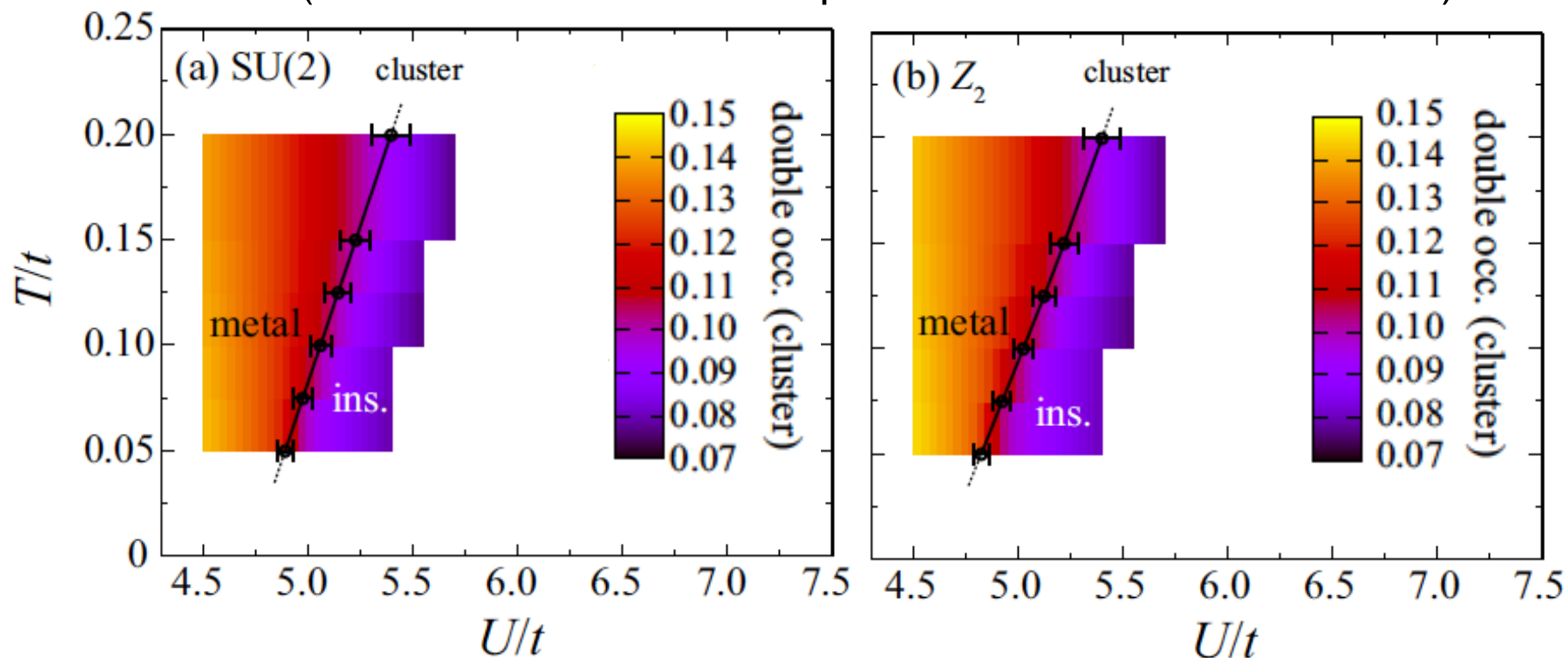
Entropy(ins,SU(2)) $\sim \ln(3)$ ($S=1, S_z = 1, 0, -1$)

Entropy(ins,Ising) $\sim \ln(2)$ ($S=1, S_z = \pm 1$)

Two-orbital Hubbard model: Mott transition

$$U=6J$$

(DMFT self-consistent loop starts with a metallic solution)



Entropy(metal) > Entropy(ins)