

Inhomogeneous noncentrosymmetric superconductors in magnetic fields

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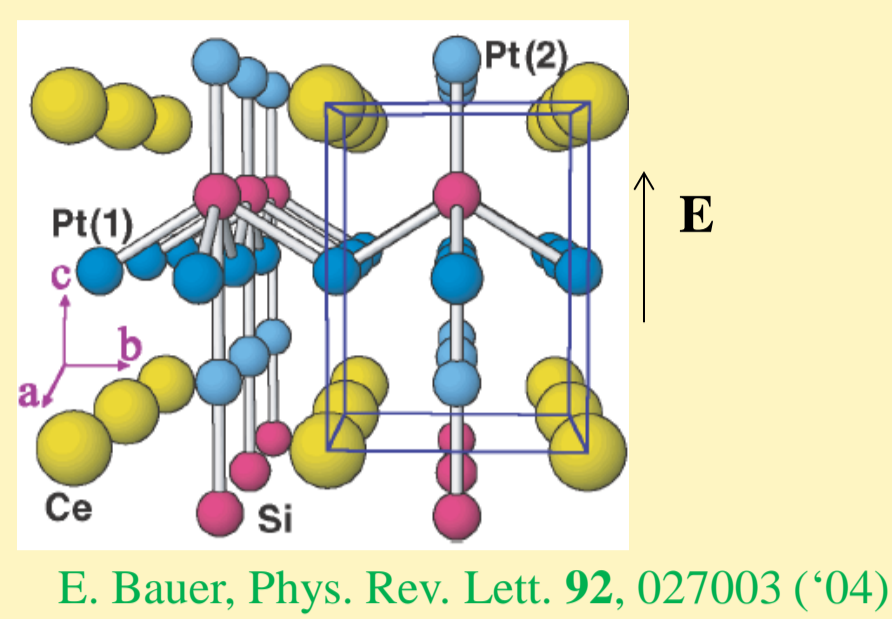
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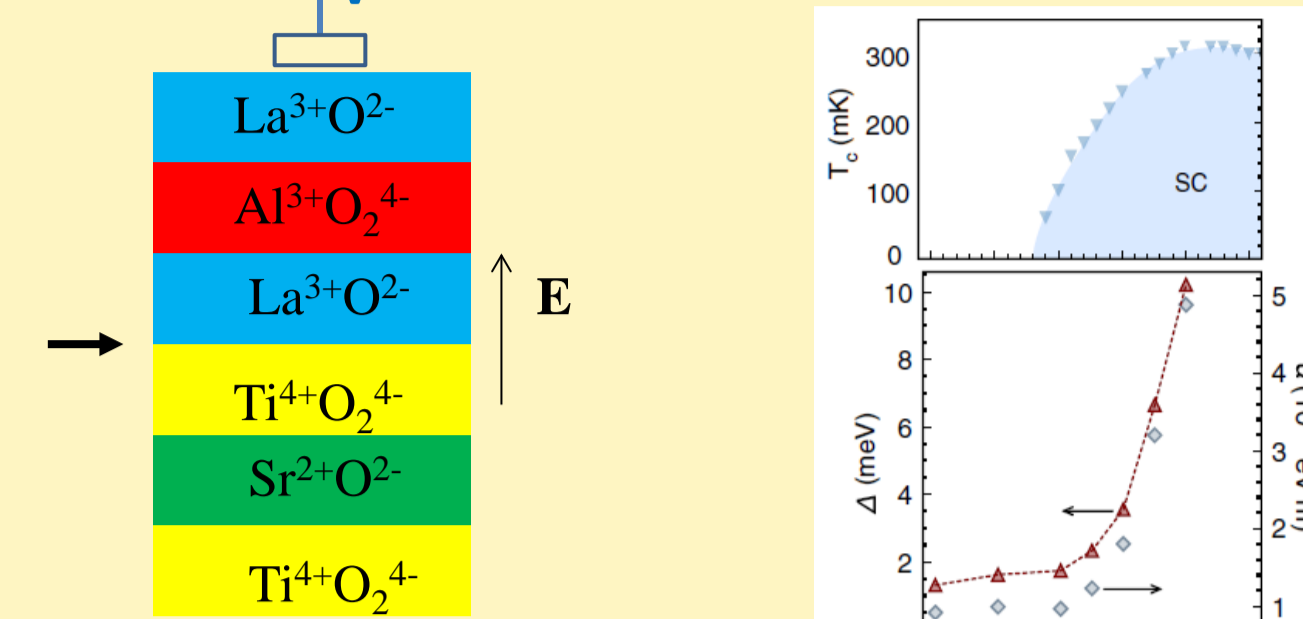
Non-centrosymmetric superconductor (NCS)

Heavy-fermion superconductor CePt₃Si



3D system

LaAlO₃-SrTiO₃ Interface superconductor



2D system

Lack of an inversion center

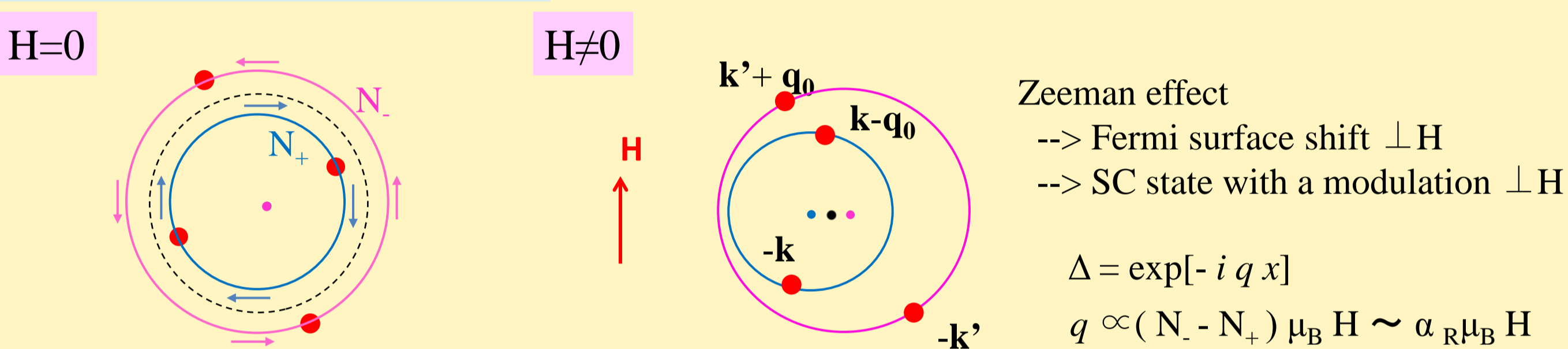
Anisotropic potential gradient: $\mathbf{E} = \nabla V$ •• Rashba spin-orbit coupling (RSOC): $\boldsymbol{\sigma} \cdot (\mathbf{k} \times \mathbf{E}) = \alpha_R \boldsymbol{\sigma} \cdot (\mathbf{k} \times \mathbf{z})$
Coupling constant $\alpha_R \propto \nabla_z V$

Sign and strength of Rashba spin-orbit coupling depend on the direction of the mirror symmetry breaking.

NCS in a magnetic field

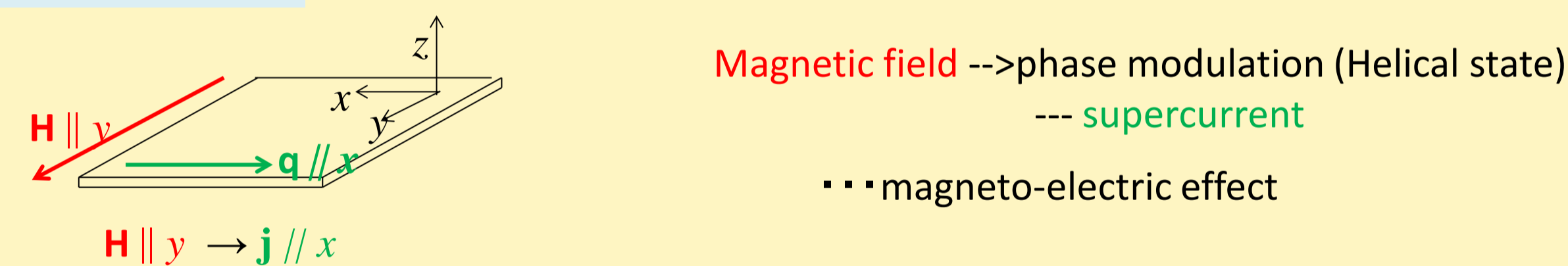
Field-induced Helical SC state

R. P. Kaur et al., PRL **94**, 137002 ('05)



Fermi surface splitting due to the Rashba spin-orbit coupling $\alpha_R \boldsymbol{\sigma} \cdot (\mathbf{k} \times \mathbf{z})$

Magneto-electric effect

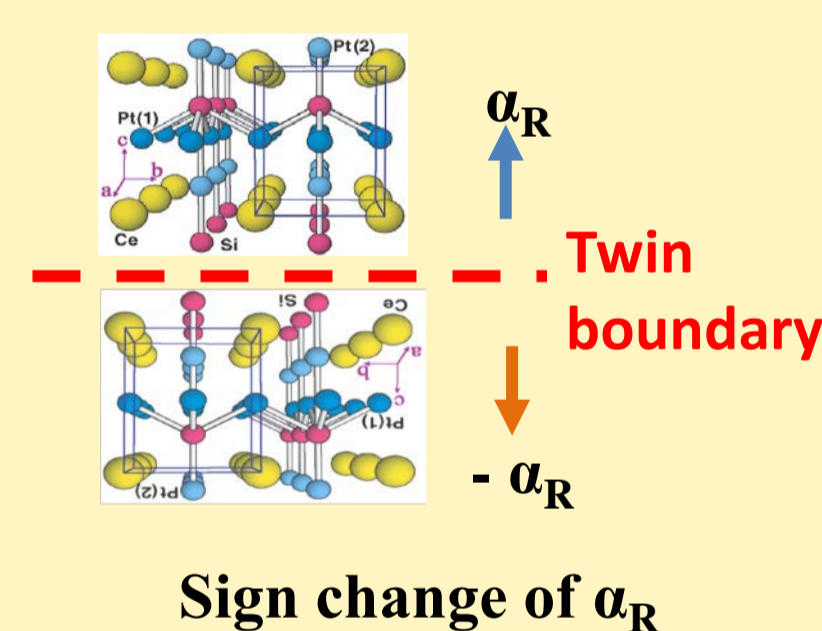


Spatially inhomogeneous NCS in a magnetic field

Spatially inhomogeneous RSOC

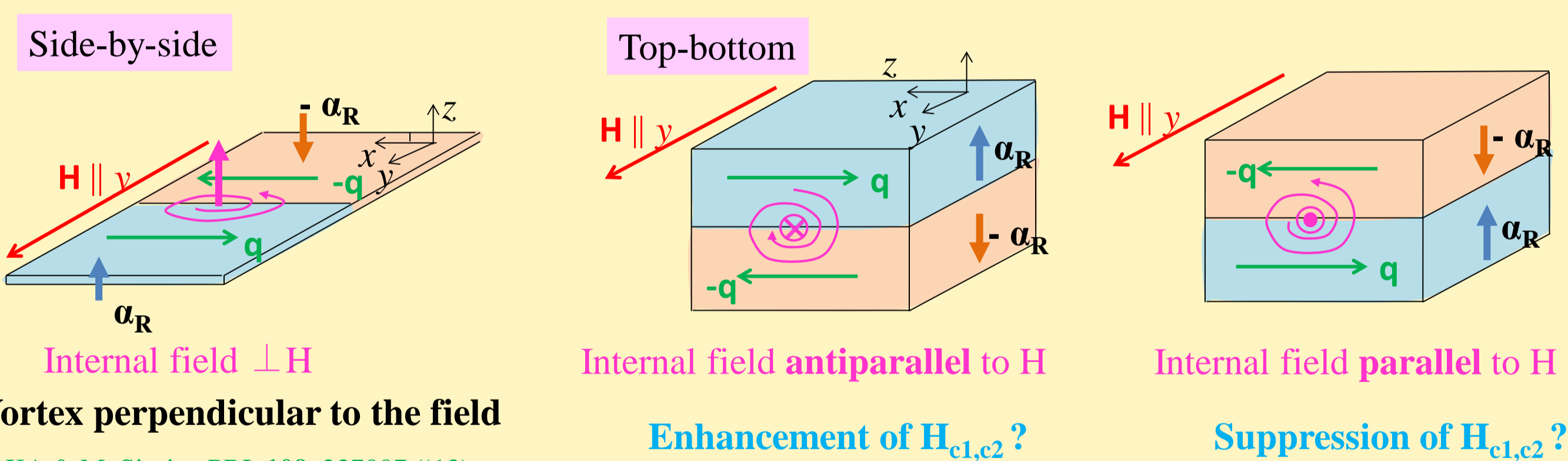
- Electric field applied to 2D SC film
- Crystalline defect ••• Twin boundary

Spatial variation of RSOC constant α_R



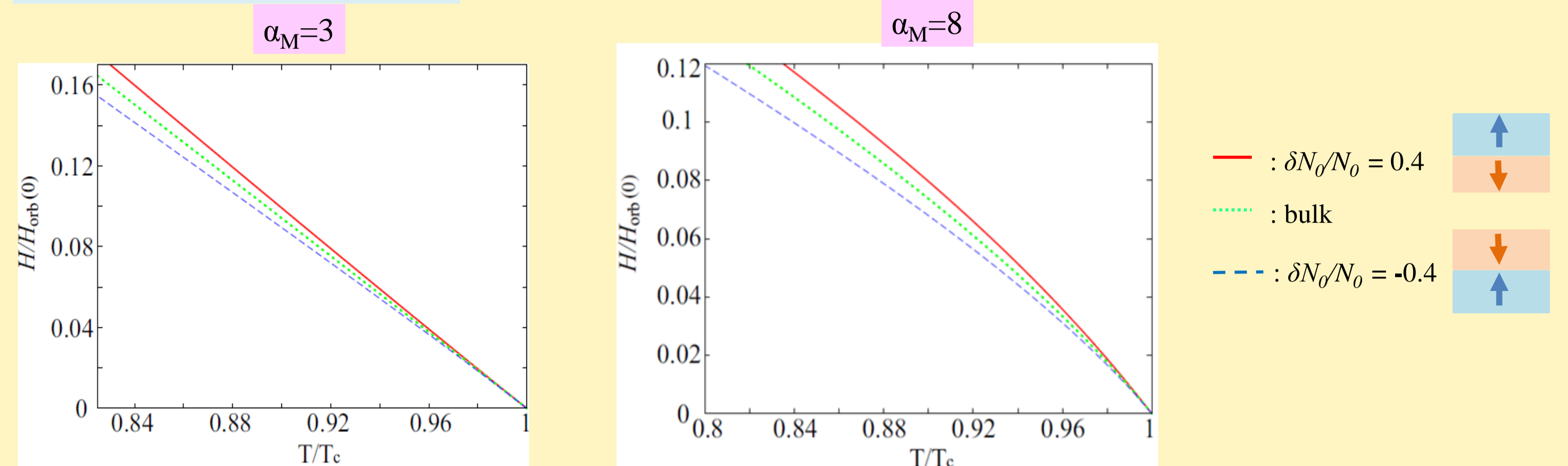
Internal field at inhomogeneity of RSOC

Two domains with opposite noncentrosymmetries α_R and $-\alpha_R$



Result and Conclusion

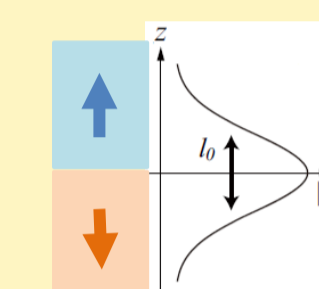
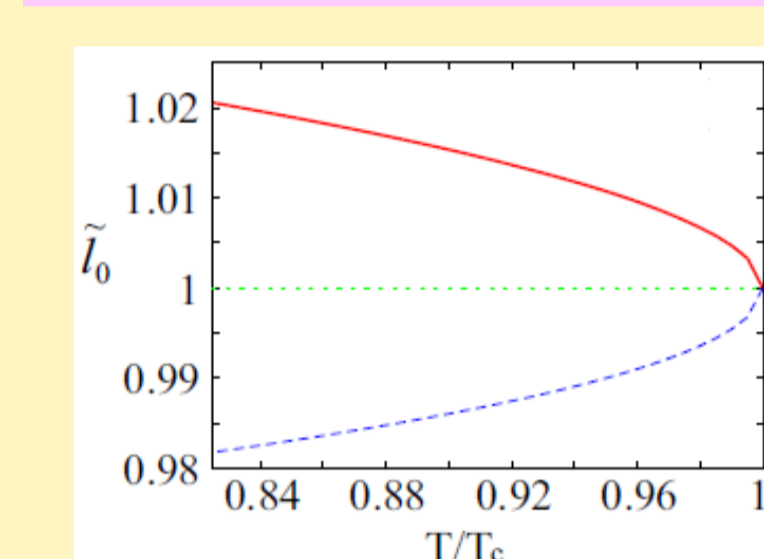
Upper critical field $H_{c2}(T)$



$H_{c2}(T)$ is enhanced (suppressed) at the twin boundary with positive (negative) $\delta N_0/N_0$.

- α_M : larger
- Shift of $H_{c2}(T)$ at the twin boundaries becomes remarkable.
- $H_{c2}(T)$ is suppressed by the paramagnetic pair-breaking effect.

Effective magnetic length l_n



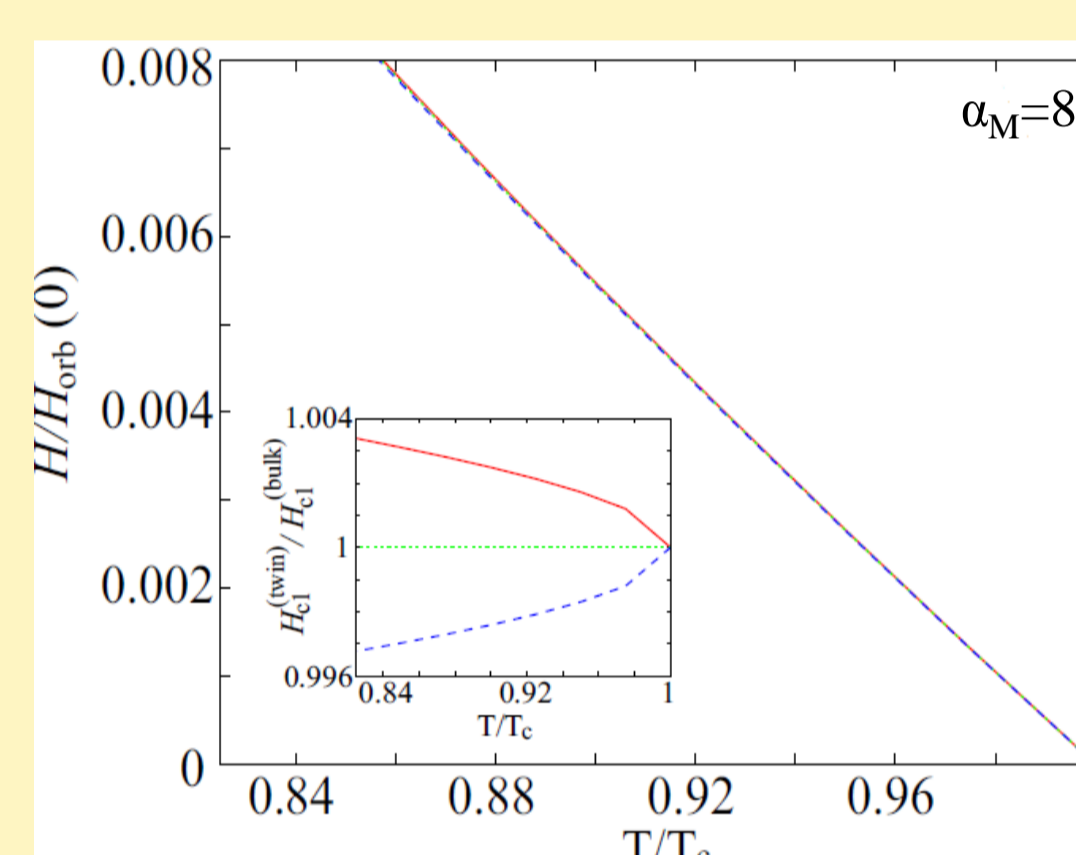
l_0 is longer (shorter) for positive (negative) $\delta N_0/N_0$.

Bulk: $l_0 = r_H = 1/\sqrt{2}eH$
Twin boundary: $l_0 \sim 1/\sqrt{2}eH_{eff}$

Internal field introduced at the twin boundary with positive (negative) $\delta N_0/N_0$ weakens (strengthens) the external field.

Lower critical field $H_{c1}(T)$

Vortex-line energy: $e_v = 2\pi \int_0^\infty r dr \left[\frac{|\Delta_0|^2 K_\perp \hbar^2}{\xi^2} f_{GL} - \left(-\frac{|a^{(2)}|^2}{4a^{(4)}} \right) \right]$ $H_{c1}(T) : \frac{H_{c1}(T)}{H_{orb}(0)} = \frac{4\pi e_r}{\Phi_0 e_r} \frac{\Phi_0}{2\pi \xi^2} = \left(\frac{\xi_0}{\xi} \right)^2 \left(\frac{\xi}{\lambda_L} \right)^2 \frac{1}{2} \int_0^\infty \tilde{r} d\tilde{r} (f_{GL} + \frac{1}{2})$



$H_{c1}(T)$ is enhanced (suppressed) at the twin boundary with positive (negative) $\delta N_0/N_0$.

But, the shift of $H_{c1}(T)$ is very small.

$$\frac{e_v}{e_{v0}} \approx 1 + R_v \frac{\delta N_0}{N_0} \frac{\alpha_M \sqrt{1 - T/T_c}}{(\lambda_L/\xi) \ln(\lambda_L/\xi)} \quad R_v = 0.215$$

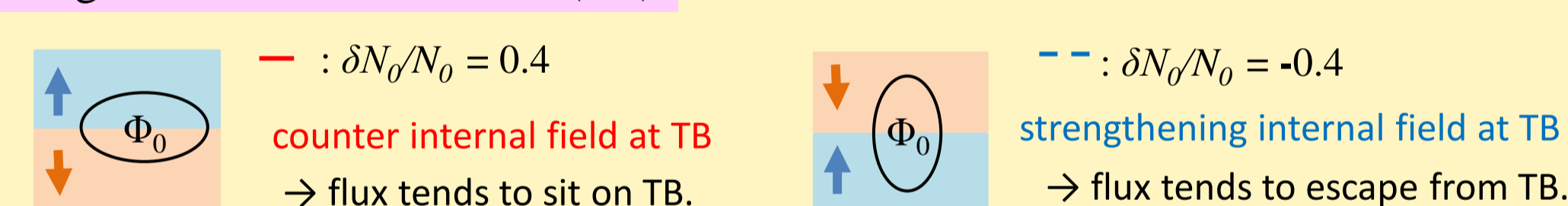
λ_L/ξ : large $\rightarrow \delta e_v$: small

Spatial dependences of c_n and a_m

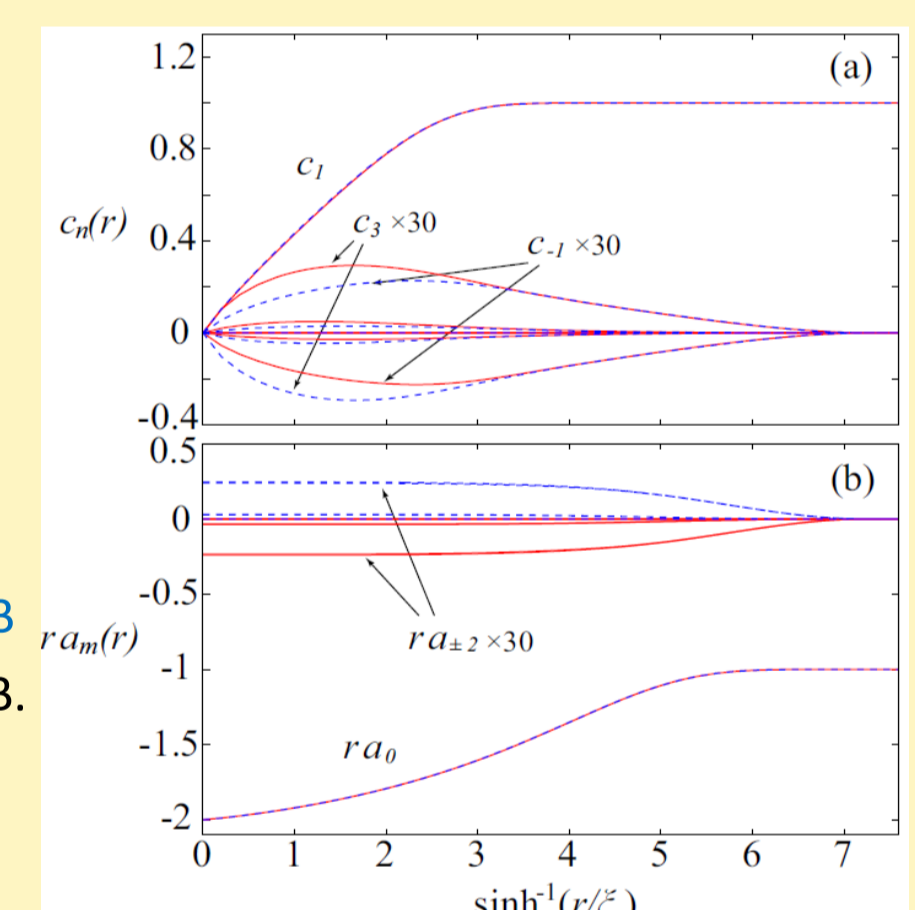
In the presence of twin boundaries, $c_{1\pm 2}$ and $a_{\pm 2}$ appear ($e^{\pm i 2\theta}$ in Δ and A).

\rightarrow occurrence of 2 fold symmetry in Δ and B

Magnetic flux distribution $B(x,z)$



--- Consistent with the previous physical picture for $H_{c2}(T)$



Conclusion

KA, L. Savary, and M. Sigrist, PRB **89**, 174518 ('14)

H_{c1} and H_{c2} at twin boundaries of a noncentrosymmetric superconductor with RSOC

Magneto-electric effect \rightarrow Shift of H_{c1} and H_{c2} at the twin boundary

In a specimen containing crystal domains, physical H_{c2} and H_{c1} are determined at 'out'- and 'in'-type twin boundaries, respectively.

H_{c2} at the 'out'-type twin boundary is higher than its bulk value.

\rightarrow The upper critical field obtained from experimental probes quite sensitive to superconductivity can be higher than H_{c2} determined by bulk measurements.

Theoretical framework

Ginzburg Landau free energy

$$\mathcal{F}_{GL} = \int dz dx_\perp \left[a^{(2)} |\Delta|^2 + a^{(4)} |\Delta|^4 + K_\perp |\Pi_\perp \Delta|^2 + K_z |\Pi_z \Delta|^2 + K_{me} (\hat{z} \times \mathbf{B}) \cdot \{ \Delta^* \Pi_\perp \Delta + \Delta (\Pi_\perp \Delta)^* \} + \frac{(\nabla \times \mathbf{A})^2}{8\pi} \right]$$

Magneto-electric effect due to RSOC

$$\Pi = -i\hbar \nabla + (2e/c)\mathbf{A} \quad \nabla \times \mathbf{A} = \mathbf{B}$$

$$a^{(2)} = N_0 \left(\ln \frac{T}{T_c} + 2\gamma g^2 \mu_B^2 B^2 \right) \quad K_{me} = \frac{\delta N_0}{N_0} g \mu_B \frac{K_\perp}{v_\perp} \quad \frac{\delta N_0}{N_0} = \frac{N_+ - N_-}{(N_+ + N_-)/2} \propto \frac{\alpha_R}{E_F}$$

Paramagnetic pair-breaking effect due to Zeeman field

(N_\pm : DOS of 2 bands split by RSOC)

Model for twin boundary system

$$K_{me} \rightarrow K_{me}(z) = \tilde{K} \text{sgn}(z)$$

$\tilde{K} > 0$ ($\delta N_0/N_0 > 0$) $\tilde{K} < 0$ ($\delta N_0/N_0 < 0$)

Length scales

$$\xi^{-2} = |a^{(2)}|/(\hbar^2 K_\perp) \quad \text{••• SC coherence length}$$

$$r_H^{-2} = 2eH/(c\hbar) \quad \text{••• magnetic length}$$

$$\lambda_L^{-2} = 32\pi K_\perp |\Delta_0|^2 c^2/\mu_B^2 \quad \text{••• London penetration depth length}$$

parameters

$$|\delta N_0/N_0| : \text{strength of RSOC}$$

$$\text{Sign of } \delta N_0/N_0 : \text{stacking order of twinning domains}$$

$$\alpha_M = \sqrt{2} H_{orb}(0)/H_p(0) : \text{strength of paramagnetic effect}$$

Upper critical field $H_{c2}(T)$

$$H \parallel y : \mathbf{A} = zH\hat{x} \quad \Delta(\mathbf{r}) = \sum_n C_n(z) e^{i 2\pi n x}$$

Variational approximation

Twin boundary effect ••• correction to harmonic potential introduced by $\mathbf{A} \rightarrow C_n(z) = C_n \frac{1}{\sqrt{l_n \sqrt{\pi}}} e^{-z^2/2l_n^2}$ l_n : variational parameter
(\times without twin boundary, $l_n = r_H = 1/\sqrt{2}eH$)

GL quadratic term

$$\mathcal{F}_{GL}^{(2)} = \sum_n |C_n|^2 \left[a^{(2)} + K_\perp \hbar^2 \left(\frac{2\pi n}{L_x} \right)^2 + \frac{K_\perp}{2} \left(\frac{2cH}{c} \right)^2 l_n^2 + \frac{K_z \hbar^2}{2} \frac{1}{l_n^2} - \frac{2}{\sqrt{\pi}} \frac{2cH}{c} \tilde{K} H l_n \right]$$

Twin boundary effect

1. Minimization of $\mathcal{F}_{GL}^{(2)}$ with respect to n and l_n
2. $\mathcal{F}_{GL}^{(2)} = 0 \rightarrow H_{c2}(T)$

$$\frac{2\tilde{K} H r_H}{\sqrt{\gamma_{FS} K_\perp K_z \hbar^2}} = \frac{1}{2c\gamma_{FS}} \frac{\delta N_0}{N_0} \alpha_M \sqrt{\frac{H}{H_{orb}(0)}}$$

Lower critical field $H_{c1}(T)$

Cylindrical coordinate system $\Delta(x,z) = |\Delta_0| \sum_n c_n(r) e^{im\theta}$
 $A_\theta(r,\theta) = \frac{c\hbar}{2e\xi} \sum_m a_m(r) e^{im\theta}$
 $B(r,\theta) = \frac{1}{r} \partial_r [r A_\theta(r,\theta)] = \frac{c\hbar}{2e\xi} \frac{1}{r} \partial_r [r \sum_m a_m(r) e^{im\theta}]$

Boundary condition

$$\Delta(r,\theta) = \begin{cases} |\Delta_0| e^{i\theta} & (r \rightarrow \infty) \\ 0 & (r \rightarrow 0) \end{cases} \quad \& \quad \Phi_0 = \int_0^{2\pi} d\theta \int_0^\infty r dr B(r,\theta) = 2\pi \frac{c\hbar}{2e\xi} \left[\lim_{r \rightarrow \infty} r a_0(r) - \lim_{r \rightarrow 0} r a_0(r) \right]$$

GL equations $\delta \mathcal{F}_{GL}/\delta c_k = 0$ & $\delta \mathcal{F}_{GL}/\delta a_k = 0 \rightarrow$ Numerical solution