Studying BEC with a New Self-Consistent Approximation

Department of physics, Hokkaido Univ. Kazumasa TSUTSUI, Takafumi KITA

Introduction and Summary

We study properties of a homogeneous dilute Bose gas based on a self-consistent perturbation expansion that satisfies Noether's theorem and Goldstone's theorem simultaneously[1]. This formalism predicts that there should be a new class of Feynman diagrams for the self-energy characteristic of BEC that has been overlooked so far, which will be shown to modify standard results based on the Bogoliubov theory[2] substantially.

First, these diagrams, which may be classified as "one-particle-reducible"(1PR), add an extra constant $c_{\rm ip}\approx O(1)$ to the well-known expressions of the ground-state energy per particle E/N and condensate density n_0/n reported by Lee, Huang, and Yang[3] as

$$\frac{E}{N}=\frac{2\pi\hbar^2 an}{m}\left[1+\frac{16}{5}\left(\frac{8}{3\sqrt{\pi}}+c_{\rm ip}\right)\sqrt{a^3n}\right],\qquad \frac{n_0}{n}=1-\left(\frac{8}{3\sqrt{\pi}}+c_{\rm ip}\right)\sqrt{a^3n}\,.$$

where a, n, and m are are the s-wave scattering length, particle density, and particle mass, respectively.

Second, the lifetime of the one-particle excitation is also affected, as clarified by our calculation of the one-particle spectral function. It is shown that each excitation should have a finite lifetime proportional to the s-wave scattering length a, instead of a^2 for the normal state. Thus, the 1PR diagrams are predicted to change the nature of the one-particle excitation of BEC substantially from the Bogoliubov[2] mode with an infinite lifetime into a "bubbling" mode with a considerable decay rate.

[1] T. D. Lee *et.al.*, Phys. Rev. **106** (1957) 1135. [2] T. Kita, PRB **80** (2009) 214502. [3] N. N. Bogoliubov, J. Phys. (USSR) **9** (1947) 23.

Conserving-Gapless theory

This theory is summarized below in two steps.

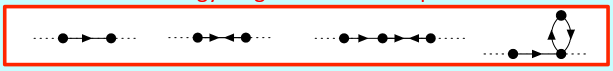
1. Construct the Luttinger-Ward functional Φ by including diagrams characteristic of condensed Bose system.
   (numerical weights of diagrams characteristic of BEC are left undetermined.)
2. Determine these weights by using Goldstone's theorem.

Self-energies

By carrying out functional differentiation for Φ, we obtain self-energies.

$$\Sigma(1,2)=-2\beta\frac{\delta\Phi}{\delta G(2,1)}\qquad \Delta(1,2)=-2\beta\frac{\delta\Phi}{\delta F(2,1)}\qquad 1\equiv(\mathbf{r}_1,t_1)$$

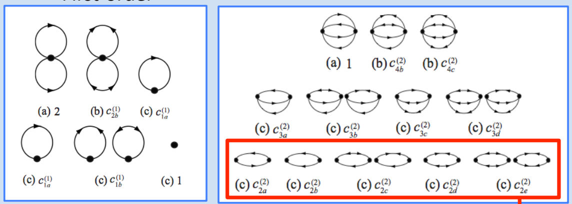
1PR self-energy diagrams should be present.



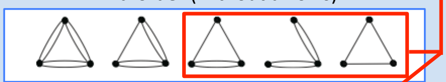
Diagrammatic structures of Φ

First-order

Second-order



Third-order (without arrows)



New class of Feynman diagrams

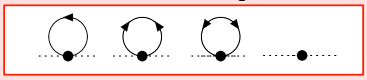
Dilute Bose gas system

Hamiltonian

$$H=\sum_{\mathbf{p}}(\epsilon_{\mathbf{p}}-\mu)c^\dagger_{\mathbf{p}}c_{\mathbf{p}}+\frac{g}{2}\sum_{\mathbf{p}_1\mathbf{p}_2\mathbf{q}}c^\dagger_{\mathbf{p}_1+\mathbf{q}}c^\dagger_{\mathbf{p}_2-\mathbf{q}}c_{\mathbf{p}_2}c_{\mathbf{p}_1}$$

First-order analysis

First-order self-energies

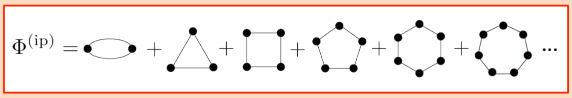


$$\Sigma^{(1)}=2gn$$
$$\Delta^{(1)}=g\left(n-\sum_{\vec p}F^{(1)}_{\vec p}\right)$$
$$n=n_0-\sum_{\vec p}G^{(1)}_{\vec p}$$
$$\mu^{(1)}=\Sigma^{(1)}-\Delta^{(1)}$$

$$\frac{n_0}{n}=1-\frac{8}{3\sqrt{\pi}}\sqrt{a^3n}$$
$$\frac{E}{N}=4\pi an\left(1+\frac{128}{15\sqrt{\pi}}\sqrt{a^3n}\right)$$

Reproduce the results by Lee *et al.*

Structure of functional $\Phi^{(\rm ip)}$ giving 1PR self-energies



→These polygonal diagrams provide dominant contribution.

Self-energies

$$\Sigma(\vec p)=\Sigma^{(1)}+\Delta_{\rm im}(\vec p)\qquad \Delta(\vec p)=\Delta^{(1)}+\Delta_{\rm im}(\vec p)$$
$$\Delta_{\rm im}(\vec p)\equiv\sum_{n=2}^{\infty}\Delta^{(n)}_{\rm im}(\vec p)$$
$$=2(gn_0)^2(G(\vec p)+G(-\vec p)-2F(\vec p))$$
$$+\frac{5}{4}(2gn_0)^3(G(\vec p)+G(-\vec p)-2F(\vec p))^2+\cdots$$

→ These self-energies add an extra constant $c_{\rm ip}$ to results by Lee *et. al.*, and give the one-particle excitation a finite lifetime proportional to a !

Applications of some approximations

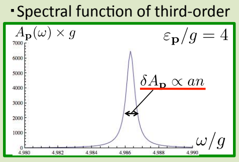
1. one-particle spectral function [4]

$$A_{\mathbf{p}}(\omega)=-2\,{\rm Im}\,G_{11}(\vec p)|_{i\varepsilon\to\omega+i\eta}$$

→ It has a sharp peak in the one-particle excitation.

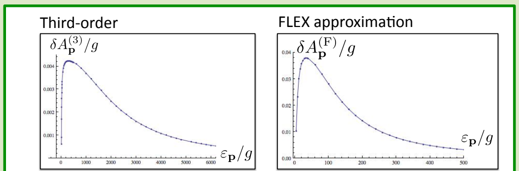
→ The width of the peak $\delta A_{\mathbf{p}}$ corresponds to the lifetime of one-particle excitation.

・Spectral function of third-order



The width proportional to a !

Momentum dependence of $\delta A_{\mathbf{p}}$



→ lifetime of one-particle excitation changes with the own momentum!

2. Estimates of $c_{\rm ip}$ [5]

$$c_{\rm ip}=\frac{4\sqrt{2}}{gn\pi^{3/2}}\int_0^\infty d\epsilon_p\,\epsilon_p^{1/2}\int_0^\infty d\varepsilon_\ell\times\frac{(\varepsilon_\ell^2-\epsilon_p^2)\Delta_{\rm im}(\vec p)}{[\varepsilon_\ell^2+\epsilon_p(\epsilon_p+2\Delta_{\rm im}(\vec p))][\varepsilon_\ell^2+\epsilon_p(\epsilon_p+2\Delta^{(1)})]}$$

Physical quantities containing the $c_{\rm ip}$

$$\frac{E}{N}=\frac{2\pi\hbar^2 an}{m}\left[1+\frac{16}{5}\left(\frac{8}{3\sqrt{\pi}}+c_{\rm ip}\right)\sqrt{a^3n}\right],$$
$$\frac{n_0}{n}=1-\left(\frac{8}{3\sqrt{\pi}}+c_{\rm ip}\right)\sqrt{a^3n}\,.$$

→ Third-order : $c_{\rm ip}\approx 0.412$ FLEX: $c_{\rm ip}\approx 0.563$

[4] K. T and T. Kita: J. Phys. Soc. Jpn. **83** (2014) 033001.
[5] K. T and T. Kita: J. Phys. Soc. Jpn. **82** (2013) 063001.