Propagation of Zero Sound in Dilute Gases

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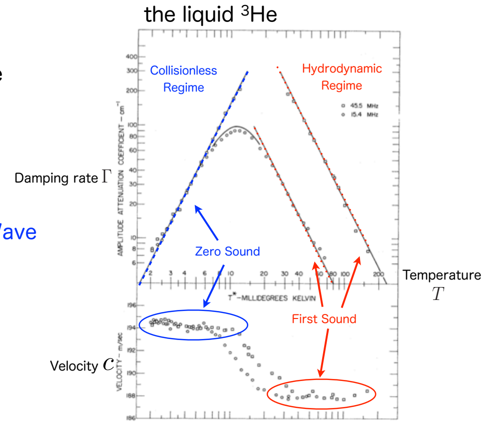
Since Landau predicted the existence of zero sound in 1957, there's been a blossoming of theoretical and experimental research on a sound propagation in normal Fermi systems, but a lot remains to be established about a sound propagation in normal Bose systems. Here, we focus on sound in normal Bose systems to calculate its damping rate as a function of coupling constant by solving the Boltzmann equation numerically. We also study sound propagation in Maxwell-Boltzmann distribution.

Sound Propagation in Normal Fermi Systems

It had been widely believed that a sound in normal Fermi systems could not propagate at the absolute zero temperature, because of the collisionless regime achieved by Pauli blocking. However, Landau predicted in 1957 based on his Fermi-liquid theory that a sound could propagate in liquid ^{3}He even at very low temperatures. L.D. Landau, Sov. Phys. JETP 5, 101 (1957).

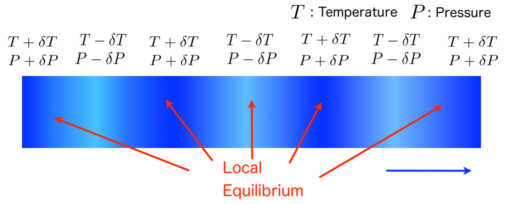
Experiment of a sound in the liquid ^{3}He



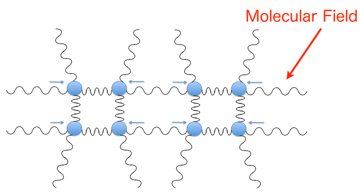
W.R. Abel, A.C. Anderson, J.C. Wheatley, Phys. Rev. Lett. 17, 74 (1966)

- First Sound →Hydrodynamic Pressure Wave

T : Temperature P : Pressure



- Zero Sound→Dynamically Elastic Wave



Recently, Watabe et al. studied a sound propagation in normal Fermi systems based on a moment method for the Boltzmann equation. It was shown that the moment method can describe the crossover between hydrodynamic and collisionless regimes at finite temperatures. S. Watabe, A. Osawa, and T. Nikuni, J. Low. Temp. Phys. 158,773 (2010)

Boltzmann Equation

$$\frac{\partial}{\partial t}f(\boldsymbol{p},\boldsymbol{r},t)+\frac{\boldsymbol{p}}{m}\cdot\boldsymbol{\nabla}_{\boldsymbol{r}}f(\boldsymbol{p},\boldsymbol{r},t)-(2s+1\pm1)\,g\boldsymbol{\nabla}_{\boldsymbol{r}}n(\boldsymbol{r},t)\cdot\boldsymbol{\nabla}_{\boldsymbol{p}}f(\boldsymbol{p},\boldsymbol{r},t)=\mathcal{I}[f]$$

Moment Method and Relaxation Time Approximation

・Linearized Boltzmann Equation and Relaxation Time Approximation

We linearize the distribution function around the static equilibrium and use a relaxation time approximation.

$$f(\boldsymbol{p},\boldsymbol{r},t)=\tilde{f}(\boldsymbol{p},\boldsymbol{r},t)+\frac{\partial f_0}{\partial\varepsilon_0}\delta\nu(\boldsymbol{p},\boldsymbol{r},t)$$

Fluctuation from Local Equilibrium

$$\mathcal{I}[f]=-\frac{1}{\tau}\frac{\partial f_0}{\partial\varepsilon_0}\delta\nu(\boldsymbol{p},\boldsymbol{r},t)\quad\tau\propto g^{-2}$$

Relaxation Time Approximation

→ We obtain the linearized Boltzmann equation for $\delta\nu$. And we expand the linearized Boltzmann equation in terms of the plane wave.

・Moment Method

1. The spherical harmonics expansion

$$\nu(\boldsymbol{p},\boldsymbol{q},\omega)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\nu_l^m(p,\boldsymbol{q},\omega)P_l^m(\cos\theta)e^{im\phi}$$

Fluctuation from Static Equilibrium

$$\nu(\boldsymbol{p},\boldsymbol{q},\omega)=\tilde{\nu}(\boldsymbol{p},\boldsymbol{q},\omega)+\delta\nu(\boldsymbol{p},\boldsymbol{q},\omega)$$
$$\tilde{\nu}(\boldsymbol{p},\boldsymbol{q},\omega)=a(\boldsymbol{q},\omega)+\boldsymbol{b}(\boldsymbol{q},\omega)\cdot\boldsymbol{p}+c(\boldsymbol{q},\omega)p^2$$

They are determined by using the conservation laws.

2. Multiplying the linearized Boltzmann equation by $p^nP_{l'}^{m'}(\cos\theta)e^{-im'\phi}$ and integrating over $\boldsymbol{p}$
3. Longitudinal wave $m'=0$

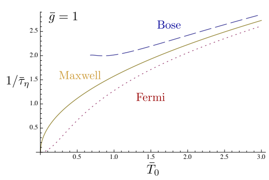
→Eigenvalue Equation (Moment Equation)

Eigenvalue ω Eigenfunction $\langle p^n\nu_l\rangle\equiv\int\frac{d^3p}{(2\pi\hbar)^3}\frac{\partial f_0}{\partial\varepsilon_0}p^n\nu_l(p,\boldsymbol{q},\omega)$

・Relaxation Time τ

The viscous relaxation contribute most to the density fluctuation.

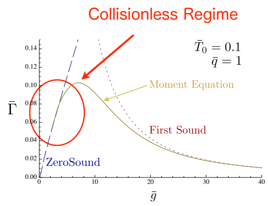
$\tau\longrightarrow\tau_\eta$ Viscous Relaxation



Results : Sound Damping Rate (Coupling Constant Dependence)

- Fermi Systems→We have confirmed the results of the preceding study.

S. Watabe, A. Osawa, and T. Nikuni, J. Low. Temp. Phys. 158,773 (2010)

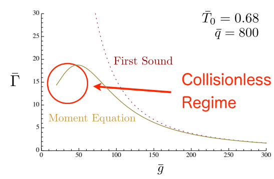


The zero and first sound modes can be explicitly distinguished by the peak of sound damping rate.

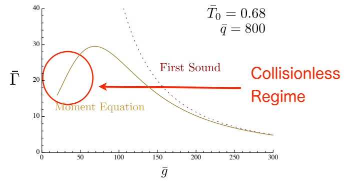
$\bar{\Gamma}$: Damping Rate

Moment Equation : Solid Line
First Sound : Dotted Line
Zero Sound : Dashed Line

- Bose Systems



- Maxwell-Boltzmann Distribution



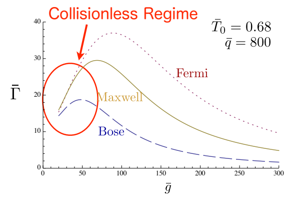
The damping rate as a function of coupling constant behaves similarly as that of normal Fermi systems.

The zero sound may also exist in normal Bose systems and Maxwell-Boltzmann distribution.

Summary

We study sound propagation in normal dilute gases obeying Bose, Fermi, and Maxwell- Boltzmann statistics based on the Boltzmann equation.

The sound attenuation rate as a function of the coupling constant exhibits a peak also in normal Bose systems and Maxwell-Boltzmann distribution.



→The zero sound may exist not only in normal Fermi systems but also in normal Bose systems and Maxwell-Boltzmann distribution at low coupling constants.