[PS-01] NHSCP2014@ISSP, Univ. of Tokyo, Kashiwa (June 26, 2014)

Symmetry-protected topological order and negative-sign problem for SO(N) bilinear-biquadratic chains

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SO(N) bilinear-biquadratic chain

$$\hat{\mathcal{H}} = \sum_i \hat{h}_{i,i+1}$$

H.H.Tu, G-M.Zhang, and T.Xiang, PRB **78**, 094404 (2008).

$$\hat{h}_{i,i+1} = \sum_{b>a} L_i^{ab} L_{i+1}^{ab} + \frac{\alpha}{N-2}\left[\left(\sum_{b>a} L_i^{ab} L_{i+1}^{ab}\right)^2 - 1\right]$$

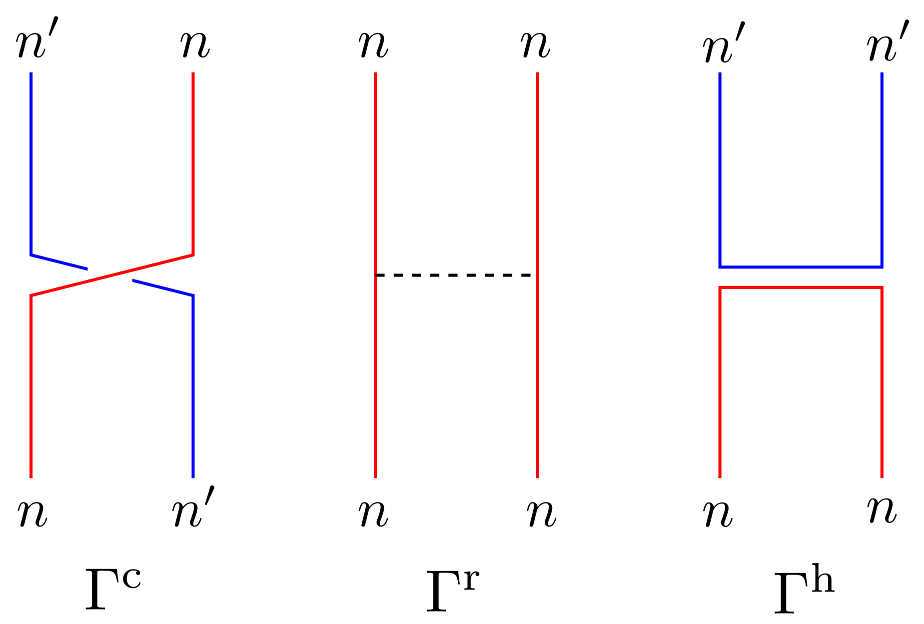
Defining representation of the SO(N) rotational group

$$(L^{ab})_{x,y} = -i(\delta_{a,x}\delta_{b,y} - \delta_{b,x}\delta_{a,y})$$

Matrix element of local Hamiltonian = N-color bosonic model

$$\hat{h}_{i,i+1} = \Gamma^{\mathrm{c}}_{i,i+1} + \alpha\Gamma^{\mathrm{r}}_{i,i+1} - (1-\alpha)\Gamma^{\mathrm{h}}_{i,i+1}$$

Negative sign!
(more than 3 colors)



Non-local unitary transformation

cf. For SU(N) Heisenberg model : L.Messio and F.Mila, PRL **109**, 205306 (2012).

$$Q = \prod_{i<j} Q_{ij}, \quad Q_{ij}|n_i n_j\rangle = (-1)^{\delta(n_i \geq n_j)}|n_i n_j\rangle$$

Negative-sign free local Hamiltonian $(\alpha \leq 1)$

$$\tilde{h}_{i,i+1} \equiv -\Gamma^{\mathrm{c}}_{i,i+1} + \alpha\Gamma^{\mathrm{r}}_{i,i+1} - (1-\alpha)\Gamma^{\mathrm{h}}_{i,i+1}$$

String correlation (symmetry protected topological order)

$$\langle L_i^{ab}\, e^{i\pi\sum_{i<k<j} L_k^{ab}}\, L_j^{ab}\rangle_{\hat{H}} = -\langle T_i^{ab} T_j^{ab}\rangle_{\tilde{H}},$$
$$(T^{ab})_{x,y} = \delta_{a,x}\delta_{b,y} + \delta_{b,x}\delta_{a,y}$$

Landau order

Non-local transformation is a topological disentangler

K.Okunishi, PRB **83**, 104411 (2011).

Relation to Kennedy-Tasaki transformation

PRB **45**, 304 (1992)

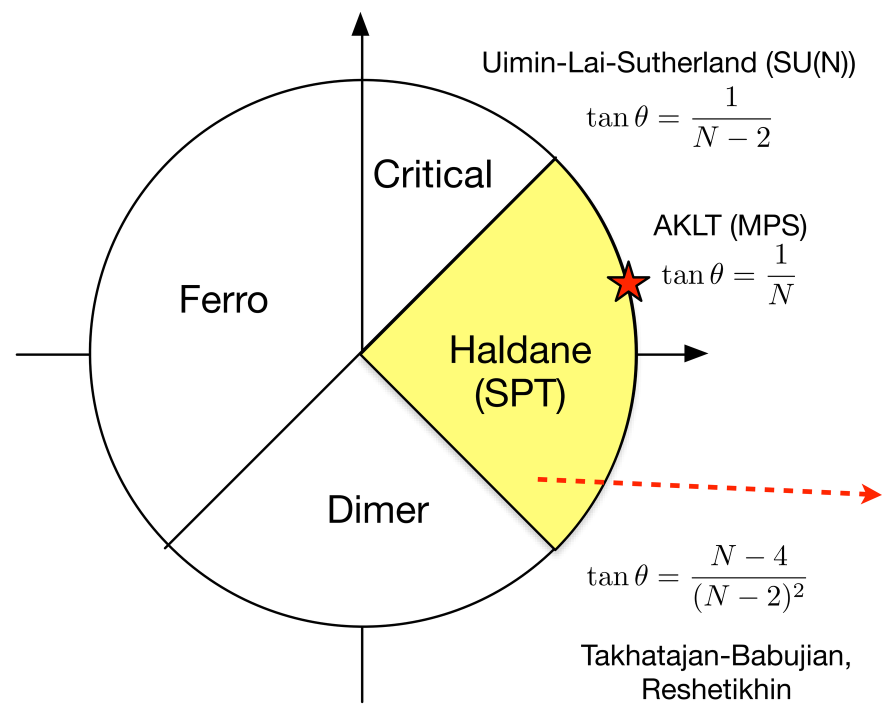
S=1 BLBQ model — SO(3) BLBQ model

$$\begin{tikzcd}[column sep=huge, row sep=huge]
\boxed{\begin{array}{c}S^z \text{ diagonal} \\ \text{(Cartan) base} \\ \mathcal{H}\end{array}} \arrow[r, leftrightarrow, "\mathcal{R}"] \arrow[d, leftrightarrow, "\mathcal{U}"', "\begin{array}{c}\text{Kennedy-Tasaki}\\ \text{transformation}\end{array}"] & \boxed{\begin{array}{c}\text{SO(3) defining} \\ \text{representation} \\ \hat{\mathcal{H}}\end{array}} \arrow[d, leftrightarrow, "\mathcal{Q}", "\begin{array}{c}\text{Jordan-Wigner}\\ \text{transformation}\end{array}"'] \\
\boxed{\begin{array}{c}\text{String order} \\ \text{diagonal base} \\ \mathcal{U}\mathcal{H}\mathcal{U}\end{array}} \arrow[r, leftrightarrow, "\mathcal{V}"'] & \boxed{\begin{array}{c}\text{Negative sign} \\ \text{removed base} \\ \tilde{\mathcal{H}}\end{array}}
\end{tikzcd}$$

Non-local transformation is a generalization of Kennedy-Tasaki transformation

Quantum Monte Carlo of SO(N) BLBQ chain

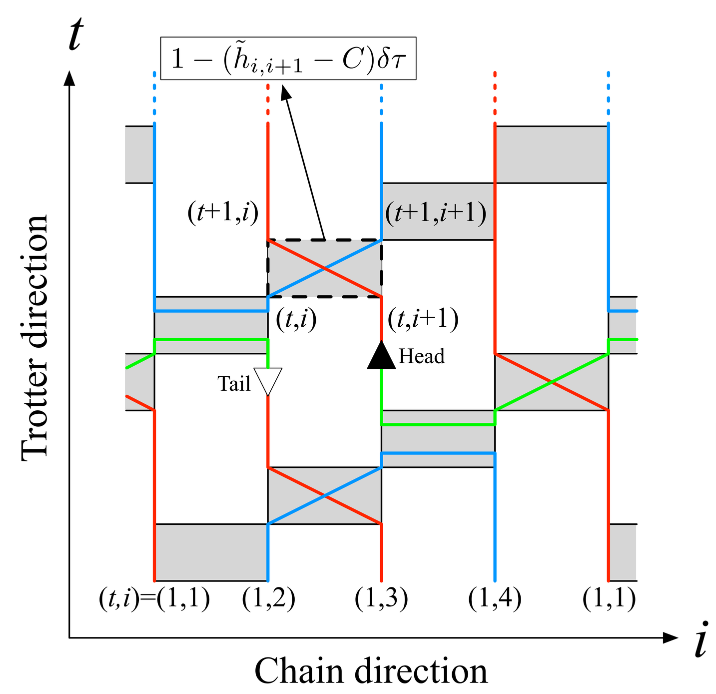
Ground state phase diagram



Another parameterization
$$\tan\theta \equiv \frac{\alpha}{N-2}$$

Symmetry breaking in new models

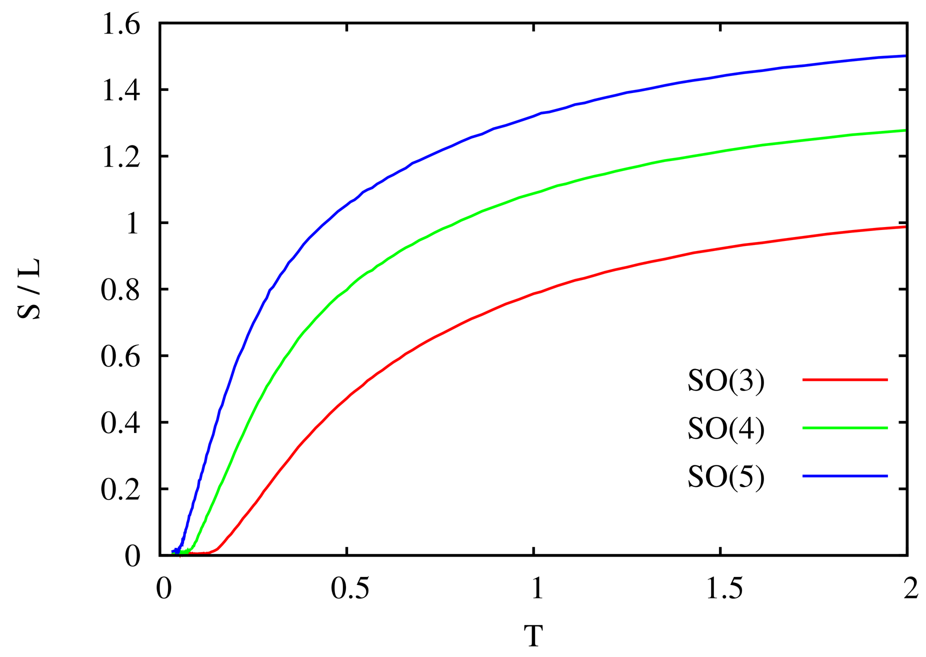
Worm algorithm



Worm changes only worldlines of randumly chosen two colors

Review of QMC: N.Kawashima and K.Harada, Journal of Physical Society of Japan **73**, 1379 (2004).

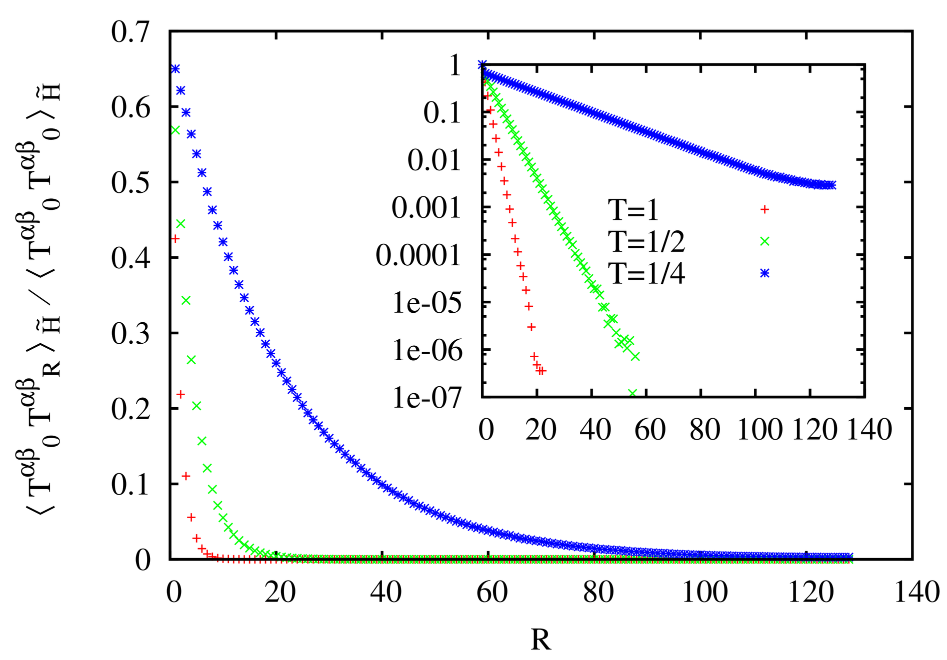
Entropy (MPS)

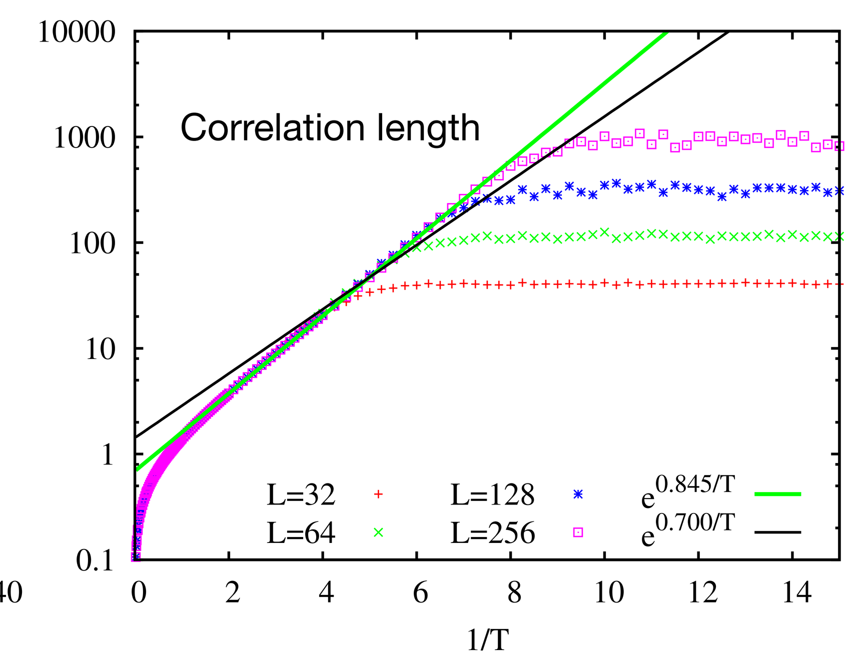


Much faster growth with SO(N) at low temperature

cf. L.Messio and F.Mila, PRL **109**, 205306 (2012).

String order parameter (AKLT)





Short range order — *Not Ising-like excitation*

Summary & discussions

An origin of negative sign is a topological order.

For the details,
Kouichi Okunishi and Kenji Harada,
Physical Review B **89**, 134422 (2014).