Current Noise in a Quantum Point Contact

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Introduction Quantum point contact (QPC) is the simplest quantum system, connecting two macroscopic reservoirs with a region shorter than the mean free path and as narrow as the Fermi wave length. QPC is an important element as a high sensitive potential meter, since its conductance (G) changes sharply with the gate voltage (potential). However, its transport characteristics have not fully been understood yet. The conductance anomaly found in a linear transport regime[1], is now believed to be originated from a many-body effect. We have been trying to explain this phenomenon as an additional electron scattering by exchange enhanced spin fluctuations localized to QPC region.[2]

Current noise spectrum, $S(\omega)$, which is a Fourier transform of symmetrized correlation function of the current fluctuations, is one of the important quantities since it manifests the dynamics/statistics of the electron transport. Low-frequency $1/f$ noise in QPC had been investigated experimentally[3], indicating the excess noise proportional to $(\frac{\partial G}{\partial \mu})^2 V^2$ where μ, V are chemical potential and applied bias, respectively. For relatively high-frequency regime, the shot noise characteristics had been predicted[4] and experimentally confirmed, where $S \propto |V| T(\mu)(1 - T(\mu))$ with transmission probability $T(\mu)$ giving Landauer's formula $G = \frac{2e^2}{h} T(\mu)$. Recently, a shot noise experiment had been conducted for the lowest one-dimensional subbands[5]. It shows suppression of shot noise near the gate bias voltages at which the conductance anomaly is observed. In this report, we study the effect of classical fluctuating field on the shot noise spectrum using time-dependent perturbation theory. By identifying this classical field to the internal thermal spin fluctuation, part of the features of the observed shot noise suppression can be explained.

Model We consider effective one-dimensional system described by a Hamiltonian, $H = -\frac{\hbar^2}{2m}\nabla^2 + U(x) + \delta U(x,t)$, with localized fluctuating potential near QPC, $\delta U(x,t)$, with properties $\overline{\delta U(x,t)} = 0$ and $W(x, x, t-t') = \overline{\delta U(x,t)\delta U(x',t')} \neq 0$ where the over-bar means statistical average. The system of strongly localized oscillating potential had been investigated[6], and a formula of average current is obtained[7]. The noise characteristics with harmonic oscillations of the barrier had been discussed in [8]. Here, we follow the scheme proposed by Levinson and Wölfle[9] where the current correlation function is decomposed into quantum mechanical and statistical parts:

$$\begin{aligned} S(z,z') &= S_q(z,z') + S_s(z,z'), \\ S_q(z,z') &= 2\sum_{nm} \overline{j_{nm}(z) j_{mn}(z')}\,[f_n(1-f_m) + f_m(1-f_n)], \\ S_s(z,z') &= 8\sum_{nm} [\overline{j_{nn}(z)\; j_{mm}(z')} - \overline{j_{nn}(z)} \cdot \overline{j_{mm}(z')}] f_n f_m, \end{aligned} \tag{1}$$

where j_{nm} is current nm matrix element with n's identifying the eigenfunctions of H and $z = (x,t)$. After the Fourier transform and taking low-frequency limit (but not as low as $1/f$ regime), we obtain the noise formula, $S = \lim_{x,x' \to \infty, \omega \to 0} S(x, x', \omega)$.

Results We execute perturbation expansion of the eigenfunctions in δU and then make statistical average. The lowest order is given by

$$S^{(0)} = 2e^2 \sum_E [2T^2\{f_L(1-f_L) + f_R(1-f_R)\} + 2TR\{f_L(1-f_R) + f_R(1-f_L)\}], \tag{2}$$

which is a well-known form[4] with $f_{L/R}$ being Fermi distribution function of the reservoir (L/R) at energy E and T, R transmission/reflection probabilities at energy E. The low-temperature condition, $k_B T \ll e|V|$, yields $S^{(0)}_{shot} = \frac{2e^2}{h} 2e|V|T(\mu)R(\mu)$. The first order vanishes and the second order is evaluated, and following is the form at low-temperature condition, $k_B T \ll e|V|$,

$$S^{(2)}_{shot} = \frac{2e^2}{h} 2e|V|[Q_a(\mu) + \frac{e|V|}{\mu} Q_b(\mu)]. \quad (3)$$

We have evaluated Q_a, Q_b as a function of μ numerically for the system with a model static potential given by $U(x) \equiv u_0/cosh^2(2x/L)$ with a spatial extension parameter L. We also assumed that the fluctuating potential $\delta U(x,t)$ is localized in the region L. The characteristic frequency of the fluctuation $W(x, x', t)$ is assumed to be smaller than the inverse transit time of the electron through the QPC, $\omega_{Thouless} \equiv v_F/L$, where v_F is the Fermi velocity. We found for $k_F L \gg 1$ that $Q_a(\mu)$ and $Q_b(\mu)$ show μ dependences roughly proportional to $\frac{\partial^3}{\partial \mu^3} T(\mu)$ and $\frac{\partial}{\partial \mu} T(\mu)$, respectively.

Discussions and Summary For low bias conditions, main contribution of the shot noise is proportional to $T(\mu)R(\mu) + Q_a(\mu)$. Since $Q_a(\mu)$ has a large negative region when μ is about equal to u_0, the total shot noise is suppressed compared with the value without fluctuating field in these region. In our previous analysis[2] in the same model, we have correction in the average current, proportional to $\frac{\partial^2}{\partial \mu^2} T(\mu)$ at the same region of chemical potential. Therefore, we expect conductance anomaly and suppression of shot noise at the same values of chemical potential. The μ dependence of Q_a has a simple explanation. Under quasi-adiabatic condition, $k_F L \gg 1$, the potential fluctuation $\delta U(x,t)$ is equivalent to the fluctuation of injection energy as $E(t) = \mu + 2a_\omega \cos \omega t$. (Here we took only one ω component since cross terms do not contribute. We need to sum over ω with relevant weights, a^2_ω.) For the validity of perturbation expansion, $a_\omega \ll \mu$ should hold. Now the shot noise correction is obtained by the Taylor expansion of S^0_{shot} in a_ω with μ replaced by $E(t)$ and the temporal average of the second order term yields, $Q_a(\mu) \sim a^2_\omega \frac{\partial^2}{\partial \mu^2}[T(\mu)R(\mu)]$. By noting that for smooth scattering potential $U(x)$, $T(\mu)R(\mu) \sim \alpha \frac{\partial}{\partial \mu} T(\mu)$ where $\alpha > 0$ characterizes the smoothness (curvature) of the potential (constriction), we obtain that $Q_a(\mu) \propto \frac{\partial^3}{\partial \mu^3} T(\mu)$. Higher bias quadratic behavior proportional to Q_b is consistent with [8,9].

In summary, we have studied the noise spectrum of the quantum point contact (QPC) with the effect of fluctuating classical field. We found *suppression* of the shot noise by this field, which is synchronized with the anomaly of the conductance. This analysis cannot account for the effect of *quantum fluctuation* of spin. Nevertheless, the effect of thermal fluctuation of spin on the transport through QPC would be related to this results.

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