Quantum Hall physics: Introduction and current affairs

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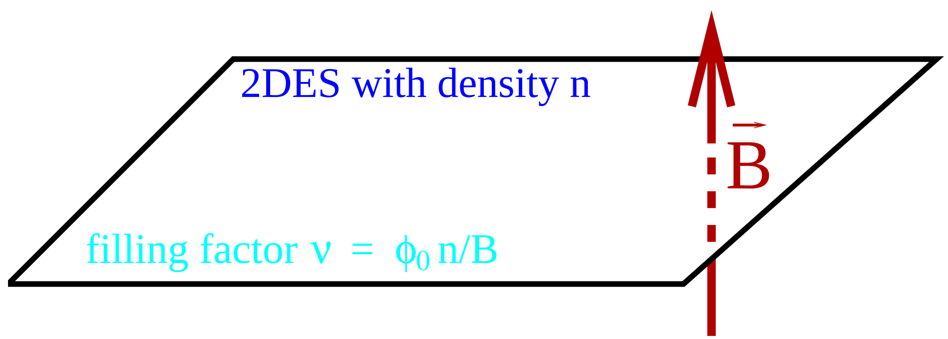
Outline

- Introduction to basics of the quantum Hall effect
  - sample geometry, measurement technique
  - incompressibility $\Rightarrow$ quantized Hall resistance
  - microsocpic origin of incompressibility
  - role of disorder
- Overview of exotic interaction effects
  quasiparticles with fractional charge and statistics; composite fermions; chiral Luttinger liquids; quantum Hall ferromagnets
- fractional quantum Hall effect in rotating atomic gases

Introduction to the basics of the quantum Hall effect

Two–dimensional electron systems

- Quantum Hall (QH) effect observed in 2D electron systems placed in a perpendicular magnetic field



important parameter: filling factor ν

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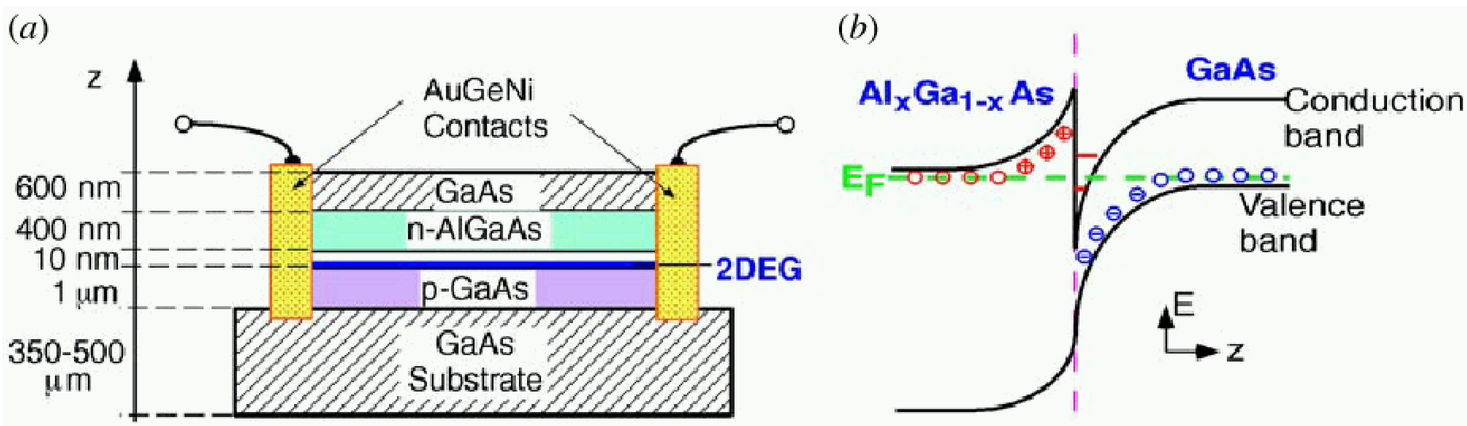

illustration taken from: Jeckelmann and Jeanneret '01

- typically realized in semiconductor heterostructures using band–gap engineering: (Ga,Al)As, (In,Al)As, . . .

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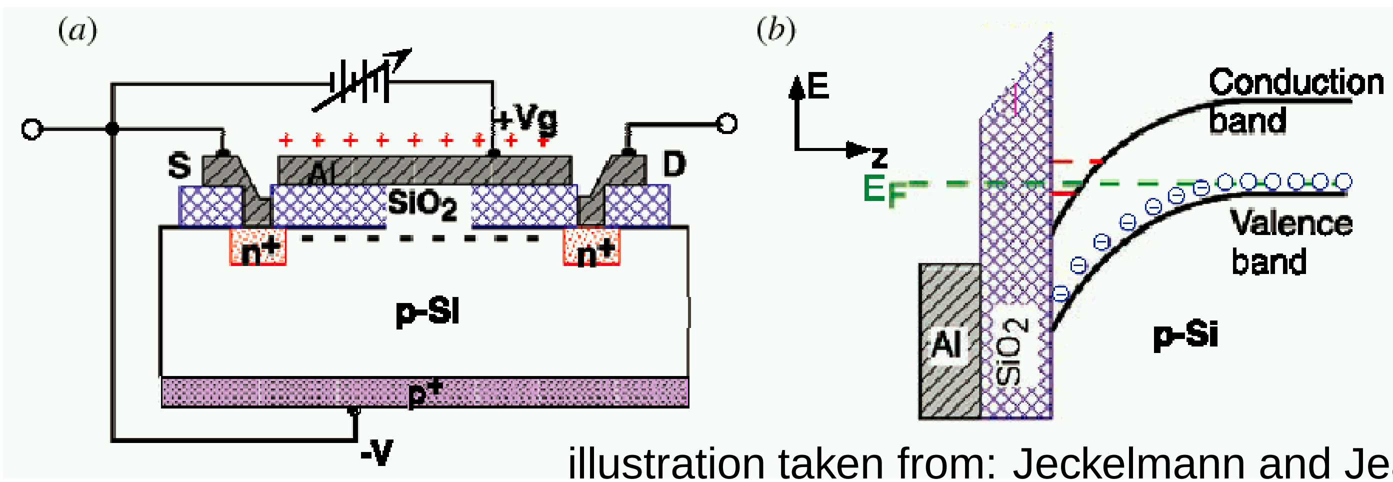

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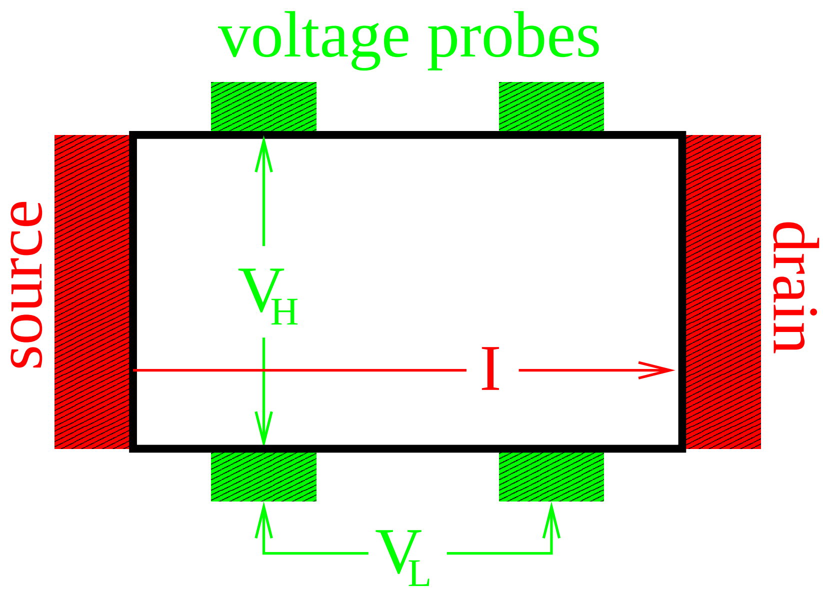
- typically realized in semiconductor heterostructures using band–gap engineering: (Ga,Al)As, (In,Al)As, . . .
- original discovery of QH effect in silicon MOSFETs

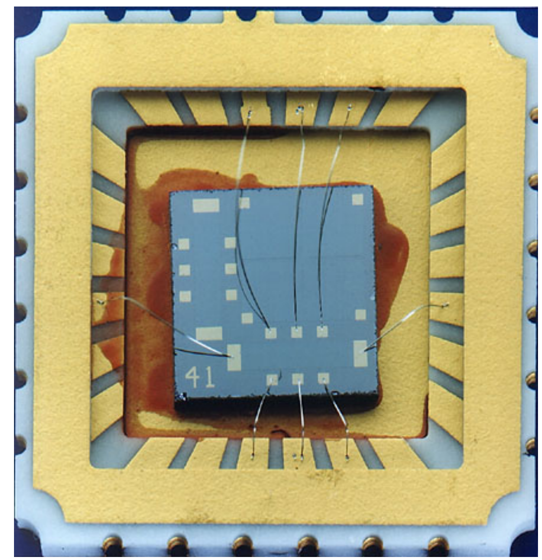
Klitzing, Dorda, Pepper '80

Setup for transport measurement

- mesoscopic current and voltage probes attached to sample, measure resistances $R = V_L/I$ and $R_H = V_H/I$



Büttiker ’86



source: PTB webpage

Setup for transport measurement

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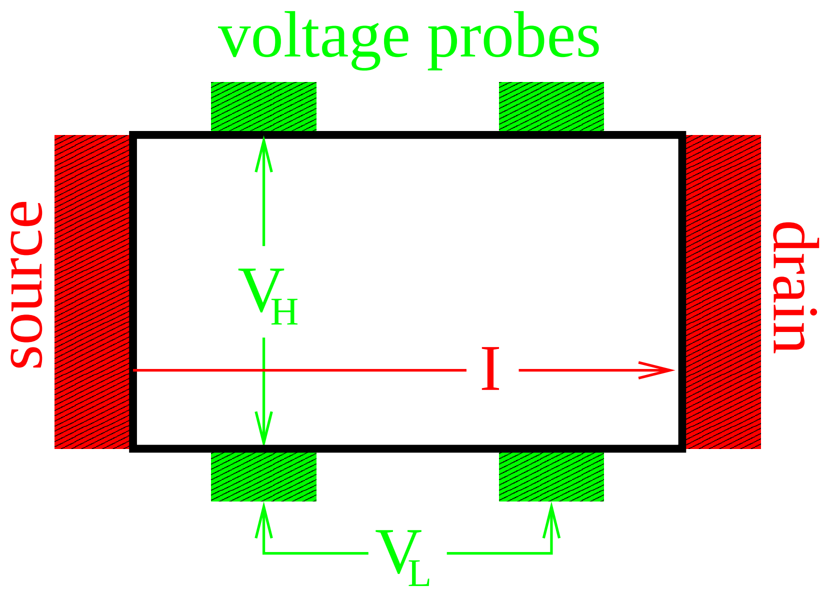
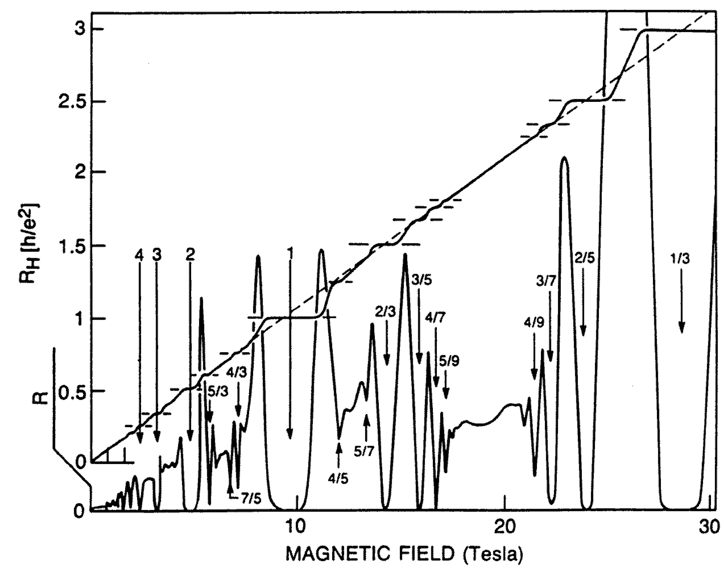




Büttiker '86

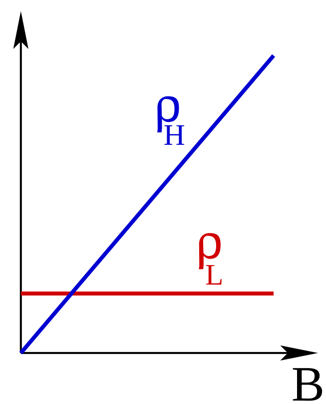
Willett et al. '87

- observe plateaux in R_{H} where, with great precision (error $< 10^{-9}$), $R_{\mathrm{H}}^{-1} = \nu_{\mathrm{c}} \frac{e^2}{h}$, concomitant with $R \to 0$

Classical magnetotransport theory

- electrodynamics in 2D: $\vec{\jmath} = \overleftrightarrow{\sigma} \cdot \vec{E}\,, \quad \vec{E} = \overleftrightarrow{\varrho} \cdot \vec{\jmath}\,, \quad \overleftrightarrow{\sigma} = \overleftrightarrow{\varrho}^{-1}$
- simple Drude theory yields (with $\omega_\mathrm{c} = eB/m$):

$$\overleftrightarrow{\varrho} = \underbrace{\frac{B}{n\,e}}_{\varrho_\mathrm{H}} \begin{pmatrix} (\omega_\mathrm{c}\tau)^{-1} & 1 \\ -1 & (\omega_\mathrm{c}\tau)^{-1} \end{pmatrix}$$



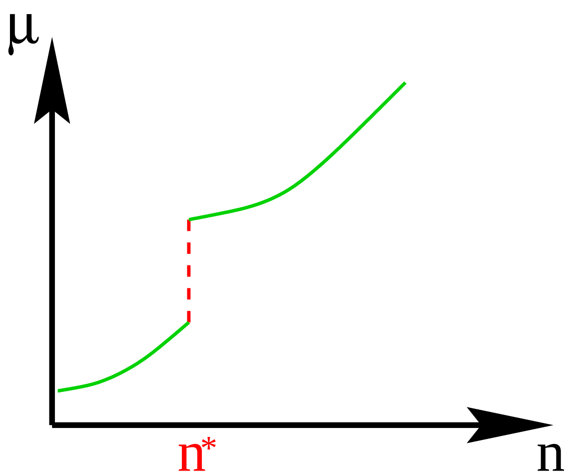
⇒ quantum Hall system not a classical conductor ⇐

- at plateau: both longitudinal resistivity and conductivity vanish (possible because ϱ_H finite)

⇒ quantum Hall system: perfect conductor or insulator? ⇐

Quantized ϱ_H: Thermodynamic argument

- reasonable hypothesis: quantum Hall system is a bulk insulator, i.e., is incompressible in the bulk: $dn/d\mu \to 0$



$$\kappa = -\frac{1}{V}\frac{\partial V}{\partial P} = \frac{1}{V}\left(\frac{\partial^2 E}{\partial V^2}\right)^{-1} \equiv \frac{1}{n^2}\frac{dn}{d\mu}$$

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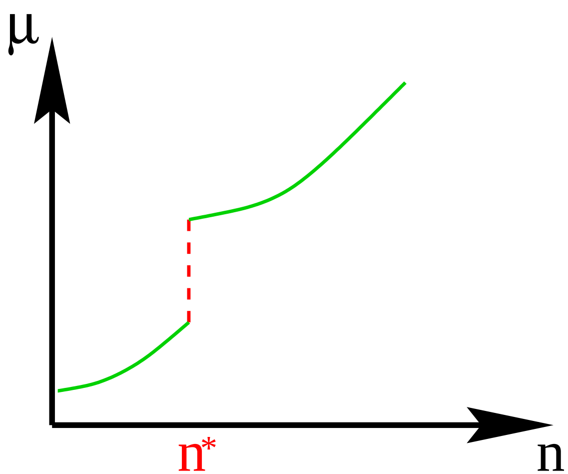

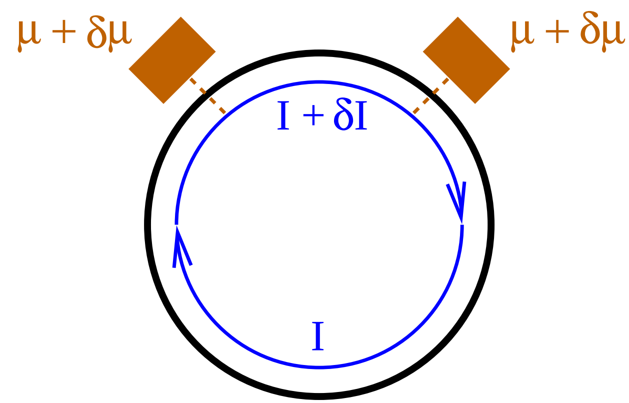

- incompressibility at magnetic–field–dependent density n^* implies quantization of Hall resistance! Widom '82

$$\delta M = \left(\frac{\partial M}{\partial \mu}\right) \delta\mu = \left(\frac{\partial N}{\partial B}\right) \delta\mu$$

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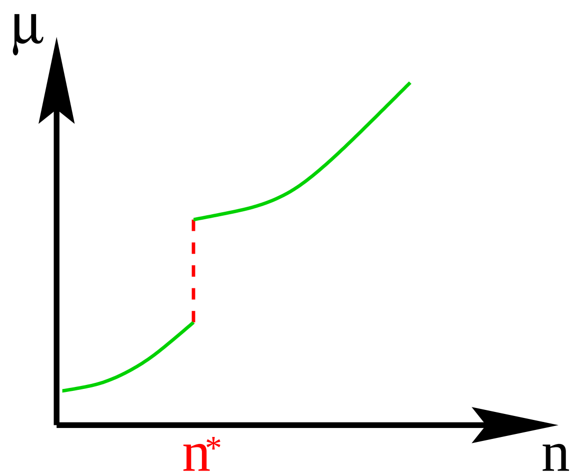

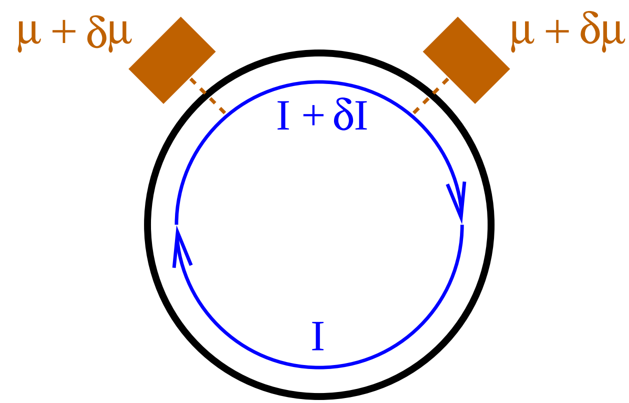

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$$\delta I = \left(\frac{\partial I}{\partial \mu}\right) \delta\mu = \left(\frac{\partial n^*}{\partial B}\right) \delta\mu$$

Quantized $\varrho_{\rm H}$: Thermodynamic argument

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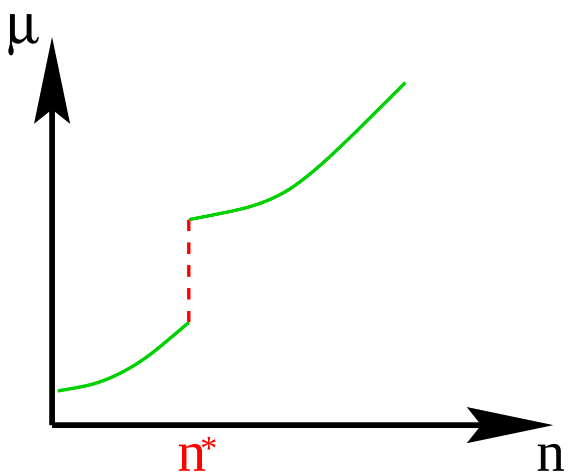

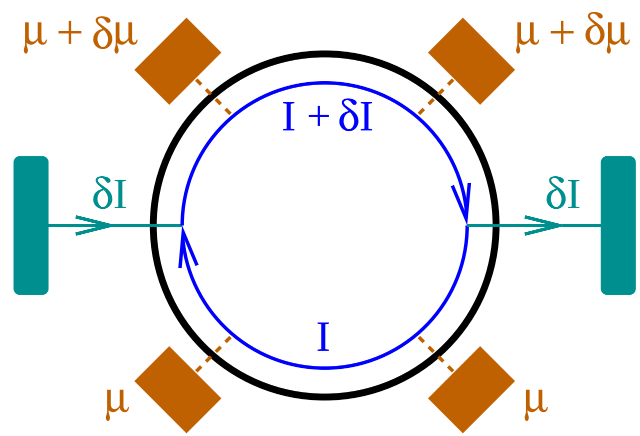

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$$\delta I = \left(\frac{\partial I}{\partial \mu}\right) \delta\mu = \left(\frac{\partial n^*}{\partial B}\right) \delta\mu \;\Rightarrow\; R_H = \frac{\delta\mu}{e\,\delta I} = \frac{h}{e^2}\left(\phi_0\,\frac{\partial n^*}{\partial B}\right)^{-1}$$

Take–home message # 1: Quantized Hall resistance due to incompressibility (mobility gap).

Incompressibility at integer ν

- Schrödinger eq. for 2D electrons in a perpendicular magnetic field reads $H_0\,\psi(\vec{r}) = E\,\psi(\vec{r})$ with

$$H_0 = \frac{\left[\vec{p} + e\vec{A}(x,y)\right]^2}{2m} = \frac{\hbar\omega_c}{2}\left\{\frac{\vec{p}^2\ell^2}{\hbar^2} + \frac{\vec{r}^2}{4\ell^2} - \frac{L_z}{\hbar}\right\}$$

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- find representation of $H_0 = \hbar\omega_{\mathrm{c}}\left(a^\dagger a + \frac{1}{2}\right)$ and angular momentum $L_z = \hbar\left(b^\dagger b - a^\dagger a\right)$ in terms of independent harmonic–oscillator ladder operators $\{a, a^\dagger\}$ and $\{b, b^\dagger\}$

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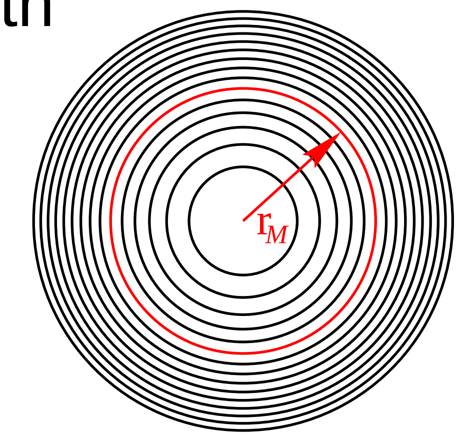

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- eigenstates of H_0 labelled by Landau–level index n and L_z quantum # M; localized at $r_M = \sqrt{2(n+M+1)}\ell$

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- finite sample: external potential $V(x, y)$ confines electrons; Hamiltonain given by $H_0 + V(x, y)$

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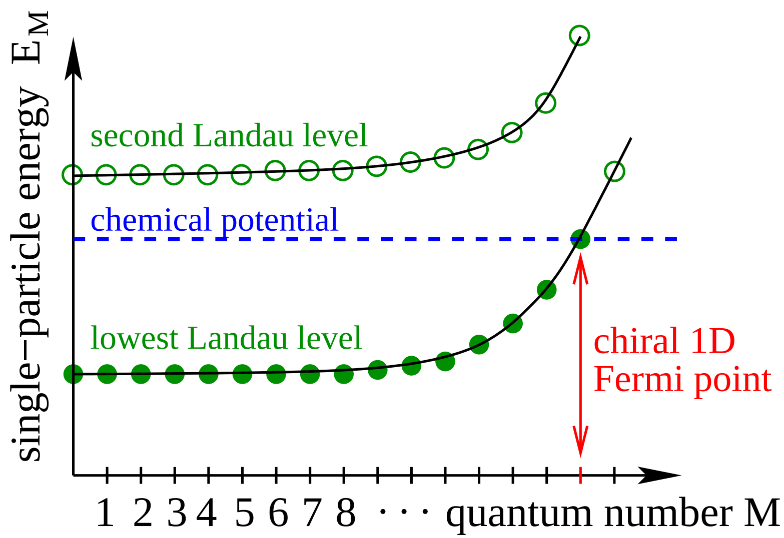
- oscillator (Landau) levels degenerate in bulk but bent upwards at sample boundary



$\Rightarrow$ bulk gap at integer ν

$\Rightarrow$ at the same time: chiral 1D edge currents flowing

$$I_M = \frac{e}{\hbar} \frac{\partial E_M}{\partial M}$$

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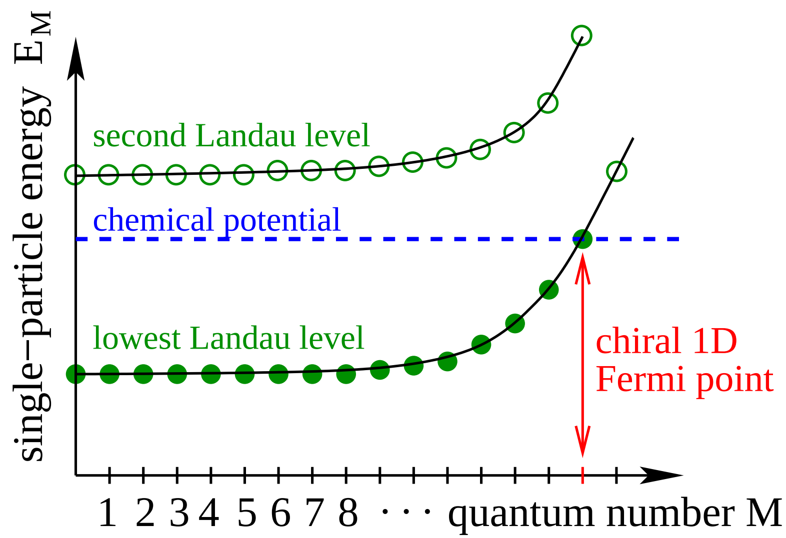

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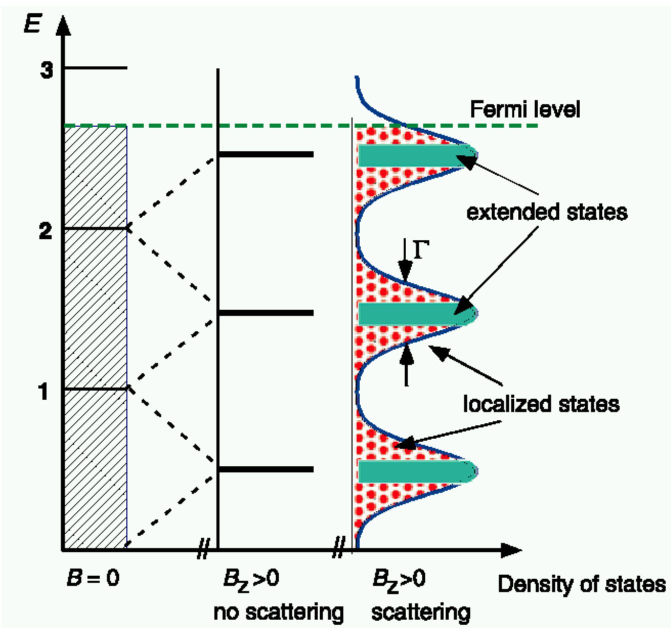
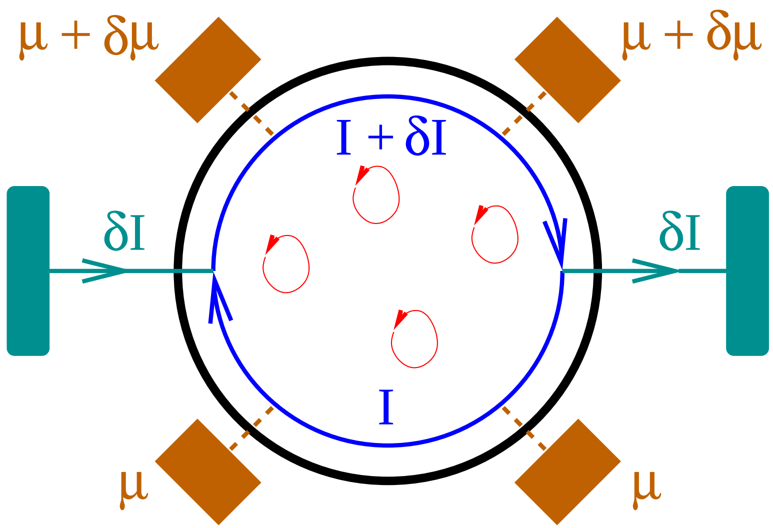
$$I_M = \frac{e}{\hbar}\frac{\partial E_M}{\partial M}$$

- Hall conductance: $\frac{\delta I}{\delta \mu} = \nu\, \frac{e^2}{h}$

Rôle of disorder

- observe quantum–Hall effect in large (mm–sized) samples with plateaux extending over $\Delta B > 1$ Tesla
- expect, however, $\frac{\Delta B}{B} \approx \frac{\ell}{\sqrt{A}} \sim 10^{-2} \ldots 10^{-4}$ typically
- resolution of puzzle: samples are disordered!





Take–home message # 2: Finite width of Hall plateaux in large samples due to disorder.

Incompressibility at fractional ν

Tsui, Gossard, Störmer '82

- fractional filling factor $\Rightarrow$ occupy only states in lowest Landau level (LLL), these are labelled by M and (except close to edge) have the same (kinetic) energy

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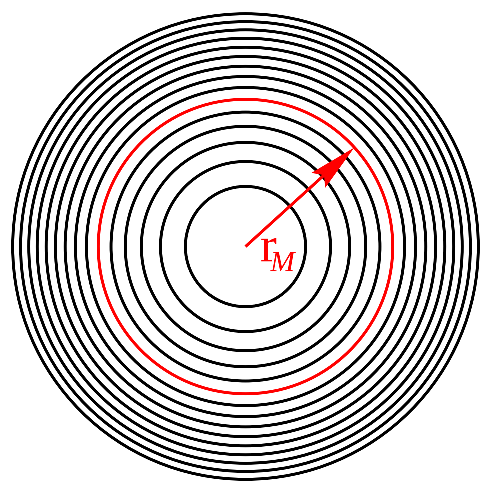
- wave function: $\psi_M(x,y) = \mathcal{N}_M\, z^M\, e^{-\frac{|z|^2}{4}}$ with $z = x + iy$,
  general LLL wave function: $\psi(x,y) = f(z)\, e^{-\frac{|z|^2}{4}}$ where $f(z) \ldots$ polynomial with, at most, $N_\phi = BA/\phi_0$ zeroes



Incompressibility at fractional ν

- two–electron wave functions classified by relative and center–of–mass angular momenta m and M are uniquely determined by analyticity requirement:
$\psi_{mM}(z_1, z_2) = (z_1 - z_2)^m (z_1 + z_2)^M \, e^{-\frac{1}{4}(|z_1|^2+|z_2|^2)}$

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- many-body Hamiltonian for electrons in LLL:

$$\mathcal{H} = \sum_{i<j} V_{\text{int}}(|z_i - z_j|) \equiv \sum_m V_m \, \mathcal{P}_m^{(1,2)}$$

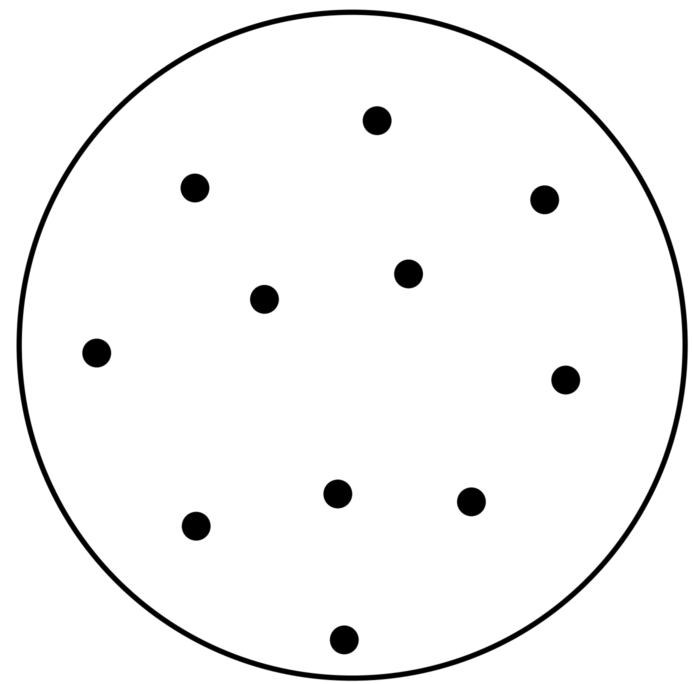
$\Rightarrow$ has discrete matrix elements in mM representation!

Incompressibility at fractional ν

$\Rightarrow$ electron wave functions characterized by their zeroes! $\Leftarrow$

consider finite sample with
N electrons, N_ϕ flux quanta

$\Rightarrow$ many–body wave function for electrons changes sign upon particle exchange



$$\Psi(z_1, \ldots, z_N) = Q(\{z_j\}) \prod_{i<j} (z_i - z_j)\; e^{-\frac{1}{4}\sum_k |z_k|^2}$$

Incompressibility at fractional ν

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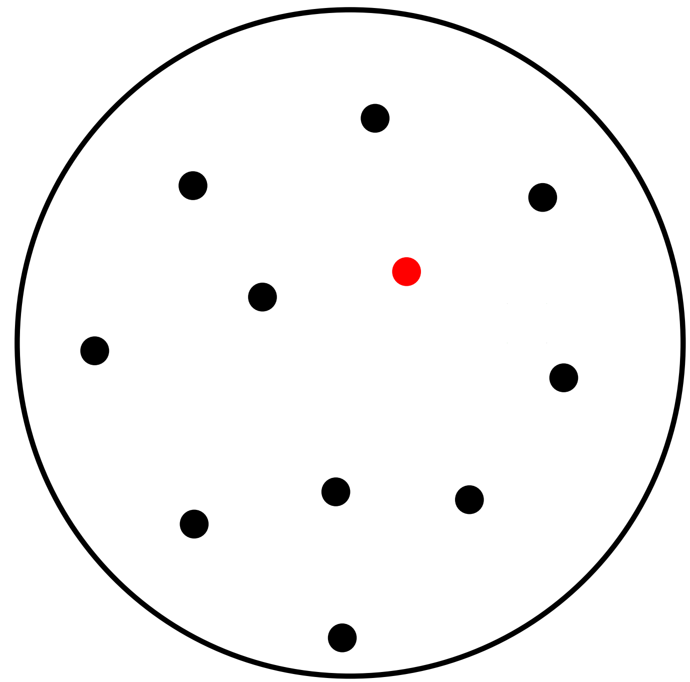
$\Rightarrow$ must be polynomial in each particle coordinate with N_ϕ zeroes: sample size!



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Incompressibility at fractional ν

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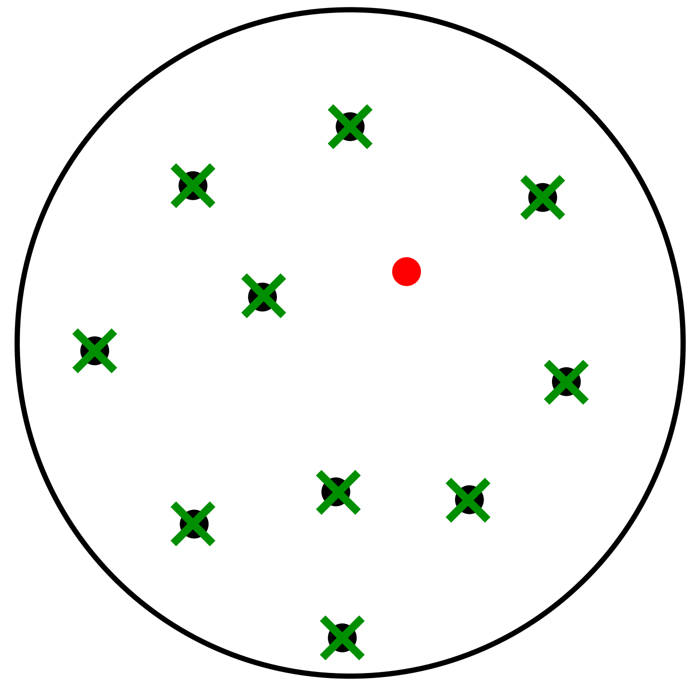
consider finite sample with N electrons, N_ϕ flux quanta

$\Rightarrow$ at least one zero at position of every other electron, due to antisymmetry



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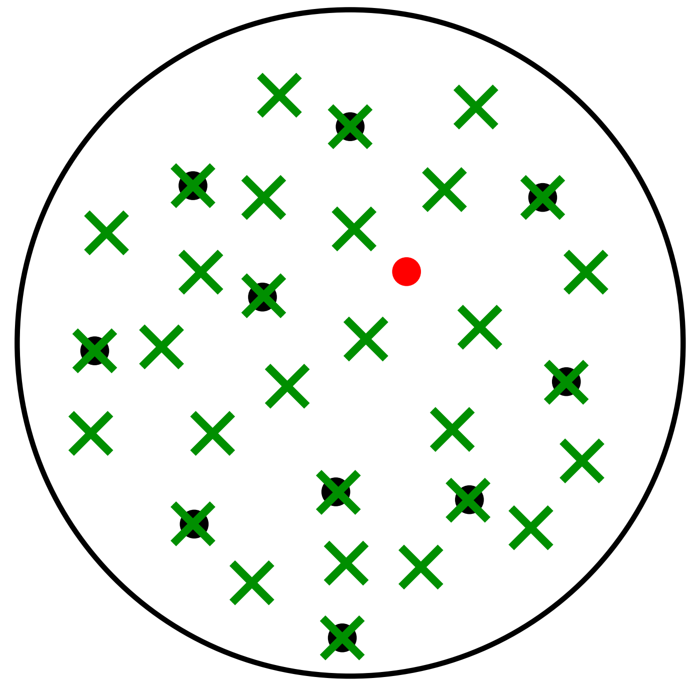
Incompressibility at fractional ν

$\Rightarrow$ electron wave functions characterized by their zeroes! $\Leftarrow$

consider finite sample with N electrons, N_ϕ flux quanta

$\Rightarrow$ position of remaining zeroes arbitrary in the absence of interactions



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Incompressibility at fractional ν

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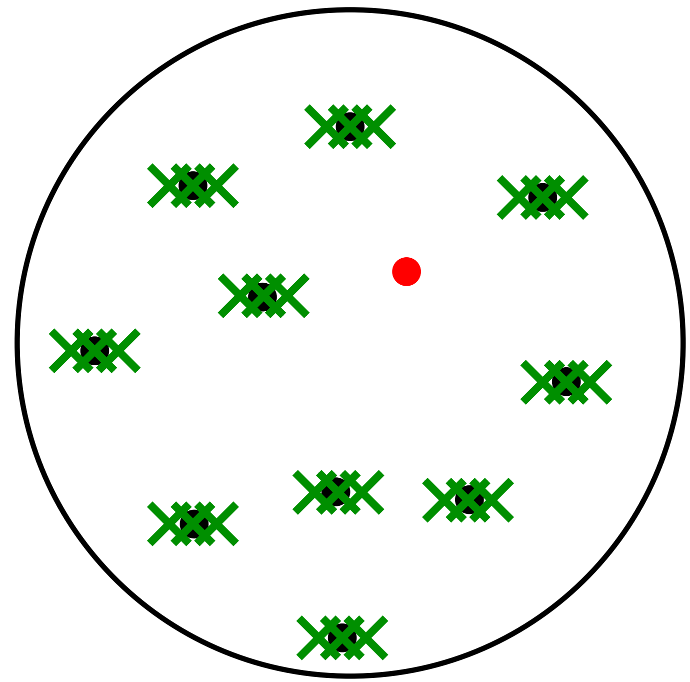
consider finite sample with N electrons, N_ϕ flux quanta

$\Rightarrow \nu \leq 1/3$: to minimize interaction energy, nucleate more zeroes at other electrons



$$\Psi(z_1, \ldots, z_N) = \prod_{i<j} (z_i - z_j)^3 \; e^{-\frac{1}{4}\sum_k |z_k|^2}$$

Laughlin '83

Incompressibility at fractional ν

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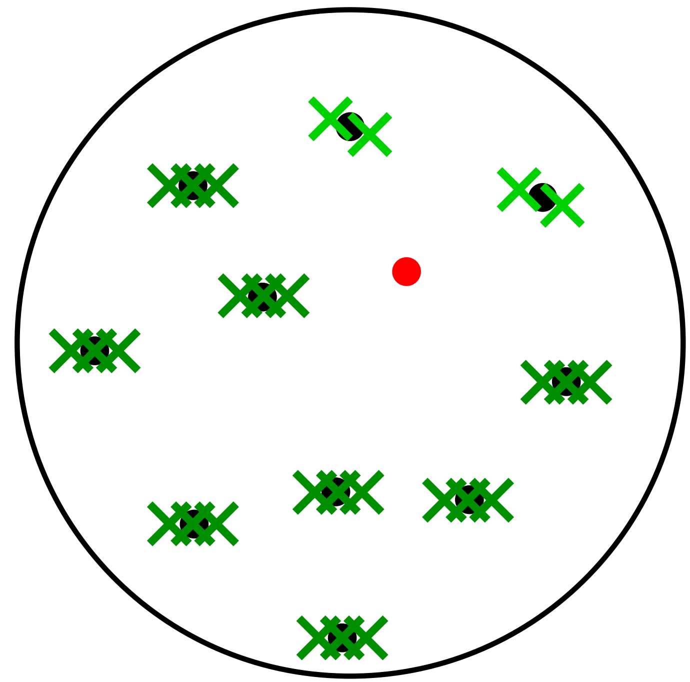
consider finite sample with N electrons, N_ϕ flux quanta



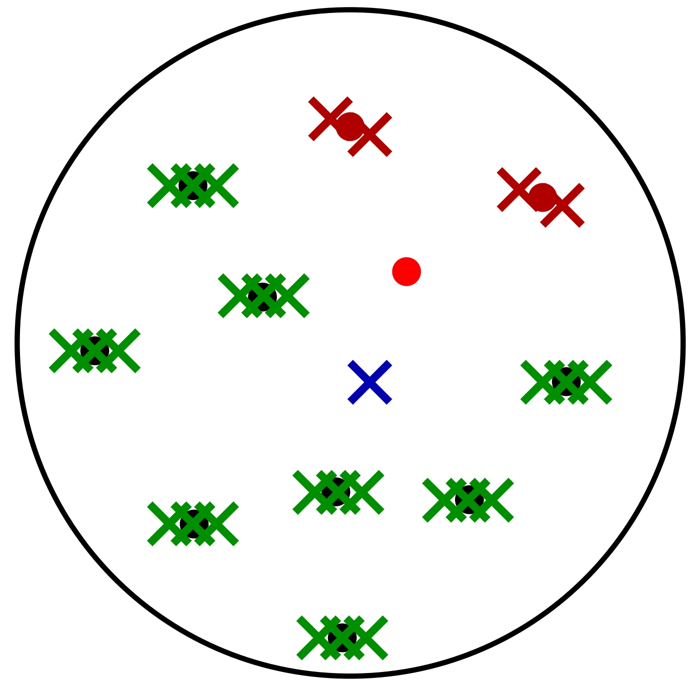
$\Rightarrow$ not enough zeroes when $\nu > 1/3$!! origin of incompressibility at $\nu = 1/3$

Incompressibility at fractional ν

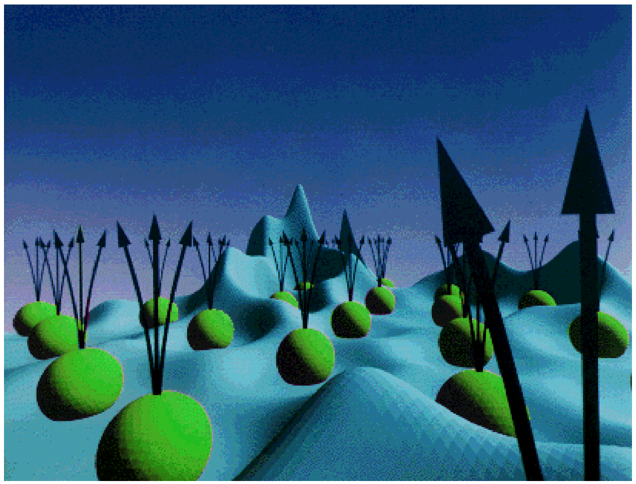
⇒ electron wave functions characterized by their zeroes! ⇐

consider finite sample with N electrons, N_ϕ flux quanta

⇒ stray/missing zeroes are quasiholes/electrons w/ fractional charge and statistics



Composite bosons / fermions



Source: Website of Bell Labs

$\Rightarrow$ zeroes of many–body wave function can be understood as flux vortices (phase winding!)

$\Rightarrow$ singular gauge transformation eliminates winding phase, introduces fictitious magnetic field

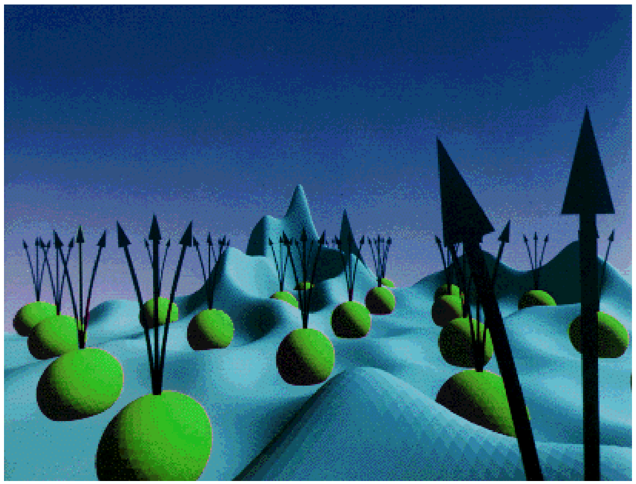
many–electron wave function for $\nu \approx 1/(p+1)$:

$$\tilde{\Psi} = \hat{Q}(\{z_j\}) \prod_{i<j}(z_i - z_j)^{p+1} \rightarrow \tilde{\Psi}_{\text{CB}} = \hat{Q}(\{z_j\})$$

many–boson wave function in zero magnetic field

Zhang, Hansson, Kivelson '89

Composite bosons / fermions



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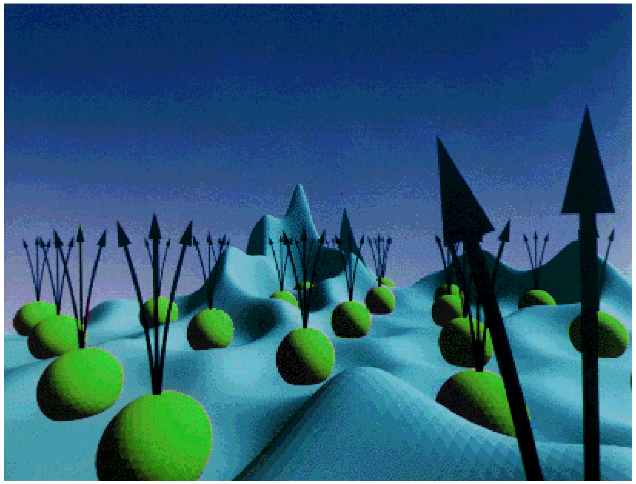
many–fermion wave function at $\nu_{CF} = 1$

Jain ’89

Composite bosons / fermions



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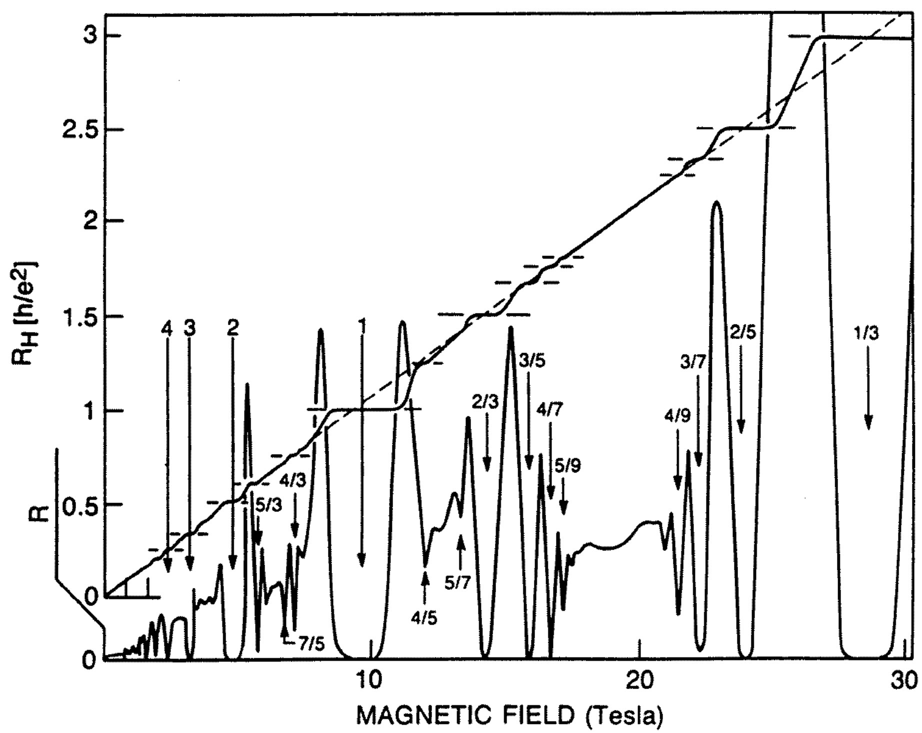
many–fermion wave function at $\nu_{\mathrm{C}F} = 1$

Jain '89

more generally: $\nu_{\mathrm{C}F}^{-1} = \nu^{-1} - p$

$\nu_{\mathrm{CF}} = n \ldots$ integer $\Rightarrow$ quantum Hall effect at $\nu = \frac{n}{n\,p+1}$

Composite bosons / fermions



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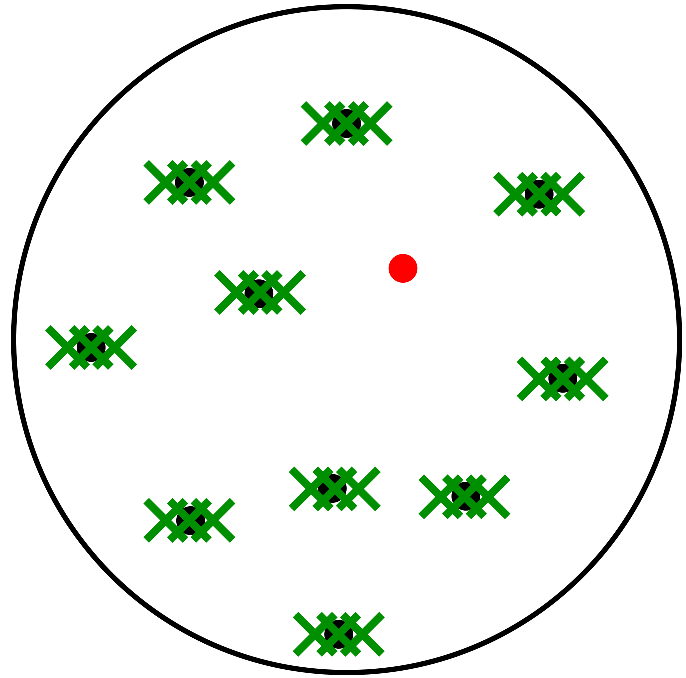
$\nu = 1/p$: composite–fermion liquid

Overview of nontrivial interaction physics in the quantum Hall regime

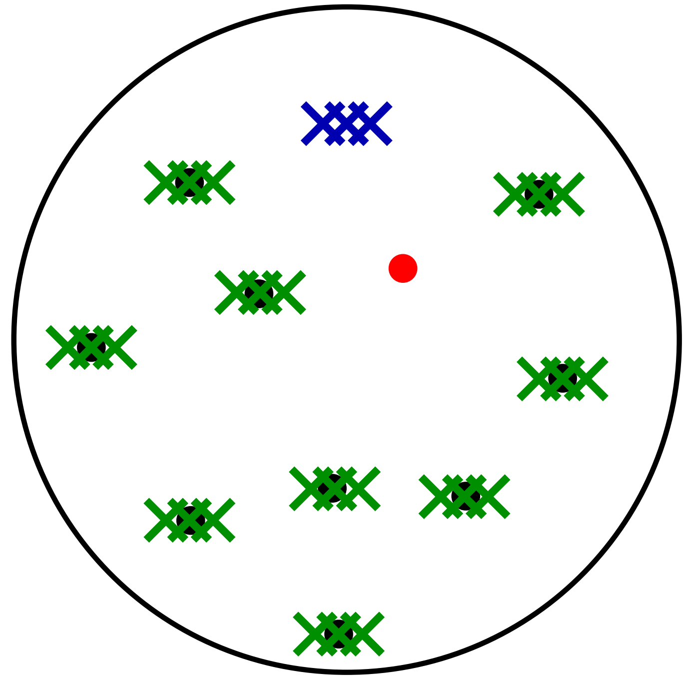
Electron fractionalization

$\Rightarrow$ Laughlin state has all electrons saturated with three flux tubes



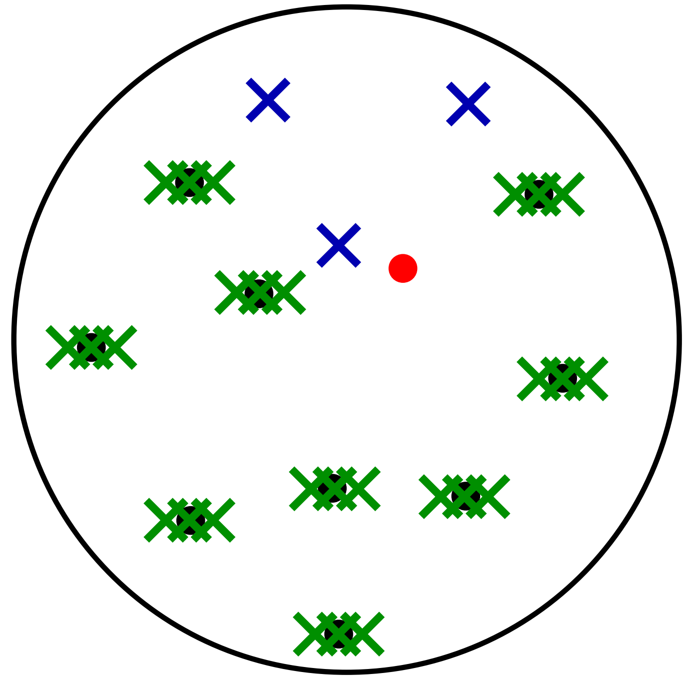
Electron fractionalization

⇒ make hole excitation, i.e., physically remove an electron: leaves 3 zeroes



Electron fractionalization

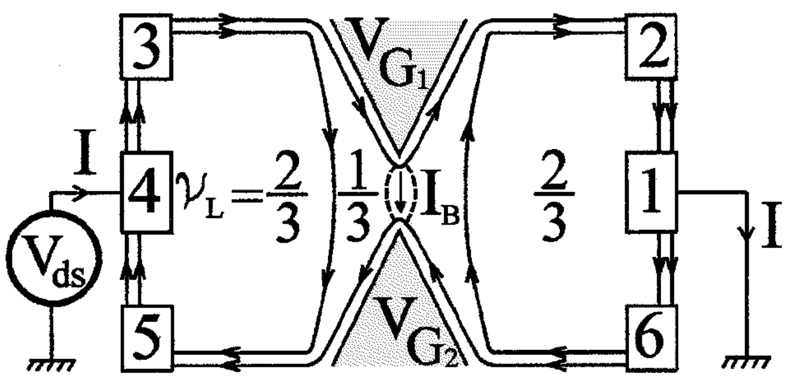
⇒ three independent fractionally charged quasi-holes generated!



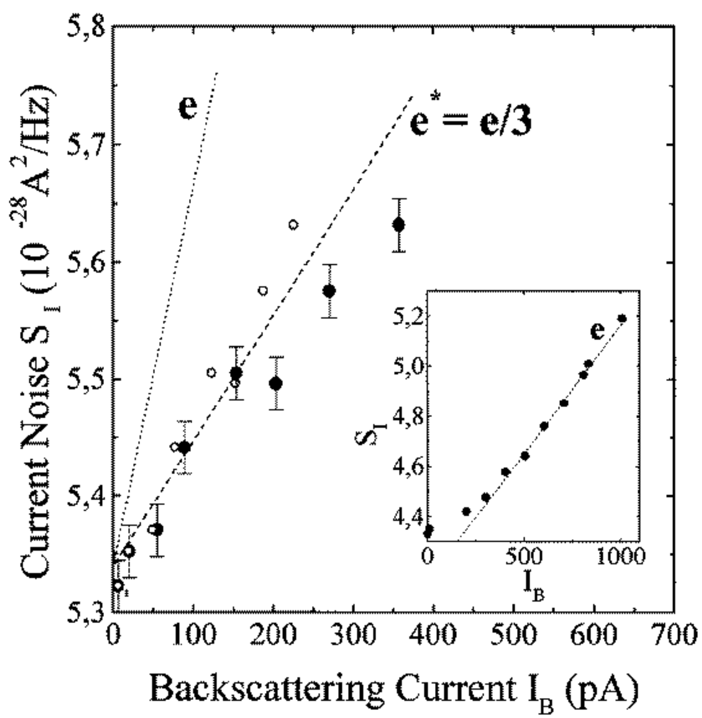
Electron fractionalization

⇒ experimental verification: shot–noise measurements

Saminadayar et al. ’97; Reznikov et al. ’97

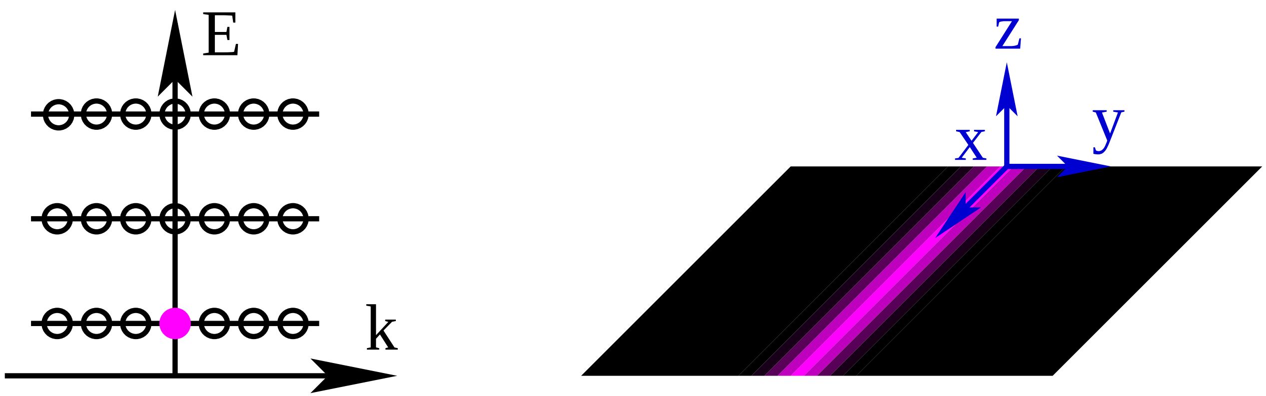


$$S = 2\, e^{*}\, I_{\mathrm{B}}$$

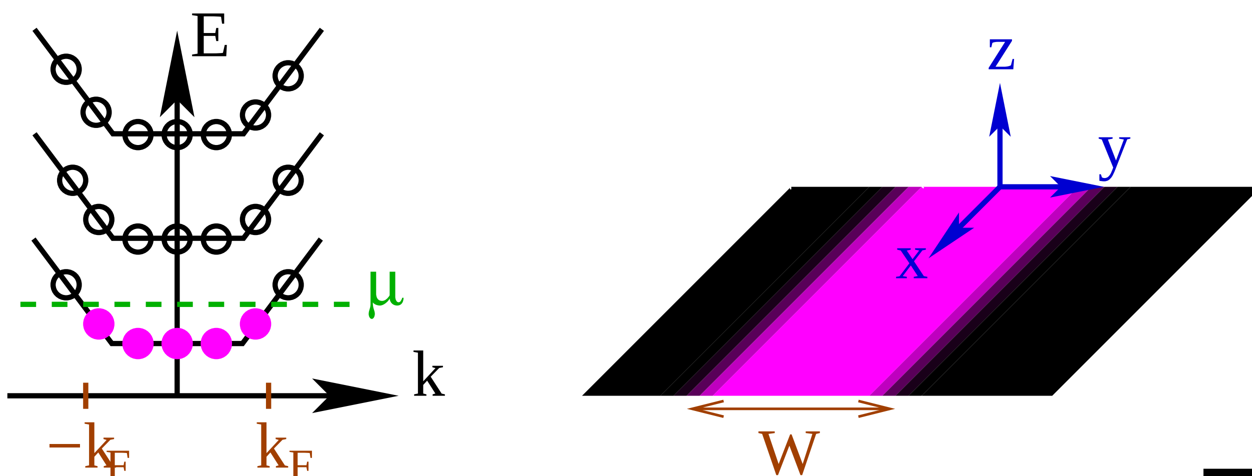


Quantum–Hall edge excitations

integer ν: energy gap due to Landau quantization



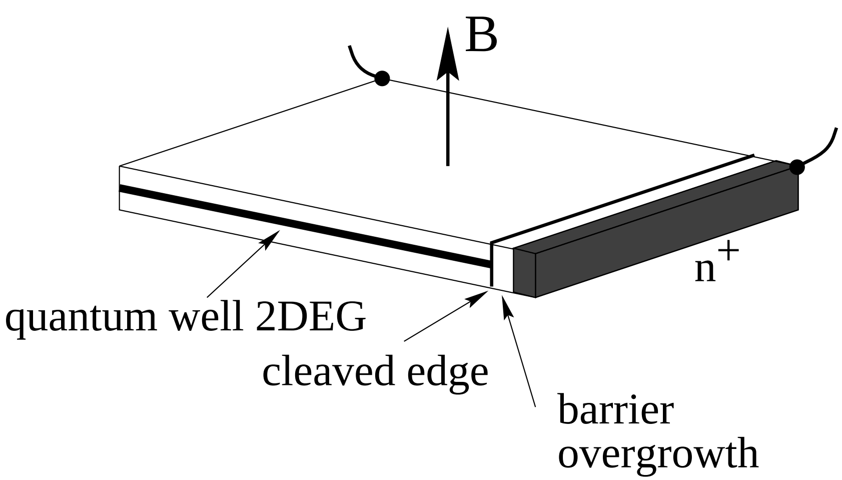
edge potential ⇒ low–lying 1D edge excitations

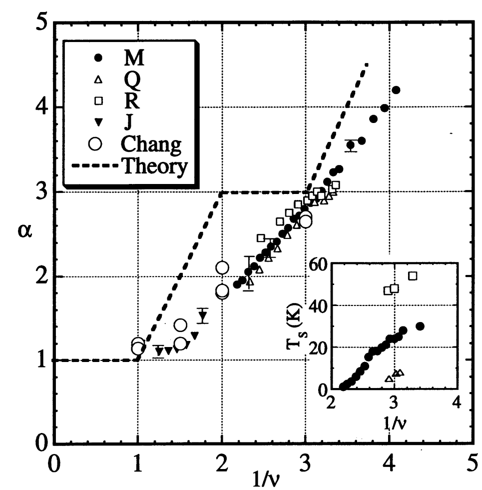


Chiral Luttinger liquids

At fractional filling factors, e.g., $\nu = \frac{n}{2n \pm 1}$: energy gap due to interactions Laughlin '83

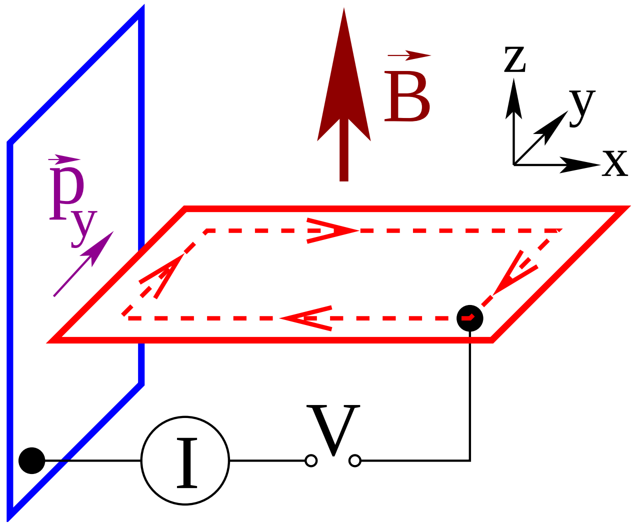
- n branches of chiral edge excitations predicted that form chiral Luttinger liquids Wen '91
- tunneling from 3D contact into edge: expect $I(V) \propto V^{\alpha}$ observe $\alpha_{\text{exp}} \neq \alpha_{\text{theo}}$





Grayson et al. '98
Grayson et al. '01
Chang et al. 01
Hilke et al. 01

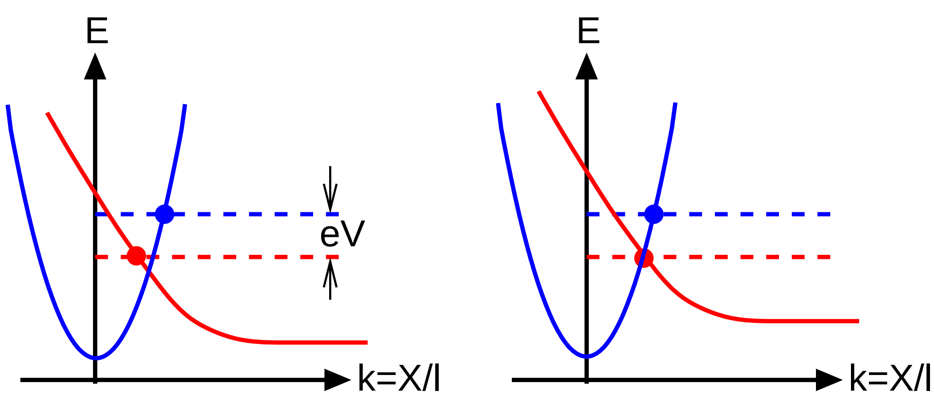
New tunneling geometry: 2D contact



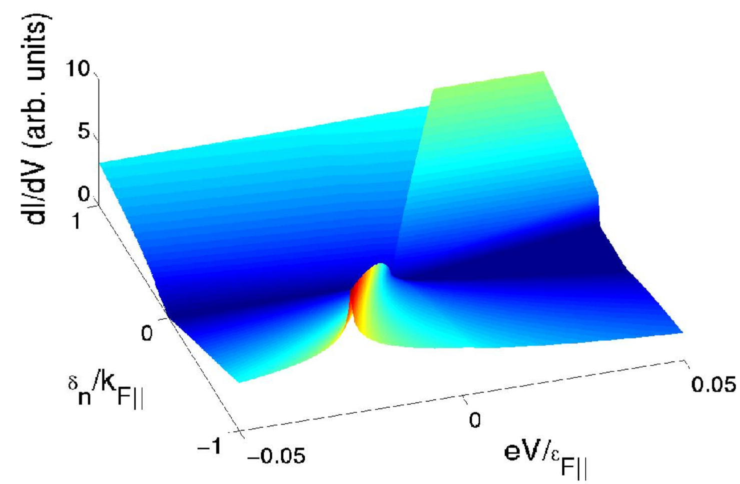
p_y conserved in tunneling!

(Huber et al. '02)

$$I(V) \sim \sum_{p_y} \int_0^{eV} d\epsilon \, A_\perp(\epsilon, p_y) \, \underbrace{\mathcal{N}_{1D}\left(\epsilon - eV - \frac{p_y^2}{2m}\right)}_{=\sum_{p_z} A_{||}(\epsilon - eV, \vec{p})}$$



Landau–level spectroscopy



chiral Luttinger liquids at $\nu = \frac{2}{3}, \frac{2}{5}$

UZ, Shimshoni, Governale '02

Quantum Hall ferromagnets

- for 2D electrons in GaAs heterostructures in a high (~ 10 Tesla) magnetic field, Zeeman splitting is small

cyclotron energy:	$\hbar\omega_{\rm c} \equiv 20$ K $\cdot\, B[$Tesla$]$	200 K
Coulomb exchange:	$e^2/\epsilon\ell \equiv 50$ K $\cdot \sqrt{B[\text{Tesla}]}$	150 K
Zeeman splitting:	$g\mu_{\rm B}B \equiv 0.5 \cdot B[$Tesla$]$	5 K

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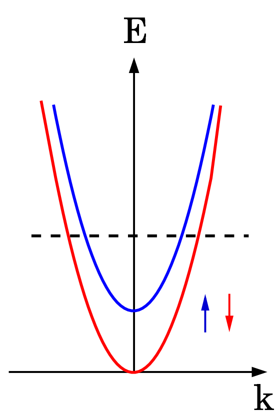
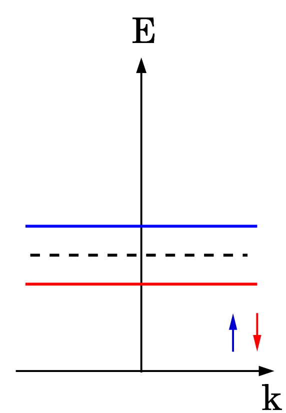
- electron system at $\nu = 1$ is 100% spin polarized even in the absence of Zeeman splitting: ideal ferromagnet





Quantum Hall ferromagnets

- low–lying excitation in typical ferromagnets: spin waves with wave vector $\vec{q}$; created by $S^-_{\vec{q}} = \sum_{\vec{k}} c^\dagger_{\vec{k}+\vec{q},\downarrow} c_{\vec{k},\uparrow}$

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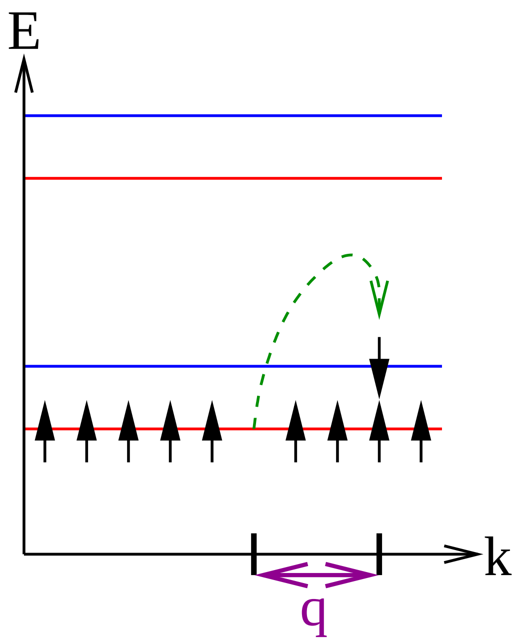

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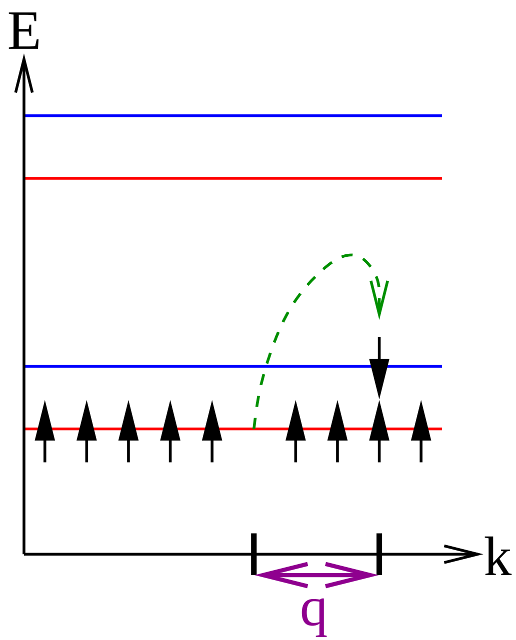
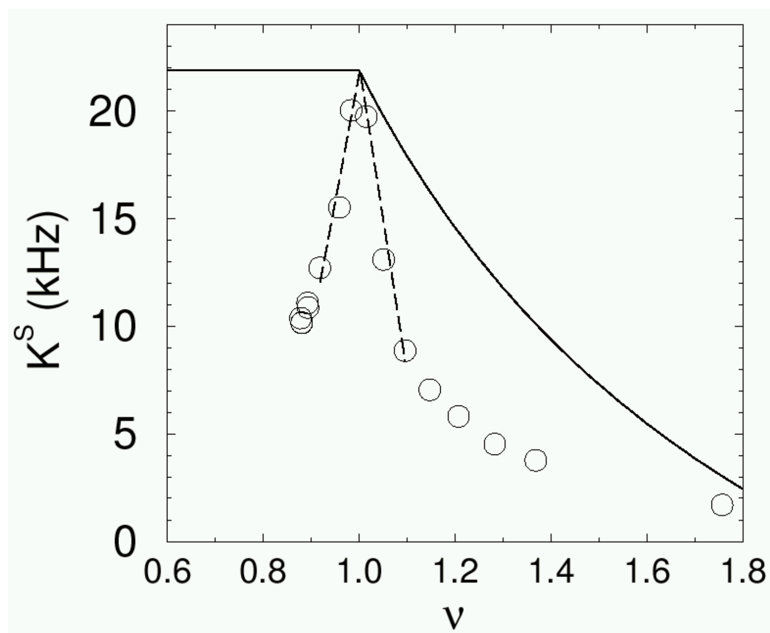
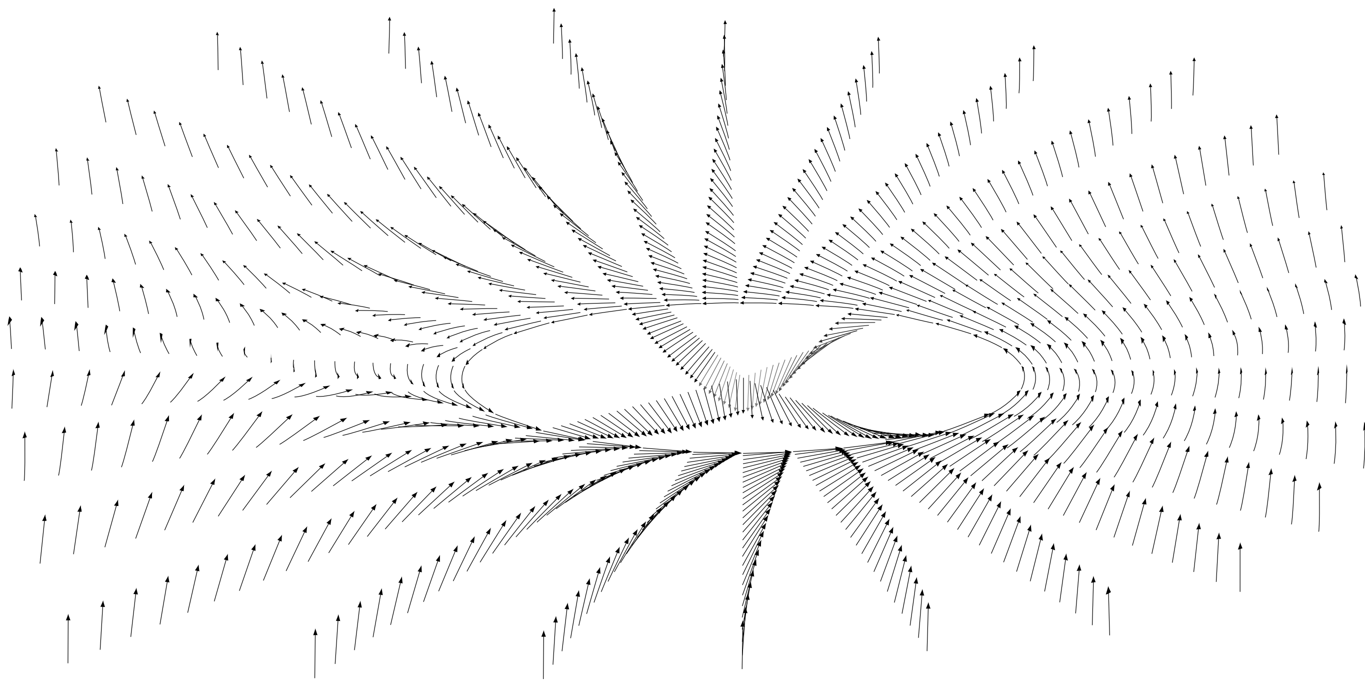




Barret et al. ’95

Skyrmions

- low–lying charged excitations in quantum Hall ferro–magnets: skyrmion spin textures $\vec{m}(\vec{r})$ Sondhi et al. '93



Skyrmions

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- spin texture results in additional (topological, Berry–phase) vector potential $\vec{A}_{\mathrm{B}}(\vec{r})$ such that

$$\vec{\nabla} \times \vec{A}_{\mathrm{B}} = \frac{\phi_0}{4\pi} \vec{m} \cdot (\partial_x \vec{m} \times \partial_y \vec{m})$$

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- skyrmion texture reduces effective magnetic flux by $\frac{1}{\phi_0} \int d^2r \; \vec{\nabla} \times \vec{A}_{\rm B} = Q[\vec{m}] \ldots$ integer, hence quantum Hall gap occurs at electron number shifted by $\nu Q[\vec{m}]$

$\Rightarrow$ skyrmion carries electric charge $\nu Q[\vec{m}]$ $\Leftarrow$

QHE in ultracold bose gases

- interacting atoms confined in a 2D harmonic trap: $H = \frac{\hbar\omega}{2}\sum_j \left\{ \left(\frac{\vec{p}_j \ell_\omega}{\hbar}\right)^2 + \left(\frac{\vec{r}_j}{\ell_\omega}\right)^2 \right\} + V_0 \sum_{i<j} \delta(\vec{r}_i - \vec{r}_j)$

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- when trap is rotating with frequency Ω: additional term; for $\Omega = \omega$: recover quantum-Hall Hamiltonian ($\omega_{\mathrm{c}} = 2\omega$)
- bose system: incompressible at $\nu = 1/2$ (minimization of contact–interaction energy possible only for $\nu < 1/2$)

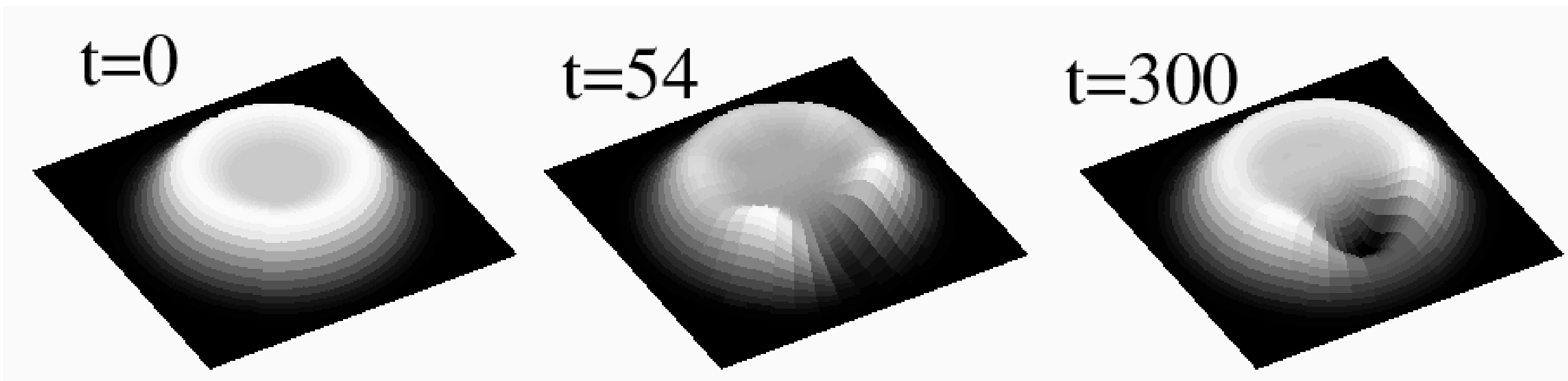
$$\Phi(z_1, \ldots, z_N) = Q(\{z_j\}) \prod_{i<j} (z_i - z_j)^2 \, e^{-\frac{1}{2}\sum_k |z_k|^2}$$

$\Rightarrow$ analog of fractional quantum Hall effect $\Leftarrow$

Cooper, Wilkin ’99; Wilkin, Gunn ’00

Creation of Laughlin quasiparticles

- intense Laser beam simulates disorder potential; create fractionally charged quasiparticle   Paredes et al. '01

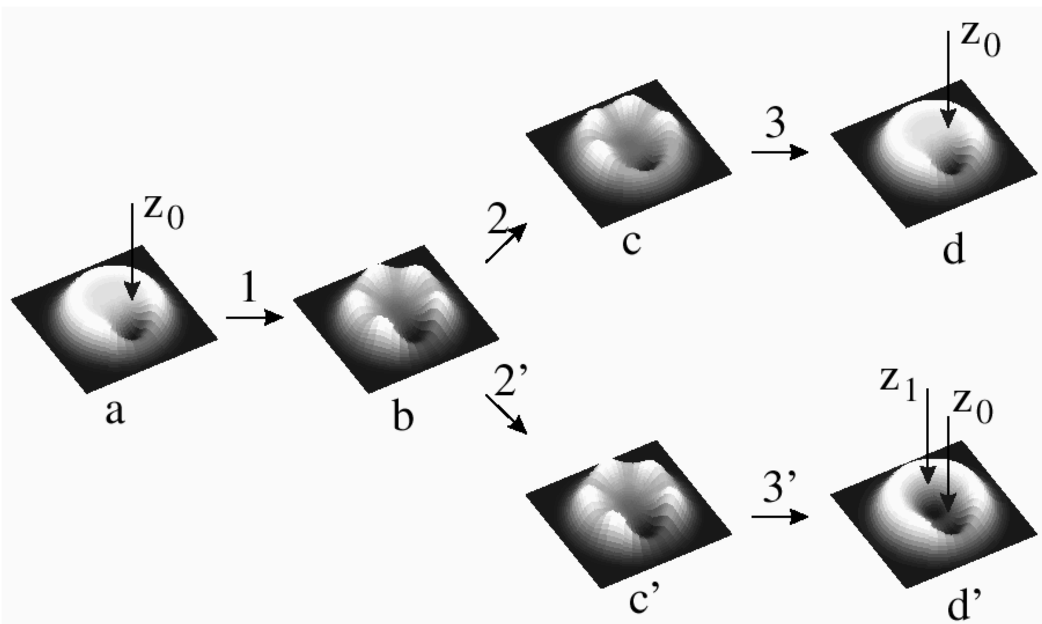


- direct spatial control: something condensed–matter experimentalists do not have!

Measurement of fractional statistics

- at intermediate Laser intensities: create superposition of initial state and state with one more quasiparticle
- use this to directly measure fractional statistics of Laughlin quasiparticles

Paredes et al. '01



Conclusions

- Basics of the quantum Hall effect
  - incompressibility $\Rightarrow$ quantized Hall resistance
  - integer effect: Landau quantization
  - fractional effect: interactions
  - finite plateau width due to disorder
- quenching of kinetic energy in the lowest Landau level gives rise to novel interaction effects: laboratory for multitude of correlated–electron states
- new avenues for the study of such effects in trapped cold atomic gases