Weak localization in Networks of diffusive wires

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1. Introduction
2. Transport theory in weakly disordered metals
3. Networks of diffusive wires
4. Conclusion

1. Introduction

- Average conductivity of a weakly disordered metal :

$$\langle \sigma \rangle = \underbrace{\sigma_0}_{\text{Drude (classical)}} + \underbrace{\langle \Delta\sigma \rangle}_{\text{quantum correction}}$$

- First measurements of weak localization correction :

  → quasi 1d wires (ex : Masden & Giordano, 1982)

  → thin films : AAS oscillations (cylinder) Sharvin & Sharvin, 1981.

  ⋮

  Bergmann, Phys.Rep. 1984.

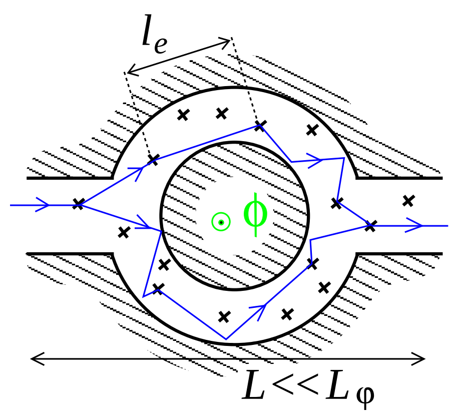
- Measurements on networks

  → From AB to AAS oscillations

  Umbach *et al*, 1986

  A weakly disordered ($k_F \ell_e \gg 1$) and phase coherent ($L \ll L_\varphi$) mesoscopic ring

h/e Aharonov-Bohm (AB) oscillations of the conductance ⇐



disorder average ⇓

$h/2e$ Altshuler-Aronov-Spivak (AAS) oscillations

→ First observation of AAS oscillations in networks :

honeycomb lattice : Pannetier *et al*, 1984.

Other experiments : rings, arrays of rings, square lattice...

Bishop, Dolan *et al* 1985, 1986.

Umbach *et al* 1986.

Chandrasekhar *et al* 1988.

⋮

- Theory for networks : Douçot & Rammal, 1985.

made more efficient by Pascaud & Montambaux, 1999.

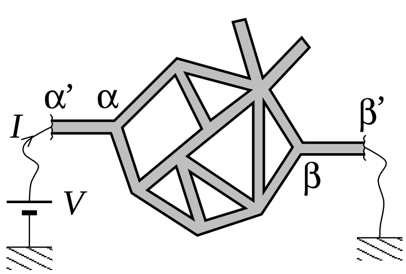
→ Quantum correction to conductivity :

$$\Delta\sigma = -\frac{e^2}{\pi}\frac{1}{\mathrm{Vol}}\int_{\mathrm{Network}} \mathrm{d}x\, P_c(x,x) \propto -\sum_{\mathrm{wire}\ (\mu\nu)} \int_{(\mu\nu)} \mathrm{d}x\, P_c(x,x)$$

Assumes the same weight for all wires.

⇓

theory of DRPM is only applicable to regular networks

- 

$$I/V = \frac{e^2}{h} T_{\alpha'\beta'}$$

$$\langle T_{\alpha'\beta'}\rangle = \overline{T}_{\alpha'\beta'} + \Delta T_{\alpha'\beta'} + \cdots$$

↓ classical

↓ quantum correction

How to take into account the correct weights ?

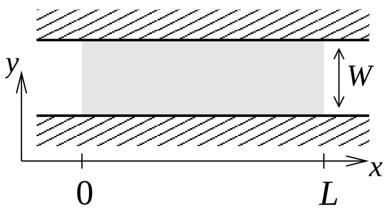
2. Transport theory of weakly disordered metals

- Kubo :

$$\sigma_{ij}(\vec{r},\vec{r}\,') = \frac{e^2}{2\pi m_e^2} G^{\mathrm{R}}(\vec{r},\vec{r}\,')\ \overset{\leftrightarrow}{\nabla}_i \overset{\leftrightarrow}{\nabla}{}'_j G^{\mathrm{A}}(\vec{r}\,',\vec{r})$$

with $\overset{\leftrightarrow}{\nabla} = \frac{1}{2}(\overset{\rightarrow}{\nabla} - \overset{\leftarrow}{\nabla})$

- Landauer :



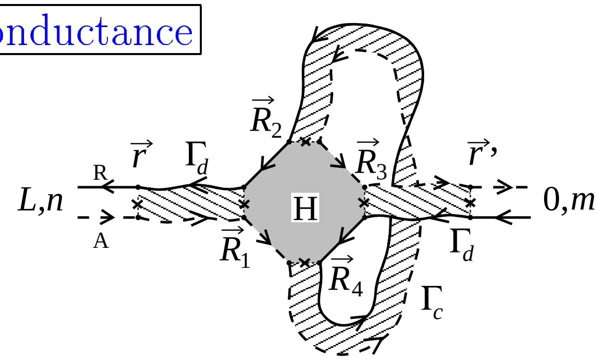
$$G = \int \mathrm{d}y\mathrm{d}y'\,\sigma_{xx}(L,y;0,y') = \frac{e^2}{h}\sum_{n,m} T_{nm}$$

with

$$T_{nm} = v_n v_m G^{\mathrm{R}}_{nm}(L,0)\, G^{\mathrm{A}}_{mn}(0,L)$$

Weak localization correction to conductance

$$\Delta T_{nm} =$$

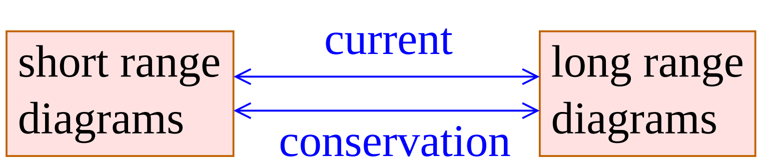


Hikami Box $\Rightarrow \Delta g = -\frac{1}{3} + \frac{1}{3} L\delta(0)$!

$\rightarrow$ A simple way to solve the problem :

Kane, Serota & Lee, 1988.

In the diagrammatic approach (classical transport, weak loc, UCF,...) :



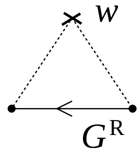
Theory of Disordered Metals

$\rightarrow$ Weakly disordered metals ($k_F \ell_e \gg 1$)

$$\langle V(\vec{r})V(\vec{r}\,')\rangle = w\,\delta(\vec{r}-\vec{r}\,')$$

- Average Green function $\overline{G}^{\mathrm{R}}$

$\frac{1}{2\tau_e} = -\,\mathrm{Im}\,\Sigma^{\mathrm{R}} =$



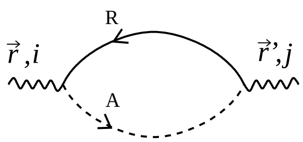
$\Rightarrow\ \overline{G}^{\mathrm{R}} \simeq G_0^{\mathrm{R}} \mathrm{e}^{-R/2\ell_e}$ (short range)

$\ell_e = v_F \tau_e$

Drude contribution $\overline{G}^{\mathrm{R}}\,\overline{G}^{\mathrm{A}}$

- Kubo :

$\langle \sigma_{ij}(\vec{r},\vec{r}\,')\rangle_{\mathrm{Drude}} =$

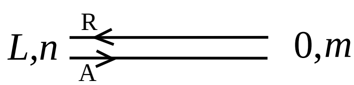


$= \sigma_0 \times \overline{\delta}_{ij}(\vec{r}-\vec{r}\,')$

$\longrightarrow \sigma_0 \times \delta_{ij}\,\delta(\vec{r}-\vec{r}\,')$

- Landauer :

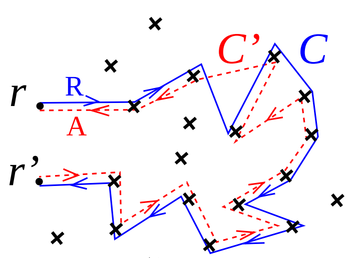
$\langle T_{nm}\rangle_{\mathrm{Drude}} =$



$\sim \mathrm{e}^{-L/\ell_e} \simeq 0$

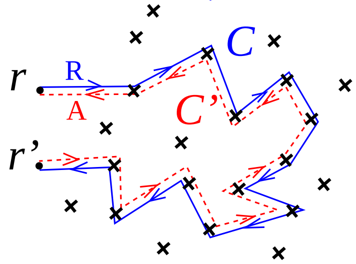
What is next in $\overline{G^{\mathrm{R}}G^{\mathrm{A}}} - \overline{G}^{\mathrm{R}}\,\overline{G}^{\mathrm{A}}$?

$C \neq C'$



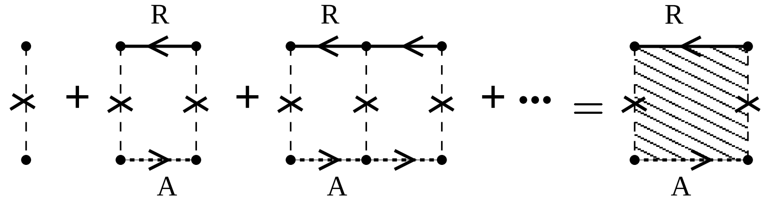
$\longrightarrow\ \overline{\mathrm{e}^{\mathrm{i}k_F[l(C)-l(C')]}} \sim \frac{1}{k_F\ell_e} \ll 1$

$C = C'$



$\longrightarrow\ \overline{\mathrm{e}^{\mathrm{i}k_F[l(C)-l(C')]}} = 1$

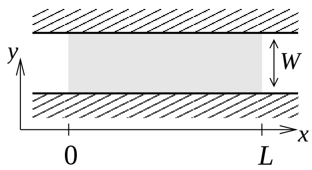
Contribution of the Diffuson

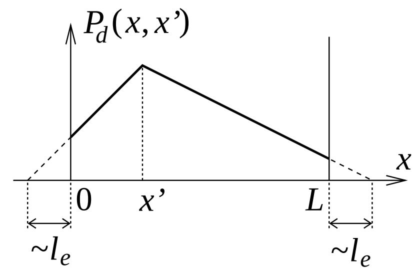
$P_d(\vec{r}, \vec{r}\,') =$ 

Diffusion approximation $\downarrow$

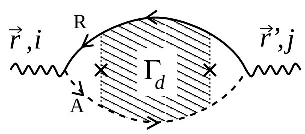
$$-\Delta P_d(\vec{r}, \vec{r}\,') = \delta(\vec{r} - \vec{r}\,')$$

boundary conditions :

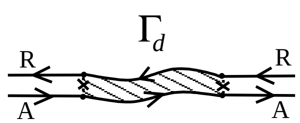




- Kubo :

$\langle \sigma_{ij}(\vec{r}, \vec{r}\,') \rangle_{\text{diffuson}} =$  $= -\sigma_0 \, \nabla_i \nabla'_j P_d(\vec{r}, \vec{r}\,')$

- Landauer :

$\overline{T}_{nm} =$ L,n  $0,m$ $= \frac{1}{\alpha_d N_c \ell_e} P_d\left(L - v_n \tau_e, v_m \tau_e\right)$

$$\Rightarrow \quad g_{\mathrm{D}} = \alpha_d N_c \frac{\ell_e}{L}$$

Current conservation :

Classical = Drude $\quad + \quad$ Diffuson

$\downarrow \qquad \downarrow$

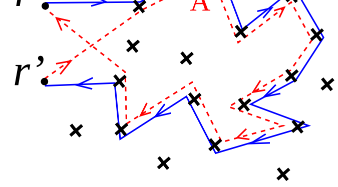
$$\langle \sigma_{ij}(\vec{r}, \vec{r}\,') \rangle_{\text{classical}} = \sigma_0 \left[\delta_{ij}\, \delta(\vec{r} - \vec{r}\,') - \nabla_i \nabla'_j P_d(\vec{r}, \vec{r}\,')\right]$$

$\downarrow \qquad \downarrow$

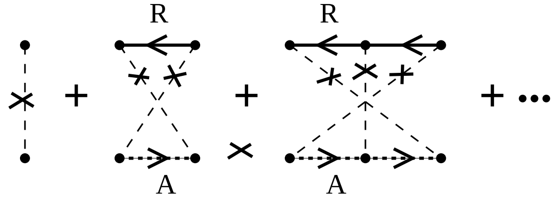
short range $\qquad$ long range

$\searrow \qquad \swarrow$

$$\sum_i \nabla_i \sigma_{ij}(\vec{r}, \vec{r}\,') = 0$$



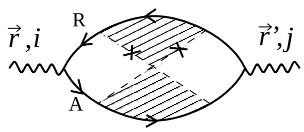
CONTRIBUTION OF THE COOPERON

$P_c(\vec{r},\vec{r}\,') =$ 

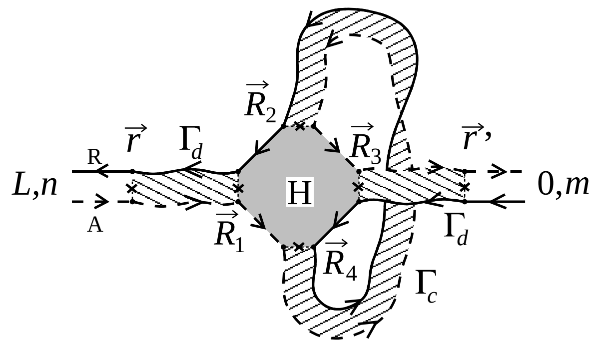
obeys $\left[\frac{1}{L_\varphi^2} - \left(\vec{\nabla} - 2\mathrm{i}e\vec{A}\right)^2\right] P_c(\vec{r},\vec{r}\,') = \delta(\vec{r}-\vec{r}\,')$

Gor'kov, A. I. Larkin & Khmel'nitzkiĭ, 1979.

- Kubo : (the short range term)

$\langle \sigma_{ij}(\vec{r},\vec{r}\,')\rangle_{\text{cooperon}} =$  $= -\frac{e^2}{\pi}\,\delta_{ij}\,\delta(\vec{r}-\vec{r}\,')\,P_c(\vec{r},\vec{r})$

- Landauer : (long range)

$\Delta T_{nm} =$ 

Hikami box : $H = \vec{\nabla}_1\cdot\vec{\nabla}_3 + \vec{\nabla}_2\cdot\vec{\nabla}_4 - \frac{1}{2}\sum_{i=1}^{4}\vec{\nabla}_i^2$

$\downarrow$

Divergency : $\vec{\nabla}_2\cdot\vec{\nabla}_4 P_c(\vec{r}_2,\vec{r}_4)|_{\vec{r}_2=\vec{r}_4} \sim \delta(\vec{0})$

- Use current conservation : Kane, Serota & Lee, 1988

$\rightarrow$ calculate the longe range term from the short range term :

$$\langle \Delta\sigma_{ij}(\vec{r},\vec{r}\,')\rangle = \sum_{i',j'}\int \mathrm{d}\vec{\rho}\,\mathrm{d}\vec{\rho}\,'\;\phi_{ii'}(\vec{r},\vec{\rho})\;\phi_{jj'}(\vec{r}\,',\vec{\rho}\,')\;\langle\sigma_{i'j'}(\vec{\rho},\vec{\rho}\,')\rangle_{\text{cooperon}}$$

where $\phi_{ij}(\vec{r},\vec{r}\,') = \delta_{ij}\,\delta(\vec{r}-\vec{r}\,') - \nabla_i\nabla'_j P_d(\vec{r},\vec{r}\,')$

corresponds to replace H by $2\vec{\nabla}_1\cdot\vec{\nabla}_3$

- Weak localization correction (Landauer) :

$\rightarrow$ Extract the long range term from $\langle \Delta\sigma_{ij}(\vec{r},\vec{r}\,')\rangle$

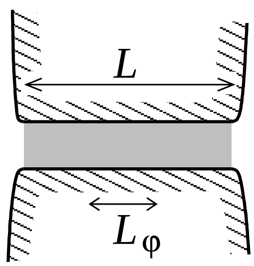
$$\Downarrow$$

$$\Delta T_{nm} = \frac{2}{(\alpha_d N_c)^2}\,\frac{1}{\ell_e^2}\int_0^L \mathrm{d}x\,\frac{\mathrm{d}}{\mathrm{d}x}P_d(L-v_n\tau_e,x)\;P_c(x,x)\;\frac{\mathrm{d}}{\mathrm{d}x}P_d(x,v_m\tau_e)$$

Conductance of the wire : $g=\frac{G}{e^2/h}=\sum_{n,m}T_{nm}$

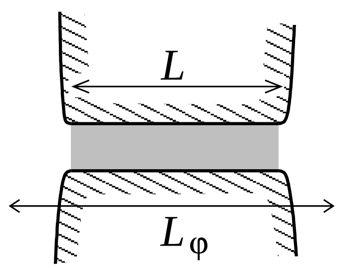
$$\Delta g = -\left(\frac{L_\varphi}{L}\right)^2\left(-1+\frac{L}{L_\varphi}\coth\frac{L}{L_\varphi}\right)$$

$\star$ Long wire



$\Rightarrow$ $\boxed{\Delta g \simeq -\frac{L_\varphi}{L}}$ $\quad (L \gg L_\varphi)$

$\star$ Short wire



$\Rightarrow$ $\boxed{\Delta g \simeq -\frac{1}{3}}$ $\quad (L \ll L_\varphi)$

Universal result : Mello & Stone, 1991.

EFFECT OF THE MAGNETIC FIELD

Altshuler & Aronov, 1981.

Solve

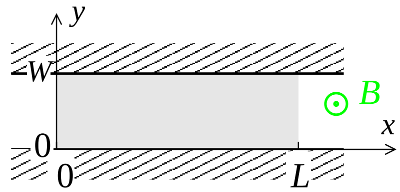
$$\left[\frac{1}{L_\varphi^2} - \left(\vec{\nabla} - 2ie\vec{A}\right)^2\right] P_c(\vec{r},\vec{r}\,') = \delta(\vec{r}-\vec{r}\,')$$

perturbatively in $\vec{A} = \vec{u}_x A_x(y)$

⋆ quasi 1d and diffusive limit : $L \gg W \gg \ell_e$.

⋆ weak magnetic field : $W \ll \sqrt{\hbar/e\mathcal{B}}$

choose the gauge $\boxed{A_x(W-y) = -A_x(y)}$:



$$\langle \vec{r}\,| \frac{1}{\gamma - (\vec{\nabla} - 2ie\vec{A})^2} |\,\vec{r}\,'\rangle$$

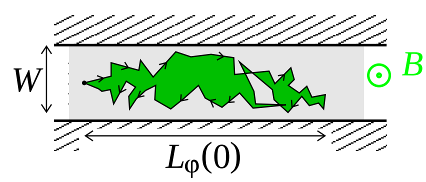
$$= \langle \vec{r}\,|\frac{1}{\gamma-\Delta}|\,\vec{r}\,'\rangle - 4e^2 \langle \vec{r}\,|\frac{1}{\gamma-\Delta}\vec{A}^2\frac{1}{\gamma-\Delta}|\,\vec{r}\,'\rangle + \cdots$$

$$\simeq \frac{1}{W}\langle x\,|\frac{1}{\gamma - \mathrm{d}_x^2}|\,x'\rangle - \frac{(e\mathcal{B}W)^2}{3}\frac{1}{W}\langle x\,|\frac{1}{(\gamma-\mathrm{d}_x^2)^2}|\,x'\rangle + \cdots$$

$$= \frac{1}{W}\langle x\,|\frac{1}{\gamma + (e\mathcal{B}W)^2/3 - \mathrm{d}_x^2}|\,x'\rangle$$

Effective phase coherence length :

$$\frac{1}{L_\varphi(\mathcal{B})^2} = \frac{1}{L_\varphi(0)^2} + \frac{1}{3}\left(\frac{e\mathcal{B}W}{\hbar}\right)^2$$



• Limit $W \ll \ell_e$ $\longrightarrow$ semiclassical methods

Dugaev & Khmel'nitzkiĭ, 1984.

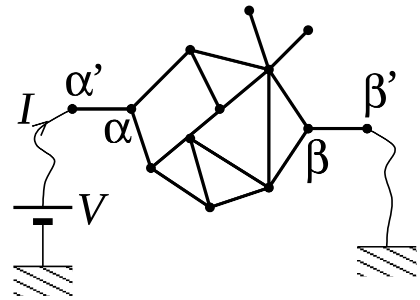
Beenakker & van Houten, 1988.

$$\frac{1}{L_\varphi(\mathcal{B})^2} = \frac{1}{L_\varphi(0)^2} + \tilde{C}e^2\mathcal{B}^2W^2\left(\frac{W}{\ell_e}\right)$$

3. Weak localization in Networks

$$\langle T_{\alpha'\beta'}\rangle = \overline{T}_{\alpha'\beta'} + \Delta T_{\alpha'\beta'} + \cdots$$

↙ classical ↘ quantum correction



- Classical transport :

$$\overline{T}_{\alpha'\beta'} = \frac{2}{\ell_e} P_d(\alpha', \beta')$$

- Weak localization correction :

$$\Delta T_{\alpha'\beta'} = \frac{2}{\ell_e^2} \int_{\text{Graph}} \mathrm{d}x \, \frac{\mathrm{d}}{\mathrm{d}x} P_d(\alpha', x) \; P_c(x, x) \; \frac{\mathrm{d}}{\mathrm{d}x} P_d(x, \beta')$$

→ Construct the solution of $(\gamma - \Delta)\, P(x, x') = \delta(x - x')$

⋆ Assume continuity of P at vertices

⋆ $\sum_\beta a_{\alpha\beta} P'_{(\alpha\beta)}(\alpha) = \lambda_\alpha P(\alpha)$

$\lambda_\alpha = 0$: internal vertex

$\lambda_\alpha = \infty$: connected vertex $\Rightarrow P(\alpha) = 0$

P_c involves

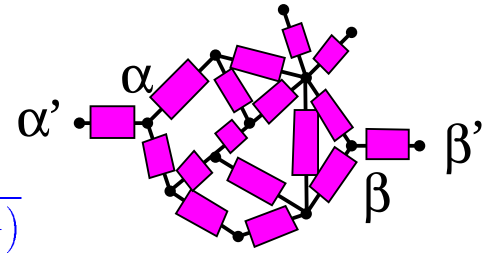
$$\mathcal{M}_{\alpha\beta} = \delta_{\alpha\beta}\left(\lambda_\alpha + \sqrt{\gamma}\sum_\mu a_{\alpha\mu}\coth(\sqrt{\gamma}l_{\alpha\mu})\right) - a_{\alpha\beta}\frac{\sqrt{\gamma}\,\mathrm{e}^{-\mathrm{i}\theta_{\alpha\beta}}}{\mathrm{sh}(\sqrt{\gamma}l_{\alpha\beta})}$$

$\downarrow$ $\gamma = 0$ & $\theta_{\alpha\beta} = 0$

P_d involves

$$(\mathcal{M}_0)_{\alpha\beta} = \delta_{\alpha\beta}\left(\lambda_\alpha + \sum_\mu a_{\alpha\mu}\frac{1}{l_{\alpha\mu}}\right) - a_{\alpha\beta}\frac{1}{l_{\alpha\beta}}$$

⋆ Classical transport : array of resistances



$$\overline{T}_{\alpha'\beta'} = \frac{2\ell_e}{l_{\alpha\alpha'}l_{\beta\beta'}}\left(\mathcal{M}_0^{-1}\right)_{\alpha\beta} = \frac{2\ell_e}{L_{\text{eff}}(\{l_{\mu\nu}\})}$$

$\rightarrow$ In $d>1 \Rightarrow \times\alpha_d N_c/2$

⋆ Weak localization correction

$$\Delta T_{\alpha'\beta'} = \frac{2}{l_{\alpha\alpha'}l_{\beta\beta'}}\sum_{(\mu\nu)}\frac{1}{l_{\mu\nu}^2}\left[-\left(\mathcal{M}_0^{-1}\right)_{\alpha\mu}+\left(\mathcal{M}_0^{-1}\right)_{\alpha\nu}+\delta_{\mu\alpha}\delta_{\nu\alpha'}\,l_{\alpha\alpha'}\right]$$
$$\times\left[-\left(\mathcal{M}_0^{-1}\right)_{\mu\beta}+\left(\mathcal{M}_0^{-1}\right)_{\nu\beta}+\delta_{\mu\beta}\delta_{\nu\beta'}\,l_{\beta\beta'}\right]$$
$$\times\int_{(\mu\nu)}\mathrm{d}y\,P_c(y,y)\,.$$

with

$$\int_{(\mu\nu)}\mathrm{d}y\,P_c(y,y) = \frac{1}{2\sqrt{\gamma}}\Bigg\{\left[\left(\mathcal{M}^{-1}\right)_{\mu\mu}+\left(\mathcal{M}^{-1}\right)_{\nu\nu}\right]\left(\coth\sqrt{\gamma}l_{\mu\nu}-\frac{\sqrt{\gamma}l_{\mu\nu}}{\mathrm{sh}^2\sqrt{\gamma}l_{\mu\nu}}\right)$$
$$+\left[\left(\mathcal{M}^{-1}\right)_{\mu\nu}\mathrm{e}^{\mathrm{i}\theta_{\mu\nu}}+\left(\mathcal{M}^{-1}\right)_{\nu\mu}\mathrm{e}^{\mathrm{i}\theta_{\nu\mu}}+\frac{\mathrm{sh}\sqrt{\gamma}l_{\mu\nu}}{\sqrt{\gamma}}\right]\frac{-1+\sqrt{\gamma}l_{\mu\nu}\coth\sqrt{\gamma}l_{\mu\nu}}{\mathrm{sh}\sqrt{\gamma}l_{\mu\nu}}\Bigg\}$$

After simplification

$$\Delta T_{\alpha'\beta'} = \frac{1}{\ell_e}\sum_{\text{wire }(\mu\nu)}\frac{\partial\overline{T}_{\alpha'\beta'}}{\partial\,l_{\mu\nu}}\int_{(\mu\nu)}\mathrm{d}y\,P_c(y,y)$$

$\rightarrow$ Incoherent networks ($l_{\mu\nu}\gg L_\varphi$) :

$$\Delta T_{\alpha'\beta'} \simeq -\frac{L_\varphi}{l_{\alpha\alpha'}l_{\beta\beta'}}\left(\mathcal{M}_0^{-1}\right)_{\alpha\beta} = -\frac{L_\varphi}{L_{\text{eff}}}$$

An heuristic argument :

$$1/T_{\alpha'\beta'} = \mathcal{R}_{\rm cl}(R_{\mu\nu}, \cdots)$$

Resistance of the wire $(\mu\nu)$: $\quad R_{\mu\nu} = \overline{R}_{\mu\nu} + \Delta R_{\mu\nu}$

- $\star$ Drude : $\overline{R}_{\mu\nu} = l_{\mu\nu}/(2\ell_e) \quad (d=1)$
- $\star$ Weak loc. : $\frac{\Delta R_{\mu\nu}}{\overline{R}_{\mu\nu}} = \frac{e^2}{\pi\sigma_0}\int_{\text{wire }(\mu\nu)} \frac{\mathrm{d}y}{l_{\mu\nu}} P_c(y,y)$.

$$\Delta R = \sum_{(\mu\nu)} \frac{\partial \mathcal{R}_{\rm cl}}{\partial R_{\mu\nu}} \Delta R_{\mu\nu} = \underset{\displaystyle 1/\ell_e \text{ (in } d=1)}{\underset{\displaystyle\downarrow}{\frac{e^2}{\pi\sigma_0}}} \sum_{(\mu\nu)} \frac{\partial \mathcal{R}_{\rm cl}}{\partial l_{\mu\nu}} \int_{(\mu\nu)} \mathrm{d}y\, P_c(y,y)$$

$\rightarrow$ Surprising that it works :

(i) starts from classical formula for transport
(no quantum interferences).

(ii) $\Delta T_{\alpha'\beta'}$ is global
contributions of the wires cannot be computed separatively :
the cooperon in a given wire depends on the whole network

Regular Networks

Douçot & Rammal, 1985.

Pascaud & Montambaux, 1999.

$\rightarrow$ start from

$$\langle \Delta\sigma \rangle = -\frac{e^2}{\pi}\frac{1}{\mathrm{Vol}}\int \mathrm{d}\vec{r}\, P_c(\vec{r},\vec{r})$$

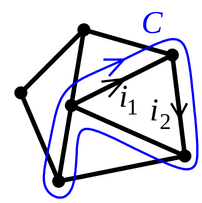
Spectral determinant : $S(\gamma) = \det(\gamma - \Delta) = \prod_n (\gamma + E_n)$,

$\{E_n\}$: eigenvalues of $-\Delta$ on the network.

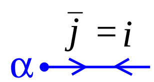
use : $\int \mathrm{d}\vec{r}\, P_c(\vec{r},\vec{r}) = \mathrm{Tr}\{\frac{1}{\gamma-\Delta}\} = \frac{\partial}{\partial\gamma}\ln S(\gamma)$

$$\boxed{\langle \Delta\sigma \rangle = -\frac{e^2}{\pi}\frac{1}{\mathrm{Vol}}\frac{\partial}{\partial\gamma}\ln S(\gamma)} \qquad \text{with} \qquad S(\gamma) = \prod_{(\alpha\beta)} \frac{\mathrm{sh}\sqrt{\gamma}\, l_{\alpha\beta}}{\sqrt{\gamma}} \det \mathcal{M}$$

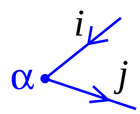
- Trace formula (Roth, 1983)



$$\epsilon_{ij} = \frac{2}{m_\alpha} - 1 \quad \text{if}$$



$$= \frac{2}{m_\alpha} \quad \text{if}$$



$$= 0 \quad \text{otherwise}$$

$$S(\gamma) = \gamma^{\frac{V-B}{2}} \mathrm{e}^{\sqrt{\gamma}\mathcal{L}} \underbrace{\prod_{\tilde{C}}}\left(1 - \alpha(\tilde{C})\mathrm{e}^{-\sqrt{\gamma}\, l(\tilde{C})}\right)$$

product over primitive orbits

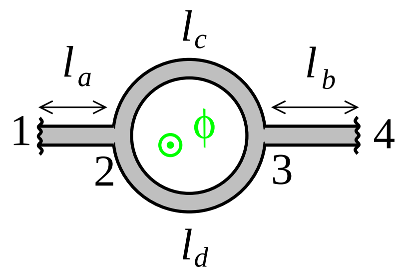
$$\alpha(C) = \epsilon_{i_1 i_2}\epsilon_{i_2 i_3}\cdots\epsilon_{i_n i_1} \ : \ \text{weight of orbit } C = (i_1, i_2, \cdots, i_n).$$

$$\frac{\partial}{\partial\gamma}\ln S(\gamma) = \frac{\mathcal{L}}{2\sqrt{\gamma}} + \frac{V-B}{2\gamma} + \frac{1}{2\sqrt{\gamma}}\sum_{\mathcal{C}} l(\tilde{\mathcal{C}})\alpha(\mathcal{C})\mathrm{e}^{-\sqrt{\gamma}\, l(\mathcal{C}) + \mathrm{i}\theta(\mathcal{C})}$$

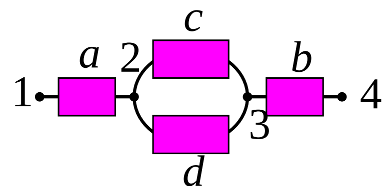
Akkermans, Comtet, Desbois, Montambaux & Texier, 2000.

APPLICATIONS

A. Ring :



$\star$ Classical transport :

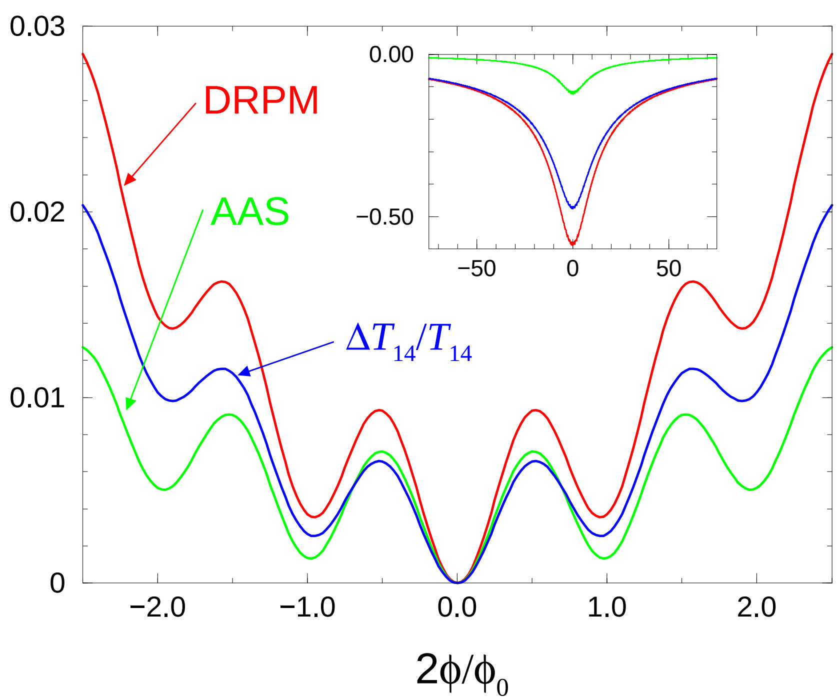


$$\overline{T}_{14} = \frac{2\ell_e}{l_a + l_{c//d} + l_b}$$

$\star$ Weak localization correction : (for $l_a = l_b$ & $l_c = l_d$)

$$\Delta T_{14} = -\frac{1}{(l_a + l_c/4)^2}\left[\int_{\mathtt{wire}\ a} \mathrm{d}y\, P_c(y,y) + \frac{1}{4}\int_{\mathtt{wire}\ c} \mathrm{d}y\, P_c(y,y)\right]$$

Comparison with theory of DRPM

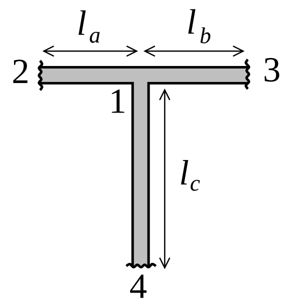


$l_c = 3.6\ \mu$m, $W = 0.19\ \mu$m, $L_\varphi(0) = 1.7\ \mu$m

(values from Chandrasekhar *et al*, 1988) and $l_a = 1\ \mu$m.

B. Wire with arms :

- one long arm



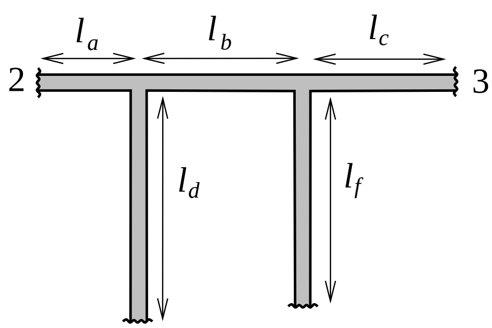
$$\overline{T}_{23} = 2\ell_e \frac{l_c}{l_a l_b + l_b l_c + l_c l_a} \qquad \Rightarrow \qquad \boxed{\frac{\partial \overline{T}_{23}}{\partial l_c} > 0}$$

→ In the fully coherent limit $L_\varphi \to \infty$

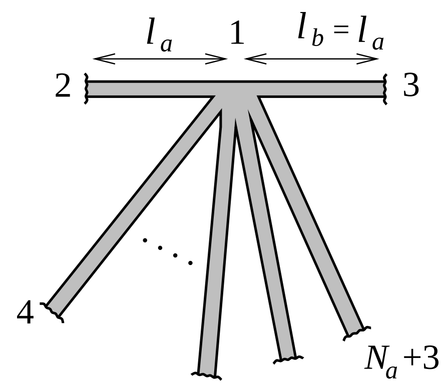
$$\Delta T_{23} = -\frac{1}{3} \frac{1 + \frac{l_{a//b}}{l_c} - \frac{l_{a//b}}{l_a + l_b}}{\left(1 + \frac{l_{a//b}}{l_c}\right)^2} \quad \xrightarrow[l_c \gg l_a, l_b]{} \quad \Delta T_{23} \simeq \frac{1}{3}\left(-1 + \frac{l_{a//b}}{l_a + l_b}\right)$$

↓ wire ↓ arm

- How to make $\Delta T_{23} > 0$?



$\Rightarrow$



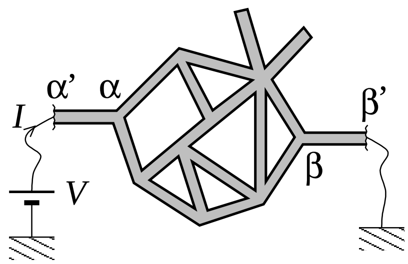
For $l_a \ll l_{\rm arm} \ll L_\varphi$:

$$\Delta T_{23} \simeq \frac{1}{3}\left(-1 + \frac{N_a}{4}\right)$$

→ Purely geometrical effect

4. Conclusion

- Weak localization corrections $\Delta T_{\alpha'\beta'}$ to the transmissions



$$\langle T_{\alpha'\beta'}\rangle = \overline{T}_{\alpha'\beta'} + \Delta T_{\alpha'\beta'} + \cdots$$

- Followed Kane, Serota & Lee to construct a $\Delta T_{\alpha'\beta'}$ free of divergency
- The weight of the wire $(\mu\nu)$ is $\frac{\partial \overline{T}_{\alpha'\beta'}}{\partial l_{\mu\nu}}$
- Comparison with theory of DRPM :

  ⋆ for a ring

  → no spectacular effect

  → extract different $L_\varphi(0)$ in experiments

  ⋆ New effect for a wire with arms : ΔT_{23} can change sign