Persistent Current and Hall effect driven by Spin Chirality

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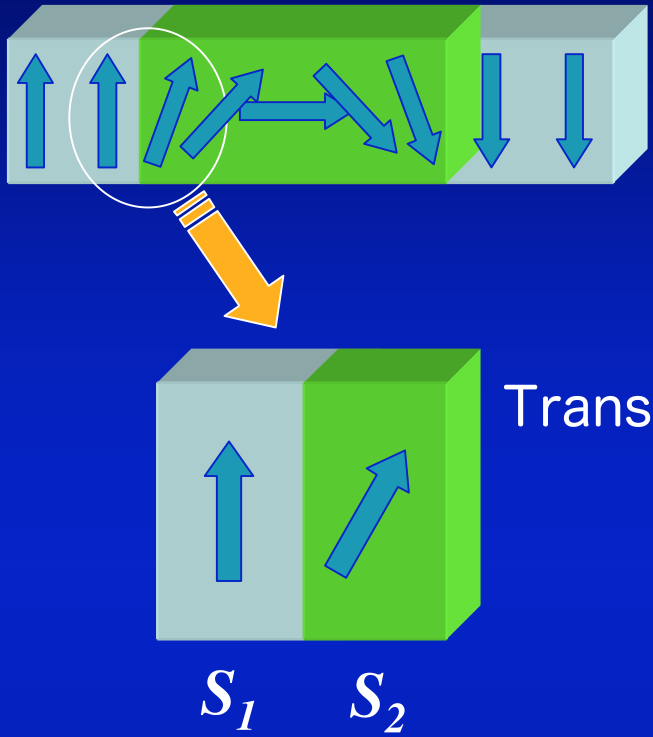
spin Josephson effect

•Electron transport through non-uniform magnetization



Transmission amplitude

modified by $S_1 \cdot S_2$ → resistance

This is not all

potential due to magnetization S $S \cdot \sigma$

amplitude of L→ R

$$(S_1 \cdot \sigma)(S_2 \cdot \sigma) = S_1 \cdot S_2 + i(S_1 \times S_2) \cdot \sigma$$

charge spin

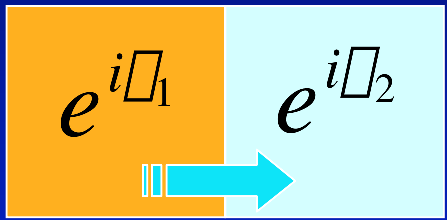
$$(S_1 \cdot \sigma)(S_2 \cdot \sigma) - (S_2 \cdot \sigma)(S_1 \cdot \sigma) = 2i(S_1 \times S_2) \cdot \sigma$$

equilibrium spin current

Konig, Bonsager & MacDonald (2001)

- Spin Josephson effect

- Superconducting junction

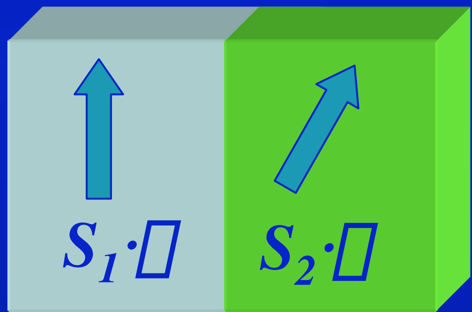


current

$$J \quad \square sin(\square_1 \square \square_2)$$

Josephson effect

- Ferromagnetic junction



Spin current

$$(S_1 \cdot \square)\,(S_2 \cdot \square) = S_1 \cdot S_2 + i(S_1 \square S_2) \cdot \square$$

$$J \quad (S_1 \square S_2) \cdot \square$$

Spin Josephson effect

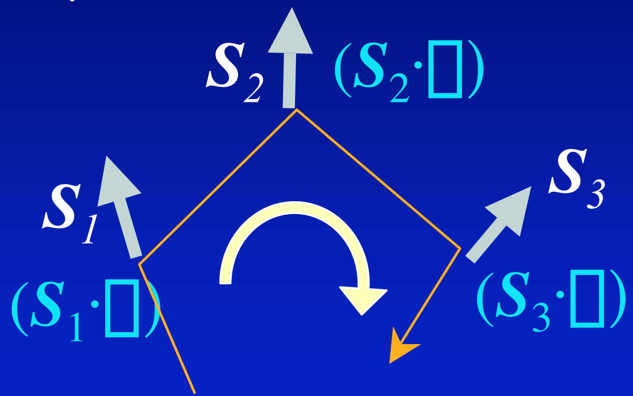
Current driven by SU(2) phase ⟹ spin and charge current

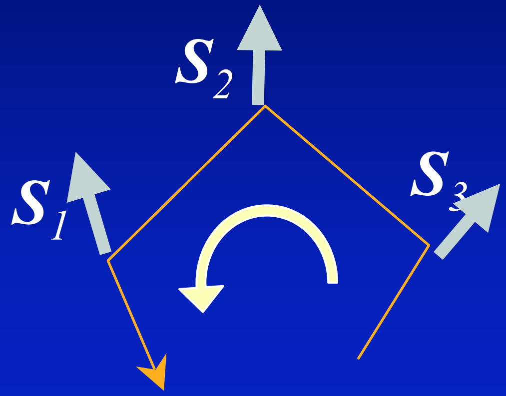
$$e^{i\square(S \cdot \square)/2}$$

- Charge current by spin Josephson effect

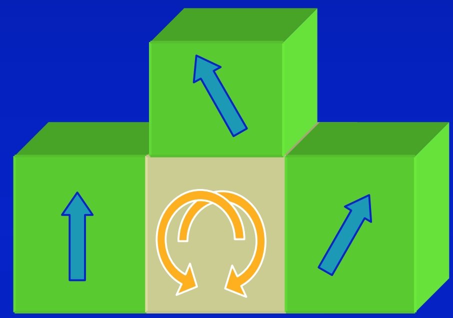
GT&Kohno, Phys. Rev. B67, 113316 (2003).

- 3 spins





$(S_1\cdot\sigma)\,(S_2\cdot\sigma)\,(S_3\cdot\sigma)$ $\qquad$ $(S_3\cdot\sigma)\,(S_2\cdot\sigma)\,(S_1\cdot\sigma)$



<u>These are not equal</u> $\quad \mathrm{tr}[\sigma_i\,\sigma_j\,\sigma_k]=2i\varepsilon_{ijk}$

$$\mathrm{tr}[(S_1\cdot\sigma)\,(S_2\cdot\sigma)\,(S_3\cdot\sigma)-(S_3\cdot\sigma)\,(S_2\cdot\sigma)\,(S_1\cdot\sigma)]=4i\,S_1\cdot(S_2\times S_3)$$

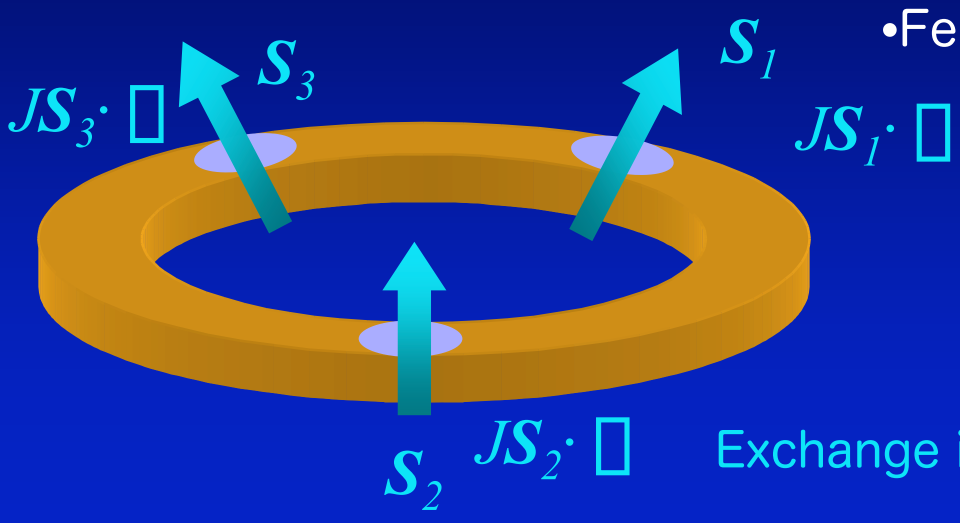
spin chirality

- spin chirality □ breaking of time reversal symmetry in orbital motion

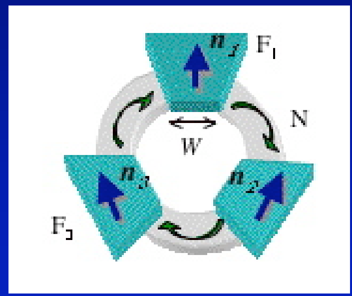
→ Spontaneous charge current $S_1\cdot(S_2\times S_3)$

<u>If electron is coherent</u>

- Persistent current in nanoscale ring



- Quantum dots
- Ferromagnets



Exchange interaction

- current

$$j = -\frac{\hbar e J^3}{m} S_1 \cdot (S_2 \times S_3) \sum_{X_i} \int \frac{d\epsilon}{2\pi} f(\epsilon) \, \mathrm{Im}(g^r_{x1} g^r_{12} g^r_{23} g^r_{3x'}) \,|_{x'=x}$$

$$j = -2e\frac{v_F}{L}\cos(k_F L)\left(\frac{J}{\epsilon_F}\right)^3 S_1 \cdot (S_2 \times S_3)$$

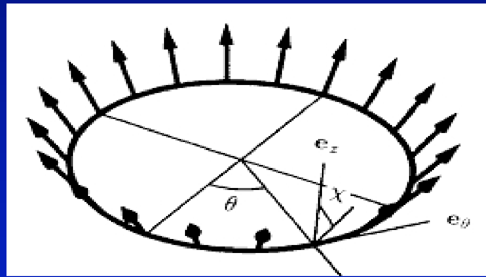
GT&Kohno PRB (2003)

•Relation to the adiabatic case

・Loss, Goldbart& Balatsky PRL (1990)

Persistent current by spin Berry phase $\Phi = \int d^2x\, \boldsymbol{S}\cdot(\partial_x \boldsymbol{S} \times \partial_y \boldsymbol{S})$

Only in adiabatic limit
(strong coupling to
a smooth spin structure)



•Our result

$$j = -2e\frac{v_F}{L}\cos(k_F L)\left(\frac{J}{\epsilon_F}\right)^3 S_1\cdot(S_2\times S_3)$$

$\boldsymbol{S}_1\cdot(\boldsymbol{S}_2\times\boldsymbol{S}_3)$ ⟹ $\int d^2x\, \boldsymbol{S}\cdot(\partial_x \boldsymbol{S}\times\partial_y \boldsymbol{S}) = \Phi$

Continuum limit

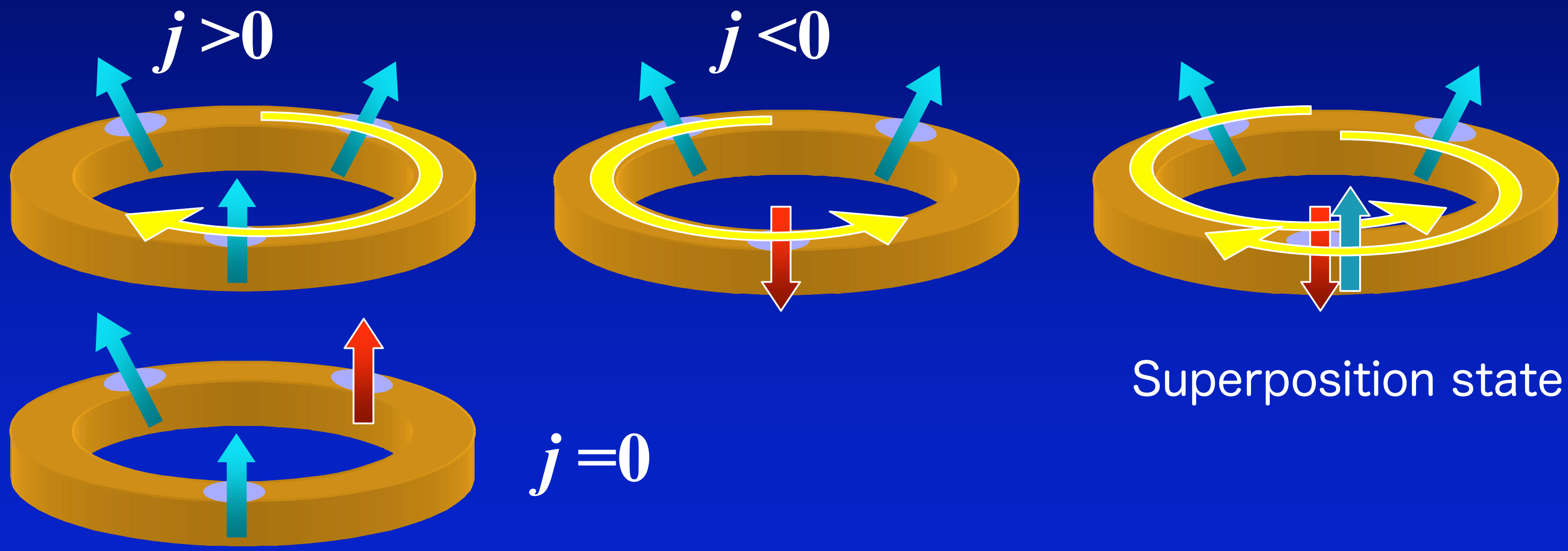
spin chirality spin Berry phase

spin chirality = non-adiabatic (perturbative) analog of Berry phase

Extension of Berry phase to non-adiabatic case

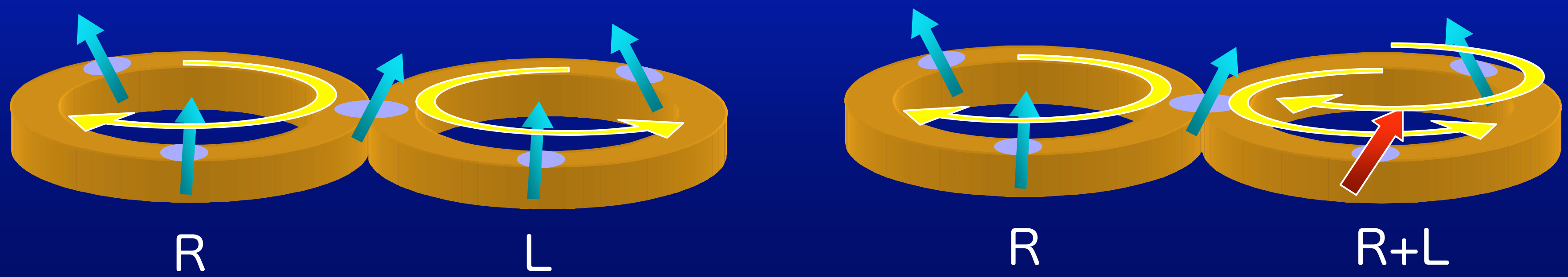
- Application to operation gates

GT and Garcia, PRL 91, 076806 (2003).



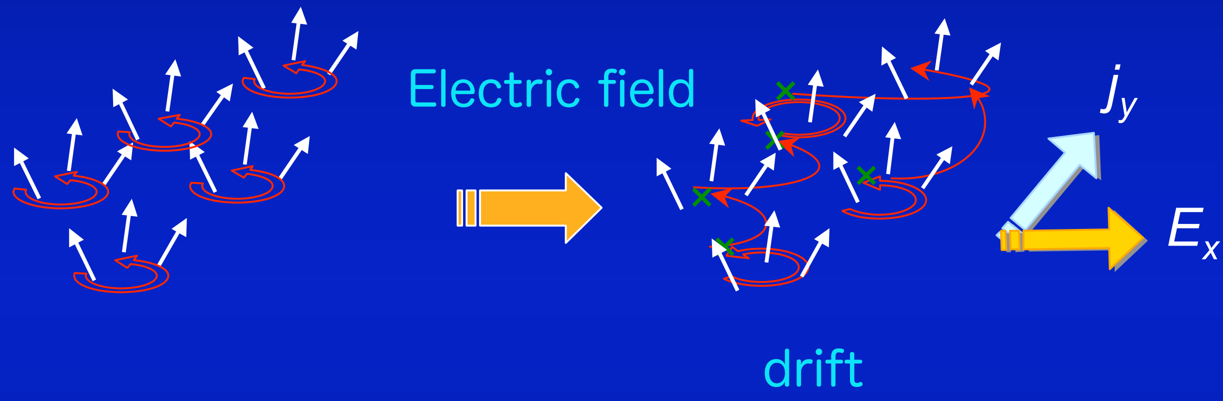
$$|j| = \mathrm{XOR}[S_2, S_3]$$

- Unitary operation



- Hall effect due to local persistent current
  - Frustrated magnets
    - finite local spin chirality

⇒ Local orbital motion of electron



Anomalous Hall effect

$$\square_{xy} \quad \langle S_1 \cdot (S_2 \square S_3) \rangle$$

• Hall conductivity

GT&Kawamura J.Phys. Soc.Jpn. (2002)

• Exchange interaction

$$H' = J \sum_X \mathbf{S}_X \cdot (c^\dagger \vec{\sigma} c)_X$$

• Kubo formula

3rd order in J

cf : Kondo'62

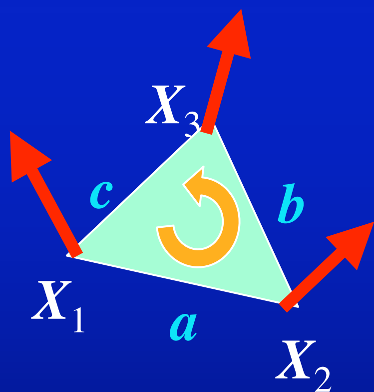
$$\sigma_{xy} \propto J^3 \tau^2 \chi_0$$

τ: lifetime (impurity)

• total chirality χ_0

$$\chi_0 = \frac{1}{3N} \sum_{\mathbf{x}_i} \mathbf{S}_{\mathbf{x}_1} \cdot (\mathbf{S}_{\mathbf{x}_2} \times \mathbf{S}_{\mathbf{x}_3}) \frac{(\mathbf{a} \times \mathbf{b})_z}{ab} I'(a) I'(b) I(c)$$

geometrical weight



$$I(r) = \frac{\sin k_F r}{k_F r} e^{-r/2\ell}$$

RKKY

Finite χ_0 ➡ Hall effect

•Adiabatic limit

Ye et al., PRL (1999).

•Magnetic vortex

$$\Phi_{xy} = \int d^2x\, \vec{S}\cdot(\partial_x \vec{S}\times\partial_y \vec{S})$$

Berry phase ➡ Hall effect

Adiabaticity was believed to be essential

•Quantiztion of Hall conductivity in the adiabatic limit

Ohgushi,Murakami & Nagaosa (2000)

•GT&Kawamura (2002)

Extension of Berry phase effect to non-adiabatic case

•Hall conductivity as a topological invariant
in the perturbative case

If spin texture is smooth (compared with mean free path l)

$$\sigma_{xy} = \frac{e^2}{h} \square \int d\mathbf{x} \frac{1}{4\pi} \mathbf{S} \cdot (\partial_x \mathbf{S} \times \partial_y \mathbf{S}) = \frac{e^2}{h} n \square$$

n: integer

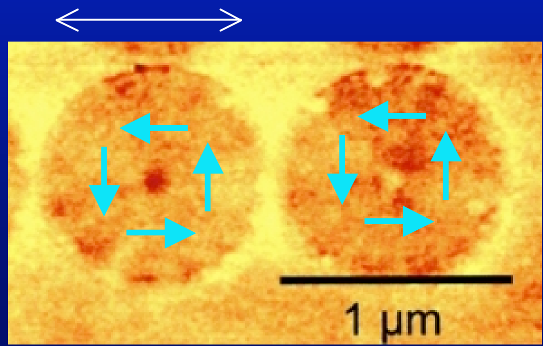
Topological invariant

Quantization occurs even in the non-adiabatic case

GT,Yamanaka &Onoda (2003)

$\square \quad (J\square)^3 (l/k_F a^2)$ Coupling factor

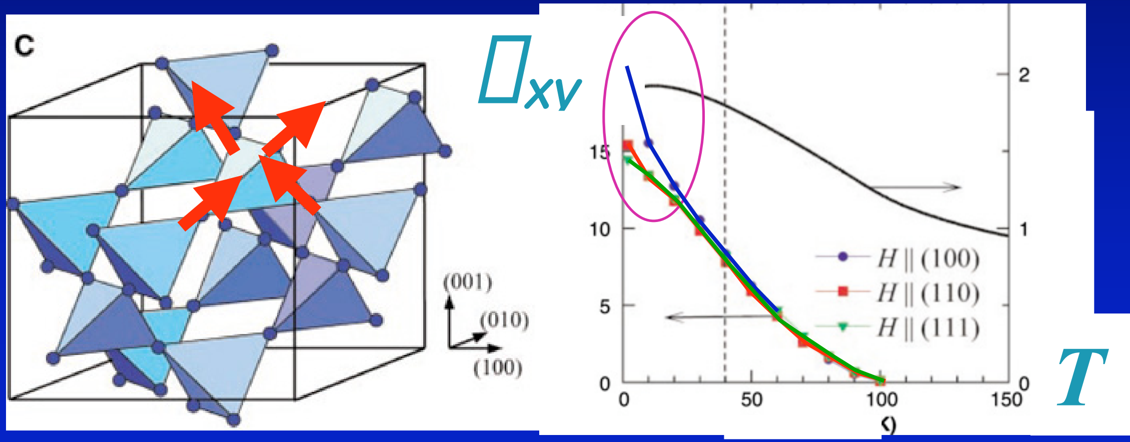
a: vortex size



Shinjo et al., (2000)

- experiment
  - Frustrated ferromagnets

    $Nd_2Mo_2O_7$ pyrochlore — Taguchi et al., *Science* (2001)

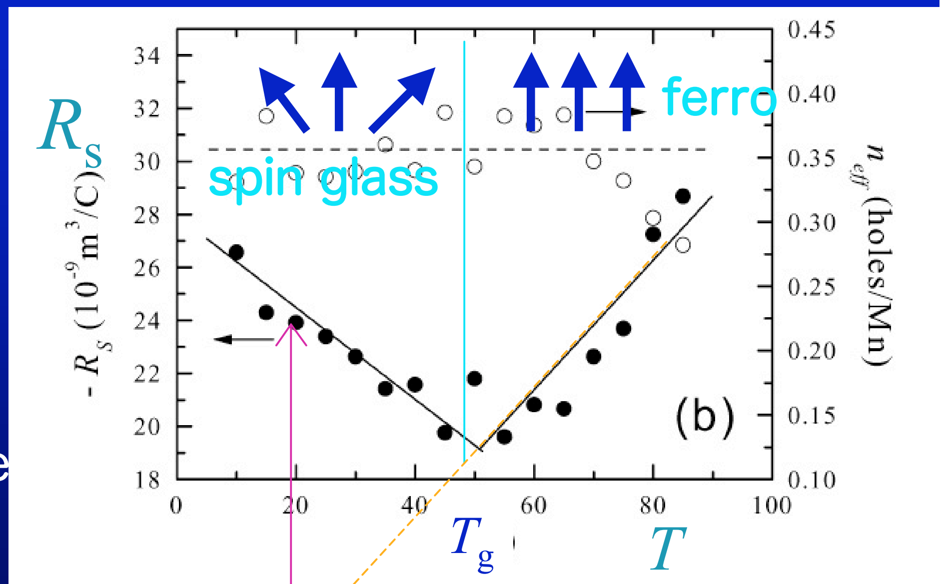


  - spin glass systems

    $La_{1.2}Sr_{1.8}Mn_2O_7$ Chun et al. ('00)

    reentrant spin glass below T_g~40K

    new contribution in spin glass phase



Summary

spin chirality →
- spontaneous persistent current
- anomalous Hall effect

spin Josephson effect

References

- G.Tatara and H. Kohno, Phys. Rev. B67, 113316 (2003).
- G.Tatara and N.Garcia, Phys. Rev. Lett. 91, 076806 (2003)
- G. Tatara and H. Kawamura J.Phys. Soc.Jpn. 71, 2613 (2002)
- G. Tatara,M. Yamanaka and M.Onoda (2003)