Magnetoresistance in parallel fields

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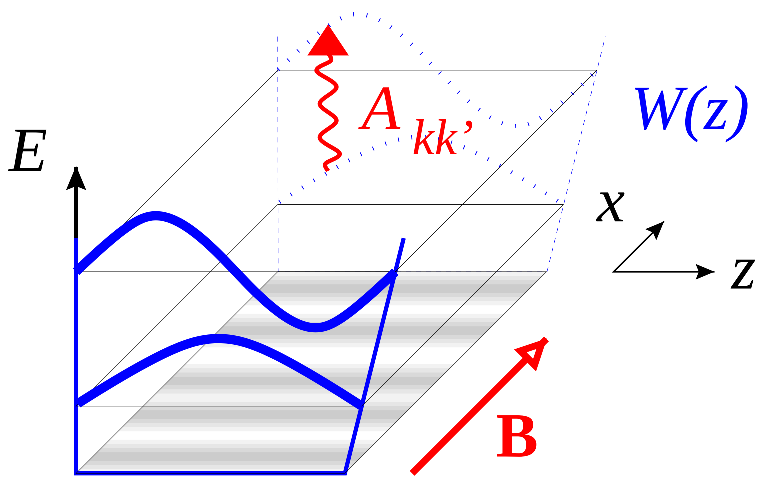


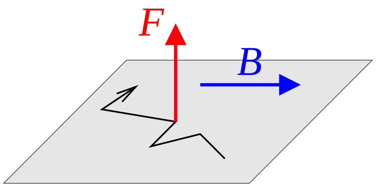
JSM, A. Altland, and B.L. Altshuler, PRL **89**, 206601 (2002);
JSM, V.I. Fal'ko, and B.L. Altshuler, cond-mat/0206024.

ISSP – August 18, 2003

Motivation I

main focus of previous works:

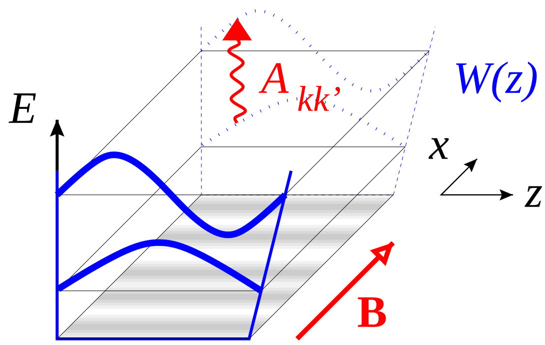
- orbital effect $\longrightarrow$ perpendicular fields
- parallel fields $\longrightarrow$ spin effects



purely $2d$ system:

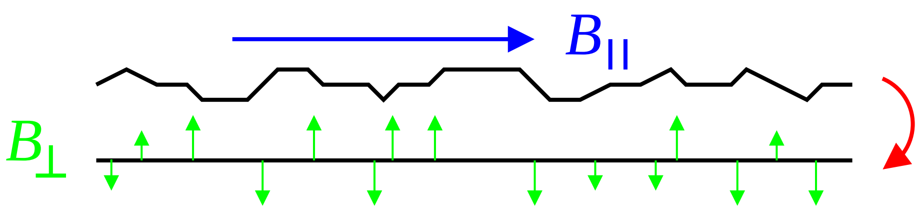
no magnetoresistance due to the orbital effect in parallel fields

$\longleftrightarrow$ 2DEGs
(subband structure)



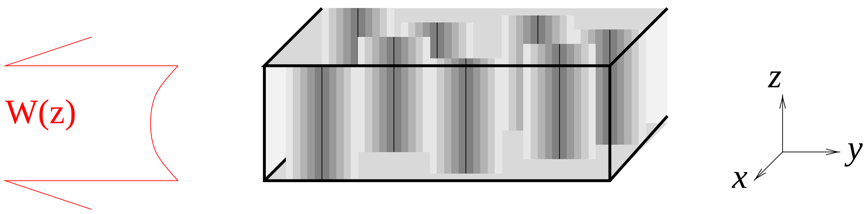
[Falko, J. Phys.: Cond. Matt. **2**, 3797 ('90);

Mathur, Baranger, Phys. Rev. B **64**, 235325 ('01).]



Motivation II

AlGaAs/GaAs heterostructures:
- clean interfaces and
- weak z-dependence of impurity scattering



$\longrightarrow$ z-inversion ($\mathcal{P}_z$) symmetry?
magnetoresistance extremely sensitive to $\mathcal{P}_z$
$\Rightarrow$ probe properties of 2DEG

special case: 1 occupied subband

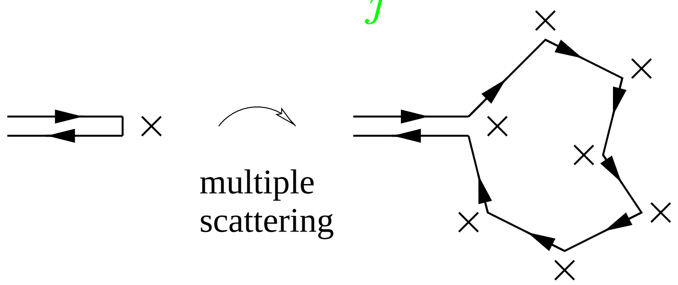
Outline

- ▷ Weak localization & magnetoresistance
- ▷ Spin effects
- ▷ Berry-Robnik symmetry effect
- ▷ Subband structure & the Cooperon matrix
- ▷ Results:
  - ▷ $M > 1$
  - ▷ $M = 1$: virtual processes
- ▷ Conclusions

REMINDER: Weak localization & magnetoresistance I

consider return probability $P(\mathbf{r},\mathbf{r})$
$\rightarrow$ sum over paths k with amplitude $A_j = a_j\, e^{i\varphi_j}$

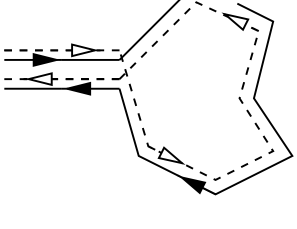
$$P(\mathbf{r},\mathbf{r}) = \Big|\sum_j A_j\Big|^2$$



due to the random phases φ_j of different paths, interference terms in general average out

$\Rightarrow$ classical result: $P_{\text{cl}}(\mathbf{r},\mathbf{r}) = \sum_j \big|A_j\big|^2$

however:
additional averaging-insensitive contribution due to (constructive) interference of time-reversed paths



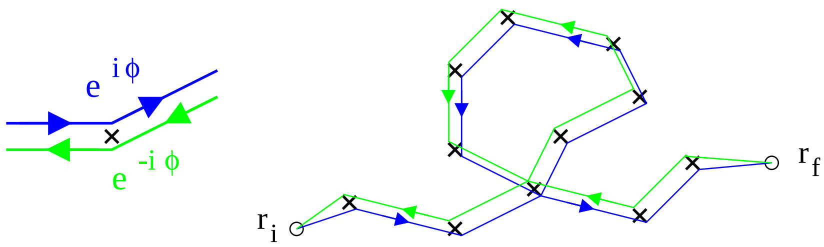
$$\begin{aligned} & |A_k + A_{\bar{k}}|^2 \\ = \; & |A_k|^2 + |A_{\bar{k}}|^2 + A_k^* A_{\bar{k}} + A_{\bar{k}}^* A_k \\ = \; & 4|A_k|^2 \end{aligned}$$

thus, $P(\mathbf{r},\mathbf{r}) = 2P_{\text{cl}}(\mathbf{r},\mathbf{r})$

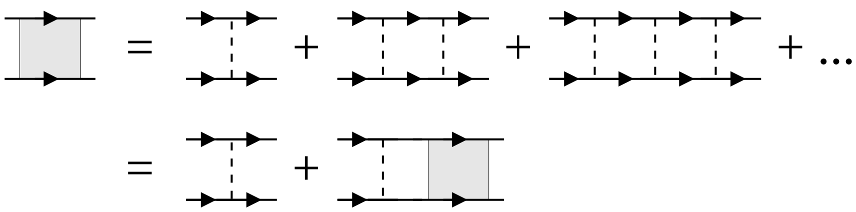
REMINDER: Weak localization & magnetoresistance II

enhanced return probability
$\leftrightarrow$ reduced conductance

corrections to classical Drude conductivity:



interference of time-reversed paths – Cooperon



$$\mathcal{C}(\mathbf{q},\omega) \sim \frac{1}{D\mathbf{q}^2 - i\omega + \frac{1}{\tau_\phi}} \quad \Rightarrow \quad \delta\sigma \sim \int d^2q\, \mathcal{C}(\mathbf{q},0)$$
$$\sim \ln(\tau/\tau_\phi)$$

D diffusion constant
τ_ϕ phase coherence time
τ elastic scattering time

temperature-dependence of $\tau_\phi \propto T^{-p}$
$\Rightarrow$ temperature-dependence of $\delta\sigma(T) \sim p \ln T$

REMINDER: Weak localization & magnetoresistance III

magnetic field breaks time-reversal invariance (electrons pick up a phase factor $\exp[\pm i \int \mathbf{A}\, d\mathbf{r}]$): phase difference proportional to enclosed flux

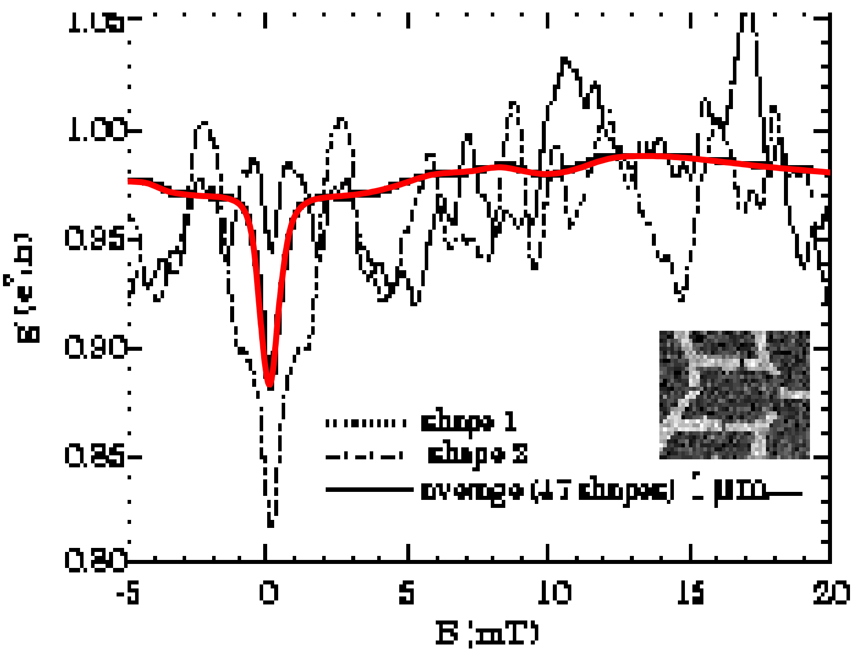
$$2 \oint \mathbf{A}\, d\mathbf{r} = 2 \int \mathbf{H}\, d\mathbf{F} = 2\Phi$$

$\Rightarrow$ limits phase coherent propagation and suppresses interference phenomena

$\Rightarrow$ Cooperons short-ranged ('massive'):

$$1/\tau_\phi \quad \rightarrow \quad 1/\tau_\phi(H) = 1/\tau_\phi + 1/\tau_H$$

$\Rightarrow$ negative magnetoresistance



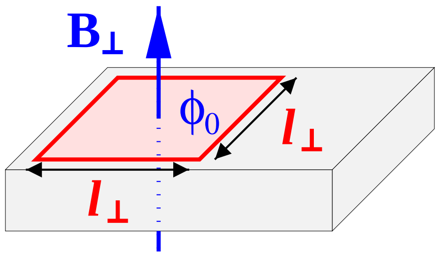
Parallel vs perpendicular fields

relevant field scale:

1 flux quantum through typical area

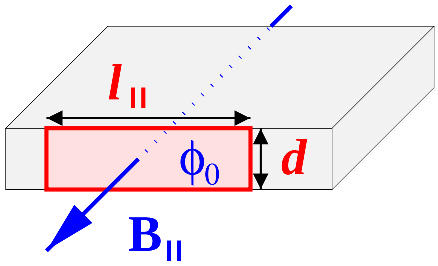
$\longrightarrow$ 'magnetic length' l_H

perpendicular magnetic field:



$$H_\perp l_\perp^2 = \phi_0 \quad \Rightarrow \quad l_\perp = \sqrt{\phi_0/H_\perp} \quad \Rightarrow \quad 1/\tau_{H_\perp} \propto H_\perp$$

parallel magnetic field:



$$H_\parallel l_\parallel d = \phi_0 \quad \Rightarrow \quad l_\parallel = \phi_0/(H_\parallel d)$$

- different from conventional magnetic length
- geometry dependent

$$\Rightarrow \quad 1/\tau_{H_\parallel} \propto H_\parallel^2$$

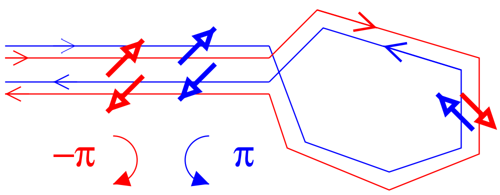
if $1/\tau_H > 1/\tau_\phi$:

$$\delta\sigma(H) \sim \ln(\tau/\tau_H) \sim \alpha \ln H \qquad (\alpha{=}1{,}2)$$

Spin effects

[REVIEW: cond-mat/0206024]

- interplay of Zeeman splitting and spin-orbit coupling ($\longrightarrow$ weak antilocalization)



Aleiner & Fal'ko, PRL **87**, 256801 (2001)

- spin-flip scattering at magnetic impurities

  $\longrightarrow$ decoherence $\longrightarrow$ suppression of WL

  magnetic field:
  polarization of impurities $\longrightarrow$ no spin-flip
  $\longrightarrow$ restoration of WL

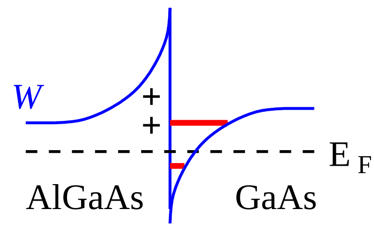
  Vavilov & Glazman, PRB **67**, 115310 (2003)

Berry-Robnik symmetry effect

(quasi-)2d system with disorder potential U and confining potential W in parallel magnetic field

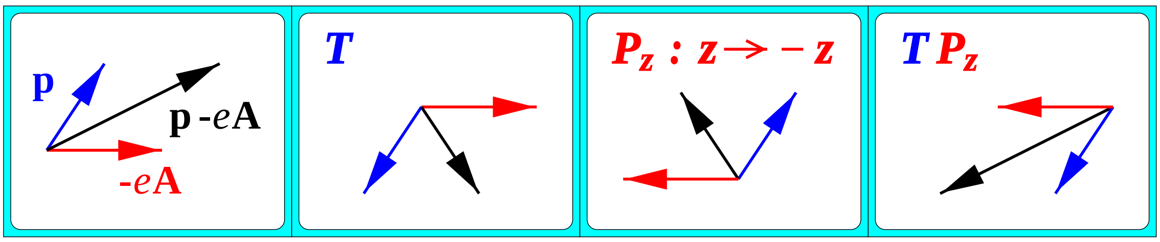
$$\mathcal{H} = \frac{1}{2m}(\mathbf{p} - e\mathbf{A})^2 + U(x, y, z) + W(z)$$

gauge choice: $\mathbf{A} = -Hz\,\mathbf{e}_y$



symmetry transformations:

- time reversal ($\mathcal{T}$)
- z-inversion ($\mathcal{P}_z$)



iff $U = U(x, y)$ and $W(z) = W(-z)$,

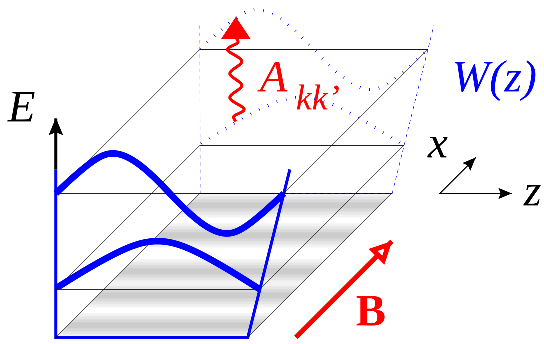
Hamiltonian invariant under $\mathcal{T}\mathcal{P}_z$: $\mathcal{T}\mathcal{P}_z\mathcal{H} = \mathcal{H}$!

Berry-Robnik ('86): unitary ↔ orthogonal spectral statistics

Berry-Robnik symmetry effect
= additional discrete symmetry compensates for time-reversal symmetry breaking

Subband structure I

consider system of finite width d:
system splits into M subbands (p_z quantized)



$$\mathcal{H} = \frac{1}{2m}(\mathbf{p} - \mathbf{A})^2 + U(x,y) + W(z)$$

- diagonalize z-dependent problem

$$\left(-\frac{\partial_z^2}{2m} + W(z) - \epsilon_k \right) \phi_k(z) = 0$$

- rewrite $\mathcal{H}$ in subband basis

$$\mathcal{H}_{kk'} = \left(\frac{1}{2m} p_x^2 + U(x,y) + \epsilon_k \right) \delta_{kk'} + \frac{1}{2m} \left(p_y - \mathbf{A} \right)^2{}_{kk'}$$

$\Rightarrow$ vector potential $\mathbf{A}_{kk'} = -H \int dz\, \phi_k(z)\, z\, \phi_{k'}(z)\, \mathbf{e}_y$

no $H_{||}$: subbands decoupled

$\Rightarrow$ each subband contributes separately to the WL correction:

$$\delta\sigma \sim \sum_{k=1}^{M} \int d^2 q\, \mathcal{C}_{kk}(\mathbf{q}) \propto M \ln \frac{\tau}{\tau_\phi}$$

Subband structure II

parallel magnetic field

1.) couples the subbands & 2.) breaks $\mathcal{T}$-invariance

coupling leads to a splitting
of formerly degenerate Cooperon modes

$$\Rightarrow \quad \boxed{\delta\sigma \sim \sum_{k=0}^{M-1} \ln \frac{\tau}{\tau_\phi^k(H)}}$$

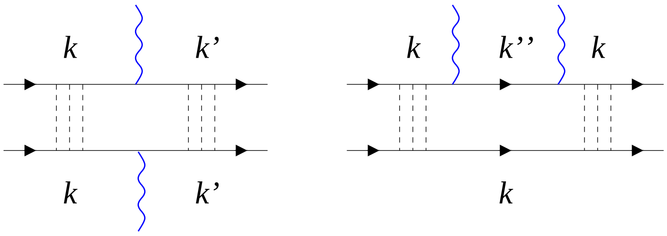
where $1/\tau_\phi^k(H) = 1/\tau_\phi + 1/\tau_H^k$ and $1/\tau_H^{k\neq 0} \neq 0$

$1/\tau_H^0 = \ldots ?$

$$(\mathcal{C}_{\mathbf{q},0}^{-1})_{kk'} = \Big[(\mathbf{q}-2\mathbf{A}_{kk})^2 + \sum_{k''} \frac{2\mathcal{X}_{kk''}}{D_k}\Big]\delta_{kk'} + \frac{2\mathcal{X}_{kk'}}{\sqrt{D_k D_{k'}}}$$

where $\mathcal{X}_{kk'} = \frac{1}{2}(1-\delta_{kk'})\frac{D_k+D_{k'}}{1+(\epsilon_{kk'}\tau)^2} A_{kk'}A_{k'k}$

(D_k diffusion constant of subband k, $\epsilon_{kk'} = \epsilon_k - \epsilon_{k'}$)



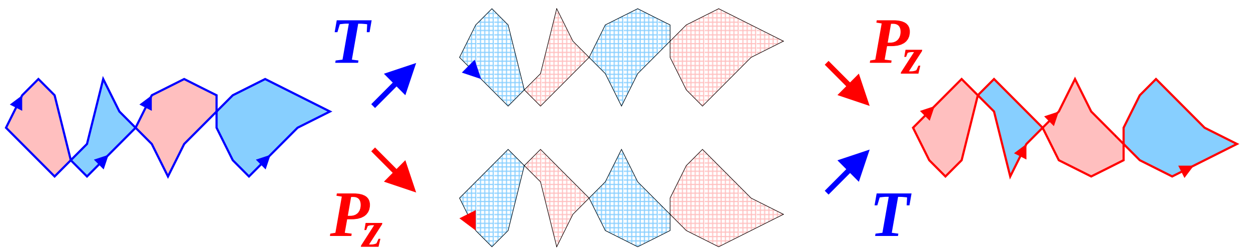
Results ($M \neq 1$)

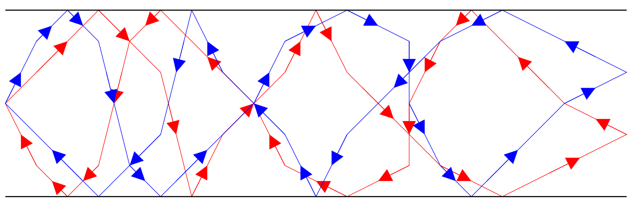
$$(\mathcal{C}^{-1}_{\mathbf{q},0})_{kk'} = [(\mathbf{q}-2\mathbf{A}_{kk})^2 + \sum_{k''} \frac{2\mathcal{X}_{kk''}}{D_k}]\delta_{kk'} + \frac{2\mathcal{X}_{kk'}}{\sqrt{D_k D_{k'}}}$$

a) $\mathcal{P}_z$-symmetry: $1/\tau_H^0 = 0$

one Cooperon mode is unaffected by the field!

$$\Rightarrow \quad \delta\sigma_{\mathrm{S}}(H,T) \sim p \ln T + 2(M-1)\ln H$$





b) no $\mathcal{P}_z$-symmetry: $1/\tau_H^0 \neq 0$

- asymmetric confining potential $W(z) \neq W(-z)$

$$1/\tau_H^{0\,(\mathrm{as})} \propto H^2$$

- z-dependent disorder $\delta U(x,y;z)$

$$1/\tau_H^{0\,(\mathrm{imp})} \propto \begin{cases} H^2 & H < H_c \\ 1/\tau' & H > H_c \end{cases}$$

where $1/\tau'$ inter-subband scattering rate

$$\Rightarrow \quad \delta\sigma_{\mathrm{as}}(H,T) \sim 2M \ln H$$

Example: $M = 2$

for simplicity: $D_0 = D_1 \equiv D$

$$\mathcal{C}^{-1} = \frac{1}{D}\begin{pmatrix} D(\mathbf{q}-\mathbf{A})^2 + 2\mathcal{X}_{01} + \frac{1}{\tau_\phi} & 2\mathcal{X}_{01} \\ 2\mathcal{X}_{01} & D(\mathbf{q}+\mathbf{A})^2 + 2\mathcal{X}_{01} + \frac{1}{\tau_\phi} \end{pmatrix}$$

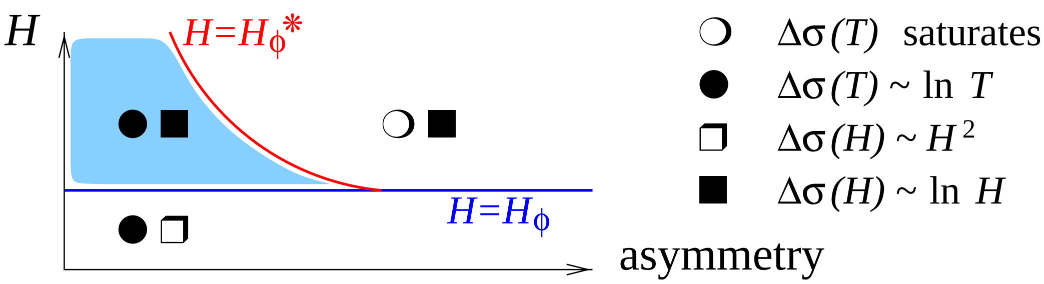
where $\mathbf{A} = \mathbf{A}_{00} - \mathbf{A}_{11}$

if $\mathbf{A} = 0$: $1/\tau_H^{(0)} = 1/\tau_\phi$ and $1/\tau_H^{(1)} = 1/\tau_\phi + 4\mathcal{X}_{01}$

if $\mathbf{A} \neq 0$: $\delta\left(1/\tau_H^{(k)}\right) = D\mathbf{A}^2$

at small H: $\delta\sigma(H) - \delta\sigma(0) \sim 2\mathcal{X}_{01}\tau_\phi$
(independent of $\mathbf{A}$)

T- and H-dependence of the conductivity:



definition of characteristic fields H_ϕ and $H_\phi{}^*$:

$$4\mathcal{X}_{01}(H_\phi) = 1/\tau_\phi \qquad D\mathbf{A}^2(H_\phi{}^*) = 1/\tau_\phi$$

note: $W(z) = W(-z) \;\Rightarrow\; \mathbf{A} = 0$

$M = 1$ occupied subband I

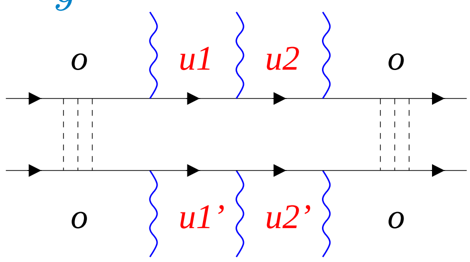
A_{00} pure gauge field $\Rightarrow$ no effect ?

residual magnetoresistance
due to virtual processes

$\longrightarrow$ modifies electron dispersion

$$\mathbf{p}^2 \quad \longrightarrow \quad \mathbf{p}^2 + \alpha\, p_y^2 + \beta\, p_y^3$$

where $\alpha \sim H^2$ and $\beta \sim H^3$



$\longrightarrow$ effective vector potential

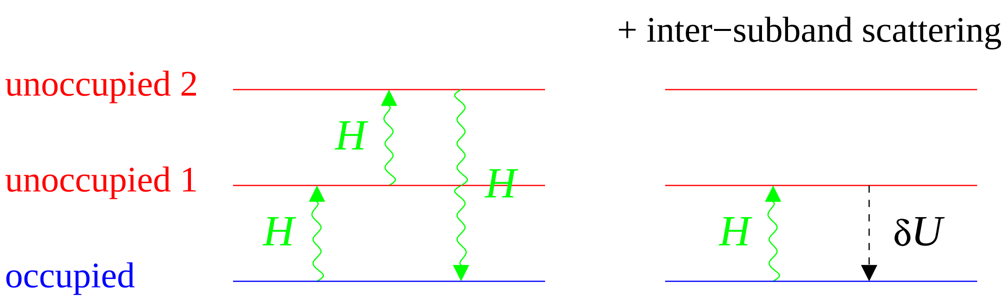
$\Rightarrow \quad 1/\tau_H \propto H^6$

(note that $\beta = 0$ in the $\mathcal{P}_z$-symmetric case!)

z-dependent impurities: $U(x, y) + \delta U(x, y, z)$

[Falko, J. Phys.: Cond. Matt. **2**, 3797 ('90).]

$\longrightarrow$ coupling to unoccupied subbands
even in the absence of a magnetic field



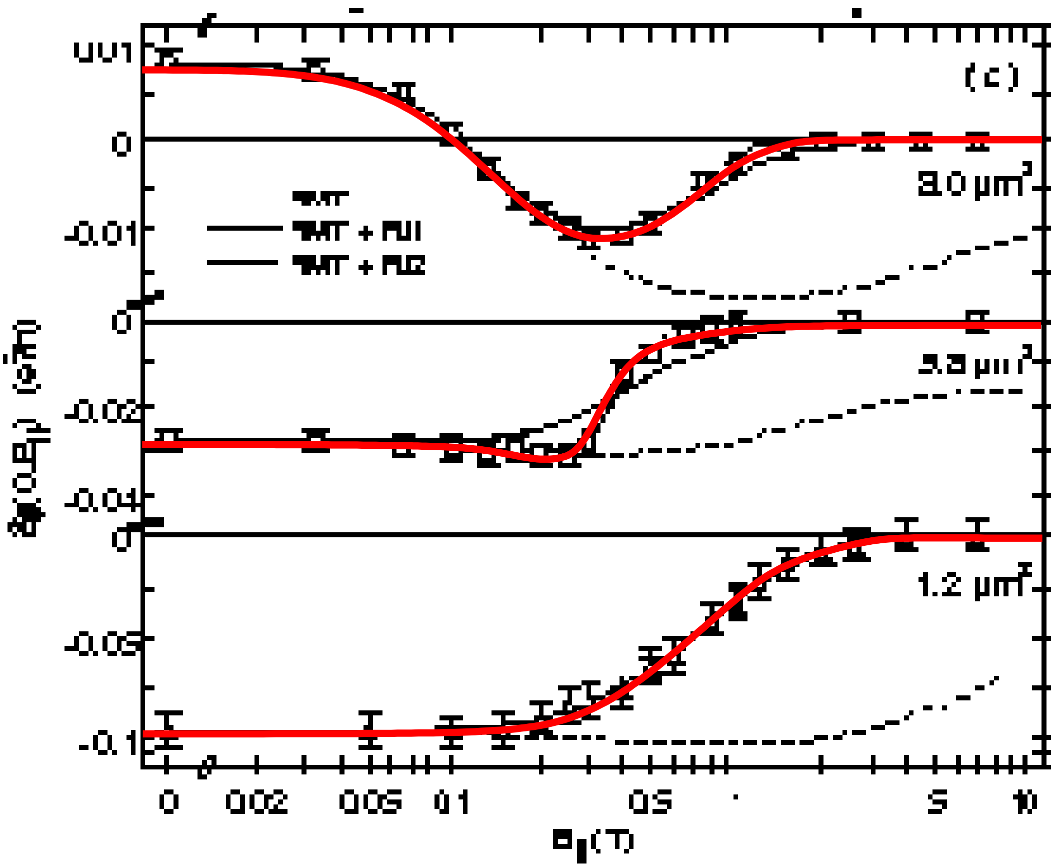
$\Rightarrow \quad 1/\tau_H \propto H^2$

($H^2 \rightarrow H^6$ crossover at $H_c^{M=1} \propto \tau'^{-1/4}$)

Experiments

quantum dots:

Zumbuhl et al., PRL **89**, 276803 (2002)



$\longrightarrow$ fit $1/\tau_H \sim a\,H^2 + b\,H^6$

Conclusions

SPIN ...

ORBITAL EFFECT:

- unconventional transport behavior
  due to interplay of
  – time-reversal ($\mathcal{T}$) symmetry,
  – z-inversion ($\mathcal{P}_z$) symmetry, and
  – subband structure

- magnetoresistance probes
  – symmetry of the confining potential
  – z-dependence of impurity scattering

$$\begin{aligned} \delta\sigma_{\mathrm{s}}(H,T) &\sim p \ln T + 2(M-1) \ln H \\ \delta\sigma_{\mathrm{as}}(H,T) &\sim 2M \ln H \end{aligned}$$

in particular $M = 1$:
residual effect due to virtual processes
($1/\tau_H \propto H^6$)