Noise and current cross-correlations in carbon nanotubes

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Over the years, the study of current noise and current cross–correlations has become a respected and useful diagnosis for transport measurements on mesoscopic conductors. Theoretically, noise was first computed mostly for non–interacting systems. However, it soon became clear that low frequency noise could be used to isolate the quasiparticle charge and to study the statistical correlations in specific quasi one–dimensional correlated electron systems, such as the edge waves in the quantum Hall effect. In these chiral Luttinger liquids, the charge of the collective excitations along the edges corresponds to the electron charge multiplied by the filling factor.

Attention is now turning towards conductors – individual nano-objects – which occur naturally, and which can be connected to current/voltage probes in order to perform a transport experiment. The crucial advantage of such nano–objects is that they are essentially free of defects and in some circumstances they have an inherent one dimensional character. Carbon nanotubes constitute the archetype of such 1D nano-objects: single wall armchair nanotubes have metallic behavior, with two propagating modes at the Fermi level. Incidentally, electronic correlations are known to play an important role in such systems. Carbon nanotubes seem to constitute good candidates to study Luttinger liquid behavior. In particular, their tunneling density of states – and thus the tunneling $I(V)$ characteristics is known to have a power law behavior in accordance with Luttinger liquid theory.

In the present work, we propose an experimental geometry which allows to probe directly the underlying charges of the collective excitations. The setup consists of a nanotube whose bulk is contacted by a scanning tunneling microscope (STM) tip which injects electrons, while both extremities of the nanotube collect the current. The current, the noise and the current cross–correlations are computed in the framework of the Luttinger theory and of the non–equilibrium Keldysh formalism, and the effective charges are determined by comparison with the Schottky formula for an “infinite” nanotube. The striking result is that current cross-correlations contribute to second order in the electron tunneling, in sharp contrast with a fermionic system which requires fourth order. The current cross–correlations are then positive, because the tunneling electron wave function is split in two counter propagating modes of the collective excitations in the nanotube.

One accepted diagnosis to detect effective or anomalous charges is to compare the noise with the associated current with the Schottky formula in mind. Despite the fact that electrons are tunneling from the STM tip to the bulk of the nanotube, we find that the zero frequency current fluctuations (noise) are proportional to the average current $\langle I(x)\rangle$ with an anomalous effective charge for an infinite nanotube:

$$S(x,x)=\langle I(x)I(x)\rangle=\frac{1+(K_{c+})^2}{2}e|\langle I(x)\rangle| \tag{1}$$

where K_{c+} is the Coulomb interaction parameter of the total charge sector.

More can be learned from a measurement of the current cross–correlations. Current cross–correlations have been proposed to detect statistical correlations in quantum transport. Indeed, our geometry can be considered as a Hanbury-Brown and Twiss correlation device. Such experiments have now been completed for photons and more recently for electrons in quantum waveguides. Here the novelty is that electronic excitations do not represent the right eigenmodes of the nanotube. For the current cross-correlations, we obtain the expression:

$$S(x,-x) = \langle I(x)I(-x)\rangle = -\frac{1-(K_{\mathsf{c}+})^2}{2} e|\langle I(x)\rangle| \qquad (2)$$

This is a priori negative. However, if the current direction is chosen to be positive from the tip to the extremities of the nanotube, the sign of the current cross–correlations is positive. Recall that the fermionic version of the Hanbury-Brown and Twiss experiment yields negative current cross–correlations. So far, positive current cross–correlations have been attributed in priority to bosonic systems. Nevertheless, there are at least two other situations where they are encountered. First, when the source of particle is a superconductor, current cross–correlations can also be positive depending on the junction configuration. Second, they also occur in systems with floating voltage probes. In the case of a superconductor, the emission of electron pairs through separate quantum dots guarantee that the current cross–correlations are always positive: a (singlet) entangled electron pair is generated outside the superconductor.

The prefactors in Eqs. (1) and (2) can be interpreted as following: an electron injected in the bulk of a nanotube is split in two counter-propagating excitations with charges $Q_+ = (1+K_{\mathsf{c}+})e/2$ and $Q_- = (1-K_{\mathsf{c}+})e/2$. Both charges $Q_\pm$ are equally likely to go right or left, and they are emitted as a pair with opposite labels. The noise and current cross–correlations are then an average over the two types of excitations:

$$\begin{aligned} S(x,x) &\sim \frac{(Q_+^2+Q_-^2)}{2} = \frac{1+(K_{\mathsf{c}+})^2}{4}e^2 && (3)\\ S(x,-x) &\sim -Q_+Q_- = -\frac{1-(K_{\mathsf{c}+})^2}{4}e^2 && (4)\end{aligned}$$

In the present case, only one electron is injected, but it is split into left and right excitations, unless one imposes one dimensional Fermi liquid leads. Here, we are dealing with entanglement between collective excitations of the Luttinger liquid. Consequently, quantum mechanical non-locality is quite explicit here. The detection of a charge $Q_\pm$ in one arm is necessarily accompanied by the simultaneous detection of a charge $Q_\mp$ in the other extremity of the nanotube. This entanglement is the direct consequence of the correlated state of the Luttinger liquid. When additional electrons are injected, these break up into the specific modes which can propagate in either direction in the nanotube. The many-body wave function which describes a Luttinger liquid with an added electron has necessarily entangled degrees of freedom. Both electrons chiralities contribute to the emission of quasiparticle pairs moving in opposite direction. This entanglement involves many particle states, unlike its electron counterpart.

For references, see:

- *Electron injection in a nanotube: noise–correlations and entanglement*
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- *Noise–correlations, entanglement, and Bell inequalities*
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