Vector spin chirality in classical & quantum spin systems

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Motivation:

A recent flurry of activities on the so-called “multiferroic” materials

As a result, a new type of coupling of spin & lattice came into focus

The key theoretical concept appears to be:

- Two noncollinear spins form a nonzero vector spin chirality (spin chirality)

\[ C_{ij} = <S_i \times S_j> \]

- The spin chirality couples to the lattice displacement or dipole moment \( P_{ij} \)
Control of ferroelectricity using magnetism

TbMnO$_3$

Magnetic control of ferroelectric polarization

T. Kimura$^1$, T. Goto$^1$, H. Shintani$^1$, K. Ishizaka$^1$, T. Arima$^2$ & Y. Tokura$^1$

Mn d orbitals are orbital ordered

Magnetic interaction contain both ferro and antiferro exchanges

Fertile ground for frustrated magnetism
Control of ferroelectricity using magnetic field

At low temperature, both magnetic & dipole order exist

Impossible without the existence of a strong coupling between magnetic order & dipole order

Magnetic field along b switches polarization from c to b axis in TbMnO$_3$ (Kimura)
Connection to Spiral Magnetism

The original work of Kimura demonstrated controllability of FE through applied field.

Connection to spiral magnetism made clear by later neutron scattering study of Kenzelman et al.
Theoretical idea

With spiral magnetism one can define a SENSE of ROTATION (chirality) given by the outer product of two adjacent spins

$$C_{ij} = \langle S_i \times S_j \rangle$$

$C_{ij}$ is odd under inversion, another order parameter of the same symmetry, $P_{ij}$ (polarization), can couple linearly to it as $C_{ij}P_{ij}$

For collinear antiferromagnetism the sense of rotation is ill-defined

$$P_{ij} \propto \hat{e}_{ij} \times (S_i \times S_j)$$
Spin–polarization coupling

Dzyaloshinskii–Moriya type:
(RMnO$_3$ and many others)

\[ H_{SP} = \lambda \sum P_{ij} \cdot \hat{e}_{ij} \times (S_i \times S_j) \]

In the DM interaction, $P_{ij}$ is a static vector, in multiferroics, $P_{ij}$ is dynamic.

Katsura, Nagaosa, Balatsky  PRL 95, 057205 (2006)
Jia, Onoda, Nagaosa, Han  arXiv:cond-mat/0701614
Multiferroics with noncollinear magnetic and ferroelectric phase

<table>
<thead>
<tr>
<th>Material</th>
<th>d-electron</th>
<th>Polarization (μC/m²): Q</th>
<th>Specifics</th>
</tr>
</thead>
<tbody>
<tr>
<td>TbMnO₃</td>
<td>d⁴ (t₂g)³ (e₉)¹</td>
<td>800; q~0.27</td>
<td>Orbital order</td>
</tr>
<tr>
<td>(Kimura et al Nature 2003; Kenzelman et al PRL 2005)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ni₃V₂O₈</td>
<td>d⁸ (t₂g)⁶ (e₉)²</td>
<td>100; q~0.27</td>
<td>Kagome</td>
</tr>
<tr>
<td>(Lawes et al PRL 2005)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ba₀.₅Sr₁.₅Zn₂Fe₁₂O₂₂</td>
<td>d⁵ [(t₂g)³ (e₉)²]</td>
<td>150 (B=1T); N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>(Kimura et al PRL 2005)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CoCr₂O₄</td>
<td>Co²⁺: d⁷ (e)⁴ (t₂)³</td>
<td>2; [qq0] q~0.63</td>
<td>ferrimagnetic</td>
</tr>
<tr>
<td>(Yamasaki et al. PRL 2006)</td>
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</tr>
<tr>
<td>MnWO₄</td>
<td>d⁵ (t₂g)³ (e₉)²</td>
<td>50; q=(-.214, .5, .457)</td>
<td>N/A</td>
</tr>
<tr>
<td>(Taniguchchi et al. PRL 2006)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>CuFeO₂</td>
<td>d⁵ (t₂g)³ (e₉)²</td>
<td>400 (B&gt;10T); 1/5&lt;q&lt;1/4</td>
<td>2D triangular; Field-driven</td>
</tr>
<tr>
<td>(Kimura et al PRB 2006)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LiCuVO₄</td>
<td>d⁹ (t₂g)⁶ (e₉)³</td>
<td>N/A; q~0.532</td>
<td>1D chain</td>
</tr>
<tr>
<td>(Naito et al cond-mat/0611659)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LiCu₂O₂</td>
<td>d⁹ (t₂g)⁶ (e₉)³</td>
<td>&lt;10; q~0.174</td>
<td>1D chain</td>
</tr>
<tr>
<td>(Park et al PRL 2007)</td>
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</tbody>
</table>

RED = magnetic ions
Often, there is the magnetic transition to COLLINEAR spin states, for which no polarization is induced.

A second transition at a lower temperature to spiral spin states cause nonzero polarization.
Abstracting away

Spiral spin phase supports an additional discrete order parameter having to do with the sense of spin rotation.

This OP can couple to uni-directional polarization P.

Can we envision a phase without magnetic order, but still has the remnant of chirality?
Chiral spin state in helical magnet?

Recent work: Ginzburg–Landau theory of spin models with SU(2)→O(2) symmetry

Onoda & Nagaosa
Cond-mat/0703064
Examples of quantum spins with chirality

Hikihara et al. considered a spin chain with nearest and next-nearest neighbour XXZ-like interactions for $S=1$

\[ H_{i,NN} = J_1 (S_{ix} S_{i+1,x} + S_{iy} S_{i+1,y} + \Delta S_{iz} S_{i+1,z}) \]

\[ H_{i,NNN} = J_2 (S_{ix} S_{i+2,x} + S_{iy} S_{i+2,y} + \Delta S_{iz} S_{i+2,z}) \]

\[ H = \sum_i \left( H_{i,NN} + H_{i,NNN} \right) \]
Long-range order of vector spin chirality

Define spin chirality operator $\hat{\kappa}_{i,j} = S_i^x S_j^y - S_i^y S_j^x$

DMRG found chiral phase for $S=1$ when $j = J_2/J_1$ is sufficiently large

$$\langle \hat{\kappa}_{i,i+1} \hat{\kappa}_{j,j+1} \rangle \to \kappa^2 \quad \text{as} \quad |i-j| \to \infty$$
Zittartz’s work

Meanwhile, Zittartz found exact ground state for the class of anisotropic spin interaction models with nearest-neighbor quadratic & biquadratic interactions

\[ I_{i,\Delta} = S_{ix}S_{i+1,x} + S_{iy}S_{i+1,y} + \Delta S_{iz}S_{i+1,z} \]

\[ H = \sum_i \left( J_1 I_{i,\Delta_1} + J_2 I_{i,\Delta_2}^2 \right) \]

Zittartz’s ground state is SU(2)→O(2) generalization of Affleck–Kennedy–Lieb–Tesaki (AKLT) state.

Zittartz’s ground state does not support spin chirality order
On one hand we have numerical evidence of spin chirality in sufficiently frustrating spin-1 chain system.

On the other hand we do not have an explicit construction of such a state

→ Can we find one?
XXZ Hamiltonian with DM

\[ H = J_1 \sum_{\langle ij \rangle} S_i^z S_j^z + J_2 \sum_{\langle ij \rangle} \left( S_i^x S_j^x + S_i^y S_j^y \right) \]

Including Dzyalonskii–Moriya (DM) interaction

\[ H = J_1 \sum_{\langle ij \rangle} S_i^z S_j^z + J_2 \sum_{\langle ij \rangle} \left( S_i^x S_j^x + S_i^y S_j^y \right) + J_3 \sum_{\langle ij \rangle} \left( S_i \times S_j \right) \cdot \hat{z} \]

\[ H = J_1 \sum_{\langle ij \rangle} S_i^z S_j^z + \left( J_2 + i J_3 \right) \sum_{\langle ij \rangle} S_i^+ S_j^- + \left( J_2 - i J_3 \right) \sum_{\langle ij \rangle} S_i^- S_j^+ \]

\[ = J_1 \sum_{\langle ij \rangle} S_i^z S_j^z + \left( J_2^2 + J_3^2 \right)^{1/2} \sum_{\langle ij \rangle} \left( e^{i\theta} S_i^+ S_j^- + e^{-i\theta} S_i^- S_j^+ \right) \]

DM interaction \( \rightarrow \) phase angle, flux
Staggered DM, arbitrary DM

Staggered oxygen shifts gives rise to “staggered” DM interaction -> “staggered” phase angle, “staggered” flux

We can consider the most general case of arbitrary phase angles:

\[ H = J_1 \sum_{\langle ij \rangle} S_i^z S_j^z + J_2 \sum_{\langle ij \rangle} (e^{i\theta_{ij}} S_i^+ S_j^- + e^{-i\theta_{ij}} S_i^- S_j^+) \]
Connecting non-chiral & chiral Hamiltonians

Define the model on a ring with N sites:

$$H[\{\theta_{ij}\}] = J_1 \sum_{\langle ij \rangle} S^z_i S^z_j + J_2 \sum_{\langle ij \rangle} (e^{i\theta_{ij}} S^+_i S^-_j + e^{-i\theta_{ij}} S^-_i S^+_j)$$

Carry out unitary rotations on spins

$$S^+_i \to U^+_i S_i U_i = S^+_i e^{-i\alpha_i}$$

Choose angles such that $$e^{i[\alpha_i - \alpha_j]} \equiv e^{i\theta_{ij}}$$

This is possible provided

$$e^{i\sum_{\langle ij \rangle} \theta_{ij}} = 1$$

Hamiltonian is rotated back to XXZ:

$$U^\dagger[\{\theta_{ij}\}] H[\{\theta_{ij}\}] U = H_{XXZ}$$
Connecting non–chiral & chiral Hamiltonians

Eigenstates are similarly connected:

\[
U^\dagger \left[ \{\theta_{ij}\} \right] H \left[ \{\theta_{ij}\} \right] U = H_{XXZ}
\]

\[
U \left[ \{\theta_{ij}\} \right] = \prod_i U_i
\]

\[
H \left[ \{\theta_{ij}\} \right] |\{\theta_{ij}\}\rangle = E |\{\theta_{ij}\}\rangle
\]

\[
H_{XXZ} |XXZ\rangle = E |XXZ\rangle
\]

\[
U |XXZ\rangle = |\{\theta_{ij}\}\rangle
\]
DM interaction and its connection to gauge transformation was noted earlier.

Shekhtman, Entin-Wohlman, Aharony PRL 69, 839 (1992)
Connecting non-chiral & chiral eigenstates

Correlation functions are also connected. In particular,

\[ \langle \{\theta_{ij}\}|S^-_i S^+_j|\{\theta_{ij}\} \rangle = e^{i[\alpha_i - \alpha_j]} \langle \text{XXZ}|S^-_i S^+_j|\text{XXZ} \rangle \]

Since

\[ \langle S^-_i S^+_j \rangle = \langle S^x_i S^x_j + S^y_i S^y_j \rangle + i\langle S^x_i S^y_j - S^y_i S^x_j \rangle \]

and

\[ \langle \text{XXZ}|S^-_i S^+_j|\text{XXZ} \rangle = \langle \text{XXZ}|S^x_i S^x_j + S^y_i S^y_j|\text{XXZ} \rangle \]

It follows that a non-zero spin chirality must exist in \(|\{\theta_{ij}\}\rangle\)

\[ \langle \{\theta_{ij}\}|S^x_i S^y_j - S^y_i S^x_j|\{\theta_{ij}\} \rangle = \langle \text{XXZ}|S^-_i S^+_j|\text{XXZ} \rangle \sin[\alpha_i - \alpha_j] \]

Eigenstates of \(H[\{\theta_{ij}\}]\) are generally chiral.
Analogy with persistent current

For $S=1/2$, Jordan–Wigner mapping gives

$$J_2 \sum_{ij} (e^{i\theta_{ij}} S_i^+ S_j^- + e^{-i\theta_{ij}} S_j^+ S_i^-)$$

$$J_2 \sum_{ij} (e^{i\theta_{ij}} f_i^+ f_j + e^{-i\theta_{ij}} f_j^+ f_i)$$

Spin chirality maps onto bond current

$$i\langle S_i^+ S_j^- - S_j^+ S_i^- \rangle \quad \Longleftrightarrow \quad i\langle f_i^+ f_j - f_j^+ f_i \rangle$$

When the flux is integral, there will be no persistent current !?

The gauge–invariant definition of fermion current is

$$i\langle e^{i\theta_{ij}} f_i^+ f_j - e^{-i\theta_{ij}} f_j^+ f_i \rangle \quad \Longleftrightarrow \quad i\langle e^{i\theta_{ij}} S_i^+ S_j^- - e^{-i\theta_{ij}} S_j^+ S_i^- \rangle$$
Generating chiral states

Given a Hamiltonian with non-chiral eigenstates, a new Hamiltonian with chiral eigenstates will be generated with non-uniform U(1) rotations:

\[ S_i^+ \rightarrow U_i S_i U_i^\dagger = S_i^+ e^{i\alpha_i} \]

\[ U[\{\alpha_i\}] H_{\text{non-\chi}} U^\dagger[\{\alpha_i\}] = H_\chi \]

\[ U[\{\alpha_i\}] = \prod_i U_i \]
Well-known Affleck–Kennedy–Lieb–Tasaki (AKLT) ground states and parent Hamiltonians can be generalized in a similar way

$$H^{AKLT} = \sum_{\langle ij \rangle} S_i \cdot S_j \left( 1 + \frac{1}{3} S_i \cdot S_j \right)$$

Using Schwinger boson singlet operators

$$A_{ij}^+ = a_{ij}^+ b_{ij}^+ - b_{ij}^+ a_{ij}^+$$

$$H^{AKLT} = \frac{1}{24} \sum_{\langle ij \rangle} (6 - A_{ij}^+ A_{ij})(4 - A_{ij}^+ A_{ij})$$

$$A_{ij}^+ A_{ij} = 2(1 - S_i^z S_j^z - S_i^+ S_j^- - S_i^- S_j^+)$$

AKLT ground state is

$$|AKLT\rangle = \prod_{\langle ij \rangle} A_{ij}^+ |0\rangle$$

Arovas, Auerbach, Haldane
PRL 60, 531 (1988)
From AKLT to chiral–AKLT

Aforementioned U(1) rotations correspond to

$$a_i^+ \rightarrow a_i^+ e^{i\theta_i/2}, \quad b_i^+ \rightarrow b_i^+ e^{-i\theta_i/2}$$

$$A_{ij}^+ = a_i^+ b_j^+ - b_i^+ a_j^+ \rightarrow A_{ij}^+ [\theta_{ij}] = e^{i\theta_{ij}/2} a_i^+ b_j^+ - e^{-i\theta_{ij}/2} b_i^+ a_j^+$$

$$H^{\chi AKLT} = \frac{1}{24} \sum_{\langle ij \rangle} (6 - A_{ij}^+ [\theta_{ij}] A_{ij} [\theta_{ij}])(4 - A_{ij}^+ [\theta_{ij}] A_{ij} [\theta_{ij}])$$

$$A_{ij}^+ A_{ij} = 2(1 - S_i^z S_j^z - e^{i\theta_{ij}} S_i^+ S_j^- - e^{-i\theta_{ij}} S_i^- S_j^+)$$

Chiral–AKLT ground state is

$$|\chi AKLT\rangle = \prod_{\langle ij \rangle} A_{ij}^+ [\theta_{ij}] |0\rangle$$
Correlations in chiral–AKLT states

Equal-time correlations of chiral–AKLT states easily obtained as chiral rotations of known correlations of AKLT states:

With AKLT: \[ \langle S_i \cdot S_j \rangle_0 = 2\delta_{ij} + 4(1 - \delta_{ij})(-1/3)^{|i-j|} \]

With chiral–AKLT:

\[ \langle \{\theta_i\}|S_i^x S_j^x|\{\theta_i\}\rangle = \frac{1}{3} \langle S_i \cdot S_j \rangle_0 \times \cos \theta_{ij} \]

\[ \langle \{\theta_i\}|S_i^y S_j^y - S_i^y S_j^y|\{\theta_i\}\rangle = \frac{2}{3} \langle S_i \cdot S_j \rangle_0 \times \sin \theta_{ij} \]

\[ \langle \{\theta_i\}|S_i^z S_j^z|\{\theta_i\}\rangle = \frac{1}{3} \langle S_i \cdot S_j \rangle_0 \]
Excitation energies in SMA

Calculate excited state energies in single-mode approximation (SMA) for uniformly chiral AKLT state: $\theta_{ij} = \theta$

With AKLT:

$$E(k) = \frac{\langle S^x_{-k} HS^x_k \rangle}{\langle S^x_{-k} S^x_k \rangle} = \frac{f(k)}{s(k)}$$

$$f(k) = (10/27)(1-\cos k), \quad s(k) = 2(1-\cos k)/(5+3\cos k)$$

With chiral-AKLT:

$$E_x(k) = \frac{\langle S^x_{-k} He S^x_k \rangle}{\langle S^x_{-k} S^x_k \rangle} = \frac{f(k+\theta) + f(k-\theta)}{s(k+\theta) + s(k-\theta)}$$

$$E_z(k) = \frac{f(k)}{s(k)}$$
Excitation energies in SMA

\[ E_x(k, \theta) \]
Higher-dimensional chiral AKLT

\[ |\chi\rangle = \prod_{\langle ij \rangle} (A^+_{ij} [\theta_{ij}])^M \]

\( M = \frac{2S}{\text{(coordination number)}} \)

For chiral Hamiltonian, replace

\[ S_i^+ S_j^- + S_i^- S_j^+ \rightarrow e^{i\theta_{ij}} S_i^+ S_j^- + e^{-i\theta_{ij}} S_i^- S_j^+ \]

In the original AKLT Hamiltonian (For proof, see our paper arXiv:0705.3993)

In higher dimensions, it is not always possible to associate \( \theta_{ij} = \theta_i - \theta_j \)
A place to look? (perhaps LaCu$_2$O$_4$)

Coffey, Rice, Zhang PRB 44, 10112 (1991)

Tilted up the plane

Tilted down the plane
A place to look? (perhaps LaCu$_2$O$_4$)

Effects of Dzyaloshinskii–Moriya interaction

Weak ferromagnetism (LaCu$_2$O$_4$)

Cheong, Thompson, Fisk PRB 39, 4395 (1989)
Quantum mechanical spin chirality is readily embodied by the DM interaction.

What is condensed by DM is more like the chiral solid, not chiral liquid.

Search for vector spin chiral liquid will be interesting.
The spins along a given axis rotates either clockwise (+1) or counterclockwise (−1). Ising variable can be associated.

Magnetic order implies chirality ordering.

But can chirality order first before magnetism does? If so, one has the classical chiral spin liquid phase.
We consider the modification of the antiferromagnetic XY model on the triangular lattice

\[ E = J_1 \sum_{\langle ij \rangle} \cos[\theta_i - \theta_j] + J_2 \sum_{\langle ij \rangle} \cos[2\theta_i - 2\theta_j] \]

For \( J_2 = 0 \) this is AFXY on triangular lattice (long history associated with chirality transition)
We consider the modification of the antiferromagnetic XY model on the triangular lattice

\[ E = J_1 \sum_{\langle ij \rangle} \cos[\theta_i - \theta_j] + J_2 \sum_{\langle ij \rangle} \cos[2\theta_i - 2\theta_j] \]

For $J_1 = 0$ this model supports nematic spin state due to equivalence of $\theta_i$ and $\theta_i + \pi$ (macroscopic degeneracy)
Classical: $J_1-J_2$ XY Model

(Classical) statistical properties of $J_2$–only model is identical to those of $J_1$–only

$$Z_2 = \int_0^{2\pi} \prod_i d\theta_i \exp \left(-\beta \sum_{\langle ij \rangle} \cos[2\theta_i-2\theta_j] \right)$$

$$= 2^N \int_0^{2\pi} \prod_i d\theta_i \exp \left(-\beta \sum_{\langle ij \rangle} \cos[\theta_i-\theta_j] \right) = 2^N Z_1$$

Two types of the transition found on triangular lattice

1. Kosterlitz–Thouless (KT) transition

2. Chirality transition
The separation of the phase transition temperatures is extremely small. The chirality-ordered phase is not well-defined.

Classical: $J_1{-}J_2$ XY Model

We find wide temperature region where chirality is condensed, when $J_1$ and $J_2$ co-exist, and $J_2 \gg J_1$
Specific heat

\[ J_2/J_1 = 9 \text{ } (L = 15, 30, 60). \]

\[ C = \frac{\langle E^2 \rangle - \langle E \rangle^2}{T^2} \]

Two phase transitions clearly identified

\[ T_1 \]

\[ T_2 \]
Magnetic order

Magnetic order undergoes transitions at lower critical temperature. A scaling plot of $M_1$ using the Ising exponents $\beta = 1/8, \nu = 1$, and $T_c = 0.177$ works very well.
**Nematic order**

**Binder cumulential**

\[ U_L = 1 - \frac{\langle N_2^4 \rangle_L}{3 \langle N_2^2 \rangle_L^2} \]

\[ T_{KT} = 0.460 \]
Helicity modulus

This $T_{KT}$ must agree with the one obtained from Binder cumulant in the previous page.

$$
\rho_s = -\frac{J_1}{2L^2} \langle \sum_{ij} \cos \theta_{ij} \rangle - \frac{2J_2}{L^2} \langle \sum_{ij} \cos 2\theta_{ij} \rangle
$$

$$
- \frac{1}{TL^2} \langle (J_1 \sum_{ij} x_{ij} \sin \theta_{ij} + 2J_2 \sum_{ij} x_{ij} \sin 2\theta_{ij})^2 \rangle
$$
Critical phase for nematic order below $T_{KT}$

We find critical dependence of $N_1$ and $N_2$ on the lattice dimension $L$ below $T_{KT}$.

$$N_1 \sim \frac{1}{L \eta_1(T)}, \quad N_2 \sim \frac{1}{L \eta_2(T)}$$
Chiral order undergoes two phase transitions. The first one at higher temperature obeys a scaling plot. A scaling plot of chirality using the $\beta = 0.15$, $\nu = 0.69$, and $T_\chi = 0.462$. This $T_\chi$ is higher than $T_{KT}$ of the nematic order.
$Z_2$ (chiral) symmetry in nematic & magnetic phase
Degenerate states are chiral counterparts

Magnetic states: \[ \chi_{ij}^{M} = (S_i \times S_j \cdot \hat{z}) = \sin[\theta_i - \theta_j] \] are opposite

Nematic states: \[ \chi_{ij}^{N} = (S_i \times S_j \cdot \hat{z})^2 = \sin[2(\theta_i - \theta_j)] \] are opposite

Magnetic order IMPLIES chirality symmetry breaking in \( \chi_{ij}^{M} \)

Nematic order IMPLIES chirality symmetry breaking in \( \chi_{ij}^{N} \)

Does nematic order IMPLY chirality symmetry breaking in \( \chi_{ij}^{M} \)??

I can give an argument that in fact it does.
In the presence of nematic ordering, each triangle supports the following configurations:

Each spin orientation takes up Ising values \(+1\) (outward) or \(-1/3\) (inward).

\[
\chi_{ijk} = \frac{1}{3} (\sigma_i \sigma_j + \sigma_j \sigma_k + \sigma_k \sigma_i)
\]

\[
\chi \sim \sum_{\langle ij \rangle} \sigma_i \sigma_j \sim \text{Ising energy}
\]
$J_2$ interaction vanishes within the nematic manifold

$J_1$ interaction lifts the degeneracy, results in the effective Hamiltonian valid within the nematic manifold

$$E = -J_1 \sum_{\langle ij \rangle} \sigma_i \sigma_j \quad \chi \sim \sum_{\langle ij \rangle} \sigma_i \sigma_j$$

At any finite temperature the average of the energy is negative $\rightarrow$ the chirality must be positive!!
Phase diagram: $J_1 - J_2$ XY model

$$H = J_1 \sum_{\langle ij \rangle} \cos[\theta_i - \theta_j] + J_2 \sum_{\langle ij \rangle} \cos[2(\theta_i - \theta_j)]$$

T

J$_2$/J$_1$=9

paramagnetic

magnetic

chiral, non-magnetic

J$_2$/J$_1$
The nematic order already breaks the chiral symmetry and opens up the possibility of the chirality condensation well above the magnetic transition.

We are trying to see if the same mechanism will result in chirality-condensed phase in quantum spin models on triangular lattice:

\[ H = J_1 \sum_{\langle ij \rangle} (S_i \cdot S_j)_\Delta + J_2 \sum_{\langle ij \rangle} (S_i \cdot S_j)^2_\Delta \]

\[ (S_i \cdot S_j)_\Delta = S^x_i S^x_j + S^y_i S^y_j + \Delta(S^z_i S^z_j) \]
\[ \chi_{ijk}^\triangle = \frac{1}{3} (\sigma_i \sigma_j + \sigma_j \sigma_k + \sigma_k \sigma_i) \]

\[ \chi \sim \sum_{\langle ij \rangle} (\sigma_i \sigma_j + \sigma_j \sigma_k + \sigma_k \sigma_i) \sim \sum_{\langle ij \rangle} \sigma_i \sigma_j \]

\[ -\frac{1}{2} J_1 \sum_{\langle ij \rangle} \sigma_i \sigma_j \]

\[ T_1 \approx 3.641 \times (J_1/2) \approx 0.18 \]
We consider the modification of the antiferromagnetic XY model on the triangular lattice.

\[ E = J_1 \sum_{\langle ij \rangle} \cos[\theta_i - \theta_j] + J_2 \sum_{\langle ij \rangle} \cos[2\theta_i - 2\theta_j] \]

In the spin language this is equivalent to putting a bi-quadratic interaction, and \( \Delta=0 \) in Zittartz’ model.

\[ H = J_1 \sum_{\langle ij \rangle} S_i \cdot S_j + J_2 \sum_{\langle ij \rangle} (S_i \cdot S_j)^2 \]

We did Monte Carlo (MC) on the classical \( J_1-J_2 \) model.
Nematic vs. Magnetic Ordering in Lattice

Nematic ordering

Magnetic ordering
$\mathbb{Z}_2$ (chiral) symmetry in nematic & magnetic phase
Degenerate states are chiral counterparts

Magnetic states: \( \chi_{ij}^M = (S_i \times S_j \cdot \vec{z}) = \sin[\theta_i - \theta_j] \) are opposite

Nematic states: \( \chi_{ij}^N = (S_i \times S_j \cdot \vec{z})^2 = \sin[2(\theta_i - \theta_j)] \) are opposite

Degenerate states are chiral counterparts

Magnetic states: are opposite

Nematic states: are opposite
Order Parameters

Order parameters referring to magnetic, nematic, and chiral orders are defined by

\[ M = \frac{3}{L^2} \left| \sum_i e^{i\theta_i} \right| \quad N = \frac{3}{L^2} \left| \sum_i e^{2i\theta_i} \right| \]

\[ \chi = \frac{1}{L^2} \sum_{\Delta_i} (\sin[\theta_{i1} - \theta_{i2}] + \sin[\theta_{i2} - \theta_{i3}] + \sin[\theta_{i3} - \theta_{i1}]) \]
Magnetic, Nematic, Chiral
The interaction strengths are parameterized as follows

\[ J_1 = 1 - x, \quad J_2 = x \quad (0 < x < 1) \]

Qualitatively the phase diagram looks like

Nematic phase also appears to be chiral
Lesson?

Perhaps a spin system with a sufficiently large frustrating interaction will support a chiral phase, and hence a ferroelectricity, without having the magnetic order.