# Observation of the Berry phase in a superconducting charge pump

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# Experimental Determination of the Berry Phase in a Superconducting Charge Pump

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We present the first measurements of the Berry phase in a superconducting Cooper pair pump. A fixed amount of Berry phase is accumulated to the quantum-mechanical ground state in each adiabatic pumping cycle, which is determined by measuring the charge passing through the device. The dynamic and geometric phases are identified and measured quantitatively from their different response when pumping in opposite directions. Our observations, in particular, the dependencies of the dynamic and geometric effects on the superconducting phase bias across the pump, agree with the basic theoretical model of coherent Cooper pair pumping.

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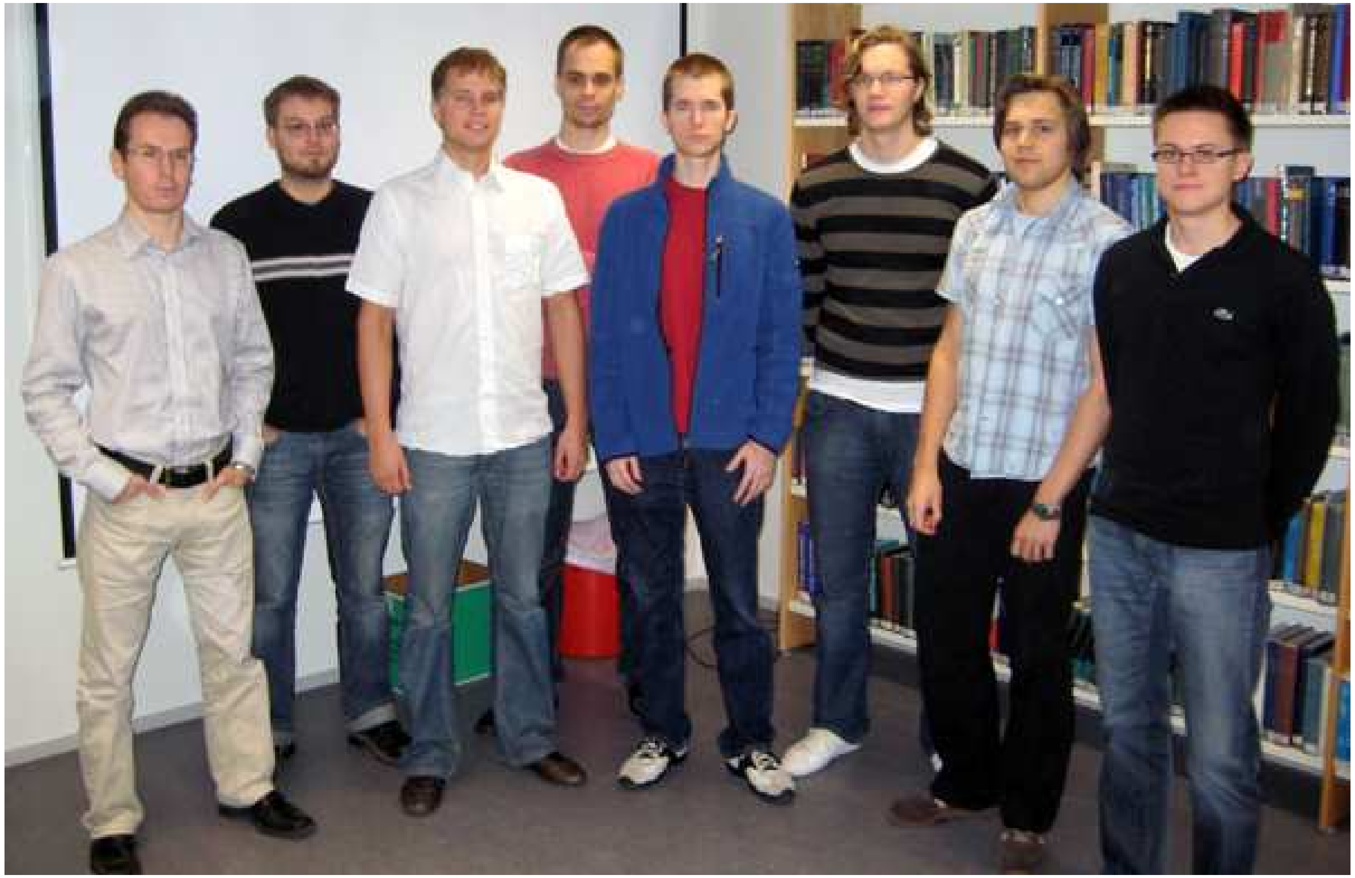
# PICO group at LTL, TKK

From left: Alexander Savin, Mikko Möttönen, Juha Vartiainen (old memeber), Antti Kemppinen (Mikes), Jukka Pekola, Matthias Meschke, Meri Helle, Kurt Baarman, Olli-Pentti Saira, Tommy Holmqvist, Joonas Peltonen, Andrey Timofeev, Francesco Giazotto (SNS Pisa).

# Quantum computing and devices group at Department of Engineering Physics, TKK



From left: Sami Virtanen (old member), Jukka Huhtamäki, Mikko Möttönen, Olli Ahonen, Ville Bergholm (old member), Juha Pirkkalainen, Pekko Kuopanportti, Ville Pietilä.
Members not in the picture: Janne Kokkala, Juha Salmilehto

# Outline

References: A.O. Niskanen, J.P. Pekola, and H. Seppä, PRL 91, 177003 (2003).
A.O. Niskanen, J.M. Kivioja, H. Seppä, and J.P. Pekola, PRB 71 012513 (2004).
M. Möttönen, J.P. Pekola, J.J. Vartiainen, V. Brosco, and F.W.J. Hekking, PRB 73 214523 (2006).
J.J. Vartiainen, M. Möttönen, J.P. Pekola, and A. Kemppinen, APL 90, 082102 (2007).
M. Möttönen, J.J. Vartiainen, J.P. Pekola, PRL 100, 177201 (2008).

- The sluice
- Side step: Nanoampere pumping
- Basics of geometric quantum computation
- Pumped charge and the Berry phase
- How to observe the Berry phase in the sluice?
- Conclusions

# Sluice Hamiltonian

$$\hat{H}_{\mathrm{sl}} = E_C(\hat{n} - n_g)^2 - E_{J1}(\Phi_1)\cos(\phi + \varphi/2) - E_{J2}(\Phi_2)\cos(\phi - \varphi/2)$$

$$\hat{n} = -i\partial_\phi$$

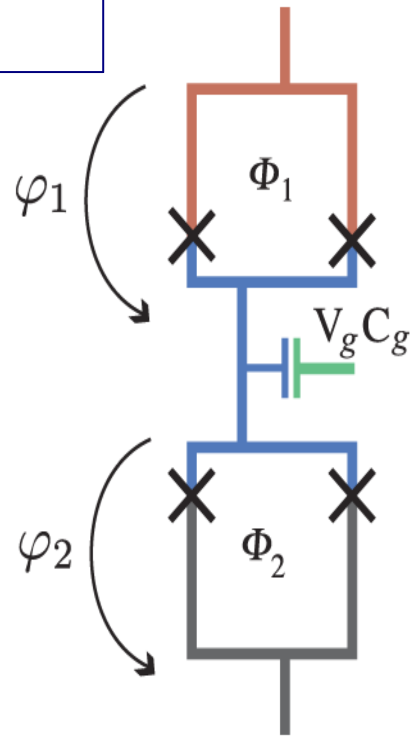
$$n_g = V_g C_g / (2e)$$

$$\phi = (\varphi_1 - \varphi_2)/2$$

$$\varphi = \varphi_1 + \varphi_2$$

$$\hat{I}_k = \frac{2eE_{\mathrm{J}k}}{\hbar}\sin(\varphi_k)$$

$$e^{\pm i\phi}|n\rangle = |n \pm 1\rangle$$

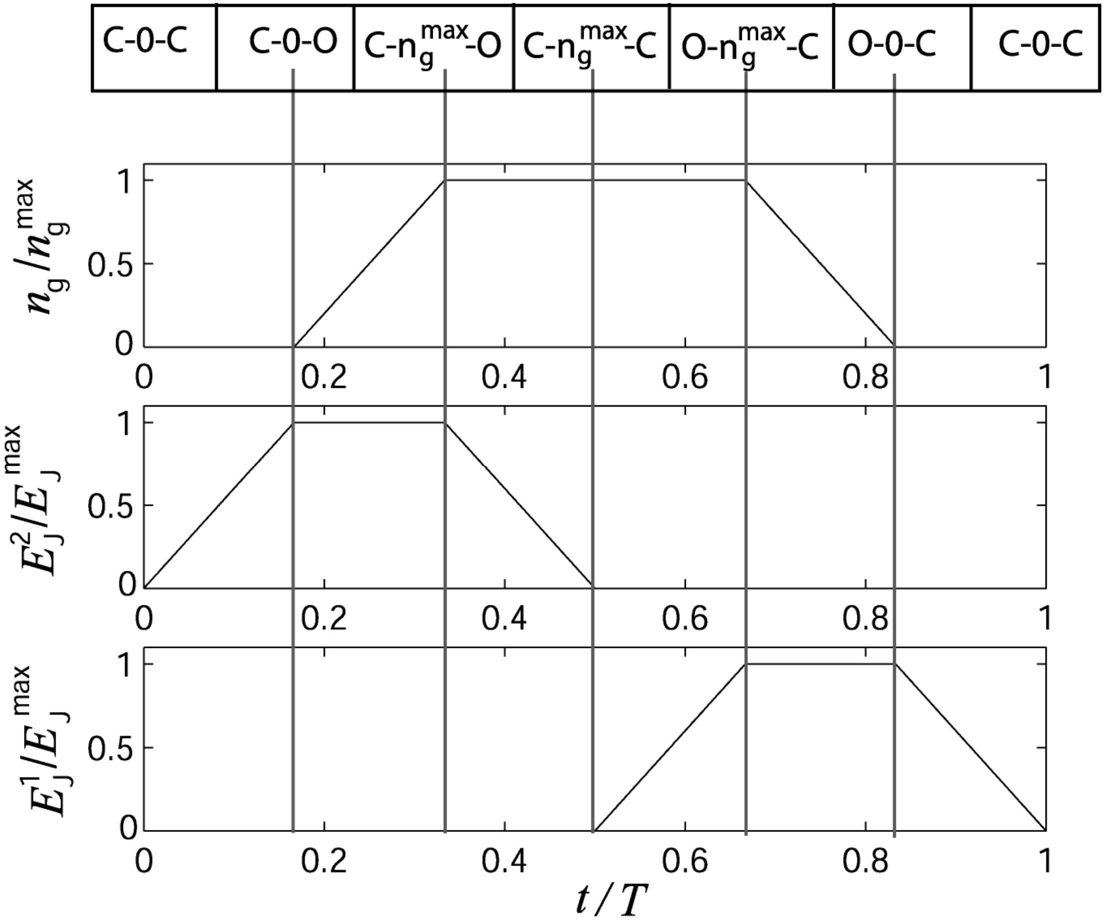


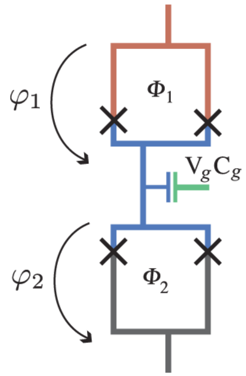
# Ideal pumping cycle

$$I_p = 2n_g^{\max} e f$$

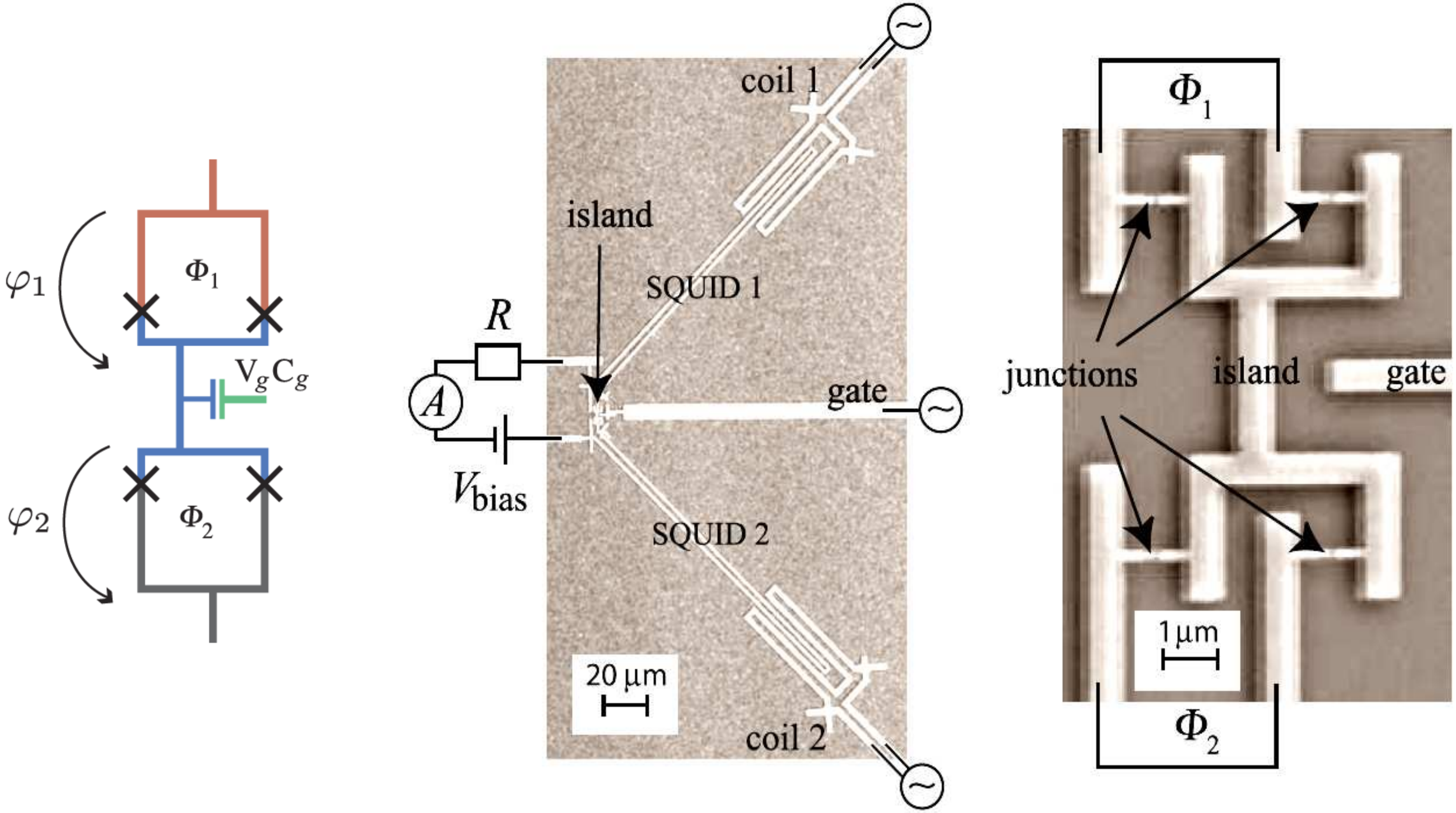
$$E_{\mathrm{C}}(\hat{n} - n_{\mathrm{g}})^2$$

$$\hat{I}_k = \frac{2eE_{\mathrm{J}k}}{\hbar}\sin(\varphi_k)$$



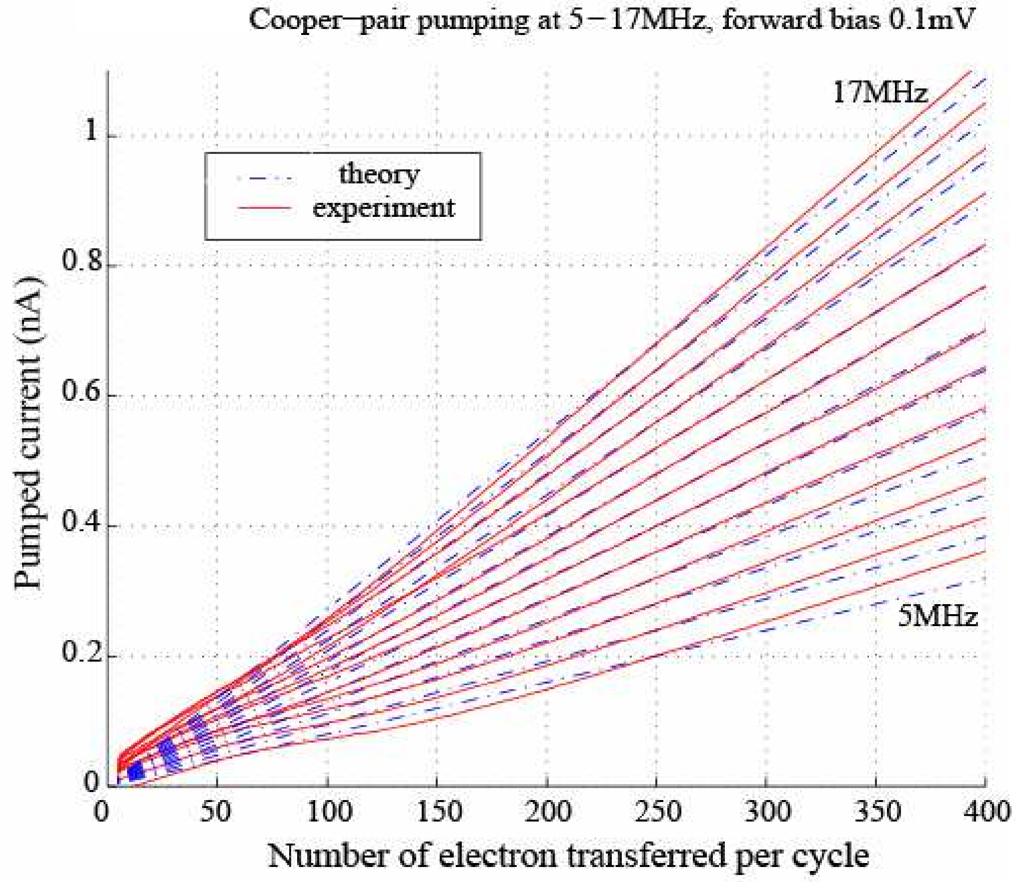



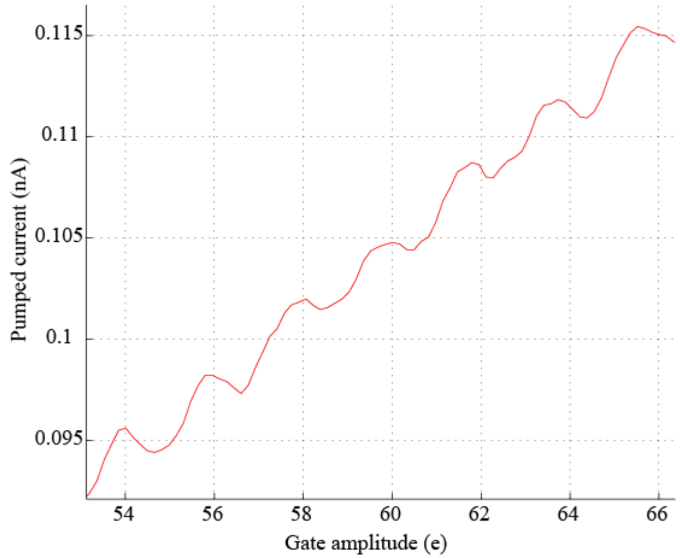
# Side step: Nanoampere pumping



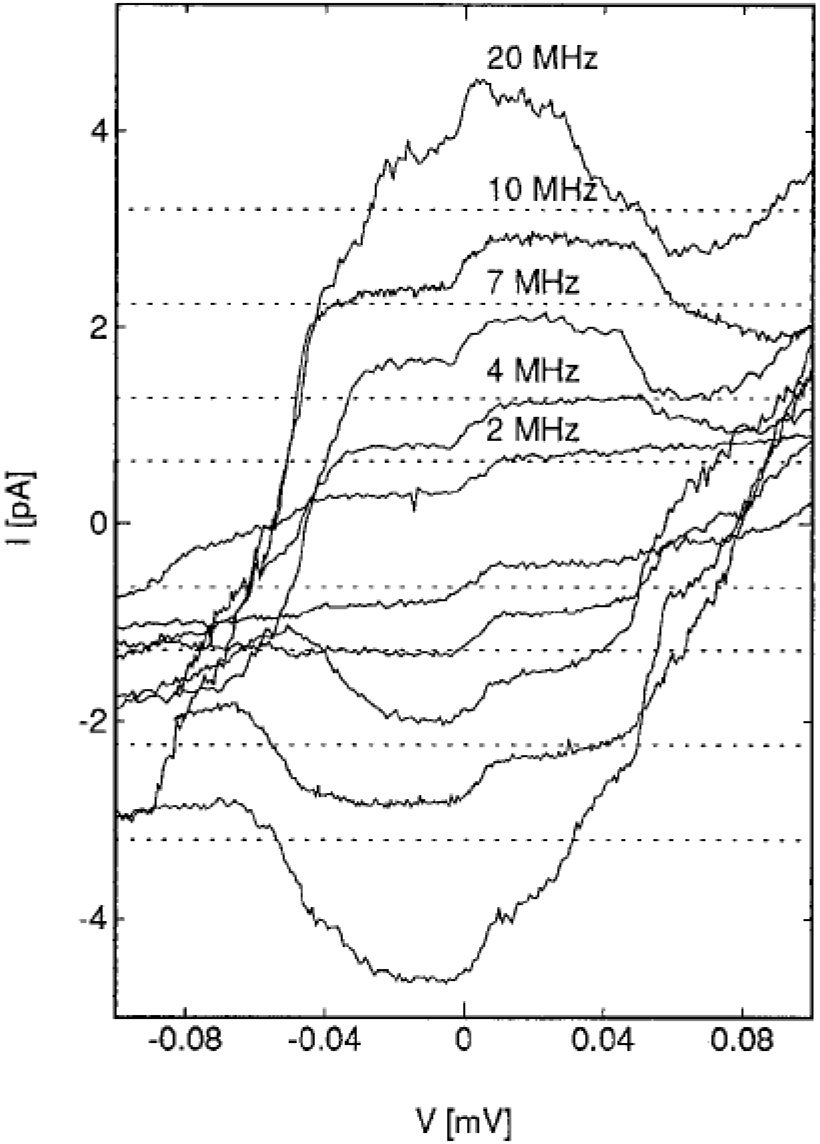
# Side step: Nanoampere pumping

- Accuracy of <2% obtained for 1 nA current

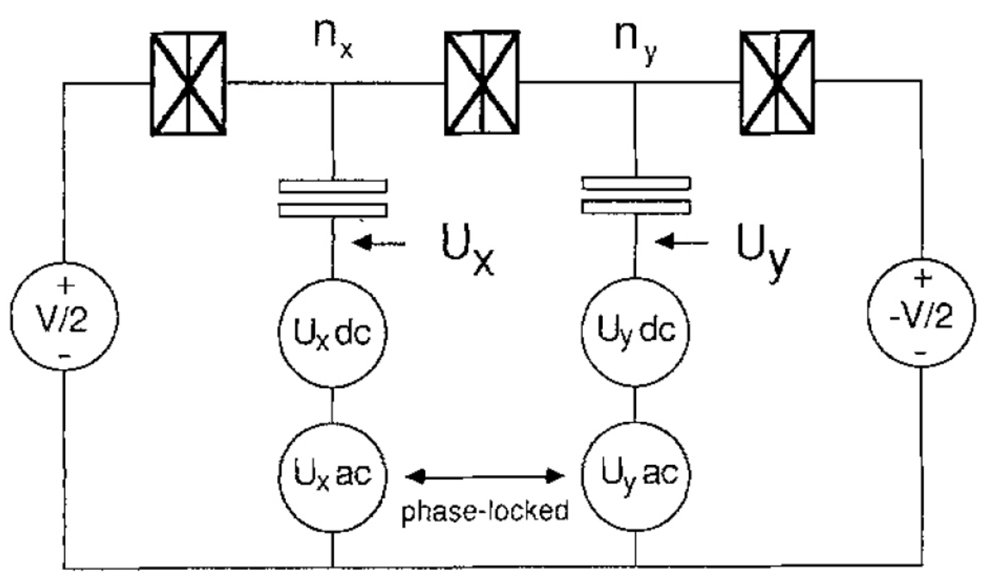


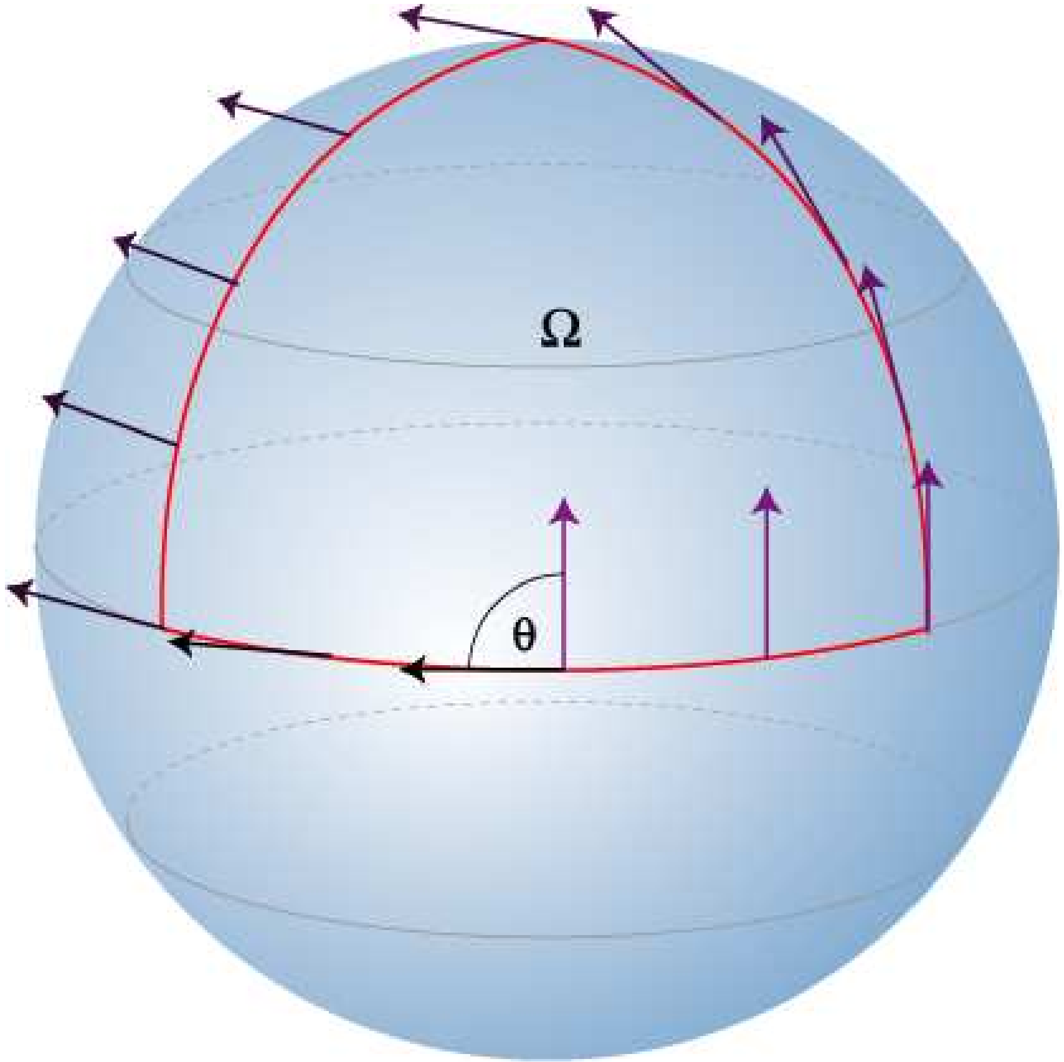
# Another side step: Cooper pair pump




L. Geerligs et al., Z. Phys. B: Condensed Matter **85**, 349 (1991)



# Geometric phases in classical parallel transport



# Appearence of geometric phases in quantum physics

- Let us assume that we are in a *g*-fold degenerate ground state of a quantum system

$$\mathcal{H}_{\mathbf{q}}|0\alpha;\mathbf{q}\rangle = \varepsilon_{\mathbf{q}}|0\alpha;\mathbf{q}\rangle \qquad (\alpha = 1, ..., g)$$

- Adiabatic cyclic evolution gives rise to

$$U(t,t_0) = e^{-i\int_{t_0}^{t}\varepsilon_{\mathbf{q}(\tau)}d\tau/\hbar}\mathcal{T}\exp\left(-\int_{t_0}^{t}\mathbf{A}(\tau)d\tau\right)$$

$$\mathbf{A}_{\beta\alpha}(t) = \langle 0\beta;\mathbf{q}(t)|\frac{d}{dt}|0\alpha;\mathbf{q}(t)\rangle \qquad \theta_{\mathrm{dyn}} = -\int_{t_0}^{t}\varepsilon_{\mathbf{q}(\tau)}d\tau/\hbar$$

# Geometric interpteration

- To observe the geometric nature of the unitary matrix we write

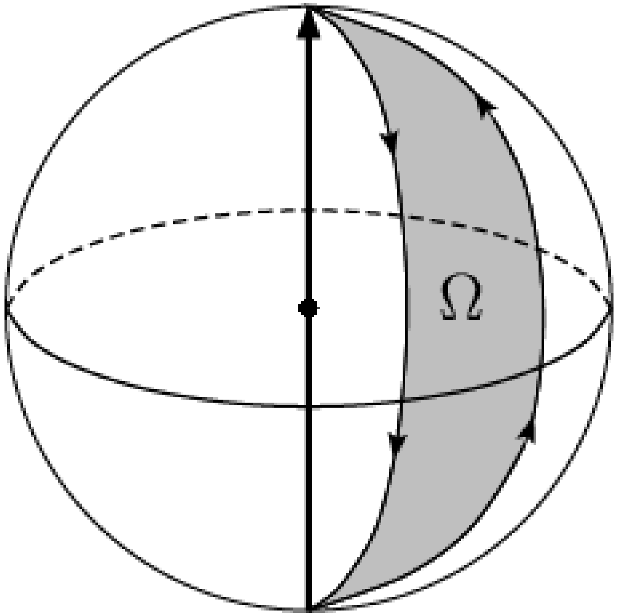
$$\mathcal{A}_{i;\beta\alpha} = \langle 0\beta; \mathbf{q}| \frac{\partial}{\partial \mathbf{q}_i} |0\alpha; \mathbf{q}\rangle$$

$$U(t, t_0) = e^{\,i\,\theta_{\text{dyn}}} \mathcal{P} \exp\left( - \oint_\gamma \mathcal{A}_i d\mathbf{q}^i \right)$$

- In the non-degenerate case we get

$$\theta_{\text{Berry}} = i \oint_\gamma \langle 0; \mathbf{q}| \frac{\partial}{\partial \mathbf{q}_i} |0; \mathbf{q}\rangle d\mathbf{q}^i = i \oint_\gamma \langle 0; \mathbf{q}(t)| \nabla_{\mathbf{q}} |0; \mathbf{q}(t)\rangle \cdot d\mathbf{q}$$

# Berry phase for an adiabatically rotated spin




$$\theta_{\mathrm{Berry}} = -m_z\Omega \qquad \varepsilon_0 = -\varepsilon_1$$

$$\theta_{\mathrm{dyn}} = -\int_{t_0}^{t} \varepsilon_{\mathbf{q}(\tau)} d\tau/\hbar$$

$$\begin{aligned}
|0\rangle &\xrightarrow{R_y(\pi)} (|0\rangle + |1\rangle)/\sqrt{2} \\
&\xrightarrow{ad.ev.} (e^{i(\theta_d+\theta_g)}|0\rangle + e^{-i(\theta_d+\theta_g)}|1\rangle)/\sqrt{2} \\
&\xrightarrow{\sigma_x} (e^{i(\theta_d+\theta_g)}|1\rangle + e^{-i(\theta_d+\theta_g)}|0\rangle)/\sqrt{2} \\
&\xrightarrow{inv.ad.ev.} (e^{i2\theta_g}|1\rangle + e^{-i2\theta_g}|0\rangle)/\sqrt{2} \\
&\xrightarrow{R_y(-\pi)} \cos(2\theta_g)|0\rangle + i\sin(2\theta_g)|1\rangle
\end{aligned}$$

- See P. Leek et al., *Science* **318**, 1889 (2007)

# The connections in phase biased pumps

- Pumped charge and the Berry phase

$$\boxed{Q_p = 2e\partial_\varphi \theta_B}$$

For a derivation, see e.g. M. Möttönen et al., PRB **73** 214523 (2006).

- Charge due to supercurrent and the dynamic phase

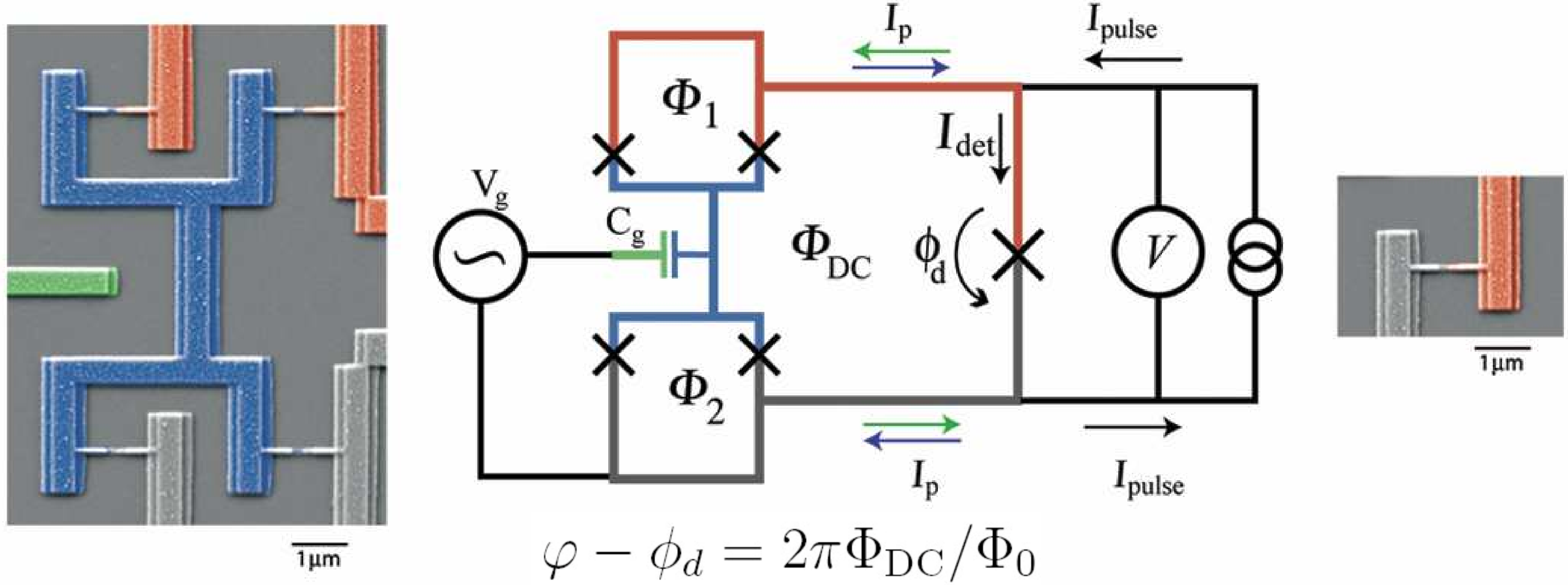
$$\boxed{Q_s = -2e\partial_\varphi \theta_d}$$

$$Q_{\text{tot}} = \int_{t_0}^{t_0+T} \langle \psi(t)|\hat{I}|\psi(t)\rangle dt \qquad Q_{sr} = \int_{t_0}^{t_0+T} \langle r;\mathbf{q}(t)|\hat{I}|r;\mathbf{q}(t)\rangle dt \qquad Q_p := Q_{\text{tot}} - Q_s$$

# Phase biasing the sluice

- Detector junction keeps the phase over the sluice constant
- Switching measurement probes the phase dependence of the pumped current

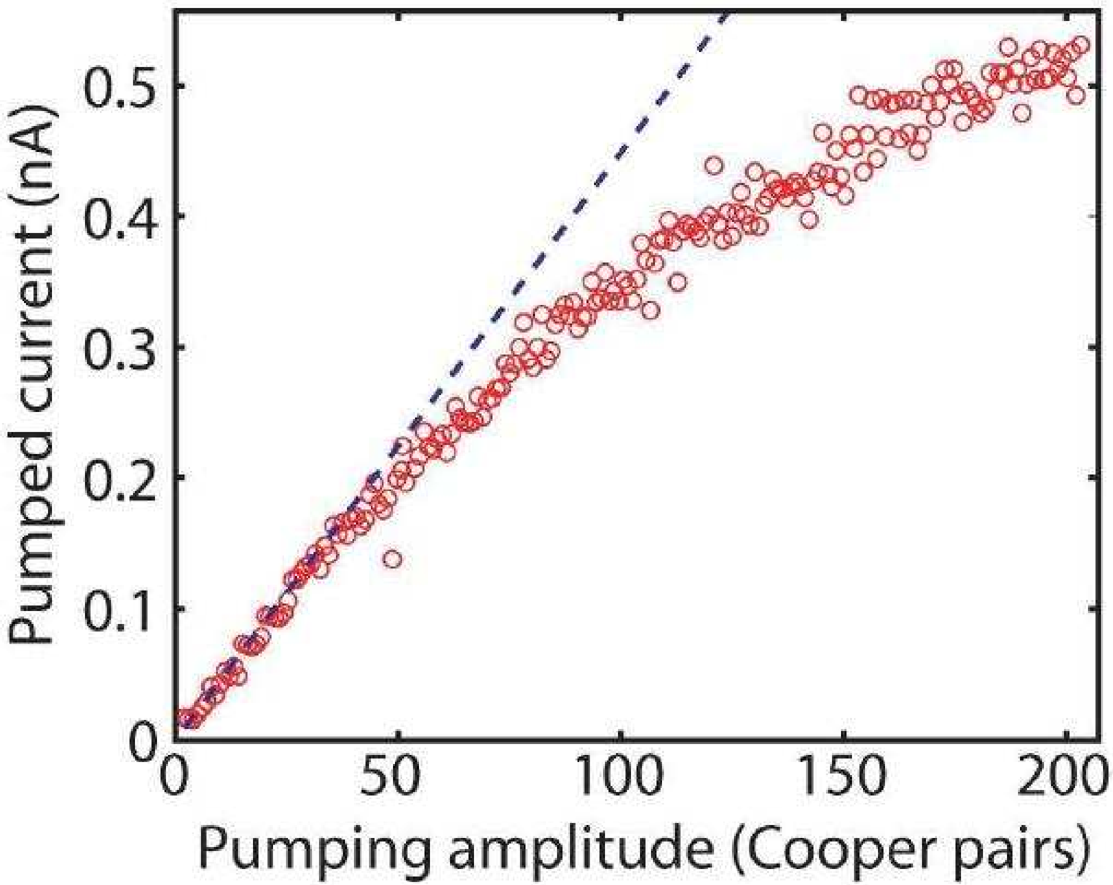
$$I_{50\%} = I_{\text{pulse}}^{\text{blue}} + I_p - I_d = I_{\text{pulse}}^{\text{green}} - I_p - I_d \Rightarrow \begin{cases} I_p = (I_{\text{pulse}}^{\text{green}} - I_{\text{pulse}}^{\text{blue}})/2 \\ I_d = (I_{\text{pulse}}^{\text{green}} + I_{\text{pulse}}^{\text{blue}})/2 - I_{50\%} \end{cases}$$




$$\varphi - \phi_d = 2\pi\Phi_{DC}/\Phi_0$$

# Pumping in a closed circuit

- Nearly ideal pumping realized at $f$=14 MHz
- $I_p = 2n_g^{max}\, ef$ for small $n_g^{max}$ (no fitting parameters)



# How to measure the Berry phase?

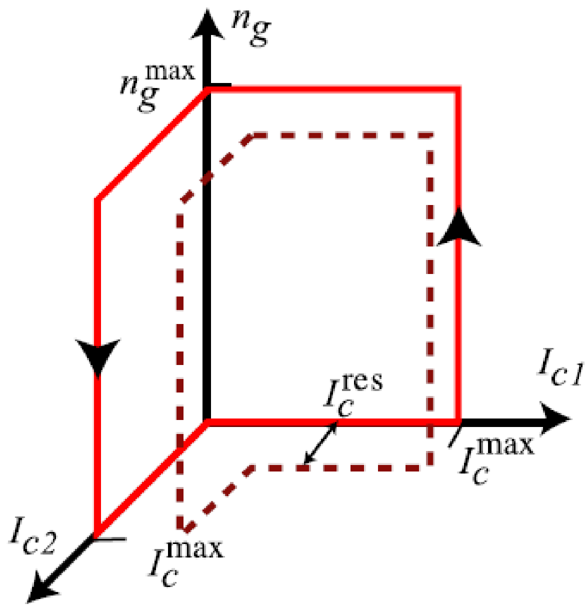
- In the two-charge-state approximation

$$Q_d \approx T I_{\mathrm{cmax}} \delta \beta \sin\varphi \qquad Q_p \approx 2e n_g^{\max}(1 - \delta\cos\varphi)$$

- Thus Eq. $Q_p = 2e\partial_\varphi \theta_B$ implies

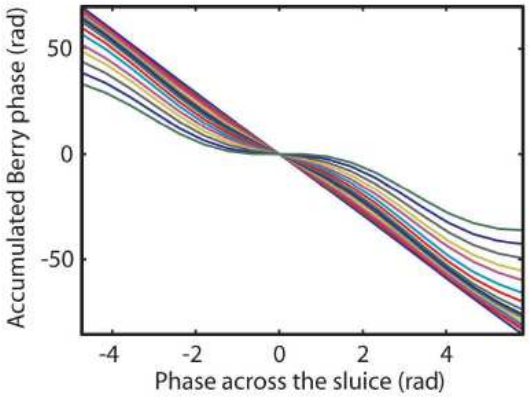
$$\boxed{\theta_B \approx -n_g^{\max}(\varphi - \delta\sin\varphi)}$$

$$\delta = 2I_c^{\mathrm{res}}/I_c^{\max}$$



# Observation of the Berry phase

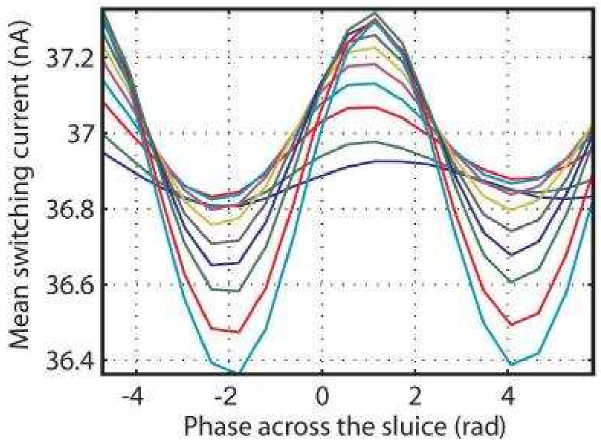
$$Q_d \approx T I_{\mathrm{cmax}} \delta\beta \sin\varphi$$

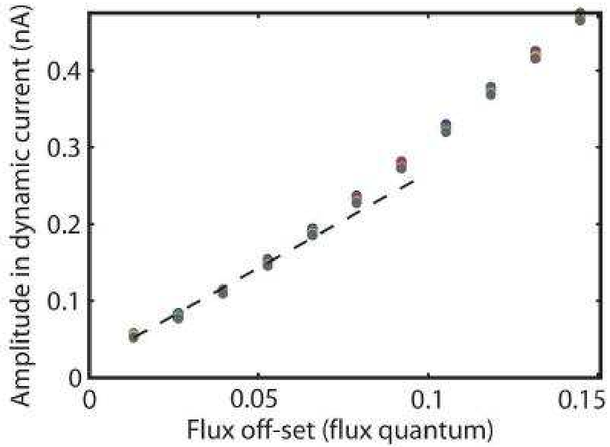


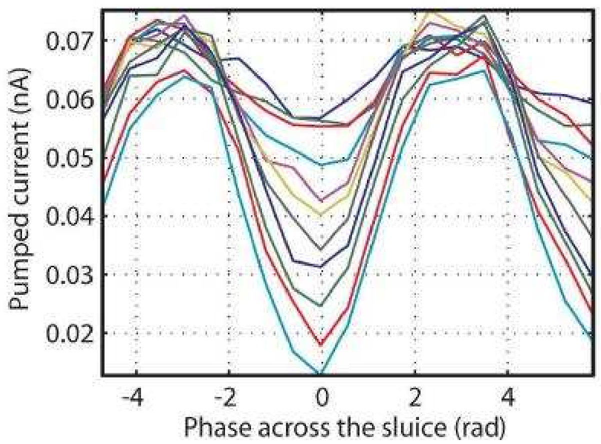

$$Q_p \approx 2e n_g^{\max} (1 - \delta \cos\varphi)$$

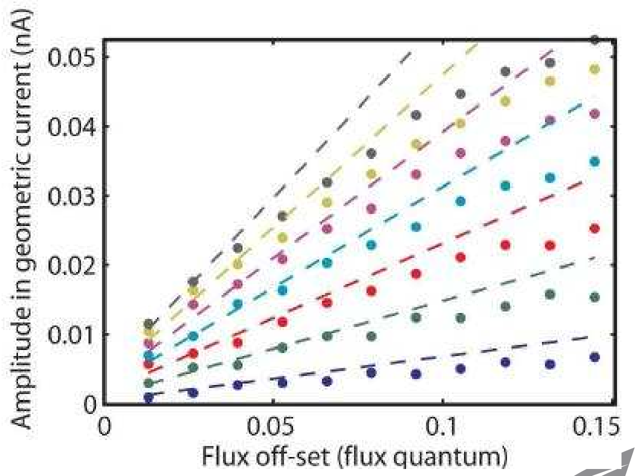
$$Q_p = 2e\partial_\varphi \theta_B$$

$$\theta_B \approx -n_g^{\max}(\varphi - \delta \sin\varphi)$$



# Conclusions

- Geometric phases can possibly be used in robust quantum computing
- We presented one of the first observations of geometric phases in superconducting circuits (see also **Leek et al., Science, Dec. 2007**)
- Dephasing of the system was not observed in the measurement, which raises discussion about the robustness of geometric phases against decoherence