Interactions and Charge Fractionalization in an Electronic Hong-Ou-Mandel Interferometer

Thierry Martin

Centre de Physique Théorique, Marseille

in collaboration with
C. Wahl, J. Rech and T. Jonckheere

Electronic quantum optics in quantum Hall systems

Quantum optics analogs with electrons, i.e. the controlled preparation, manipulation and measurement of single excitations in ballistic conductors.

**INGREDIENT LIST**

- Photons
- Light beam
- Chiral edge QHE
- Beam-splitter
- Point contact
- Coherent light source
- Single electron source

**Diagram:**

1. Photons
2. Light beam
3. Chiral edge QHE
4. Beam-splitter
5. Point contact
6. Coherent light source
7. Single electron source
Single electron sources

Lorentzian voltage pulses

Charge pumps, quantum turnstiles

Flying electrons on surface acoustic waves

Mesoscopic capacitor

flows opens the way to all sorts of interference experiments!

[Dubois et al., Nature ('13)]

[Giblin et al., Nature Comm. 3, 930 ('12)]

[Hermelin et al., Nature 477, 435 ('11)]

[McNeil et al., Nature 477, 439 ('11)]

[Fève et al., Science 316, 1169 ('07)]
Single electron source: the LPA mesoscopic capacitor

- **Setup** [Fève et al., Science 316, 1169 ('07); Mahé et al., PRB 82, 201309 ('10)]
  - quantum dot coupled to edge
  - discrete levels spaced by $\Delta$
  - tunable dot transmission via $V_g$
    - sets level broadening

- Operating the source
  - time-dependent excitation voltage $V_{\text{exc}}(t)$  
    - well described through Floquet scattering theory
  - emission of one electron + one hole per period

- Injected wave-packet?  
  - exponential shape

**Figure 1**: Single charge injection. A) Schematic of single charge injection. Starting from an antiresonant situation where the Fermi energy lies between two energy levels of the dot (step 1), the dot potential is increased by bringing an occupied level above the Fermi energy (step 2). One electron then escapes the dot at the same time. The dot potential is then brought back to its initial value (step 3) where one electron can enter it, leaving a hole in the Fermi sea. Inset: The quantum R\&C circuit: one edge channel is transmitted inside the submicron dot with transmission tuned by the QPC gate voltage. The dot potential is varied by a radio frequency excitation applied on a macroscopic gate located on top of the dot. The electrostatic potential can also be tuned due to the electrostatic coupling between the dot and the QPC. B) Time-domain measurement of the average current (black curves) on one period of the excitation signal (red curves) at three values of the transmission.
Hong-Ou-Mandel interference experiment

- **Two-photon interferences**
  - two identical photons sent on a beam-splitter
  - necessarily exit by the same output channel
  - signature of bosonic statistics

- **Interference experiment**
  - [Hong, Ou and Mandel, PRL 59, 2044 ('87)]
  - non-linear crystal generates photon pairs
  - measure the coincidence rate

- **Coincidence rate**
  - counts occurrences of photons present in the two output channels
  - dip is observed when photons arrive at the same time
  - signatures of incoming wave packets
HOM with electrons: general principle

- **Setup**
  - 2 single electron sources
  - counter-propagating channels coupled at QPC
  - measure output currents

- Zero-frequency cross-correlations of output currents

\[
S_{RL}^{\text{out}} = \int dt dt' \left[ \langle I_{R}^{\text{out}}(x, t) I_{L}^{\text{out}}(x', t') \rangle - \langle I_{R}^{\text{out}}(x, t) \rangle \langle I_{L}^{\text{out}}(x', t') \rangle \right]
\]

- Differences with photons
  - they obey fermionic statistics
    - existence of a Fermi sea, hole-like excitations, ...
  - thermal effects do matter
  - they interact via Coulomb interaction
HOM with electrons: theoretical results for $\nu = 1$

[Jonckheere et al. Phys. Rev. B 86, 125425 ('12)]

- Collision of identical wave-packets

When electrons arrive independently
- $S_{12} < 0$ : sum of the partition noise
- flat background contribution
  ➞ Hanbury-Brown and Twiss

When electrons arrive simultaneously
- $S_{12} = 0$ ➞ HOM/Fermi/Pauli dip
- signatures of injected object

- More exotic situations

Different packets ➞ asymmetry

Electron-hole collision ➞ peak
HOM with electrons: experimental results

- **Experimental setup**
  - two independent sources
  - synchronized electron emission, and collision at the beamsplitter
  - observe two-particle interferences?

[Bocquillon et al., Science 339, 1054 ('13)]

- **Main results**

As expected
  - Flat background contribution (random partitioning)
  - Pauli dip for simultaneous injection

But... How come it does not reach 0?

→ decoherence

Something special happens when we go beyond the simple $\nu = 1$ picture
Beyond $\nu = 1$: interactions!

- Different possible types of interactions
  - between counter-propagating channels, near the QPC
  - strong interactions within a channel: Fractional QHE
  - between co-propagating channels at filling $\nu = 2$

- Formalism and methods
  - injection
  - propagation
  - tunneling

For $\nu = 2$
- prepared state $|\varphi\rangle$
- bosonized $H$ (+ diagonalization)
- scattering matrix

Bosonization, Keldysh
Injection

- **Simplified model of injection: prepared state**
  - injection in the past at \( t = -T_0 \): \( |\varphi\rangle = \mathcal{O}^\dagger(-T_0) |0\rangle \)
  - preparation operator \( \mathcal{O}^\dagger = \mathcal{O}_R^\dagger \mathcal{O}_L^\dagger \) with

\[
\mathcal{O}_{R,L}^\dagger = \int dk \varphi_{R,L}(k) \psi_{R,L}^\dagger(k; t = -T_0)
\]

- True one shot injection of electron or hole
- Versatile: any wave-packet

- **Exponential wave-packets**

\[
\varphi_{R,L}(x) = \sqrt{\frac{2\Gamma}{v_F}} e^{\pm(i\epsilon_0 + \Gamma)x/v_F} \theta(\mp x)
\]

Tunable resolution \( \gamma = \epsilon_0/\Gamma \)

\[
\begin{align*}
\epsilon_0 &= 0.175K \\
\Gamma &= 0.175K \\
\gamma &= 1
\end{align*}
\]

\[
\begin{align*}
\epsilon_0 &= 0.7K \\
\Gamma &= 0.0875K \\
\gamma &= 8
\end{align*}
\]
Propagation

- \( \nu = 2 \text{ QHE } \Rightarrow \text{ Two channels: outer and inner } (j = 1, 2) \)

- Bosonization identity: \( \psi_{j,r}(x) = \frac{U_{j,r}}{\sqrt{2\pi a}} \exp (i \phi_{j,r}(x)) \)

- Hamiltonian \( H = H_0 + H_{\text{intra}} + H_{\text{inter}} \)

- Propagation along the edge

\[
H_0 = \frac{\hbar}{\pi} \sum_{j=1,2} v_j^{(0)} \sum_{r=R,L} \int dx (\partial_x \phi_{j,r})^2
\]

- Intra-channel interaction

\[
H_{\text{intra}} = \frac{\hbar}{\pi} U \sum_{j=1,2} \sum_{r=R,L} \int dx (\partial_x \phi_{j,r})^2
\]

- Inter-channel interaction

\[
H_{\text{inter}} = 2\frac{\hbar}{\pi} u \sum_{r=R,L} \int dx (\partial_x \phi_{1,r}) (\partial_x \phi_{2,r})
\]

Typically

\( 0 \leq u \leq U \)

\( U \gtrsim v_F \)
Charge fractionalization

- Hamiltonian: \[ H = \frac{\hbar}{\pi} \sum_{r=R,L} \int dx \left[ v_+ (\partial_x \phi_+)^2 + v_- (\partial_x \phi_-)^2 \right] \]

- Diagonalization \( \Rightarrow \) mixing angle \( \theta \) | \( \tan \theta = \frac{2u}{v_1 - v_2} \)

- Rotated fields and eigen-velocities
  \[
  \begin{align*}
  \phi_1 &= \cos \theta \ \phi_+ + \sin \theta \ \phi_- \\
  \phi_2 &= \sin \theta \ \phi_+ - \cos \theta \ \phi_-
  \end{align*}
  \]
  and \( v_\pm = v_1 + v_2 \pm \sqrt{\left( \frac{v_1 - v_2}{2} \right)^2 + u^2} \)

- Propagating excitations

- Average charge density \( q_{s,r}(x, t) = \frac{e}{\pi} \langle \partial_x \phi_s, r(x, t) \rangle \phi \)

- Strong interaction: \( \theta = \pi/4 \)

- Excitations characterized by the charge they carry \( \oplus/\ominus \)

Free propagation of two modes: fast charged \( \phi_+ \) and slow neutral \( \phi_- \)
Tunneling

- QPC couples counter-propagating channels $\rightarrow$ two possibilities
- Two setups $s = 1, 2$

SETUP 1

- **Tunneling Hamiltonian** $H_{\text{tun}} = \Gamma \left[ \psi_{s,R}^\dagger(0)\psi_{s,L}(0) + \psi_{s,L}^\dagger(0)\psi_{s,R}(0) \right]$
- **Scattering matrix:**

\[
\begin{pmatrix}
\psi_{s,R} \\
\psi_{s,L}
\end{pmatrix}^{\text{outgoing}} = \begin{pmatrix}
\sqrt{T} & i\sqrt{R} \\
i\sqrt{R} & \sqrt{T}
\end{pmatrix}
\begin{pmatrix}
\psi_{s,R} \\
\psi_{s,L}
\end{pmatrix}^{\text{incoming}}
\]

$T$ is the transmission and $R$ the reflexion probability
Performing the calculation

Zero-frequency crossed correlations of outgoing currents

\[ S_{RL}^{\text{out}} = \int dt dt' \left[ \langle I_{s,R}^{\text{out}}(x, t) I_{s,L}^{\text{out}}(x', t') \rangle_{\varphi} - \langle I_{s,R}^{\text{out}}(x, t) \rangle_{\varphi} \langle I_{s,L}^{\text{out}}(x', t') \rangle_{\varphi} \right] \]

\[ I_{s,r}^{\text{out}}(x, t) \]

- Linear dispersion → \[ \int dt \ I_{j,r}^{\text{out}}(x, t) = \int dt \ I_{j,r}^{\text{out}}(0, t) \]

- Electric current: \[ I_{j,r}^{\text{out}}(0, t) = -ev : \psi_{j,r}^{\dagger}(0, t) \psi_{j,r}^{\text{out}}(0, t) : \]

- Scattering matrix: \[ \begin{pmatrix} \psi_{j,R}(0, t) \\ \psi_{j,L}(0, t) \end{pmatrix}^{\text{out}} = S \times \begin{pmatrix} \psi_{j,R}(0, t) \\ \psi_{j,L}(0, t) \end{pmatrix}^{\text{in}} \]

- Bosonization: \[ \psi_{j,r}^{\text{in}}(0, t) = \frac{U_{j,r}}{\sqrt{2\pi a}} \exp(i\phi_{j,r}^{\text{in}}(0, t)) \]

- Diagonalization:
  \[ \begin{cases} 
  \phi_{1,r}^{\text{in}}(0, t) = \cos \theta \phi_{+,r}^{\text{in}}(0, t) + \sin \theta \phi_{-,r}^{\text{in}}(0, t) \\
  \phi_{2,r}^{\text{in}}(0, t) = \sin \theta \phi_{+,r}^{\text{in}}(0, t) - \cos \theta \phi_{-,r}^{\text{in}}(0, t)
  \end{cases} \]

Quantity of interest: \[ S_{RL}^{\text{out}} \left[ I_{s,r}^{\text{out}}(x, t) \right] \rightarrow S_{RL}^{\text{out}} \left[ \phi_{\pm,r}^{\text{in}}(0, t) \right] \]
Performing the calculation: final expression

- Focus on the following situation
  - different setups $s = 1, 2$
  - strong interaction $\theta = \frac{\pi}{4}$
  - symmetric injection $\pm L$
  - identical packets $\varphi(x)$
  - time delay $\delta T$

- Final expression of noise for the Hong-Ou-Mandel experiment

$$S_{\text{HOM}} = -2S_0 \Re \left\{ \int d\tau \Re [g(\tau, 0)^2] \right. \right.$$

$$\times \int dy_R dz_R \frac{\varphi_R(y_R)\varphi_R^*(z_R)}{(2\pi a)^2 N_R} g(0, y_R - z_R) \int dy_L dz_L \frac{\varphi_L(y_L)\varphi_L^*(z_L)}{(2\pi a)^2 N_L} g(0, z_L - y_L)$$

$$\times \int dt \left[ \frac{h_s(t; y_L + L, z_L + L)h_s(t + \tau - \delta T; L - y_R, L - z_R)}{h_s(t + \tau; y_L + L, z_L + L)h_s(t - \delta T; L - y_R, L - z_R)} - 1 \right] \right.$$}

$$g(t, x) = \left[ \frac{\sinh \left( i \frac{\pi a}{\beta v_+} \right) \sinh \left( i \frac{\pi a}{\beta v_-} \right)}{\sinh \left( \frac{ia + v_+ t - x}{\beta v_+ \pi} \right) \sinh \left( \frac{ia + v_- t - x}{\beta v_- \pi} \right)} \right]^{\frac{1}{2}}$$

$$h_s(t; x, y) = \left[ \frac{\sinh \left( \frac{ia - v_+ t + x}{\beta v_+ \pi} \right)}{\sinh \left( \frac{ia + v_- t - y}{\beta v_- \pi} \right)} \right]^{\frac{1}{2}} \left[ \frac{\sinh \left( \frac{ia - v_- t + x}{\beta v_- \pi} \right)}{\sinh \left( \frac{ia + v_+ t - y}{\beta v_+ \pi} \right)} \right]^{s - \frac{3}{2}}$$
Interference pattern: expected structures

Time delay $\delta T = 0$

Interference of excitations with same charge and velocity

Time delay $\delta T = \pm L \frac{v_+-v_-}{v_+ v_-}$

Interference of excitations with different velocity and possibly different charge

+ flat background contribution from non-interfering excitations
Results: setup 1

Central dip
- noise reduction $\rightarrow$ destructive interference of $\oplus/\ominus$ excitations
- loss of contrast due to interactions, strong dependence on resolution

Side dips
- $\oplus$-excitations with different velocities
- destructive interference
- velocity mismatch: asymmetry + smaller than half central dip

3-dip structure + flat background contribution (no interference)

\[ |\mathcal{S}_{\text{HBT}}(\delta T)|/(e^2RT) \]

\[ L = 2.5 \mu m, L = 5 \mu m \]

\[ \epsilon_0 = 175 \text{mK}, \gamma = 1 \]

\[ \text{setup 1} \]

\[ |\mathcal{S}_{\text{HOM}}(\delta T)|/(e^2RT) \]

\[ \Delta t = \frac{1}{L} \text{, } L = 2.5 \mu m, L = 5 \mu m \]

\[ \epsilon_0 = 0.7 \text{K}, \gamma = 8 \]

\[ \Delta t = \frac{1}{L} \text{, } L = 2.5 \mu m, L = 5 \mu m \]
Results: setup 2

Central dip identical for the 2 setups

- interference independent of the charge carried by the excitations, both in sign and amplitude

Side peaks
- excitations with opposite charge
  - constructive interference
- vanish as the resolution increases
- velocity mismatch: asymmetry

peak-dip-peak structure + flat background contribution (no interference)
Central dip vs. packet broadening
- bottom of dip sinks deeper for wider packets

Contrast $\eta = 1 - \frac{S_{\text{HOM}}(\delta T=0)}{2S_{\text{HBT}}}$
- dramatic reduction as resolution increases

Electron-hole collision
- confirms the pattern
  $\oplus/\oplus$ or $\ominus/\ominus \rightarrow$ destructive
  $\oplus/\ominus \rightarrow$ constructive
- side peaks smaller than dips
  $\rightarrow \oplus/\ominus$ interference is weaker
Refined model for the source

- More experimental results are now becoming available for setup 1
- Injection problematic for spatially extended packets

From the SES

In our model

- Refined setup and contrast

All these are given by the experiment No adjustable parameters! Allows for a more careful comparison with experimental results
Comparing with experimental results

- **Contrast**

![Graph showing contrast vs. $\tau_e$ (ps)]

- **Possible sources of decoherence?**
  - Differences between emitters
  - Environmental noise
  - Coulomb interaction

Parameters:
- $\epsilon_0 = 0.7$ K
- $1/(k_B \beta) = 100$ mK
- $\tau_s = 70$ ps
- $f = 1$ GHz

Only the last scenario can account for all observed data!
Conclusions

- Our interacting model recovers the main experimental features
  ➡️ Detailed quantitative comparison is under way!
- Strong coupling between channels accounts for a sensible loss of contrast of the HOM central dip
- The contrast strongly depends on the energy resolution of the injected wave-packet
- Fast and slow modes interfere and produce, depending on the charge carried by the colliding excitations, smaller asymmetric dips or peaks

*Interactions and charge fractionalization in an electronic HOM interferometer*
Claire Wahl, Jérôme Rech, Thibaut Jonckheere, Thierry Martin
Floquet scattering theory

- **Stationary scattering matrix** \( \hat{b}(\epsilon) = S(\epsilon) \hat{a}(\epsilon) \) relates outgoing fermionic operators \( \hat{b}(\epsilon) \) to incoming ones \( \hat{a}(\epsilon) \):

- **Dynamic scattering matrix** \( S(\epsilon) \rightarrow S(\epsilon_1, \epsilon_2) \)
  - two energy arguments! The energy of incoming and scattered electron can be different

- **Floquet scattering matrix** extends this to time-periodic problems, by expressing the absorption/emission of a quantized number of energy quanta \( \hbar \Omega \) (\( \Omega \) is the frequency of the periodic potential):

  \[
  \hat{b}(\epsilon) = \sum_m U_m(\epsilon) \hat{a}(\epsilon_m)
  \]

  \[
  U_m(\epsilon) = \sum_n c_n c_{n+m}^* S(\epsilon - n)
  \]

  with \( \epsilon_{\pm m} = \epsilon \pm m\hbar \Omega \), and \( c_n \) the Fourier coeff. of the periodic potential

Back to **TALK**
Tunneling and refermionization

- **Tunnel Hamiltonian**
  \[
  H_{\text{tun}} = \Gamma \left[ \psi_{1,R}^\dagger(0) \psi_{1,L}(0) + \psi_{1,L}^\dagger(0) \psi_{1,R}(0) \right]
  \]
- **Refermionization**
  \[
  \psi_p^{\pm}(x) = \frac{U_p^{\pm}}{\sqrt{2\pi a}} e^{i\phi_p^{\pm}x}
  \]

  where

  \[
  \phi_A^{\pm} = \pm \left( \phi_{1,R} - \phi_{1,L} \right) \pm \left( \phi_{2,R} - \phi_{2,L} \right) \frac{2}{2}
  \]

  \[
  \phi_S^{\pm} = \pm \left( \phi_{1,R} + \phi_{1,L} \right) \pm \left( \phi_{2,R} + \phi_{2,L} \right) \frac{2}{2}
  \]
- **Full Hamiltonian is now quadratic!**

  \[
  H = -i\hbar \sum_{p,\sigma} v_\sigma \int dx \psi_p^{\sigma \dagger}(x) \partial_x \psi_p^\sigma(x) - \Gamma \psi_{A+}^\dagger(0) \psi_{A-}(0)
  \]
- **Scattering matrix**

  \[
  r_0 = \cos \varphi \text{ and } t_0 = \sin \varphi \text{ with } \varphi = -\Gamma/(\hbar \sqrt{v_+ v_-})
  \]

  \[
  \begin{pmatrix}
  \psi_{A+} \\
  \psi_{A-}
  \end{pmatrix}^{\text{outgoing}} = \begin{pmatrix}
  t_0 & -i r_0 \\
  -i r_0 & t_0
  \end{pmatrix} \begin{pmatrix}
  \psi_{A+} \\
  \psi_{A-}
  \end{pmatrix}^{\text{incoming}}
  \]
- **Outgoing current?**

  \[
  \rightarrow \text{ exact same expression up to defining } r_0 = \sqrt{R} \text{ and } t_0 = \sqrt{T}
  \]