Mesoscopic topological insulator

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Outline

Finite-size effects:

STI vs. WTI

- STI: Berry phase π
- WTI: even/odd effects

real space geometries:

- nanofilm
- nanowire
- nanoparticle

Response to disorder: localization properties

- spectral flow

Focus on the cases of

- WTI nanofilm: nontrivial/delocalized
- STI nanowire: trivial/localized

Introduction

topological insulator

bulk: insulating/gapped

edge/surface: metallic/gapless

semiconductor

narrow-gap band insulator

inverted gap: $\Delta E < 0$

$\nu = 0, 1$

BiSe, BiTe, …

$\mathbb{Z}_2$ topological insulator

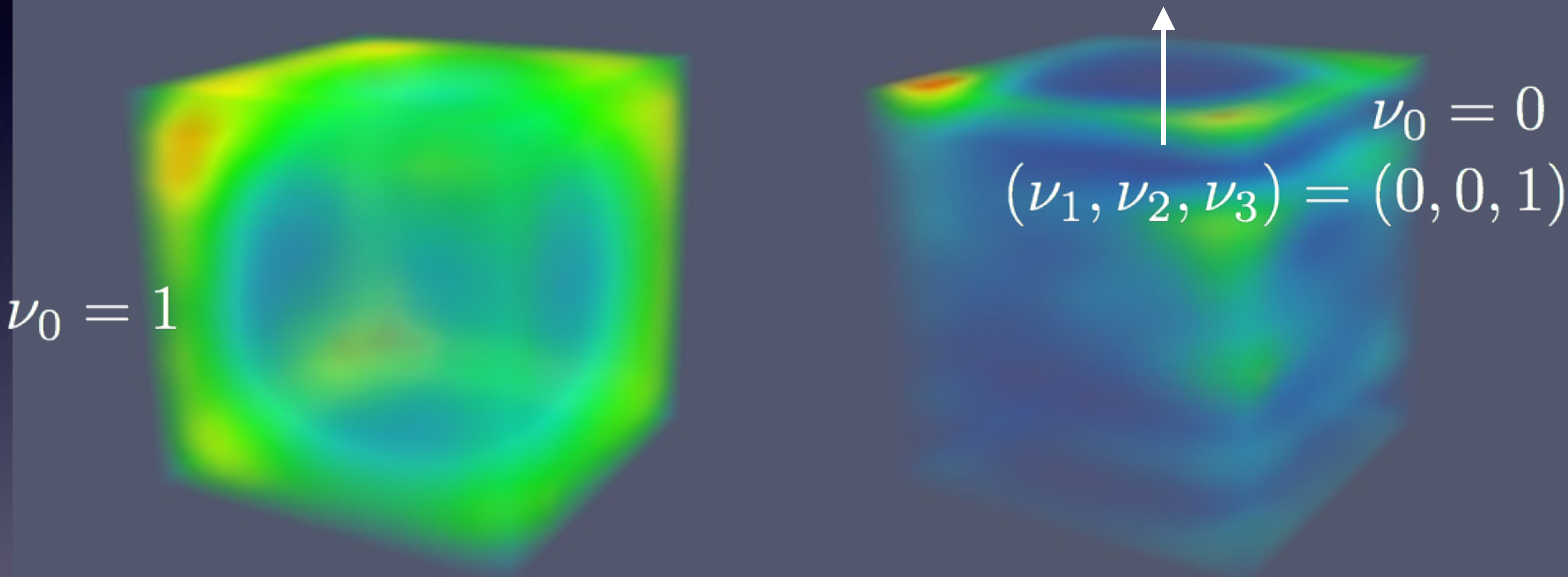
time-reversal symmetry

Kramers degeneracy

spin-orbit coupling

STI vs. WTI

- two types of topological insulators: strong vs. weak



STI: all the surfaces are metallic

single Dirac cone

WTI: there are gapped surfaces

double Dirac cones

Model: Wilson-Dirac type effective Hamiltonian

Tight-binding form:

$$H(\boldsymbol{k}) = \tau_z m(\boldsymbol{k}) + \tau_x \sum_{\mu=x,y,z} \sigma_\mu A_\mu \sin k_\mu$$

where

$$m(\boldsymbol{k}) = m_0 + 2 \sum_{\mu=x,y,z} m_{2\mu}(1 - \cos k_\mu)$$

In the continuum limit:

$$H_\Gamma(\boldsymbol{k}) = \tau_z m(\boldsymbol{k}) + \tau_x \sum_{\mu=x,y,z} \sigma_\mu A_\mu k_\mu$$

where

$$m(\boldsymbol{k}) = m_0 + \sum_{\mu=x,y,z} m_{2\mu} k_\mu^2$$

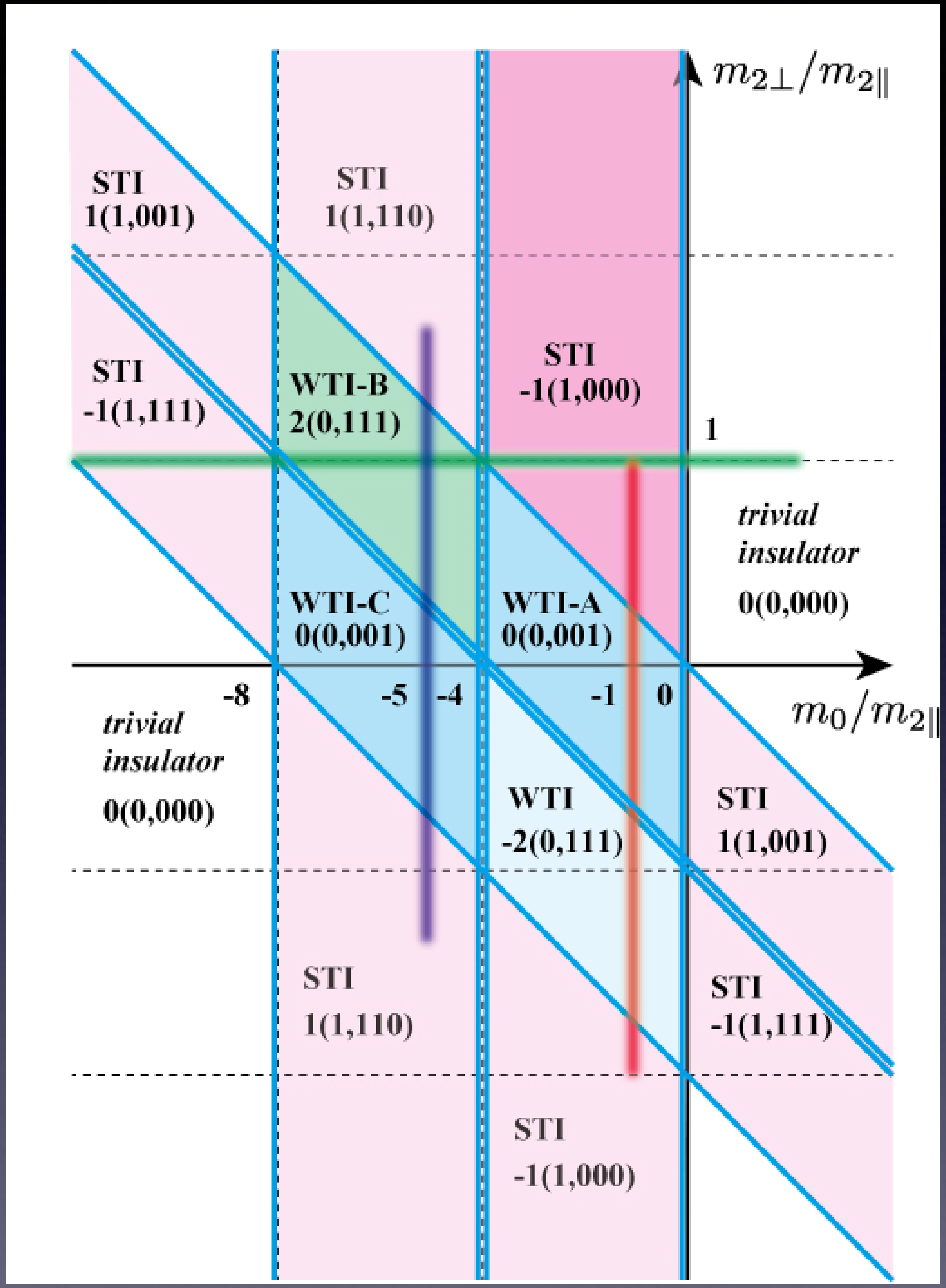
Wilson terms

Phase diagram
in the clean limit

case of uniaxial anisotropy

$$m_{2x} = m_{2y} = m_{2\parallel}$$
$$m_{2z} = m_{2\perp}$$

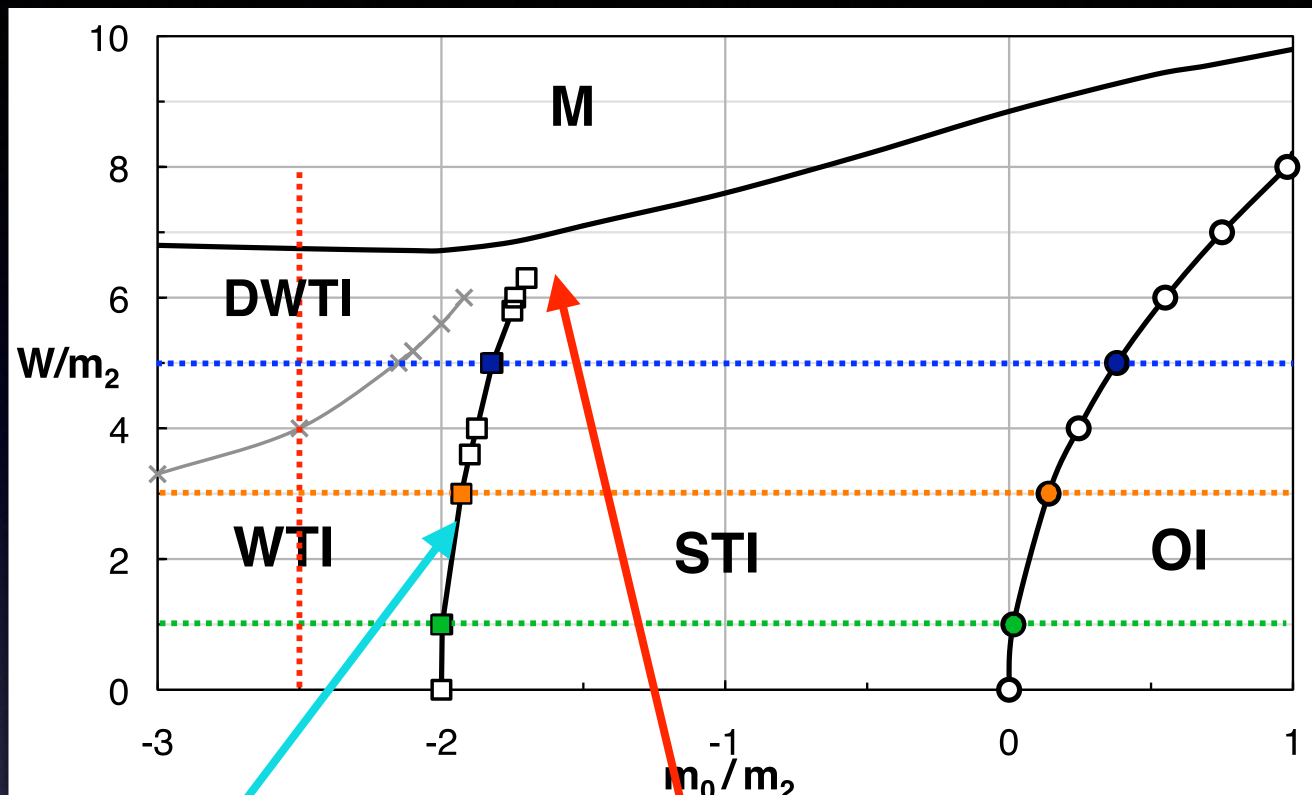
various STI & WTI phases

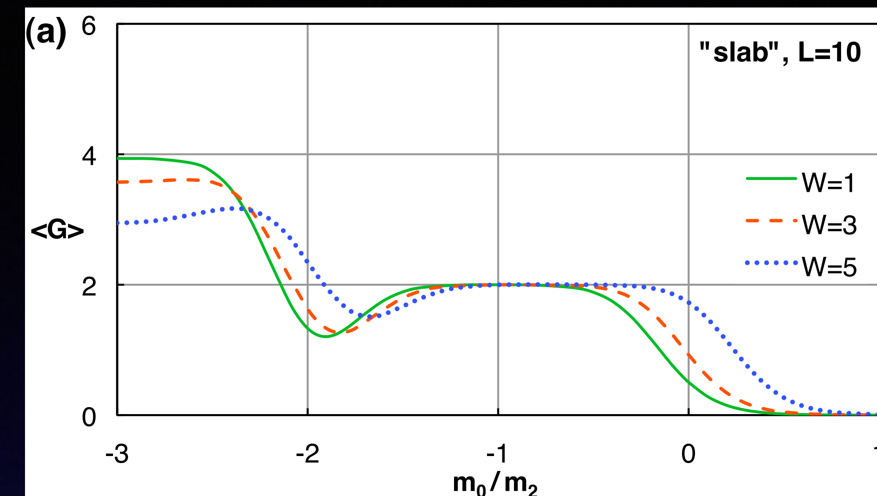


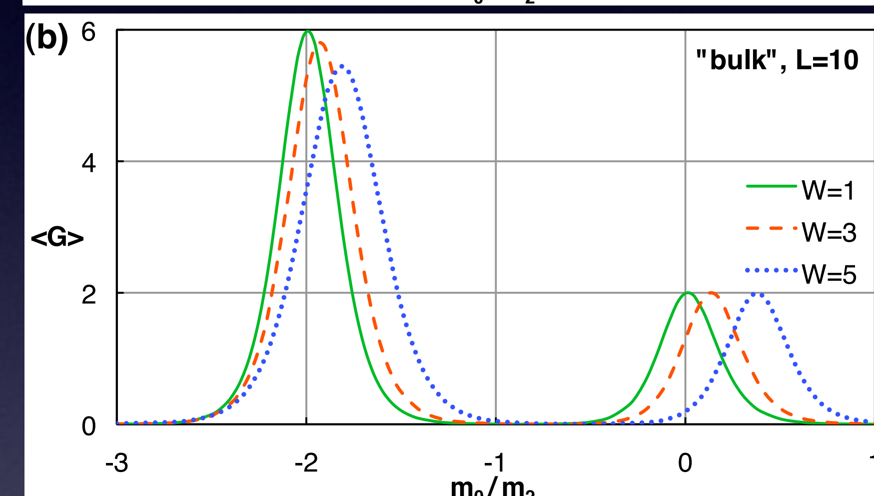
KI, Okamoto, Yoshimura, Takane & Ohtsuki,
PRB 86, 245436 (2012)

Phase diagram of disordered STI & WTI

Kobayashi, KI & Ohtsuki,
PRL 110, 236803 (2013)

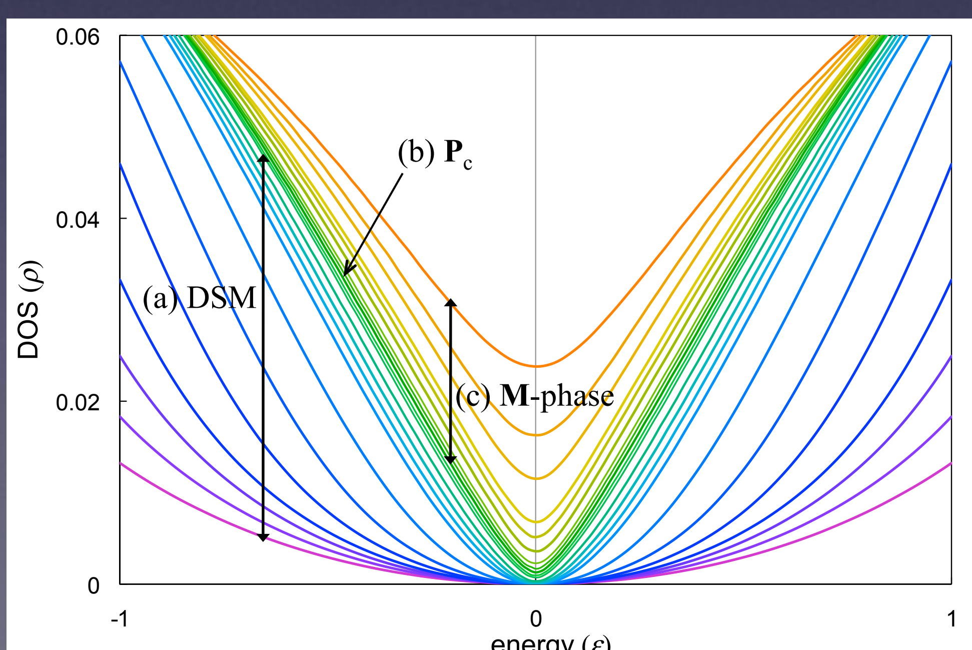


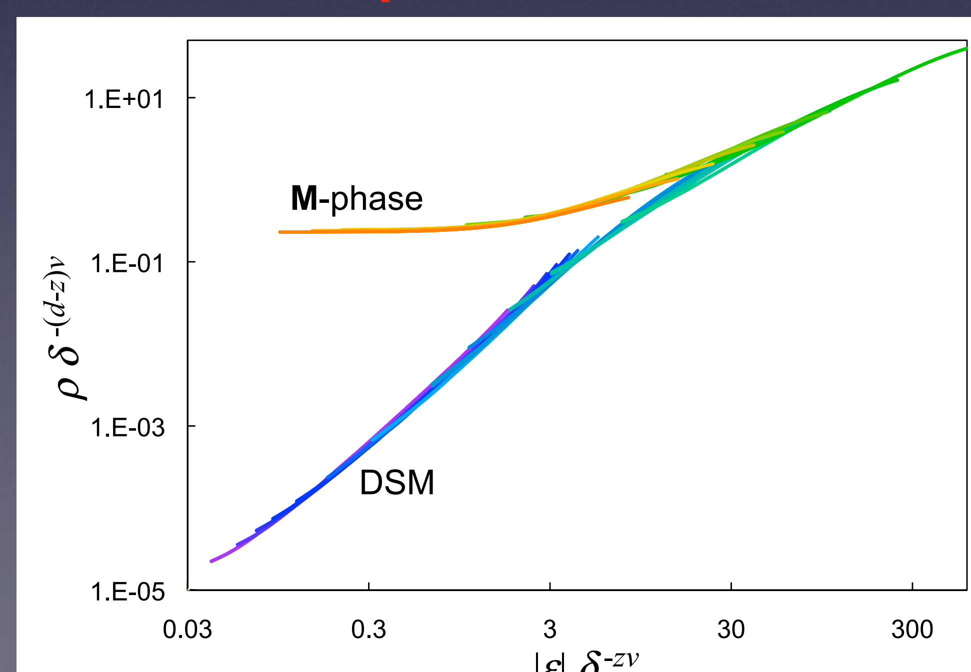




Dirac semimetal

tricritical point





- quadratic density of states
- one-parameter scaling

Kobayashi, Ohtsuki,KI & Herbut,
PRL 112, 016401 (2014)

"Density of states scaling" in disordered Dirac semimetal

Act I
WTI nanofilm: even/odd feature

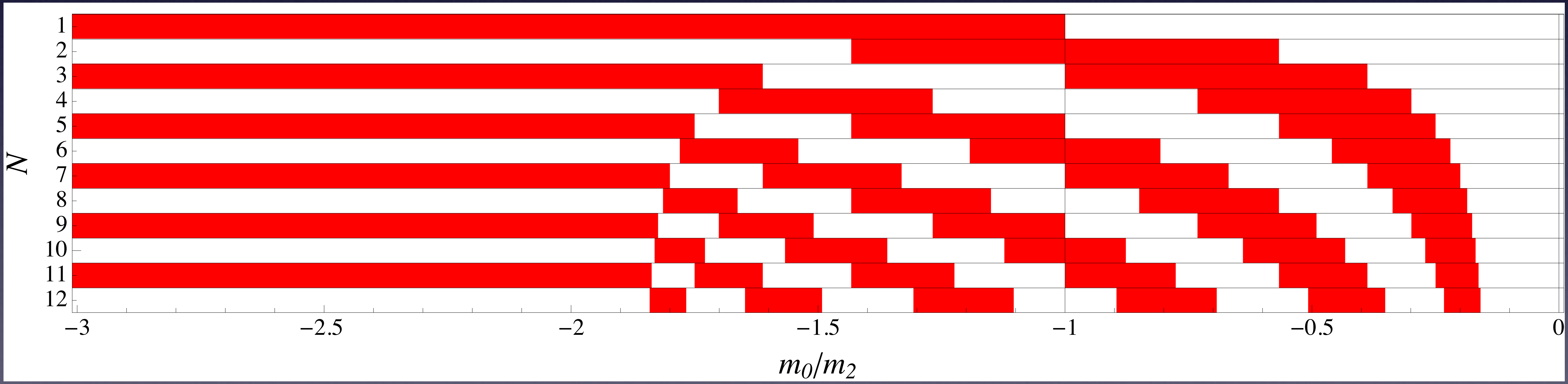
TI nanofilms

topological classification: 2D Z2 topological number

$$\nu = \sum_{\boldsymbol{k} \in \mathcal{B}_{1/2}} n_{12}(\boldsymbol{k}) \mod 2$$

Fukui & Hatsugai, JPSJ 76, 053702 (2007)

where $$F_{12}(\boldsymbol{k}) = \Delta_1 A_2(\boldsymbol{k}) - \Delta_2 A_1(\boldsymbol{k}) + 2\pi i n_{12}(\boldsymbol{k})$$

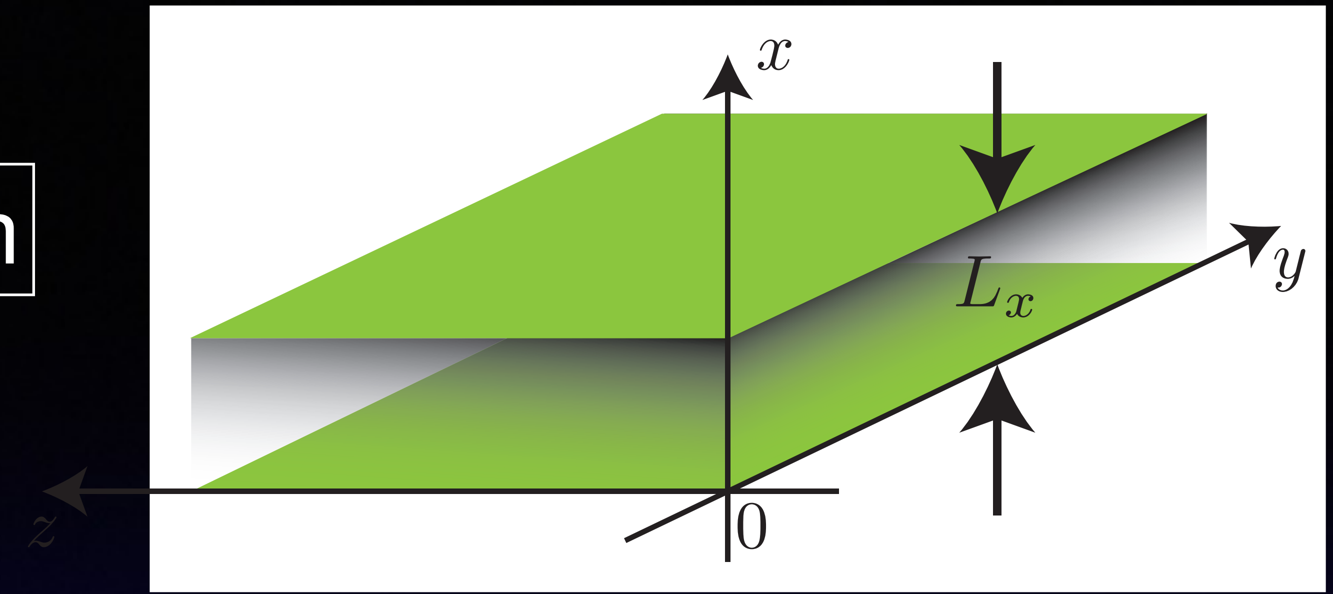


Kobayashi, Yoshimura, KI & Ohtsuki, arXiv:1409.1707

TI nanofilm = (effective) 2D TI

or QSHI: quantum spin Hall insulator

Even/odd feature in spectrum



Nano-film/flake geometry

- finite thickness
- side surfaces (edges)

A typical situation in WTI

two Dirac cones at $k_x = 0, \pi$

Boundary conditions:

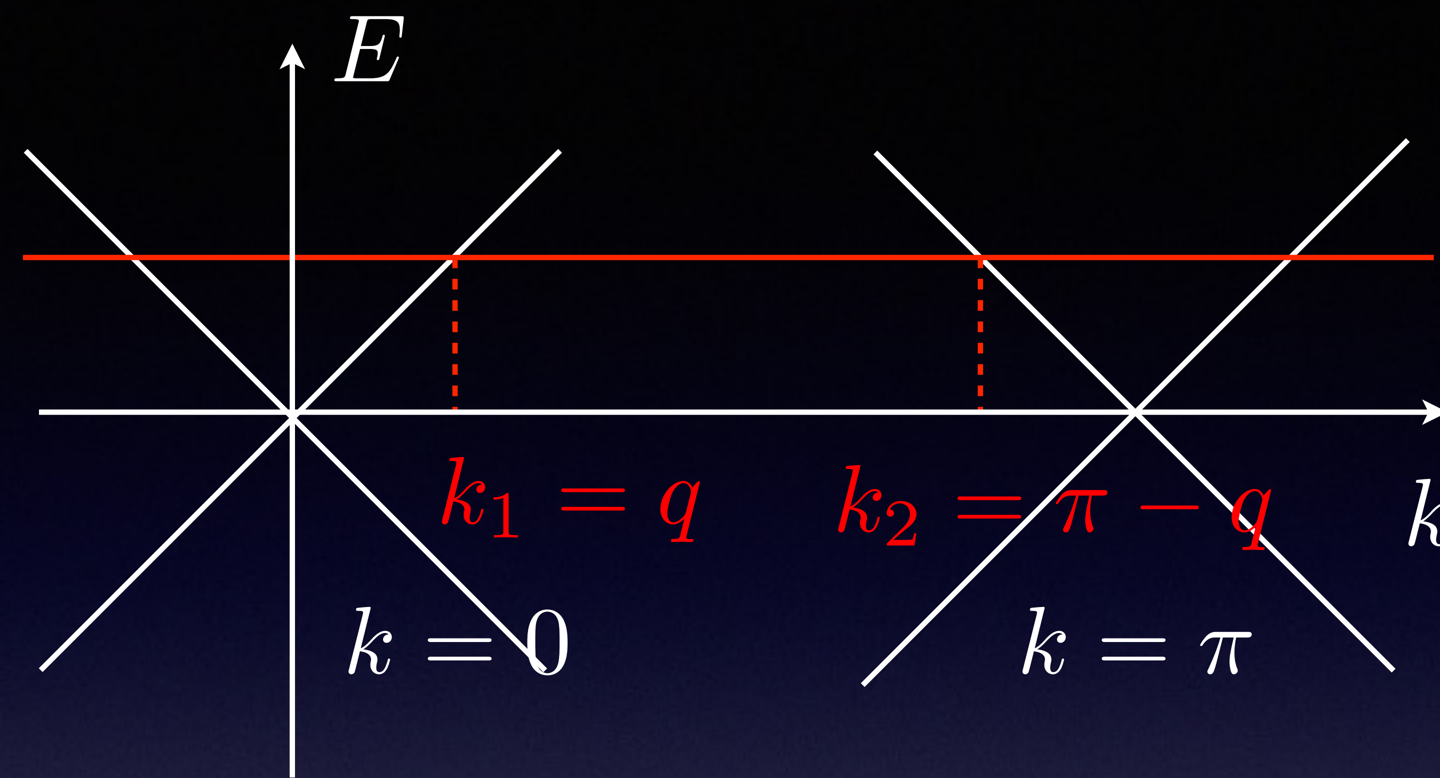
$$\psi(x=0)=0, \quad \psi(x=N+1)=0$$

$$\psi(x) = e^{ik_1 x} - e^{ik_2 x}, \qquad k_1 = 0, k_2 = \pi$$

this is compatible with b.c.
if $N = 1,3,5,\ldots$ ⟶ $E = 0$

if $N = 2,4,6,\ldots$

→ finite-size energy gap

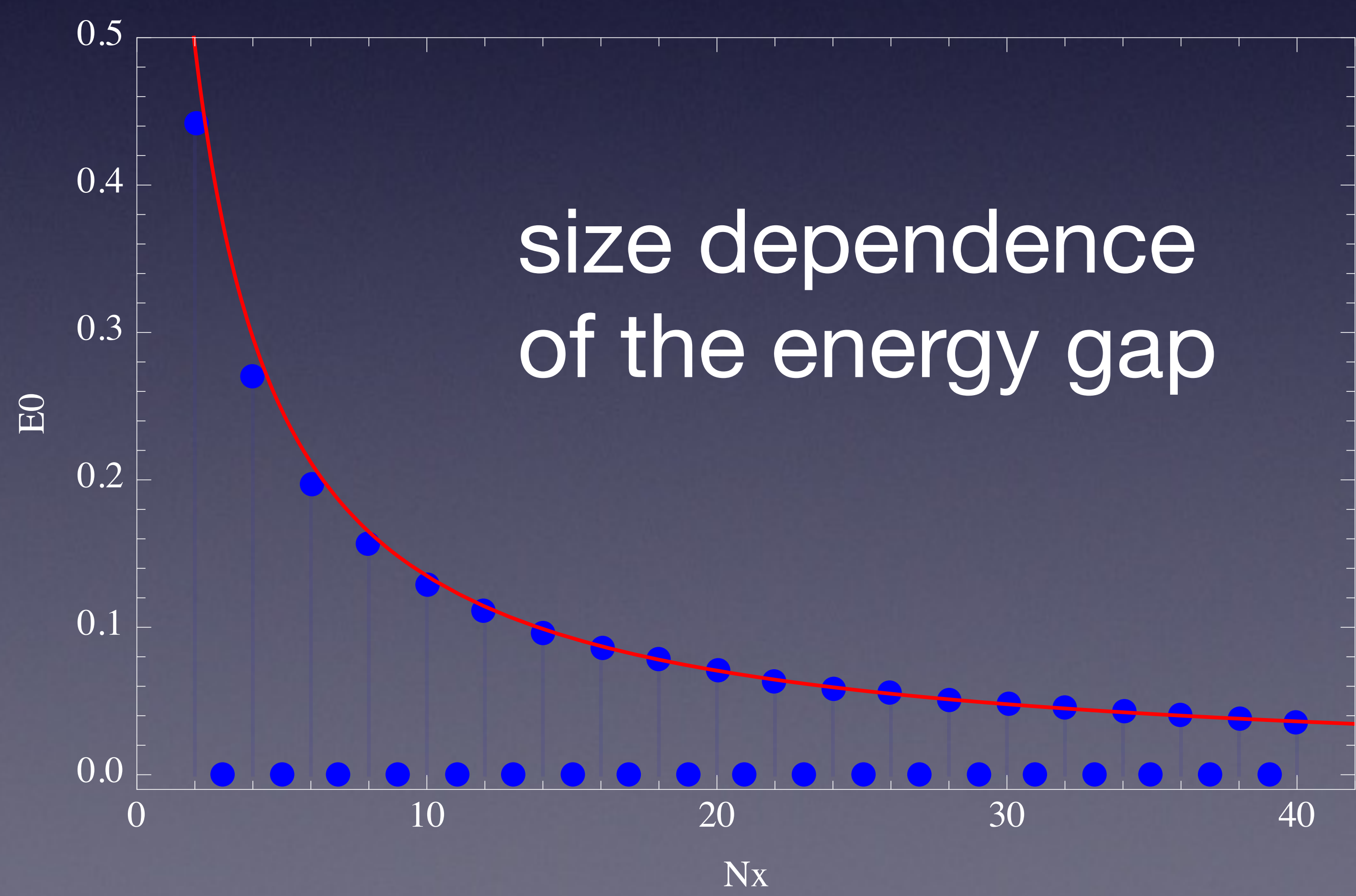


$$\psi(x) = e^{ik_1 x} - e^{ik_2 x}$$

$$k_1 = q,\ k_2 = \pi - q$$

$$q = \frac{1}{2(N+1)}\pi$$

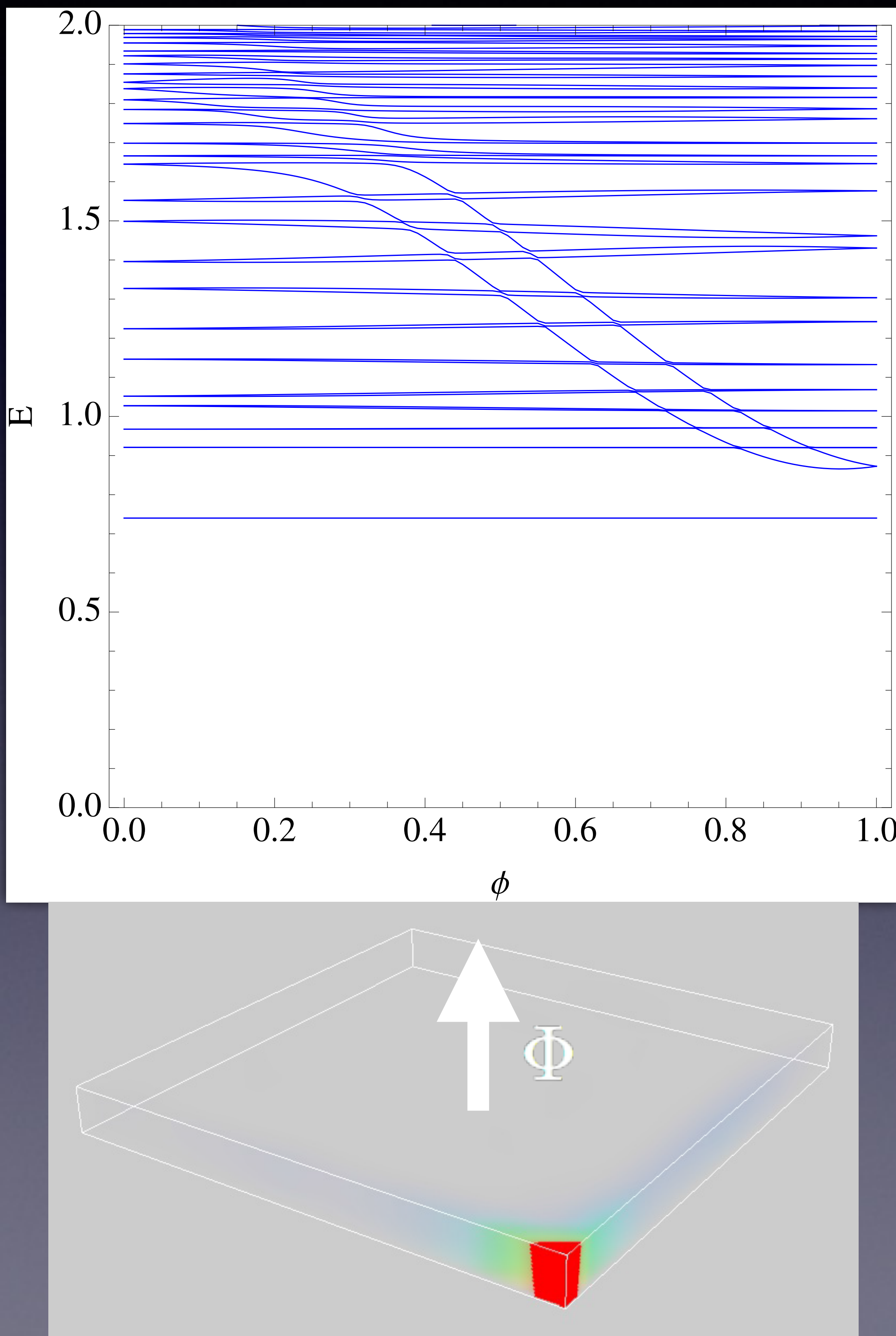
$$E = \begin{cases} 0 & \textit{if N is odd} \\ \dfrac{A}{2(N+1)}\pi & \textit{if N is even} \end{cases}$$



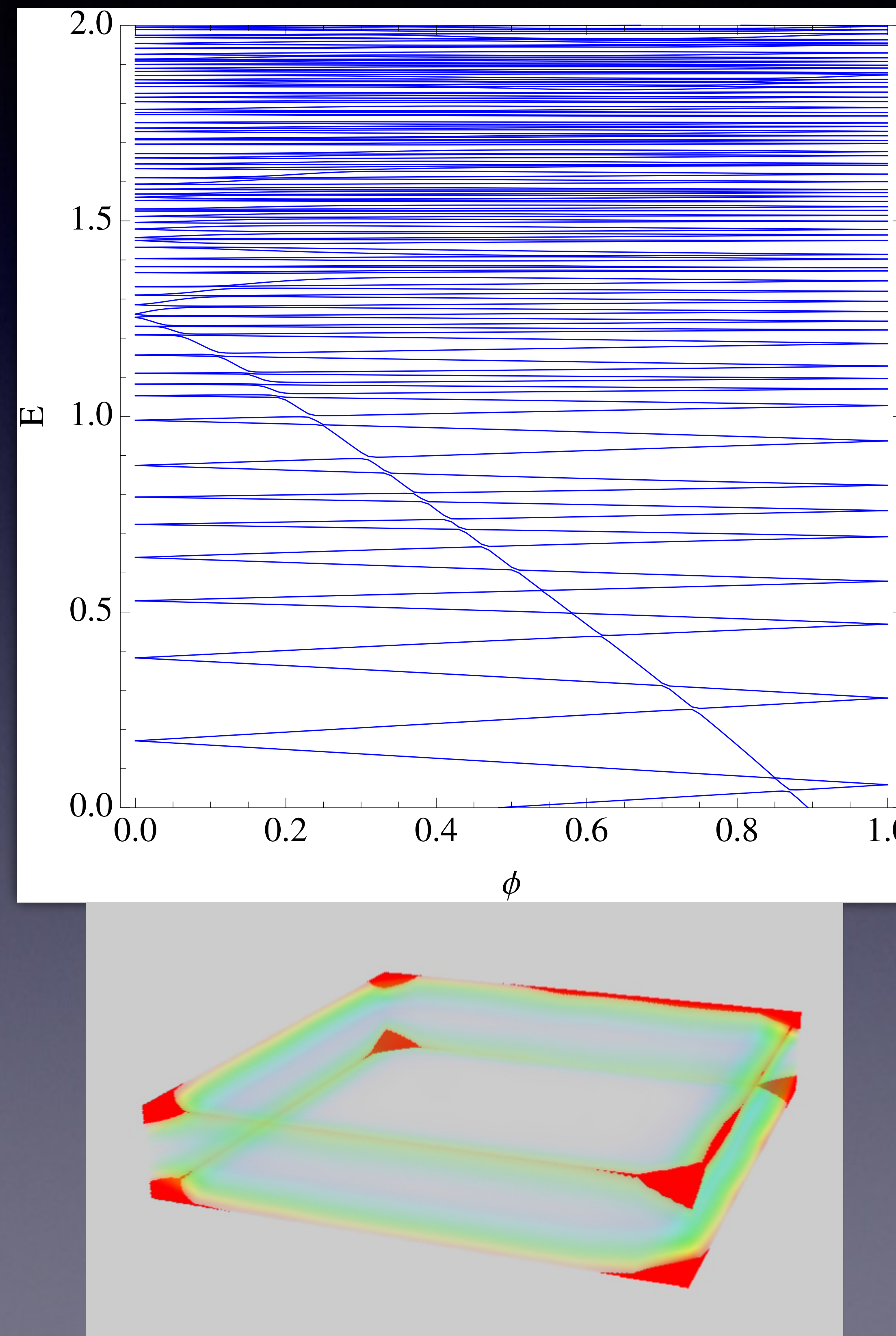
KI, Okamoto, Yoshimura, Takane & Ohtsuki, PRB 86, 245436 (2012)

Even/odd feature in localization properties: spectral flow

N=even



N=odd



Act II
case of STI nanowire

STI: a single Dirac cone

Is a system of single Dirac cone always protected?

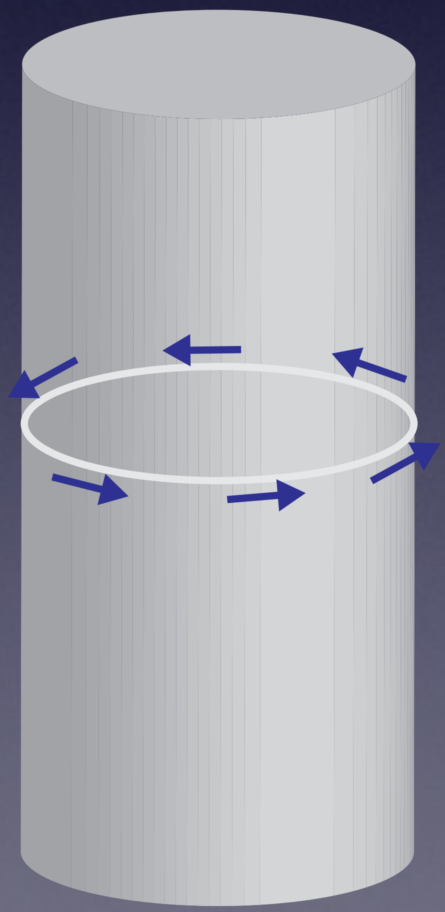
The *cylindrical* topological insulator

- case of *cylindrical* topological insulator

"Berry phase π"

$$H_{\rm surf} = \begin{bmatrix} 0 & -ip_z + \frac{1}{R}\left(-i\frac{\partial}{\partial\phi} + \frac{1}{2}\right) \\ ip_z + \frac{1}{R}\left(-i\frac{\partial}{\partial\phi} + \frac{1}{2}\right) & 0 \end{bmatrix}$$

An electron on the cylindrical surface behaves as if an *imaginary solenoid* pierces the cylinder.



"intrinsic" Aharonov-Bohm effect

KI, Takane & Tanaka, Phys. Rev. B ***84****, 195406 (2011)*

Gapped surface states on the cylinder

- spin Berry phase
- spin-to-surface locking

→ Half-odd integral quantization → Finite-size energy gap

$$e^{iL_z(\phi+2\pi)} \times \underline{(-1)} = e^{iL_z\phi}$$

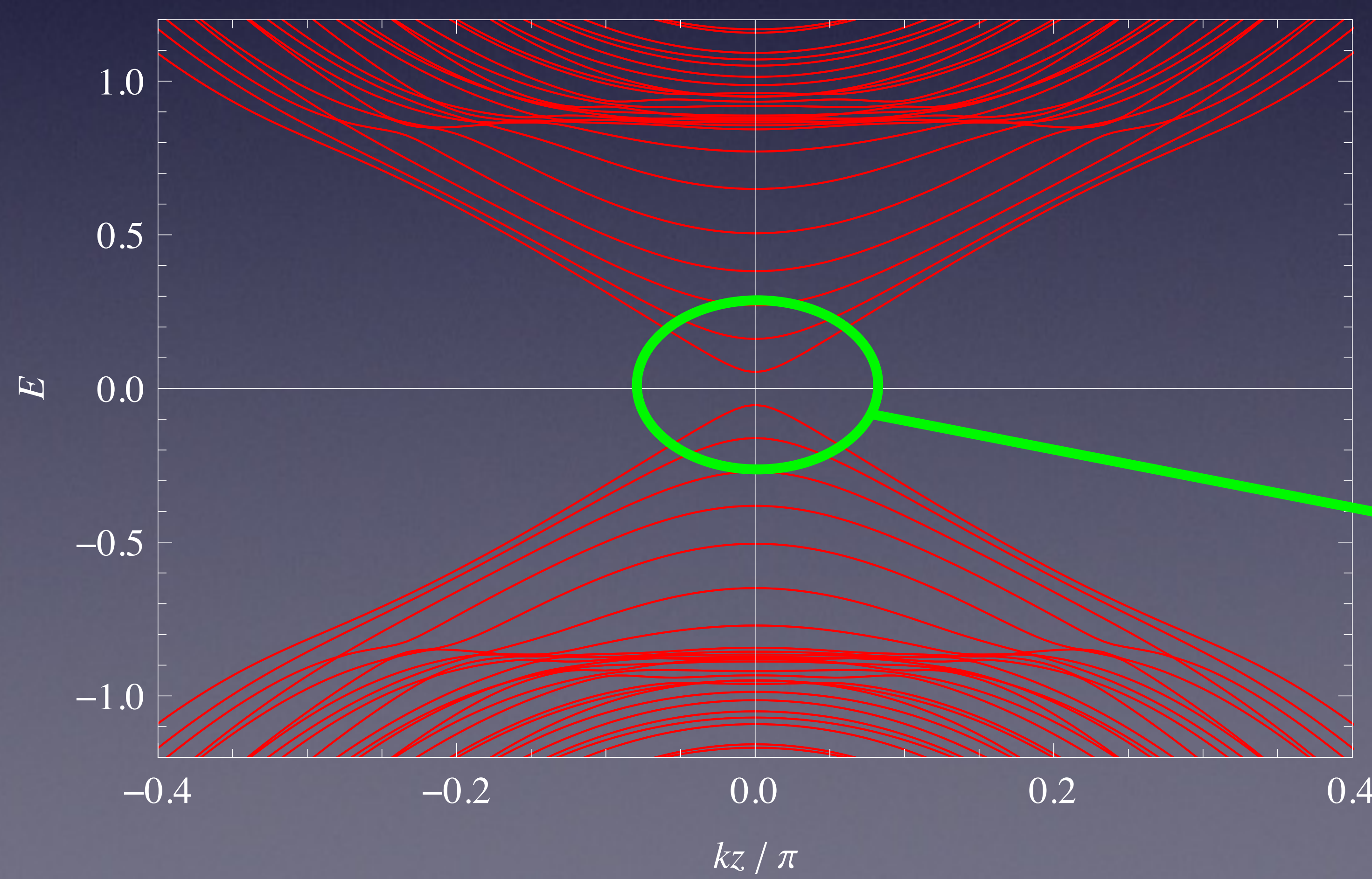
anti-periodic

$$L_z = \pm\frac{1}{2}, \pm\frac{3}{2}, \cdots$$

$$L_z = p_\phi R$$

$$E = \pm\sqrt{p_\phi^2 + p_z^2}$$

$$\Delta E \simeq R^{-1}$$



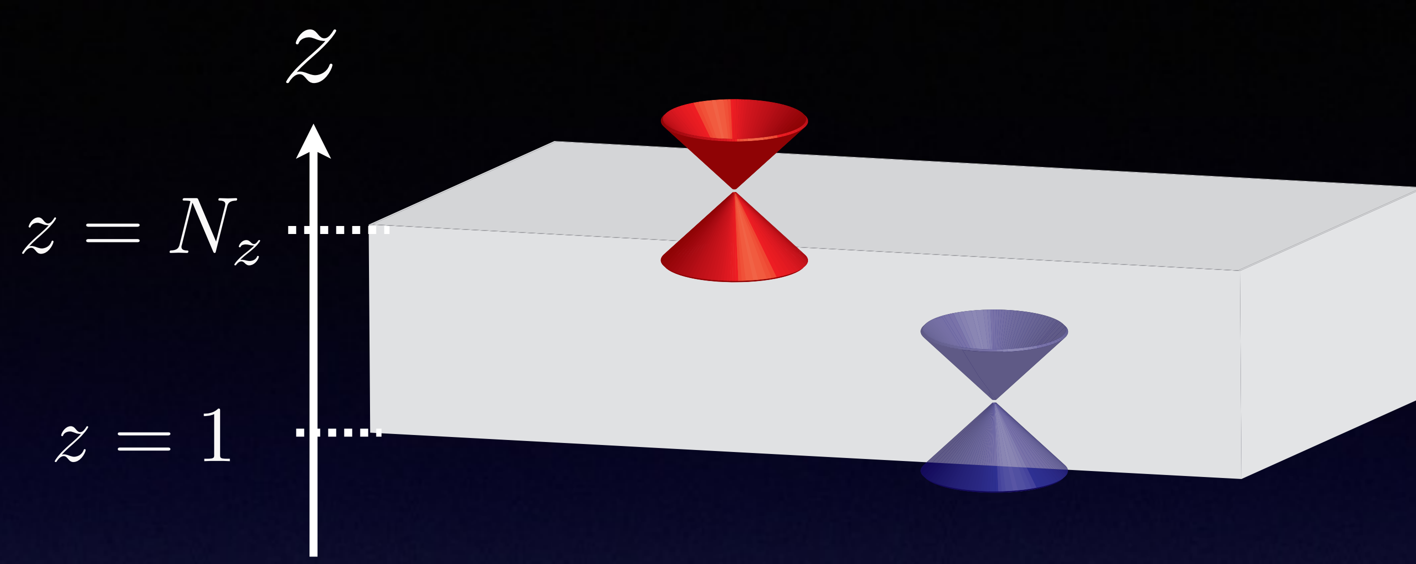
Finite-size energy gap decays slowly (only *algebraically*)

$$\Delta E \propto (N_x + N_y)^{-1}$$

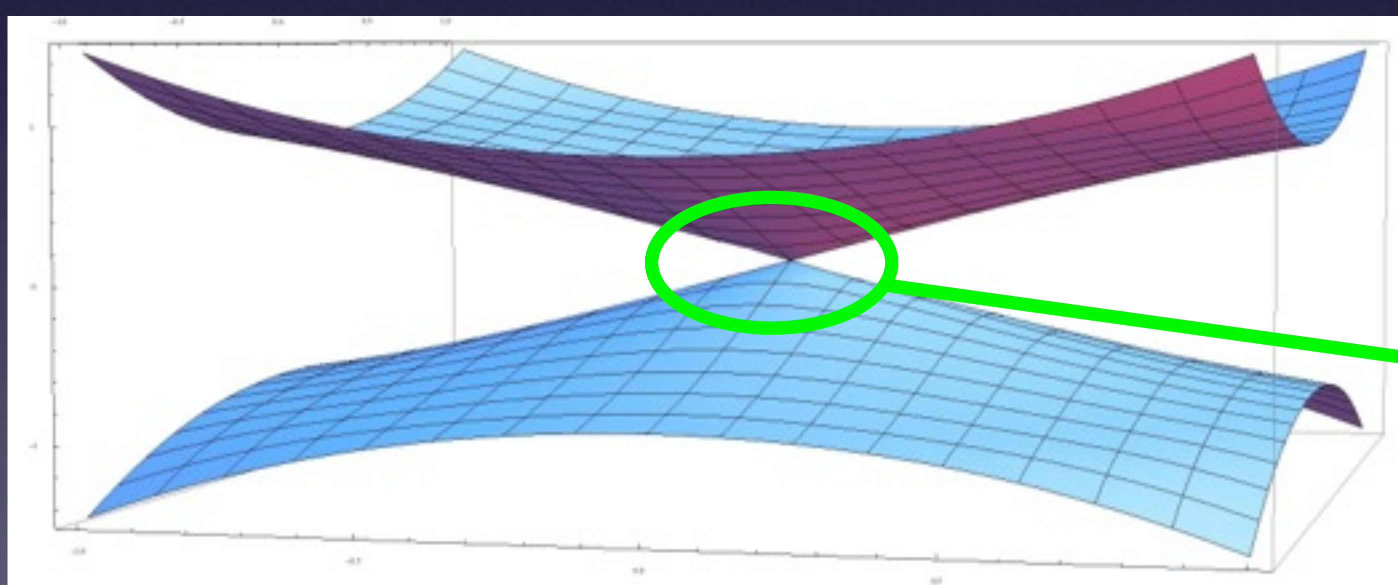
prism: $N_x \times N_y$

cf. case of TI slab

The magnitude of the gap is determined by the *overlap* of the two wave functions.



a system of infinitely large slab



$$\Delta E \propto e^{-\kappa N_z}$$

exponentially small

Linder, Yokoyama & Sudbo, PRB '09; Lu et al., PRB '10

Conclusions

Mesoscopic topological insulator

- importance of real space geometry
- peculiar finite-size effects

case 1: WTI nano-film/flake

- even/odd feature
- spectral flow: connected

  protected/delocalized edge state

in spite of a system of

double Dirac cones

case 2: STI nanowire

- spectral flow: disconnected

  surface states: localized

in spite of a system of

single Dirac cone