Non-adiabatic effect on the quantum heat flux control

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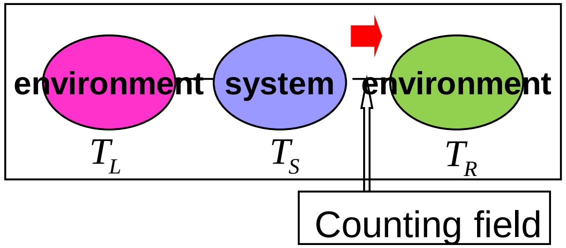
(C.Uchiyama, PRE,89 (2014) 052108)

1.Overview of this talk

- Transport of quantum particles between two environments
  - Cyclic and out-of-phase control of environmental temperatures
  - Use of full counting statistics
  - Based on the quantum master equation under Born-Markov approximation
    - factorized initial condition
    - system-environment interaction
      - weak
      - infinitely short correlation time (memory-less)



Non-adiabatic effect shows…

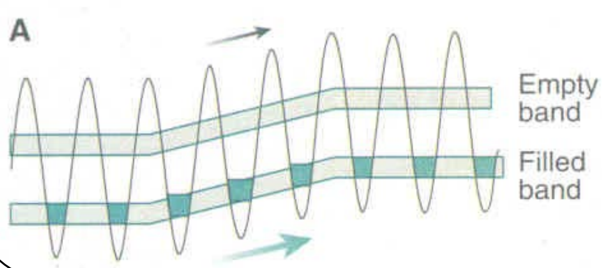
"memory" of initial condition of the relevant system

⇒Reminiscent effect

1. Backgrounds

- **Quantum pumping:** (quantum transport between two environments)
  - Adiabatic electron pump : Thouless, PRB27(1983)6083

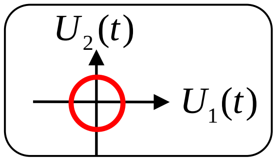
Slow modulation of potential



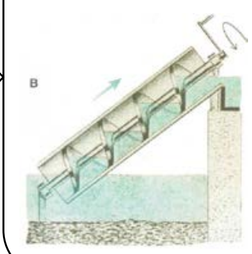
out-of-phase modulation of two-kinds of standing waves

$$U(x,t)=\sum_{k=1}^{2} U_k(t)\,\boxed{f_k(x)}$$

Standing wave



Archmedian screw

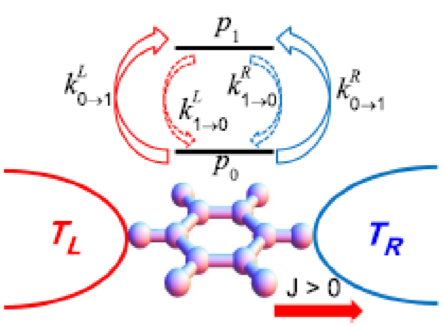


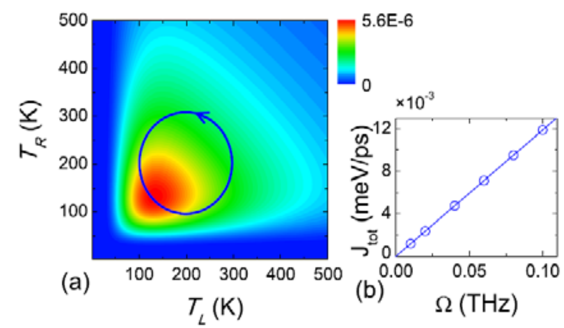
Altshuler and Glazman, Science 283 (1999) 1864

Experimental demonstration by Switkes et. al. Science 283(1999)1905

  - Adiabatic heat pump : Ren, Li, Hänggi, PRL104(2010)170601

Anharmonic junction model





3. Objective of this study

- **Berry-phase induced heat pump** (Ren, Li, Hänggi, PRL104 (2010) 170601)

Anharmonic junction model

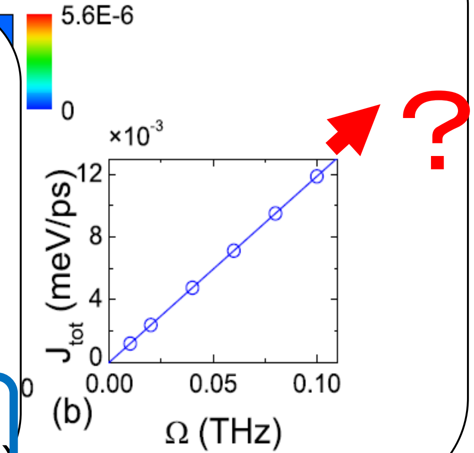
Pumping current

$$\tilde{j}^{\nu} = \frac{1}{\mathcal{T}} J^{\nu} = \frac{\hbar\omega_0}{\mathcal{T}} \left(\boxed{\mathcal{G}_1^{\nu}} + \boxed{\mathcal{G}_2^{\nu}} \right) = \frac{1}{\mathcal{T}} \mathcal{G}_{ad}^{\nu}$$

$(\nu = L, \text{ or } R)$

Dynamical phase

Berry phase
(Geometrical phase)



Adiabatic approximation

(Sufficiently slow modulation)

$\tau_r \ll$ period of modulation

(τ_r = relaxation time of junction system)

Pumping current obtained in this study

$$\tilde{j}^{\nu} = \frac{1}{\mathcal{T}} J^{\nu} = \frac{1}{\mathcal{T}} \left(\boxed{\mathcal{G}_{ad}^{\nu}} + \boxed{\mathcal{G}_{nad}^{\nu}} \right) = \frac{\hbar\omega_0}{\mathcal{T}} \left(\boxed{\mathcal{G}_1^{\nu} + \mathcal{G}_2^{\nu}} + \boxed{\mathcal{G}_3^{\nu}} \right)$$

Dynamical phase

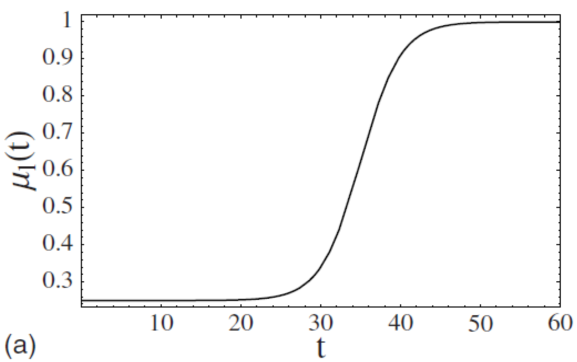
Berry phase

4. Motivation

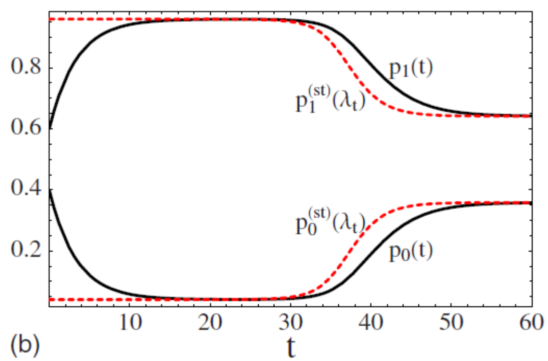
- Non-adiabatic effect on entropy production

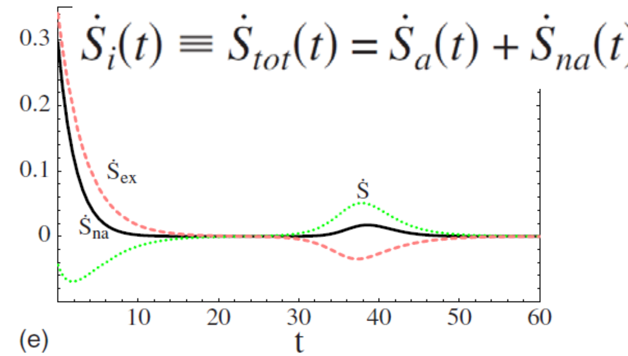
  electron transfer via quantum dot (Esposito,Harbola,Mukamel,PRE76(2007)031132)

Modulation protocol



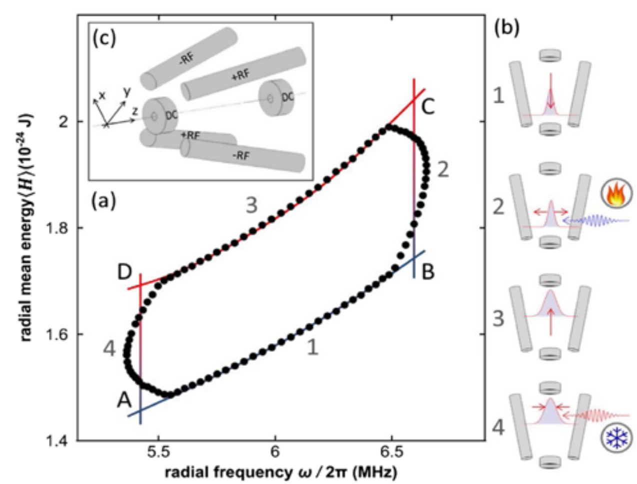
Population in quantum dot





- **Microscopic heat engine with ion trap**

  (O.Abah et.al. PRL109 (2012) 203006)



5.Conventional studies on quantum pumping

Stochastic pump	（interaction coefficient）	Sinitsyn -Nemenman (2007)
Heat pump	（temperature）	Ren-Li-Hänggi (2010)
Electron pump	（chemical potential）	Yuge-Sagawa-Sugita-Hayakawa(2012)

Full counting statistics

M. Esposito, U. Harbola, and S. Mukamel, Rev. Mod. Phys. 81, 1665 (2009).

master equation under counting field

Adiabatic approximation

Eigenvalues of time evolution (non-Hermitian) matrix

0 stationary state

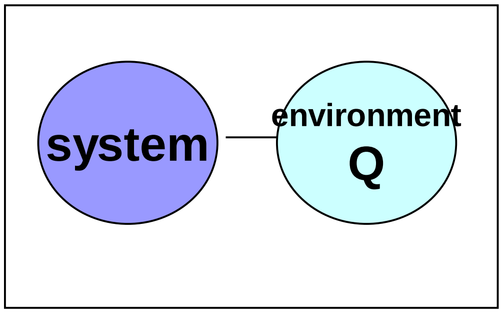
~~non-zero non-stationary state~~

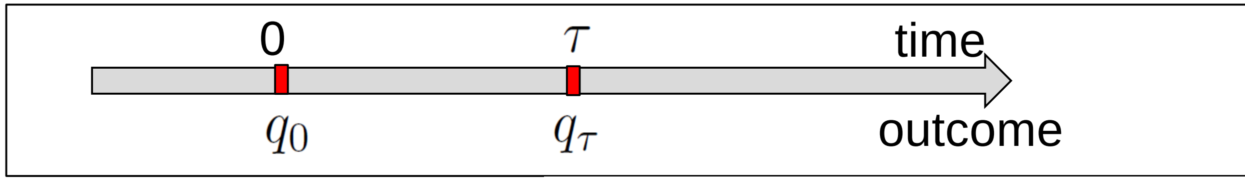
6. Formulation -1

Full counting statistics

M. Esposito, U. Harbola, and S. Mukamel, Rev. Mod. Phys. 81, 1665 (2009).

Two-point projective measurements on variable Q





Probability density of $\Delta q = q_\tau - q_0$: $P_\tau(\Delta q) = \sum_{q_\tau, q_0} \delta(q_\tau - q_0) P[q_\tau, q_0]$

Generating function

$$S_\tau(\chi) = \ln \int P_\tau(\Delta q) e^{i\chi\Delta q} d\Delta q$$

(χ : counting field)

$\Downarrow$

$$S_\tau(\chi) = \ln \mathrm{Tr}_s \rho^\chi(\tau)$$

$$\rho^\chi(\tau) = \mathrm{Tr}_E U_{\chi/2}(\tau,0) W_0 U_{-\chi/2}(\tau,0)$$

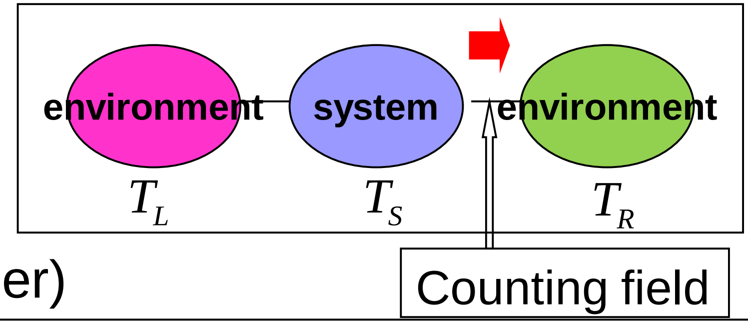
$$U_\chi(\tau,0) = e^{i\chi Q(\tau)} U(\tau,0) e^{-i\chi Q(0)}$$

cumulants of Q

$$\langle \Delta q^n \rangle_c = \partial^n S_\tau(\chi)/\partial(i\chi)^n|_{\chi=0}$$

6. Formulation -2

- Full counting statistics



Time-convolutionless master eq. (2nd order)

$$\frac{d}{dt}\rho(\chi,t) = -\frac{i}{\hbar}[\mathcal{H}_S, \rho(\chi,t)] + \sum_{\nu=L,R} (\frac{i}{\hbar})^2 \int_0^t d\tau \mathrm{Tr}_{\mathrm{E}}[\mathcal{H}_{SE}^{\nu}[\mathcal{H}_{SE}^{\nu}(-\tau), \rho_{\mathrm{E}}\rho(\chi,t)]_{\chi}]_{\chi}]$$

Transformation into the Hilbert-Schmidt space

c.f. V.I.Manko,G.Marmo,E.C.G.S.Sudarshan and F.Zaccaria, Phys. Lett. A 327(2004)353.

$$\frac{d}{dt}\left|\rho(\chi,t)\right\rangle = \Xi^{\chi}(t)\left|\rho(\chi,t)\right\rangle \qquad |\rho(\chi,t)\rangle = (\rho_{00}^{\chi}(t), \rho_{01}^{\chi}(t), \rho_{10}^{\chi}(t), \rho_{11}^{\chi}(t))^T$$

Formal solution in the Hilbert-Schmidt space

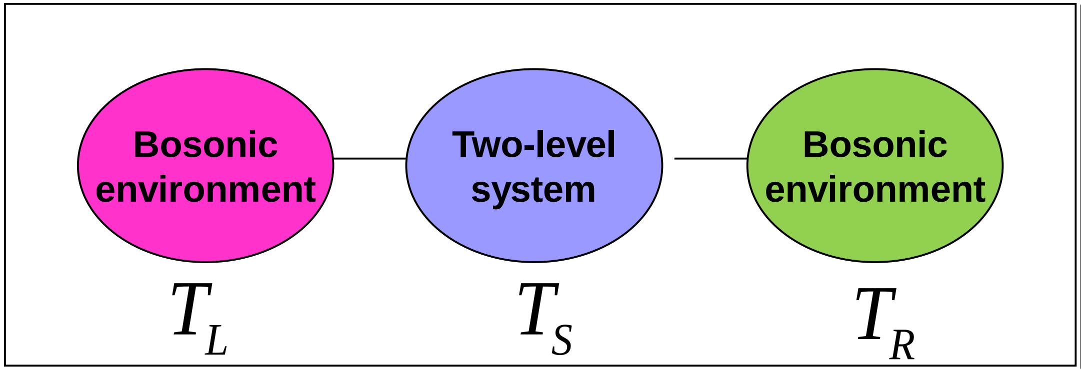
$$|\rho(\chi,t)\rangle = T_+ \exp[\int_0^t dt' \Xi^{\chi}(t')]|\rho(\chi,0)\rangle$$

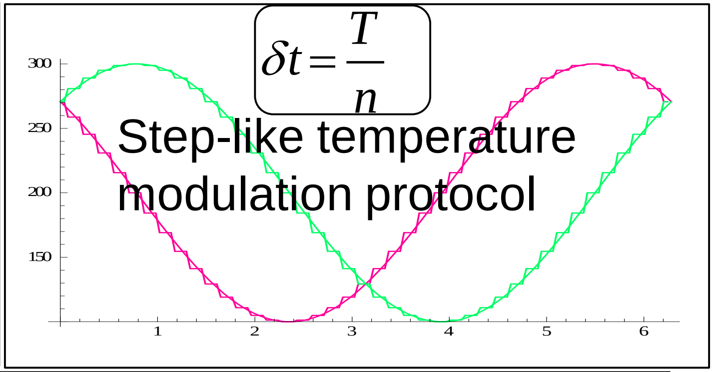
First moment of exchanged quantity between system and environment

$$\langle \Delta q \rangle_t = \langle 1| \int_0^t dt' \left[\frac{\partial\, \Xi^{\chi}(t')}{\partial (i\chi)}\right]_{\chi=0} \rho(0,t')\rangle$$

7. Application

- Anharmonic junction model





$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{int}$$

$$\mathcal{H}_S = \sum_{m=0.1} \varepsilon_m |m\rangle\langle m|$$

$$\mathcal{H}_0 = \mathcal{H}_S + \sum_{\nu=L,R} \mathcal{H}_{E,\nu}$$

$$\mathcal{H}_{E,\nu} = \sum_k \hbar\omega_{k,\nu} b_{k,\nu}^\dagger b_{k,\nu}$$

$$\mathcal{H}_{int} = \sum_{\nu=L,R} \mathcal{H}_{1,\nu}$$

$$\mathcal{H}_{1,\nu} = X_\nu(|0\rangle\langle 1| + |1\rangle\langle 0|)$$

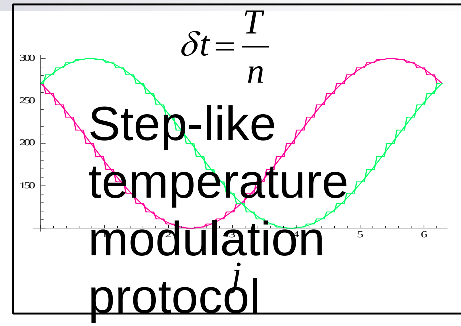
$$X_\nu = \sum_k \hbar g_{k,\nu}(b_{k,\nu}^\dagger + b_{k,\nu})$$

Ohmic spectral density

$$h_\nu(\omega) \equiv \sum_k g_{k,\nu}^2 \delta(\omega - \omega_{k,\nu}) = s_\nu \omega \exp[-\omega/\omega_{c,\nu}]$$

7. Application -2

- Full counting statistics



Time-convolutionless master eq. (2nd order)

$$\frac{d}{dt}\rho(\chi,t) = -\frac{i}{\hbar}[\mathcal{H}_S,\rho(\chi,t)] + \sum_{\nu=L,R}\left(\frac{i}{\hbar}\right)^2\int_0^t d\tau\,\mathrm{Tr}_{\mathrm{E}}[\mathcal{H}^{\nu}_{SE}[\mathcal{H}^{\nu}_{SE}(-\tau),\rho_{\mathrm{E}}\rho(\chi,t)]_{\chi}]_{\chi}]$$

Formal solution in Hilbert-Schmidt space

$$|\rho(\chi,t)\rangle = T_+\exp\left[\int_0^t dt'\,\Xi^{\chi}(t')\right]|\rho(\chi,0)\rangle \qquad |\rho(\chi,t)\rangle = (\rho^{\chi}_{00}(t),\rho^{\chi}_{01}(t),\rho^{\chi}_{10}(t),\rho^{\chi}_{11}(t))^T$$

$$\Xi^{\chi}(t) = -\int_0^t d\tau \begin{bmatrix} \boxed{V_+(\tau)} & 0 & 0 & \boxed{W_+{}^{\chi}(\tau)} \\ 0 & -i\omega_0+Y_+(\tau) & Z^{\chi}(\tau) & 0 \\ 0 & Z^{\chi}(\tau) & i\omega_0+Y_-(\tau) & 0 \\ \boxed{W_-{}^{\chi}(\tau)} & 0 & 0 & \boxed{V_-(\tau)} \end{bmatrix}$$

Decouple between diagonal and off-diagonal elements

First moment of exchanged quantity between system and environment

$$\langle\Delta q\rangle_t = -\int_0^t dt'\left\{\left[\frac{\partial W^{\chi}_+(t')}{\partial(i\chi)}\right]_{\chi=0}\boxed{\rho_{00}(t')} + \left[\frac{\partial W^{\chi}_-(t')}{\partial(i\chi)}\right]_{\chi=0}\boxed{\rho_{11}(t')}\right\}$$

Diagonal elements

Markovian（long time) approximation

$$\frac{d}{dt}\rho(\chi,t) = -\frac{i}{\hbar}[\mathcal{H}_S,\rho(\chi,t)] + \sum_{\nu=L,R}\left(\frac{i}{\hbar}\right)^2 \int_0^{t \to \infty} d\tau \,\mathrm{Tr}_{\mathrm{E}}[\mathcal{H}^{\nu}_{SE}[\mathcal{H}^{\nu}_{SE}(-\tau),\rho_{\mathrm{E}}\rho(\chi,t)]_\chi]_\chi]$$

Formal solution in Hilbert-Schmidt space

$$|\rho(\chi,t)\rangle = T_+ \exp[\int_0^t dt' \Xi^\chi(t')]|\rho(\chi,0)\rangle \qquad |\rho(\chi,t)\rangle = (\rho^\chi_{00}(t),\rho^\chi_{01}(t),\rho^\chi_{10}(t),\rho^\chi_{11}(t))^T$$

$$\Xi^\chi(t) = -\int_0^{t \to \infty} d\tau \begin{bmatrix} \boxed{V_+(\tau)} & 0 & 0 & \boxed{W_+{}^\chi(\tau)} \\ 0 & -i\omega_0 + Y_+(\tau) & Z^\chi(\tau) & 0 \\ 0 & Z^\chi(\tau) & i\omega_0 + Y_-(\tau) & 0 \\ \boxed{W_-{}^\chi(\tau)} & 0 & 0 & \boxed{V_-(\tau)} \end{bmatrix}$$

$$\Xi^\chi_{d,M} = -\sum_{\nu=L,R}\begin{bmatrix} \Gamma_\nu n_\nu(\omega_0) & -\Gamma_\nu(1+n_\nu(\omega_0))e^{i\chi_\nu\hbar\omega_0} \\ -\Gamma_\nu n_\nu(\omega_0)e^{-i\chi_\nu\hbar\omega_0} & \Gamma_\nu(1+n_\nu(\omega_0)) \end{bmatrix}$$

$$\Gamma_\nu = 2\pi h(\omega_0)$$

First moment of exchanged quantity between system and environment

$$\langle \Delta q \rangle_t = \langle 1 | \int_0^t dt' \left[\frac{\partial \Xi^\chi(t')}{\partial(i\chi)} \right]_{\chi=0} \rho(0,t') \rangle$$

Markovian（long time) approximation

$$\left\langle \Delta q^\nu \right\rangle_t = \left\langle 1 \right| \left[\frac{\partial \Xi^\chi_{d,M}}{\partial (i\chi_\nu)} \right]_{\chi_\nu=0} \int_0^t dt' \rho(0,t') \left. \right\rangle$$

$$\Xi^\chi_{d,M} = -\sum_{\nu=L,R} \begin{bmatrix} k_d^\nu & -k_u^\nu e^{i\chi_\nu \hbar\omega_0} \\ -k_d^\nu e^{-i\chi_\nu \hbar\omega_0} & k_u^\nu \end{bmatrix}$$

$$\begin{cases} k_d^\nu = \Gamma_\nu n_\nu(\omega_0) \\ k_u^\nu = \Gamma_\nu (1 + n_\nu(\omega_0)) \end{cases}$$

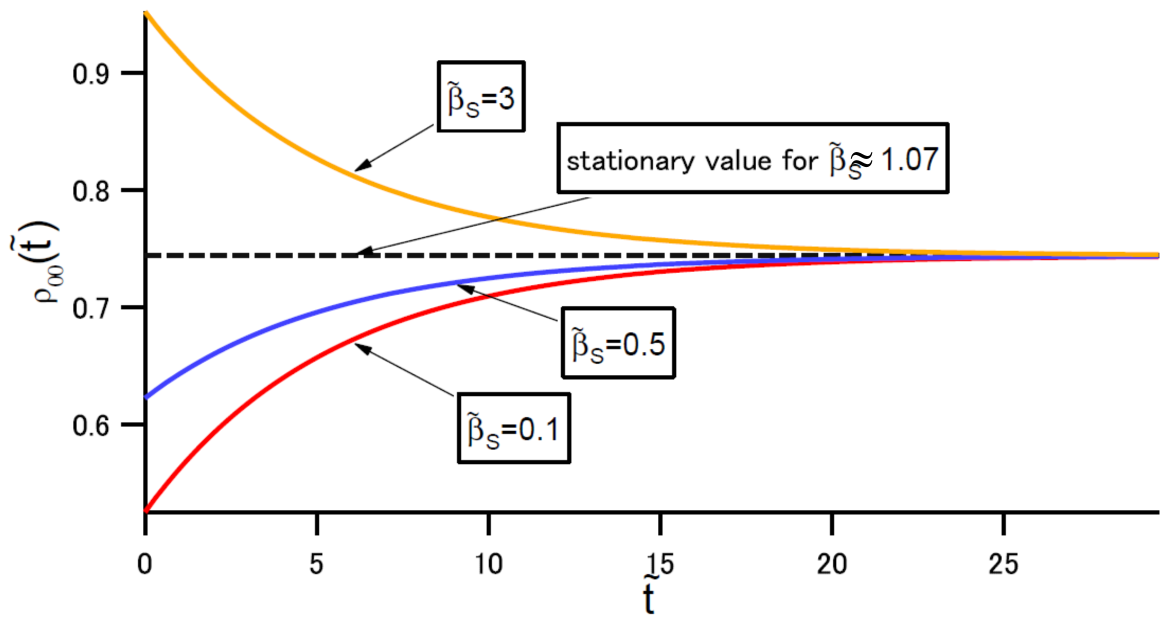
$$\left\langle \Delta q^\nu \right\rangle_t = \hbar\omega_0 \left\{ A^\nu \int_0^t dt' \rho_{00}(t') - B^\nu t \right\}$$

$$A^\nu = -(k_d^\nu + k_u^\nu), \quad B^\nu = -k_u^\nu$$

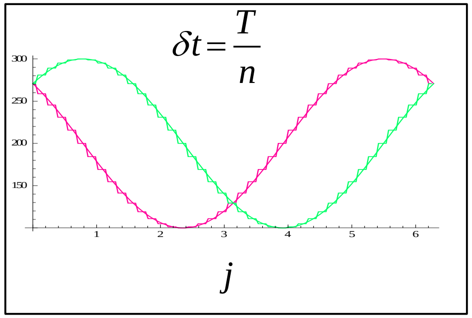
First moment of exchanged quantity between system and environment

$$\left\langle \Delta q^{\nu} \right\rangle_t = \hbar\omega_0 \left\{ A^{\nu} \int_0^t dt' \, \rho_{00}(t') - B^{\nu} \, t \right\} \qquad A^{\nu} = -(k_d^{\nu} + k_u^{\nu}), \;\; B^{\nu} = -k_u^{\nu}$$

dynamics of $\rho_{00}(t)$ (population of lower level)



→ Step-like temperature modulation protocol is necessary to be considered



Pumping current with Non-adiabatic term

$$\tilde{J}^{\nu} = \frac{1}{\mathcal{T}} J^{\nu} = \frac{1}{\mathcal{T}}(\mathcal{G}^{\nu}_{ad} + \mathcal{G}^{\nu}_{nad}) = \frac{\hbar\omega_0}{\mathcal{T}}\left(\boxed{\mathcal{G}^{\nu}_{1}} + \boxed{\mathcal{G}^{\nu}_{2}} + \boxed{\mathcal{G}^{\nu}_{3}}\right)$$

Dynamical phase Geometrical phase

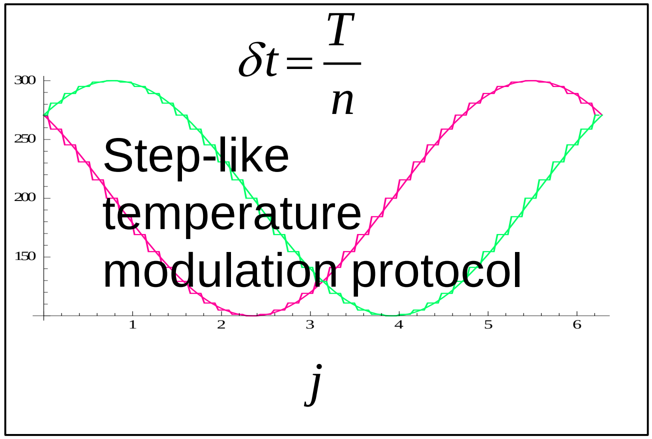
$$\boxed{\mathcal{G}^{\nu}_{3}} = \sum_{j=1}^{n} \phi^{\nu}_{0}(j) + \sum_{j=2}^{n-1} \boxed{\boxed{\left(\rho_s(j-1) - \rho_s(j)\right)}\psi^{\nu}(j)}$$

Reminiscent effect

$$\phi^{\nu}_{0}(j) = \boxed{\boxed{\left(\rho_{00}(0) - \rho_s(1)\right)} f^{\nu}(1,j)}$$

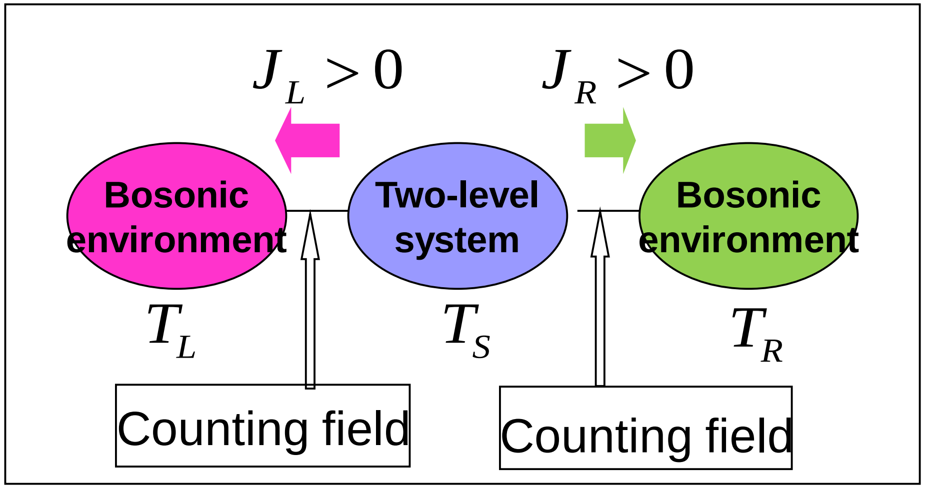
$$f^{\nu}(p,q) = \frac{A^{\nu}(q)}{\Lambda_1(q)} e^{\sum_{\kappa=p}^{q-1}\Lambda_1(\kappa)\delta t}\left(e^{\Lambda_1(q)\delta t} - 1\right)$$

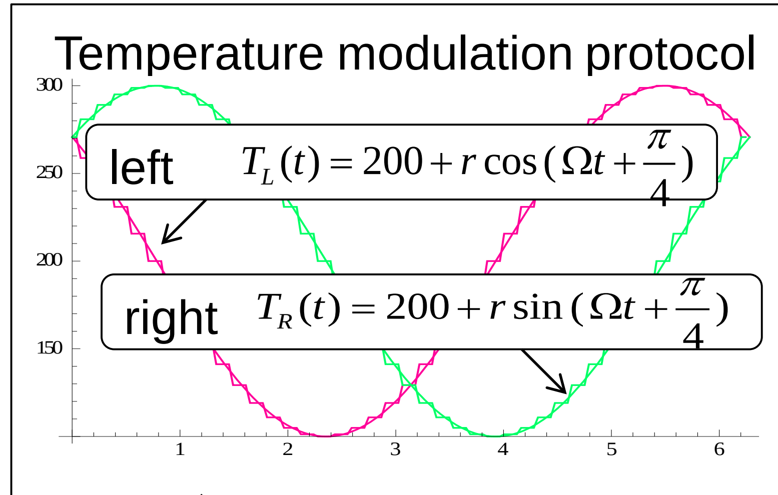
$$\psi^{\nu}(j) = \frac{A^{\nu}(j)}{\Lambda_1(j)} e^{\Lambda_1(j)\delta t} + \sum_{m=j}^{n-1} f^{\nu}(j, m+1)$$



$$\boxed{\Lambda_1(j) = \sum_{\nu} A^{\nu}(j)} \qquad \boxed{A^{\nu}(j) = -\Gamma_{\nu}(1 + 2N_{\nu}(j))}$$

8. Numerical evaluation





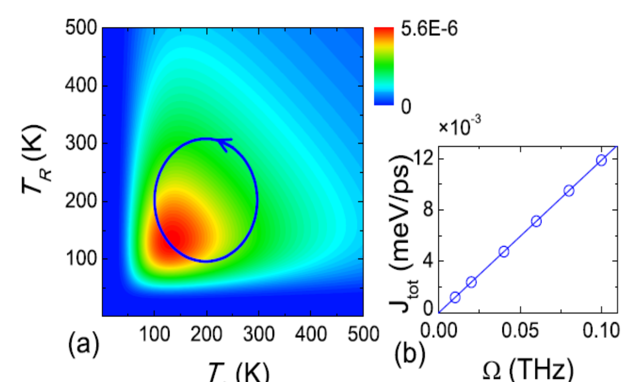
⇕ changing T with keeping n

system - environment coupling

weak : $s_\nu = 0.01$

Ohmic spectral density: $\omega_{c,\nu} = 3\omega_0$

Energy level difference (25meV) as in Ren-Li-Hänggi (2010)



initial population of two-level system

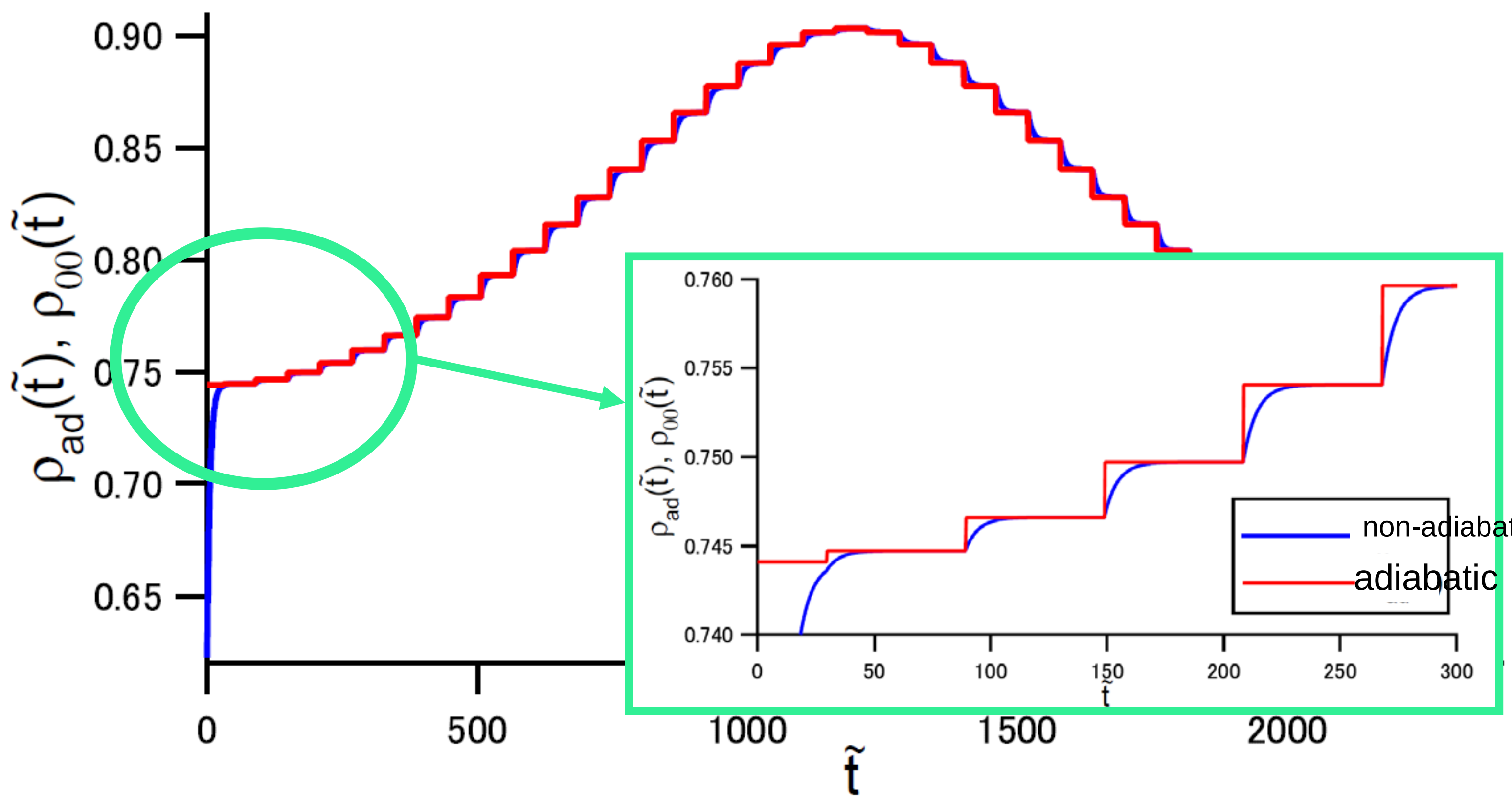
$$\tilde{\beta}_S = \frac{\hbar\omega_0}{k_B T_s} = 0.1,\ 0.5,\ \tilde{\beta}(0),\ 3$$

⇨ Non-adiabatic effect under Markovian approximation

Time evolution of lower population -1

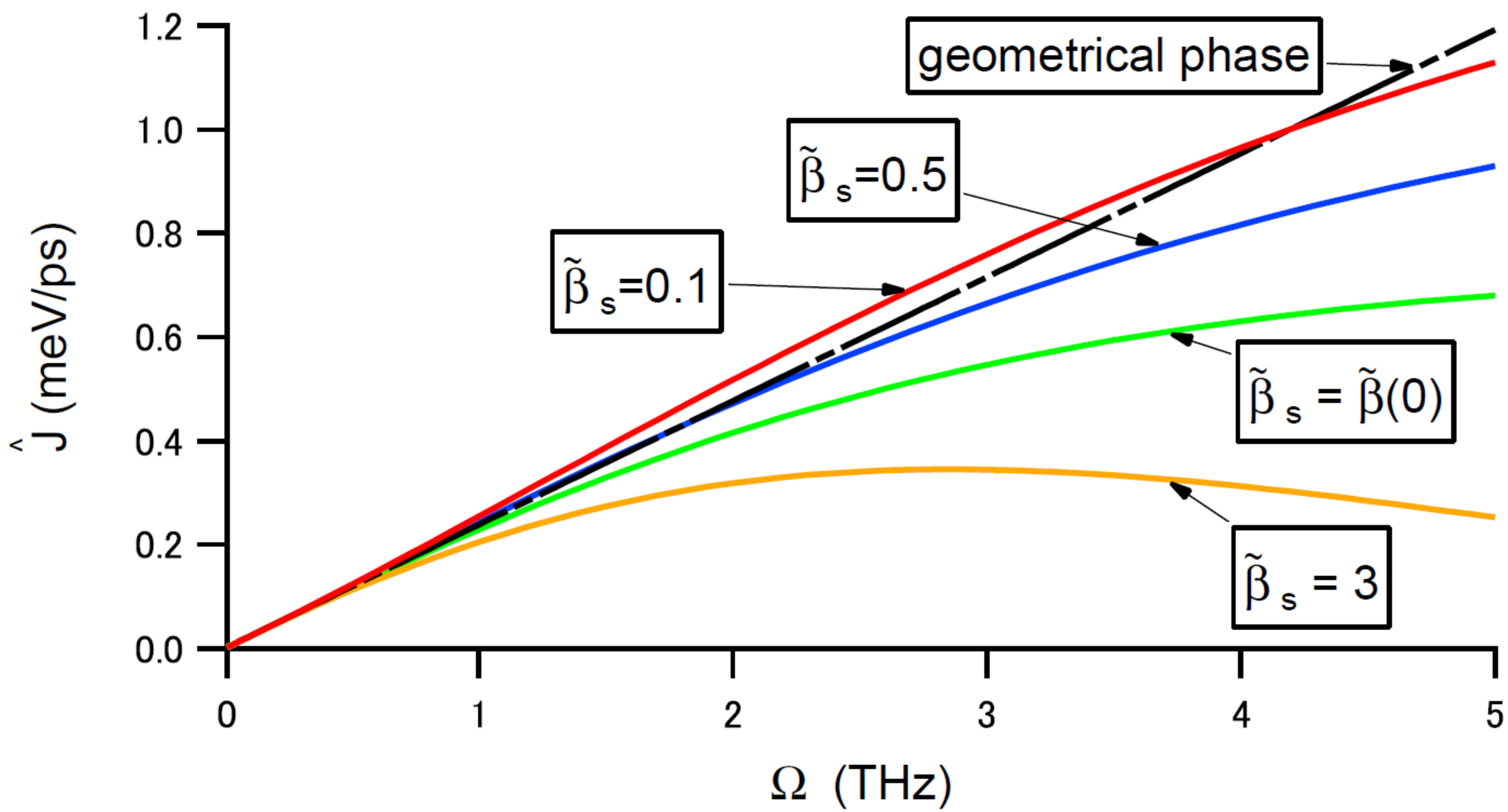
long interval case (lower frequency case) : Ω=0.1 THz

initial population of two-level system : $\tilde{\beta}_S = \dfrac{\hbar\omega_0}{k_B T_s} = 0.5$

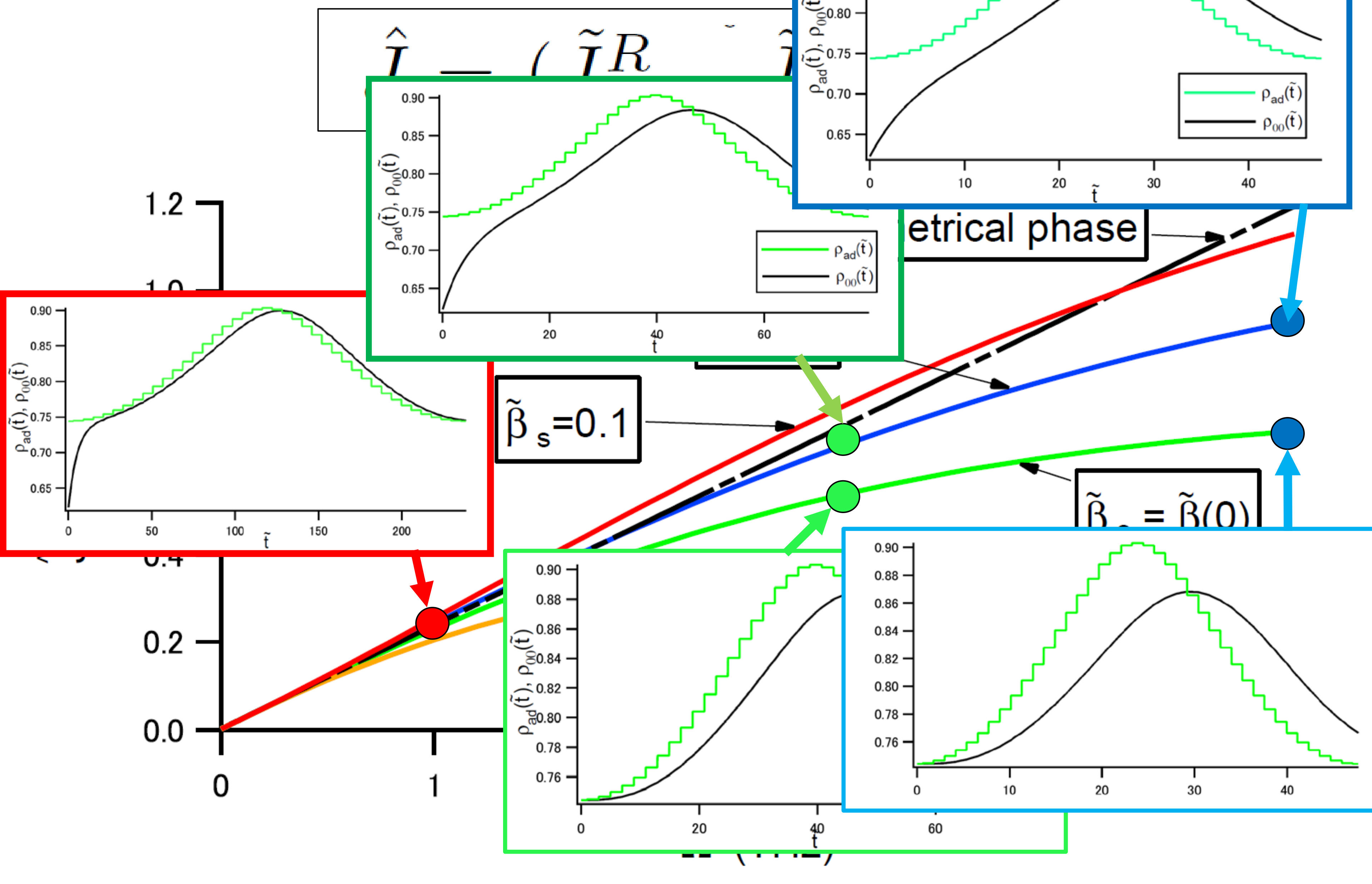


Frequency dependence of net pumping current

$$\hat{J} = (\tilde{J}^R - \tilde{J}^L)/\hbar\omega_0$$



Frequency dependence of net



9. Conclusion and issues for future

- Formulation for quantum transport between system and environment including non-adiabatic effect (also non-Markovian effect)
- Non-adiabatic effect on quantum pumping under Markovian approximation
  - Additive effect to adiabatic term : $J^{\nu} = \mathcal{G}^{\nu}_{ad} + \mathcal{G}^{\nu}_{nad}$
  - Reminiscent effect of past modulation
  - Dependence on initial condition

(C.Uchiyama,PRE,89 (2014) 052108)

Issues for future

1. Non-Markovian effect
2. Effect of initial correlation