Luttinger liquid universality after a quantum quench

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The interaction quench

protocol to be studied — questions to be addressed:

- consider a closed (isolated) quantum many-body system
- prepare the system in ground state of noninteracting $\hat{H}_{\mathrm{i}}$
- perform time evolution with respect to interacting Hamiltonian $\hat{H}_{\mathrm{f}}$
- do expectation values of certain observables become stationary at large t?
- if so, how can the 'steady state' be characterized?
- what is the leading long-time dependence?

possible extensions:

- consider thermal state (density matrix) as initial state
- consider interacting state as initial state and quench interaction strength
- change interaction gradually (Dóra et al. '11, Pollmann et al. '13)

is a fundamental question, but why of interest now?

$\rightarrow$ cold atomic gases are 'isolated' and can (still) be controlled

(Bloch, Dalibard, Zwerger '08; Polkovnikov et al. '11)

Here: 1d correlated Fermi systems

why 1d models:

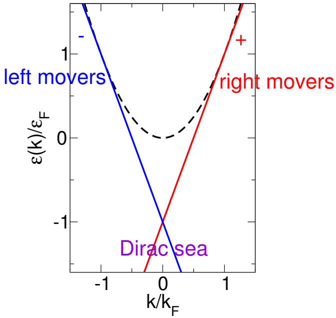
- specific equilibrium many-body methods available
  - bosonization
  - density-matrix renormalization group (DMRG)
  - RG methods
  - Bethe ansatz
  - conformal field theory
- equilibrium physics is well understood (see e.g. Giamarchi '03)
  - metallic system: Luttinger liquid — not a Fermi liquid!
  - Mott insulator, CDW, SDW,...
- extension of (some of) the methods to nonequilibrium time evolution
- Luttinger liquid universality after quench?

what is equilibrium Luttinger liquid universality?

Consider different models of 1d interacting fermions (spinless)

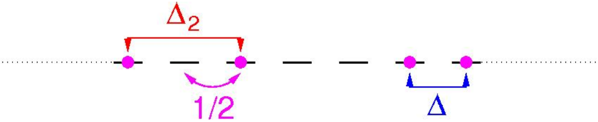
- electron gas with linearized dispersion: Tomonaga-Luttinger model

$$\hat{H} = \sum_n \Big[v_{\mathrm{F}}(k_n - k_{\mathrm{F}})\, \hat{c}^\dagger_{n,+}\hat{c}_{n,+} + (-v_{\mathrm{F}})(k_n + k_{\mathrm{F}})\, \hat{c}^\dagger_{n,-}\hat{c}_{n,-} \Big] + \frac{1}{2L}\sum_{n\neq 0}\Big[g_4(q_n)\left(\hat{\rho}^\dagger_{n,+}\hat{\rho}_{n,+} + \hat{\rho}^\dagger_{n,-}\hat{\rho}_{n,-}\right) + g_2(q_n)\left(\hat{\rho}^\dagger_{n,+}\hat{\rho}_{n,-} + \hat{\rho}^\dagger_{n,-}\hat{\rho}_{n,+}\right)\Big]$$



- lattice model

$$\hat{H} = \sum_j \left[-\frac{1}{2}\hat{c}^\dagger_j \hat{c}_{j+1} + \text{H.c.} + \Delta \hat{n}_j \hat{n}_{j+1} + \Delta_2 \hat{n}_j \hat{n}_{j+2} \right] \;, \quad \hat{n}_j = \hat{c}^\dagger_j \hat{c}_j - 1/2$$



- and other (electron gas, lattice models with additional terms, . . .)

Equilibrium Luttinger liquid universality

version I:

a large class of 1d metallic Fermi systems shows the same ‘universal’ low-energy physics (thermodynamics, correlation functions)

version II (emphasizes the importance of the TL model):

the TL model is the low-energy fixed point model under RG flow of a large class of 1d fermion models

(Haldane ’80; Sólyom 79)

crucial:

low-energy physics of the TL model can be computed exactly using bosonization

also crucial:

it is not always straightforward to show that a model belongs to this class

First type of observables: low-energy 'thermodynamic' ones

specific heat

- low-temperature $C_{\mathrm{V}}(T)$ from temperature dependence of energy:

$$C_{\mathrm{V}}(T) = \frac{d\left\langle \hat{H} \right\rangle}{dT} = \gamma T \; , \quad \frac{\gamma}{\gamma^0} = \frac{v_{\mathrm{F}}}{v}$$

compressibility

- κ from ground state energy E_0 (at $T = 0$):

$$\kappa \sim \left(\partial^2 E_0 / \partial \left\langle \hat{\mathcal{N}} \right\rangle^2 \right)_L^{-1} \Rightarrow \frac{\kappa}{\kappa^0} = K \, \frac{v_{\mathrm{F}}}{v}$$

charge stiffness

- D from ground state energy E_0 (at $T = 0$):

$$D \sim \left(\partial^2 E_0 / \partial \left\langle \hat{\mathcal{J}} \right\rangle^2 \right)_L \Rightarrow \frac{D}{D^0} = K \, \frac{v}{v_{\mathrm{F}}}$$

observation

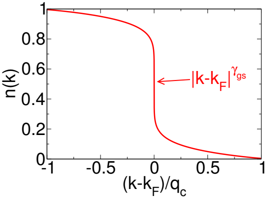
- interaction only via K and v; need two observables to determine K and v

Second type of observables: correlation functions

momentum distribution function

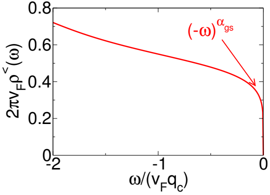
$$\left|n(k) - \tfrac{1}{2}\right| \sim |k - k_{\mathrm{F}}|^{\gamma_{\mathrm{gs}}},$$

$$\gamma_{\mathrm{gs}} = \tfrac{1}{2}\left(K + K^{-1} - 2\right)$$



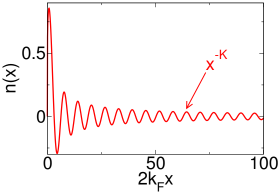
k-integrated spectral function

$$\rho(\omega)^{<} \sim (-\omega)^{\gamma_{\mathrm{gs}}}\,\theta(-\omega)$$



Friedel oscillations

$$n(x) \sim x^{-K}\,\sin\left(2k_{\mathrm{F}}x\right)$$



K and v for our models

TL model

$$K = \sqrt{\frac{1+\hat{g}_4(0)-\hat{g}_2(0)}{1+\hat{g}_4(0)+\hat{g}_2(0)}}, \quad v = v_{\mathrm{F}}\sqrt{[1+\hat{g}_4(0)]^2-\hat{g}_2^2(0)}, \quad \hat{g}_{2/4} = \frac{g_{2/4}}{2\pi v_{\mathrm{F}}}$$

lattice model

1. $\Delta_2 = 0$, filling $\nu = 1/2$:

$$K = \frac{\pi}{4\eta}, \quad v = \frac{\pi \sin(2\eta)/2}{\pi - 2\eta}, \quad 2\eta = \arccos(-\Delta)$$

2. $\Delta_2 = 0$, other fillings: solve Bethe ansatz integral equations numerically
3. $\Delta_2 \neq 0$: compute some of the above observables numerically using DMRG

in general (U stands for interaction $g_{2/4}$ or Δ, Δ_2)

$K = 1 - \frac{U}{U_{\mathrm{c}}} + \mathcal{O}(U^2) \Rightarrow$ exponent of $n(x)$ is $\mathcal{O}(U)$, others $\mathcal{O}(U^2)$

Luttinger liquid universality in nonequilibrium?

steady state

- concept of RG irrelevance for the nonequilibrium steady state?
- first step, TL model: observables expressible in K and v?
- other models: compute infinite time expectation values for as many observables as possible

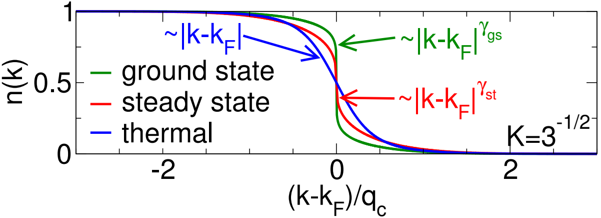
time evolution

- transient times — no universality can be expected
- first step, TL model: asymptotic t dependence expressible in K and v?
- other models: compute large time expectation values for as many observables as possible

Momentum distribution function in steady-state — universality?

TL model and bosonization

$$\lim_{t\to\infty}\left|n(k,t)-\frac{1}{2}\right| \sim |k-k_{\mathrm{F}}|^{\gamma_{\mathrm{st}}},\ \gamma_{\mathrm{st}}(K)=\frac{1}{4}(K^2+K^{-2}-2),\ 2\gamma_{\mathrm{gs}}-\gamma_{\mathrm{st}}=\mathcal{O}(U^4)$$



(Cazalilla '06; Uhrig '09; Rentrop, Schuricht, V.M. '12)

universality — study lattice model with TDMRG

t reachable not large enough for steady state $\Rightarrow$ resort to 'asymptotic' t behavior

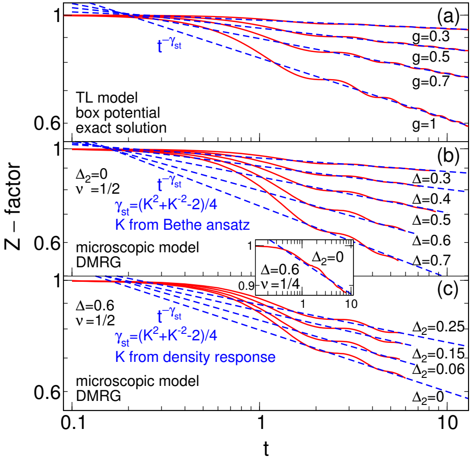
note: observable must be 'rather steady' — we want to read off a power law!

Quench dynamics of momentum distribution — universality?

quasi-particle weight TL model (Cazalilla '06; Uhrig '09; Rentrop, Schuricht, V.M. '12)

$$Z(t) = \lim_{k \nearrow k_F} n(k,t) - \lim_{k \searrow k_F} n(k,t) \sim t^{-\gamma_{st}}$$

compare to lattice model (TDMRG) — universality!?



question: is this the asymptotics? (Karrasch, Rentrop, Schuricht, V.M. '12)

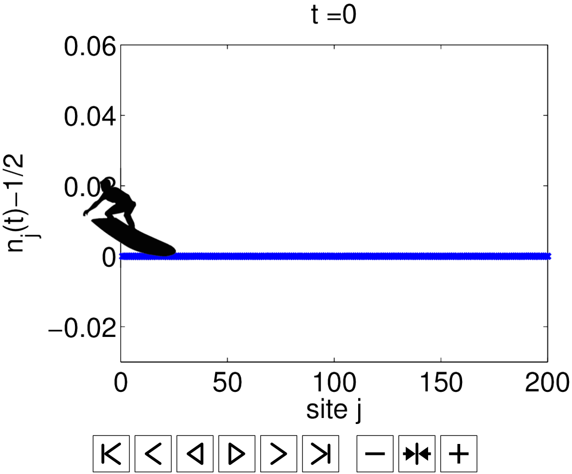
Alternative approaches to reach the steady state for lattice model?

functional RG (Metzner, Salmhofer, Honerkamp, V.M., Schönhammer '12)

- extend existing scheme to closed systems (Kennes, Jakobs, Karrasch, V.M. '12)
- price to pay: approximate and thus reliable only for small to intermediate U
- in equilibrium: Luttinger liquid power laws with leading order exponents
- therefore: consider inhomogeneous system with open boundaries

Can the steady state be reached?

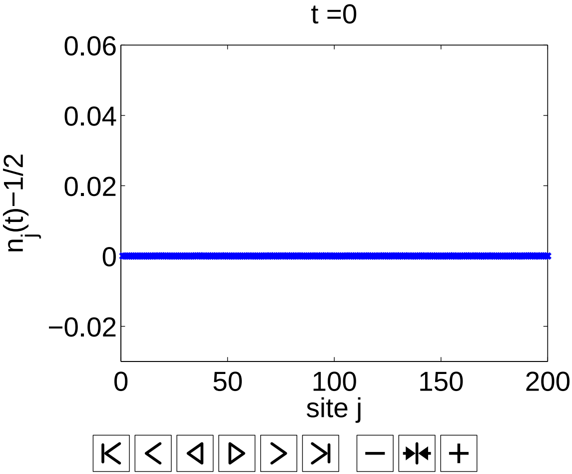
time evolution of density



times reachable $t \approx 300$ compared to $t \approx 30$ in TDMRG (Kennes, V.M. '13)

Can the steady state be reached?

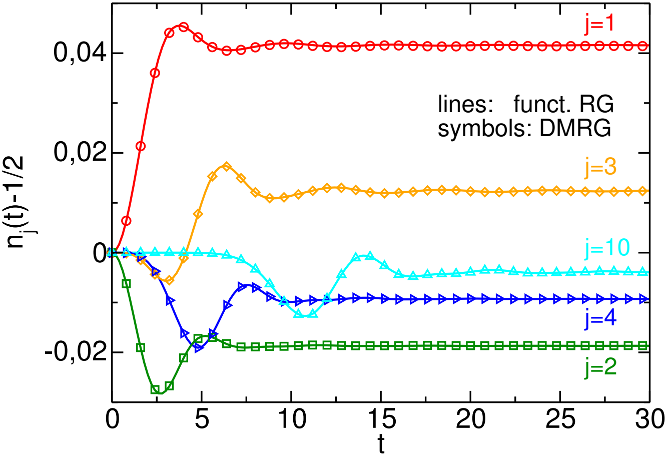
time evolution of density



times reachable $t \approx 300$ compared to $t \approx 30$ in TDMRG (Kennes, V.M. '13)

Are the results accurate? — comparison to TDMRG

functional RG controlled for small U thus $\Delta = 0.1$ (and $\Delta_2 = 0$)



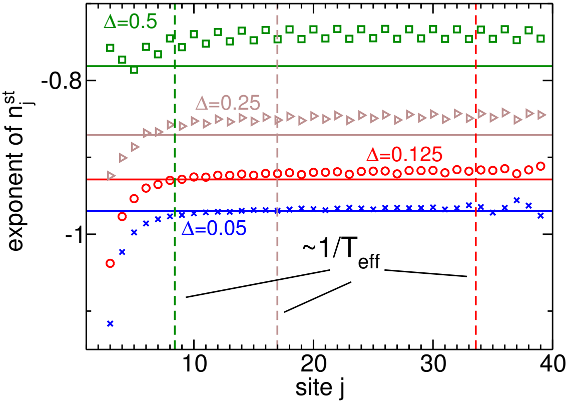
(Kennes, V.M. '13)

Friedel oscillations in steady state — universality?

TL model and bosonization

$$n^{\mathrm{st}}(x) = \lim_{t\to\infty} n(x,t) \sim \frac{\sin(2k_{\mathrm{F}}x)}{x^{(1+K^2)/2}}$$

universality!? — study lattice model with functional RG ($\Delta_2 = 0$)



question: higher order (in U) corrections?

(Kennes, V.M. '13)

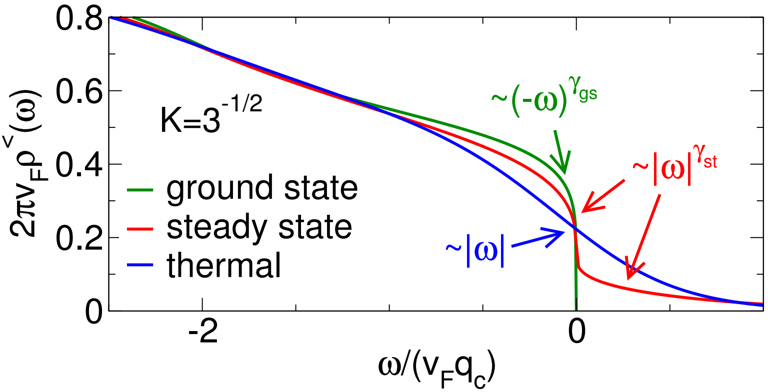
This is all nice, but . . .

. . . are there more striking differences than modified exponents?

- consider dynamical correlation functions in the steady state
- two-time correlation functions with relative time $t - t'$ and $t + t' \to \infty$
- here: single particle spectral function (k-resolved and -integrated)

Momentum-integrated spectral function

TL model and bosonization

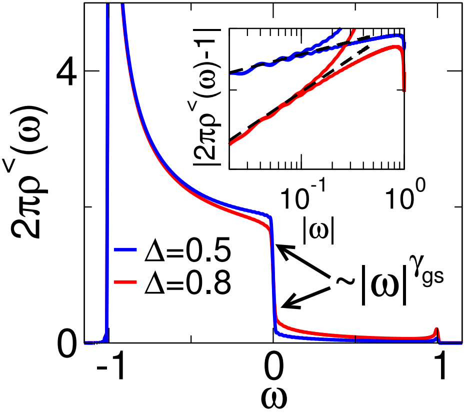


- $\rho^{<}_{\mathrm{st}}(0) \neq 0$ (nonuniversal!) (Kennes, Klöckner, V.M. '14)
- power law with γ_{st} on both sides

Momentum-integrated spectral function

universality!? — study lattice model with TDMRG

- here the inverse quench
- start from $\Delta > 0$ (and $\Delta_2 = 0$) and turn off the interaction
- large times can be reached with TDMRG (steady state, Fourier transformation)

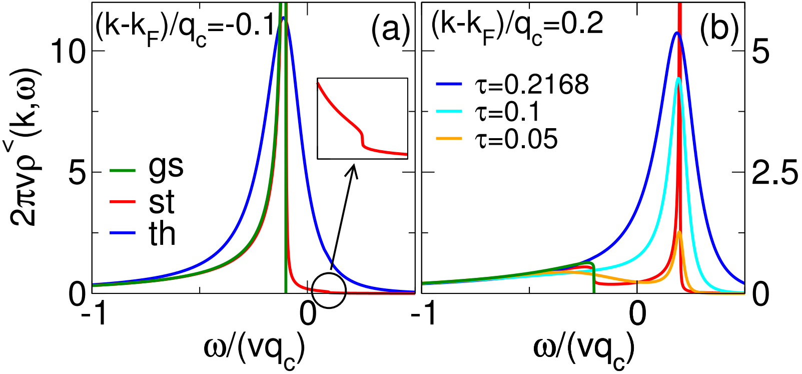


weight at $\omega > 0$, power laws but γ_{gs}

(Kennes, Klöckner, V.M. '14)

Momentum-resolved spectral function

TL model and bosonization



- narrow peak at $\omega > 0$ for $k > k_{\mathrm{F}}$ (Kennes, Klöckner, V.M. '14)
- universality!? — δ-peak in lattice model for inverse quench

Summary

- Luttinger liquid universality in equilibrium
- physics of Tomonaga-Luttinger model is central
- asymptotic t dependence after quench: indications of universality
- steady state after quench: indications of universality
- striking features of steady state in spectral function

Refs.:

- Rentrop, Schuricht, V.M., NJP **14**, 075001 (2012)
- Karrasch, Rentrop, Schuricht, V.M., PRL **109**, 126406 (2012)
- Kennes, V.M., PRB **88**, 165131 (2013)
- Kennes, Klöckner, V.M., PRL **113**, 114401 (2014)