Failure of thermalization and the Generalized Gibbs Ensemble Hypothesis in strongly interacting quantum systems

Gergely Zarand

Budapest Univ. Technology and Economics

Collaborators: **B. Pozsgay, M. A. Werner, M. Kormos**

M. Mestyán, G. Takács

Discussions: **F. Pollmann** (Dresden)

BME PHYSICS

ISSP Workshop, 2015, Tokyo

exotic quantum phases group

Outline

- **Introduction:** Ergodicity, thermalization and the GGE
- **Quantum quenches in the XXZ spin chain**

  Real time (TEBD) simulations

  GGE vs. overlap TBA
- **Conclusions I**
- **Semiclassical theory of quantum quench in sine-Gordon model**
- **Conclusions II**

INTRODUCTION

Ergodicity

Hamiltonian system

Phase space Γ : $(q_1, \ldots, q_N, p_1, \ldots, p_N)$
Dynamics: Hamiltonian H

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \qquad \dot{p}_i = -\frac{\partial H}{\partial q_i}$$

Time average:

$$\bar{A} = \lim_{T\to\infty} \frac{1}{T}\int_0^T A(p(t), q(t))$$

Microcanonical average:

$$\langle A \rangle_{\mathrm{mc}} = \frac{1}{\mathrm{vol}\ (\Gamma)} \int_{\Gamma(E,N)} A(p,q)$$

Boltzmann's ergodic hypothesis (1871):

$$\boxed{\bar{A} = \langle A \rangle_{\mathrm{mc}}}$$

Ergodicity in QM

prepare initial state (ground state) $\Rightarrow$ change Hamiltonian

$\Rightarrow$ measure ...

$$|\psi(t=0)\rangle = \sum_\alpha C_\alpha |\psi_\alpha\rangle \quad \Rightarrow \quad |\psi(t)\rangle = \sum_\alpha C_\alpha e^{-\frac{i}{\hbar}E_\alpha t}|\psi_\alpha\rangle$$

$$\Rightarrow \langle A(t)\rangle = \sum_\alpha C^*_\alpha C_\beta e^{-\frac{i}{\hbar}(E_\alpha - E_\beta)t} A_{\alpha\beta}$$

time average: $\bar{A} = \lim_{T\to\infty} \frac{1}{T}\int_0^T dt\, \langle A(t)\rangle = \sum_\alpha |C_\alpha|^2 A_{\alpha\alpha}$

Is described by diagonal ensemble: $\rho_{\mathrm{DE}} = \sum_\alpha |C_\alpha|^2 |\psi_\alpha\rangle\langle\psi_\alpha|$

(caution: $L \to \infty$ and $T \to \infty$ imits not interchangable...)

Eigenstate thermalization hypothesis

Deutsch (1991) : "quantum ergodicity"

$$\sum_\alpha |C_\alpha|^2 A_{\alpha\alpha} = \frac{1}{N(\langle E\rangle, \Delta E)} \sum_{\alpha: E_\alpha \in I} A_{\alpha\alpha}$$

Srednicki (1994), Huse: Eigenstate thermalization hypothesis

$$A_{\alpha\alpha} = \langle A\rangle_{\mathrm{mc}}(E_\alpha) \qquad \forall\alpha$$

Do they hold ???? $\Rightarrow$ for many systems yes

For what observables ??? $\Rightarrow$ local, few-body

Exceptions

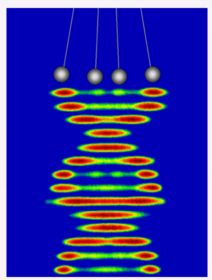
- **Spontaneous symmetry breaking**
- **Localization, glassy systems**

  $\Rightarrow$ Many-body localization...

- **Integrable systems**

A quantum Newton's cradle

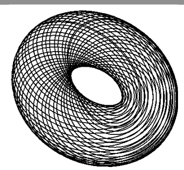
T. Kinoshita, T. Wenger and D.S. Weiss,
Nature 440 (2006) 900-903.



Integrability and the Generaized Gibbs Ensemble

- **Classical picture: ergodicity on tori**

$$[Q_i, Q_j] = 0 \qquad H \in \{Q_i\}_{i=1,\dots,N}$$



- **Quantum case: GGE**

(M. Rigol, V. Dunjko, and M. Olshanii, 2008)

assumption: ergodicity holds on common subspace of constants of motion

$\Rightarrow$ small subsystem is described by

$$\rho_{\mathrm{GGE}} = \frac{1}{Z} e^{-\sum_i \beta_i Q_i} \qquad Z = \mathrm{Tr}\, e^{-\sum_i \beta_i Q_i}$$

Generalized Gibbs Ensemble (GGE)

$$\mathrm{Tr}(\rho_{\mathrm{GGE}} Q_i) = \langle\psi(0)|Q_i|\psi(0)\rangle \quad \Leftrightarrow \quad \beta_i$$

GGE has been tested for many simple systems... What about a truely interacting system ??

Failure of the GGE

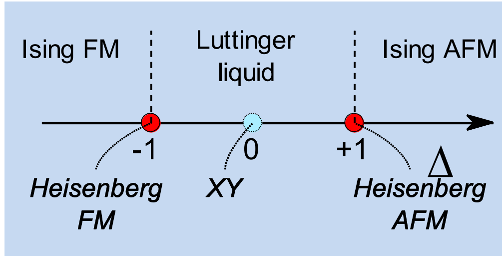
B. Pozsgay, M. Mestyán, M.A. Werner, M. Kormos, G.Z., and G.Takács, Phys. Rev. Lett. 113, 117203 (2014)

[B. Wouters et al., Phys. Rev. Lett. 113, 117202 (2014)
G. Goldstein and N. Andrei, PRA 90, 043625 (2014)]

Phase diagram of XXZ chain

XXZ chain $$H=\sum_i \left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z\right)$$

phase diagram



Testing GGE

- Initial state: Néel state / dimer state

$$|\uparrow\downarrow\uparrow\downarrow\dots\uparrow\downarrow\rangle \qquad \left(\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)\right)\otimes\cdots\otimes\left(\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)\right)$$

- Time evolution (TEBD)

Time-evolving block decimation (TEBD)

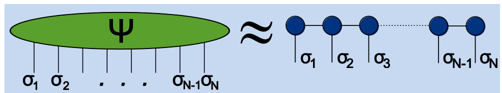
- Test local observables against exact results (GGE and overlap-thermodynamic Bethe Ansatz)

Time evolvong block decimation (TEBD)

Time-evolving block decimation (TEBD) (Vidal):

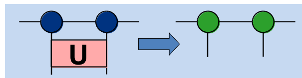
Matrix Product States:

$$|\Psi\rangle = \sum_{\sigma_1\dots\sigma_N} \Psi_{\sigma_1\dots\sigma_N} |\sigma_1\rangle\dots|\sigma_N\rangle \quad \Rightarrow \quad \sum_{\sigma_1\dots\sigma_N} \underline{\underline{A}}^{[1]\sigma_1}\underline{\underline{A}}^{[2]\sigma_2}\dots\underline{\underline{A}}^{[N]\sigma_N}|\sigma_1\rangle\dots|\sigma_N\rangle$$



Time evolution:

SVD



G. Vidal, PRL 93, 040502 (2004); G. Vidal, PRL 98, 070201 (2007).

Testing GGE

- Initial state: Néel state / dimer state

$|\uparrow\downarrow\uparrow\downarrow\ldots\uparrow\downarrow\rangle$ $\quad$ $\left(\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle)\right)\otimes\cdots\otimes\left(\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle)\right)$

- Time evolution

Time-evolving block decimation (TEBD)

- **Test local observables against exact results (GGE and overlap-thermodynamic Bethe Ansatz)**

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Bethe Ansatz for XXZ chain

XXZ chain is integrable

eigenstates

$$|\Omega\rangle = |\ldots\uparrow\uparrow\uparrow\ldots\rangle \qquad |\lambda_1,\ldots,\lambda_N\rangle = B(\lambda_1)\ldots B(\lambda_M)|\Omega\rangle$$

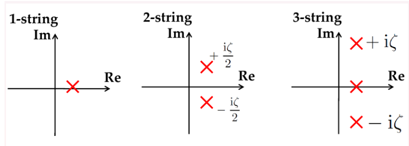
Bethe ansatz equations

$$\left(\frac{\sin(\lambda_j+i\eta/2)}{\sin(\lambda_j-i\eta/2)}\right)^L = \prod_{k\neq j}^{M}\frac{\sin(\lambda_j-\lambda_k+i\eta)}{\sin(\lambda_j-\lambda_k-i\eta)}$$

Additive, local charges

$$Q_n|\lambda_1,\ldots,\lambda_N\rangle = \sum_{i=1}^{n} q_n(\lambda_i)|\lambda_1,\ldots,\lambda_N\rangle$$

Strings (bound states):



Thermodynamical limit: String densities:

$$\rho_n(\lambda)d\lambda$$

overlap TEBD

$\bar{A} = \lim_{T\to\infty}\frac{1}{T}\int_0^T dt\,\langle A(t)\rangle = \sum_\alpha |C_\alpha|^2 A_{\alpha\alpha} \qquad C_\alpha = \langle\psi(0)|\alpha\rangle$

Replace sum: $\sum_\alpha \longrightarrow \int\prod_{n=1}^{\infty} D\rho_n(\lambda)e^{Ls[\{\rho_n(\lambda)\}]}$

Average of an observable

$$\bar{A} = \int\prod_{n=1}^{\infty} D\rho_n(\lambda)e^{-L\left(-\frac{2}{L}\mathrm{Re}\ln\langle\psi(0)|\{\rho_n(\lambda)\}\rangle - s[\{\rho_n(\lambda)\}]\right)}\langle\{\rho_n(\lambda)\}|A|\{\rho_n(\lambda)\}\rangle$$

(overlap)^2 $\qquad$ entropy

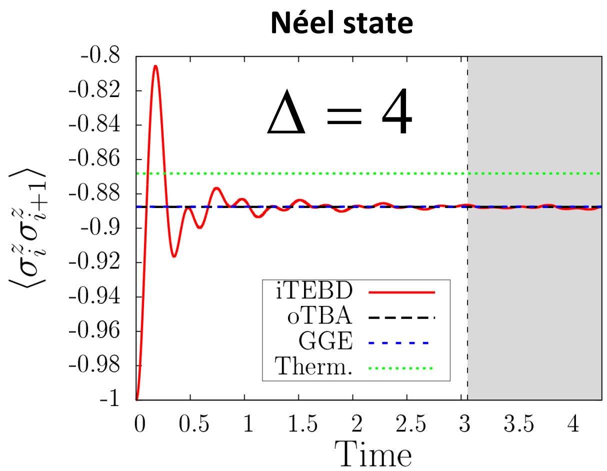
Assumption:

diagonal ensemble reached is the *maximum entropy* state that *overlaps* with the initial state (~ saddle point)

⇨ **overlap-thermodynamic Bethe ansatz !**

J-S Caux and F H L Essler, PRL 110, 257203 (2013)

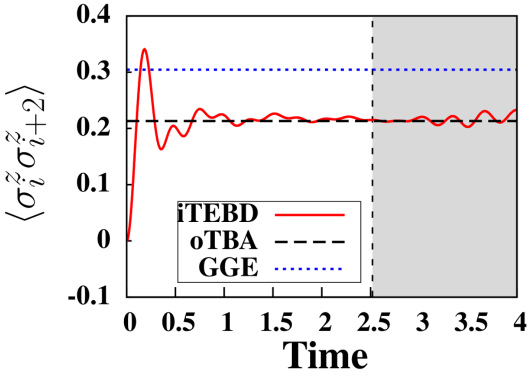
Correlation functions: Néel state



Pozsgay, et al, Phys. Rev. Lett. 113, 117203 (2014)

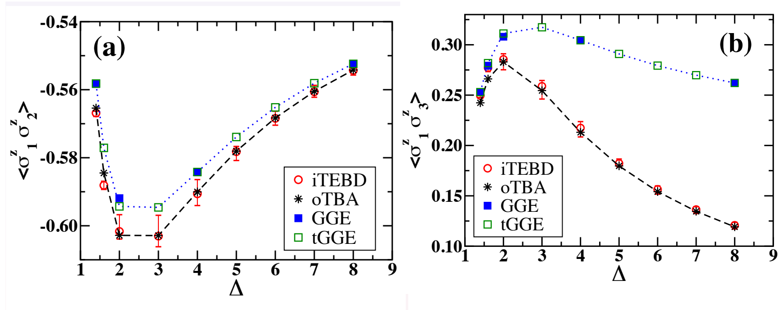
Correlation functions: Dimer state

Dimer state



Comparison

Dimer states



Pozsgay, et al, Phys. Rev. Lett. 113, 117203 (2014)

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Conclusions I

- First serious test for a truly interacting system...

- GGE fails for the XXZ chain, some states are more sensitive

⟺ no ergodicity on torus

reasons: ???? what next ??

follow up papers: GGE should fail for most systems with strings ...

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SEMICLASSICAL THEORY OF QUANTUM QUENCH IN THE SINE-GORDON MODEL

M. Kormos and G.Z. unpublished

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sine-Gordon model

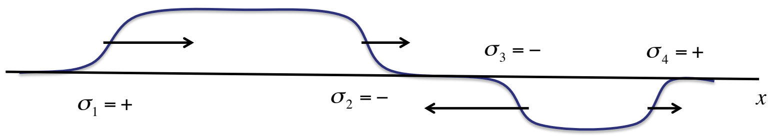
Imaginary time action

$$\mathcal{S} = \frac{c}{16\pi}\int \mathrm{d}x\mathrm{d}\tau \left[(\partial_x\Phi)^2 + \frac{1}{c^2}(\partial_t\Phi)^2 - \lambda\cos(\beta\Phi)\right]$$

$0 < \beta < 1/\sqrt{2},$ attractive regime ⇨ solitons / antisolitons + breathers

$1/\sqrt{2} < \beta < 1$ repulsive regime ⇨ solitons / antisolitons

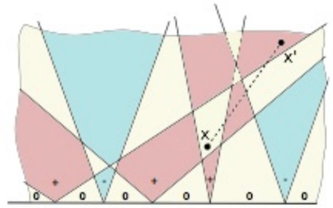
Soliton gas:



Interaction quench

interaction quench: initial state consits of soliton-antisoliton pairs

$$|\psi_0\rangle = \exp\left\{\int_0^\infty \frac{\mathrm{d}\theta}{2\pi} K_{ab}(\theta)\hat{Z}_a^\dagger(-\theta)\hat{Z}_b^\dagger(\theta)\right\}|0\rangle$$



small quench ⇨ small velocity (gap !)

⇨
1. semiclassical theory works
2. S-matrix is universal

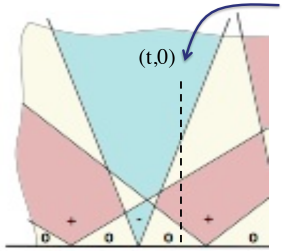
$$S^{m_1,m_2}_{m_1',m_2'} = (-1)\,\delta_{m_1,m_2'}\delta_{m_2,m_1'}$$

Phase relaxation

$$\left\langle e^{i\alpha\Phi(0,t)}\right\rangle = ?$$

Phase jumps each time a soliton line is cut !

$\Phi = -2\pi\beta$



$\Phi = 0$

left cuts $n_- = 2$

ight cuts $n_+ = 1$

⇨
$$\left\langle e^{i\alpha\Phi(x,t)}\right\rangle \sim \sum_{n_+,n_-=0}^{\infty} p(n_+,n_-)\left(\frac{1+(-1)^{n_+-n_-}}{2} + \frac{1-(-1)^{n_+-n_-}}{2}\cos(2\pi\alpha/\beta)\right)$$

$$p(n_+,n_-) = \frac{1}{n_+!}\frac{1}{n_-!}Q^{n_++n_-}e^{-2Q} \qquad Q = t\rho\int_0^\infty \mathrm{d}v\,v f(v)$$

Phase relaxation

$$\left\langle e^{i\alpha\Phi(0,t)}\right\rangle = ?$$

Phase jumps each time a soliton line is cut !

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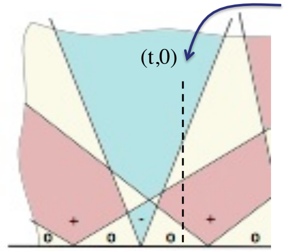


left cuts $n_- = 2$

ight cuts $n_+ = 1$

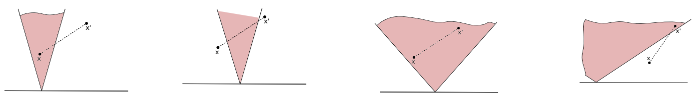
$$\boxed{\left\langle e^{i\alpha\Phi(x,t)}\right\rangle \sim \cos^2(\pi\alpha/\beta) + \sin^2(\pi\alpha/\beta)e^{-t/\tau},} \qquad \tau^{-1} = 4\rho\int_0^\infty \mathrm{d}v v f(v)$$

- does not relax to 0 !
- Bertini, Schuricht, Essler: $\left\langle e^{i\alpha\Phi(x,t)}\right\rangle \sim 1 - t\,\sin^2(\pi\alpha/\beta)/\tau + \ldots$

Correlation function

$$C_\alpha(x,x';t,t') = \left\langle e^{i\alpha\Phi(x,t)} e^{-i\alpha\Phi(x',t')} \right\rangle$$

Many more cases ...



$$\bar{C}_\alpha(x,x';t,t') = \sum_{n_R,n_L,n_A,n_D,n_I} \frac{1}{n_R! n_L! n_A! n_D! n_I!} Q_R^{n_R} Q_L^{n_L} Q_A^{n_A} Q_D^{n_D} Q_I^{n_I} e^{-Q_R-Q_L-Q_A-Q_D-Q_I}$$
$$\left(\delta_{s,0}\cdot 1 + (1-\delta_{s,0})\cdot\left\{\frac{1-(-1)^s}{2}\cos\left(\frac{2\pi\alpha}{\beta}\right) + \frac{1+(-1)^s}{2}\left[\frac{1+(-1)^l}{2}\cdot 1 + \frac{1-(-1)^l}{2}\cos^2\left(\frac{2\pi\alpha}{\beta}\right)\right]\right\}\right)$$

$$q_L = \frac{1}{L}\int_0^{\tilde{v}} \mathrm{d}v 2vtf(v) + \frac{1}{L}\int_{\tilde{v}}^{v_s} \mathrm{d}v[x'-x-v(t'-t)]f(v) + \Theta(t-t')\frac{1}{L}\int_{v_s}^{\infty} \mathrm{d}v[x'-x-v(t'-t)]f(v)$$

Correlation function: imiting cases

Local correlations:

$$\bar{C}_\alpha(0;t,t') = \cos^4(\pi\alpha/\beta) + \sin^4(\pi\alpha/\beta)\, e^{-\Delta t/\tau}$$
$$+ \sin^2(\pi\alpha/\beta)\cos^2(\pi\alpha/\beta)\left[e^{-t/\tau}(1+e^{-\Delta t/\tau}) + 2e^{-\Delta t/(2\tau)}(1-e^{-t/\tau}) I_0\left(\frac{\Delta t}{2\tau}\right)\right],$$

Diffusive decay !

Equal time correlations

$$\bar{C}_\alpha(\Delta x;t,t) = \cos^4(\pi\alpha/\beta) + \sin^4(\pi\alpha/\beta)\, e^{-4\rho\Delta x + 4Q_D}$$
$$+ 2\sin^2(\pi\alpha/\beta)\cos^2(\pi\alpha/\beta)\left[e^{Q_D - 2\rho\Delta x} + e^{-t/\tau}(1-e^{-Q_D}])\right]$$

$$Q_D = \rho\int_0^{\tilde{v}} \mathrm{d}v(\Delta x - 2vt) f(v)$$

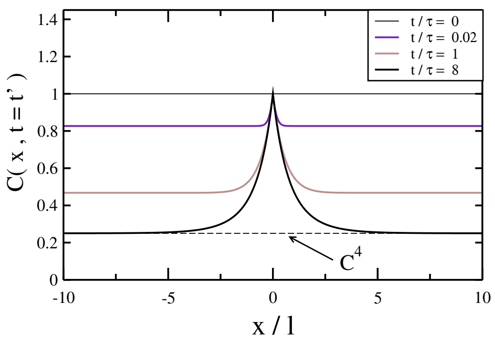
Exponential decay...

Equal time correlations

Equal time correlations
$$\bar{C}_\alpha(\Delta x;t,t) = \cos^4(\pi\alpha/\beta) + \sin^4(\pi\alpha/\beta)\, e^{-4\rho\Delta x + 4Q_D}$$
$$+ 2\sin^2(\pi\alpha/\beta)\cos^2(\pi\alpha/\beta)\left[e^{Q_D - 2\rho\Delta x} + e^{-t/\tau}(1-e^{-Q_D}])\right]$$

with $Q_D = \rho\int_0^{\tilde{v}} \mathrm{d}v(\Delta x - 2vt) f(v)$

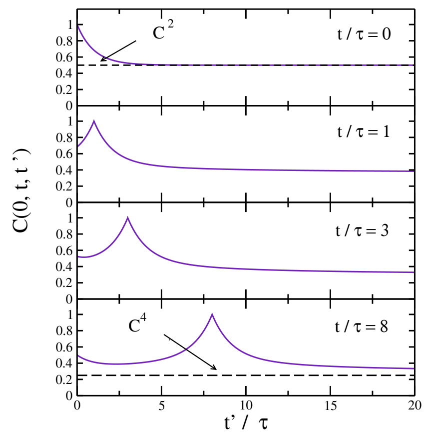
Exponential decay...



Local correlations

Local correlations:
$$\bar{C}_\alpha(0;t,t') = \cos^4(\pi\alpha/\beta) + \sin^4(\pi\alpha/\beta)\, e^{-\Delta t/\tau}$$
$$+ \sin^2(\pi\alpha/\beta)\cos^2(\pi\alpha/\beta)\left[e^{-t/\tau}(1+e^{-\Delta t/\tau}) + 2e^{-\Delta t/(2\tau)}(1-e^{-t/\tau}) I_0\left(\frac{\Delta t}{2\tau}\right)\right]$$

Diffusive decay !



Conclusions II

- Universal semiclassical correlation functions in sine-Gordon model
- Diffusive phase correlations
- universal decay $\Leftrightarrow$ $\lim_{t\to\infty}\left\langle e^{i\alpha\Phi(x,t)}\right\rangle \neq 0$!?
- "Prethermalized state", non-universal S-matrix needed...
- Applicable for non-integrable systems!

Addition

Our works on quantum quenches and work statistics in Luttinger liquids

Loschmidt echo in a Qubit-Coupled Luttinger Liquid
[Phys. Rev. Lett. 111, 046402 (2013)]

Work statistics in a quenched Luttinger liquid
[Phys. Rev. B 86, 161109(R) (2012)]

Crossover from Adiabatic to Sudden Interaction Quench in a Luttinger Liquid
[Phys. Rev. Lett. 106, 156406 (2011)]