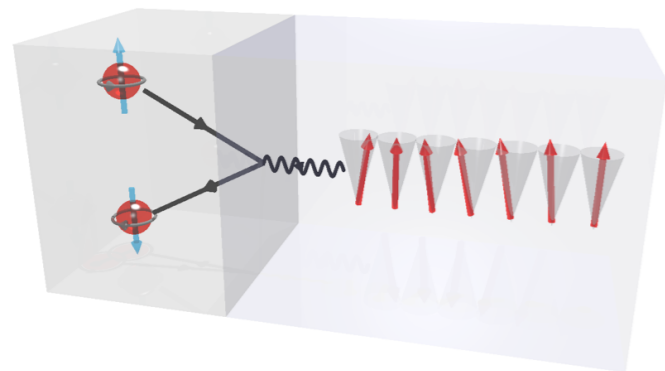


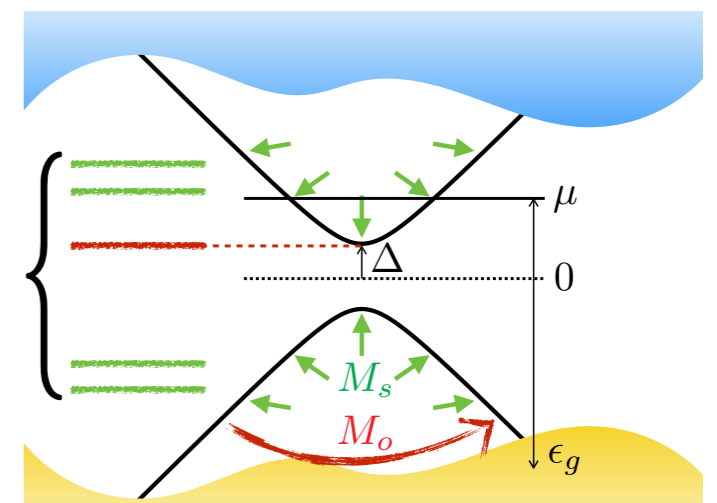
Spin and orbital magnetic response on the surface of a topological insulator

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in collaboration with D. Loss ([Basel](#)), D. Pesin ([Utah](#)), S. Bender and K. Wang ([UCLA](#))



YT and Loss, *PRL* (2012)
Pesin and MacDonald, *PRL* (2013)
YT and Bender, *PRB* (2014)
Fan et al., *Nature Mat.* (2014)
YT, Pesin, and Loss, *PRB* (2015)

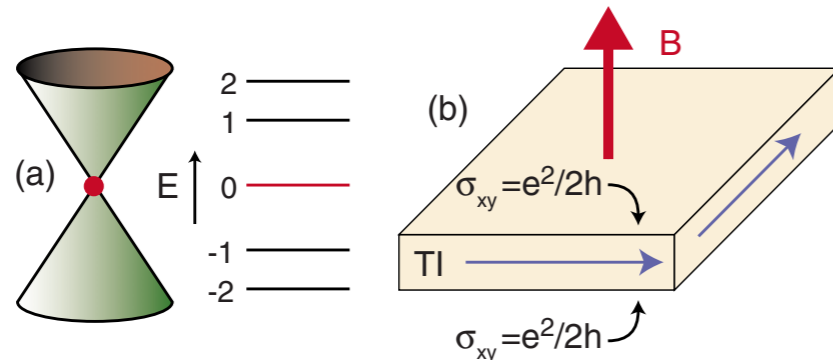


Outline

- ◆ Equilibrium spin and orbital magnetic response of Dirac electrons
- ◆ Spin Hall phenomenology of out-of-equilibrium dynamics
- ◆ UCLA experiments on Cr-doped (magnetic) BiSbTe
- ◆ Dynamics (theory) of a domain-wall/chiral-mode composite

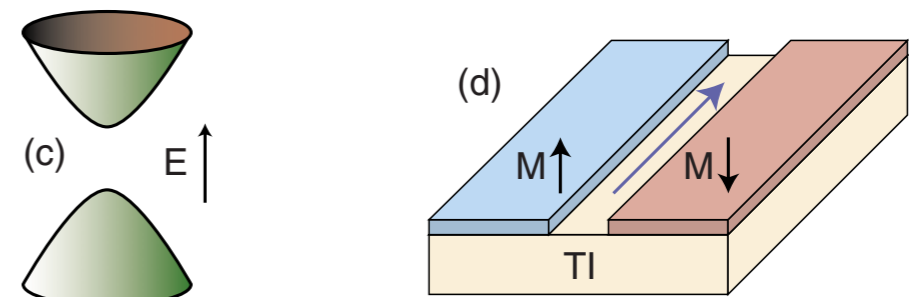
Overview (magnetoelectric phenomena)

◆ Magnetic field



0th Landau level

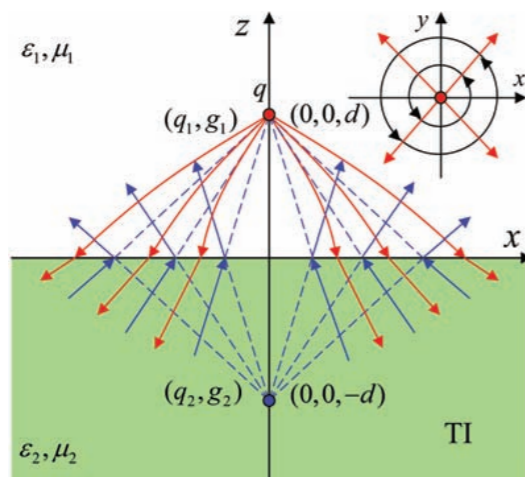
◆ Magnetic exchange



chiral "anomaly"

Hasan and Kane, *RMP* (2010)

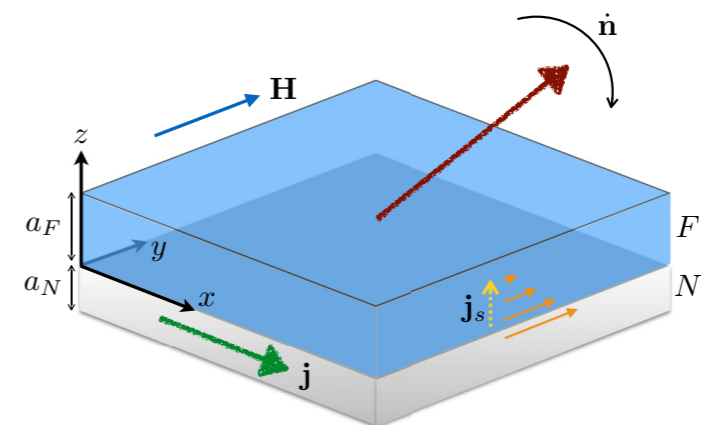
◆ Equilibrium



axion electrodynamics

Qi et al., *Science* (2009)

◆ Nonequilibrium



spin Hall phenomenology

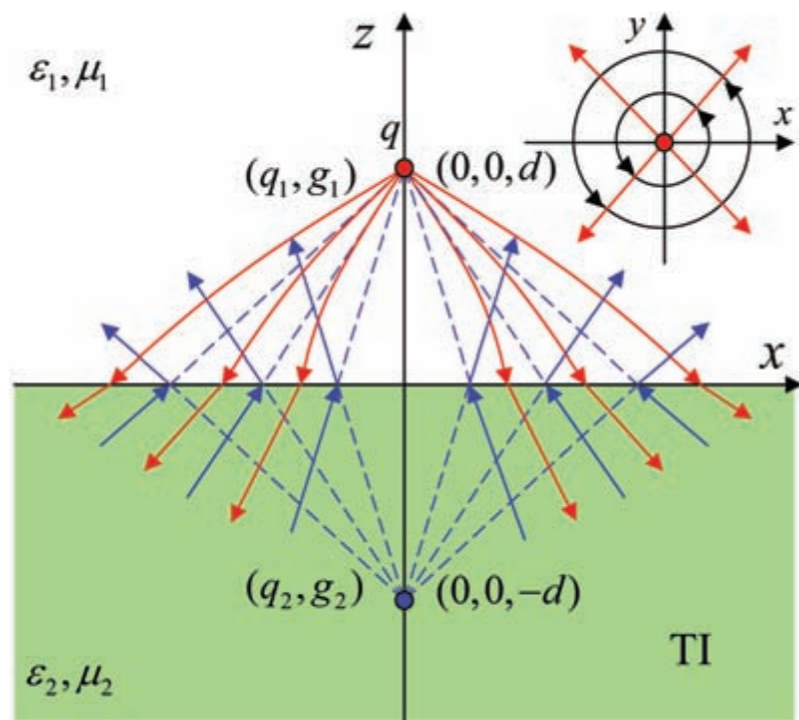
YT and Bender, *PRB* (2014)

Topological magnetoelectric effect

- ◆ Axion electrodynamics: $\Delta\mathcal{L} = \theta\mathbf{E} \cdot \mathbf{B}$ $\theta = \pm e^2/2h \rightarrow \sigma_{xy}$
(sign determined by the magnetically induced gap)

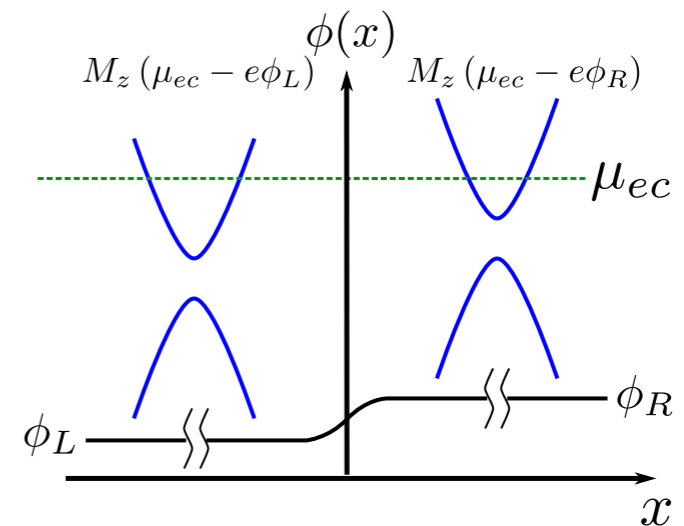
Wilczek, PRL (1987)

- ◆ dyonic screening:



Qi et al., Science (2009)

- ◆ longitudinal conductance leads to a receding monopole
- ◆ no monopoles in equilibrium for a metallic surface

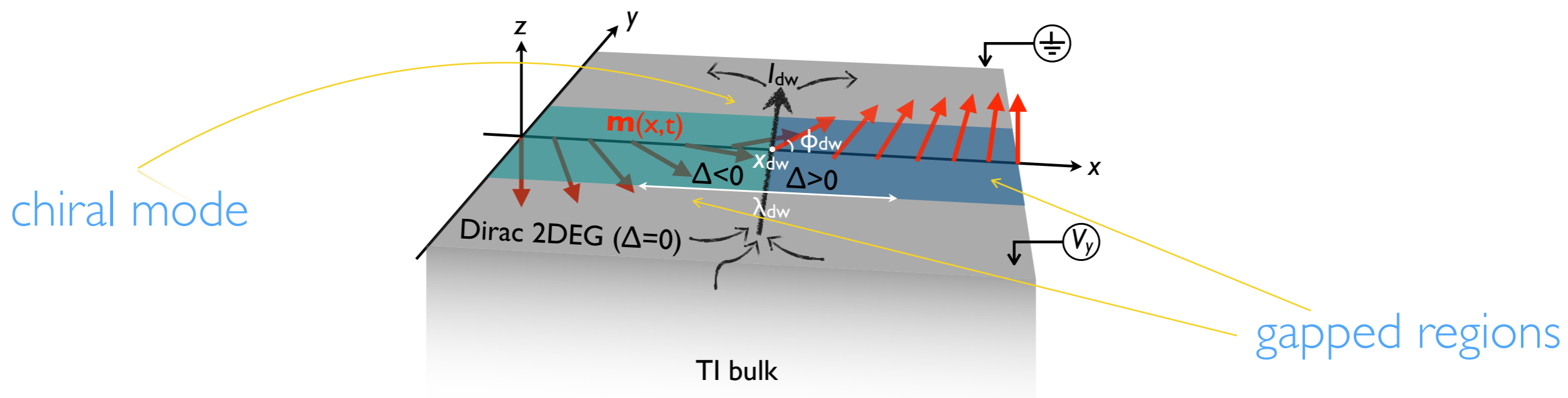


Pesin and MacDonald, PRL (2013)

no direct correspondence between Hall effect and equilibrium currents

Beyond axion electrodynamics

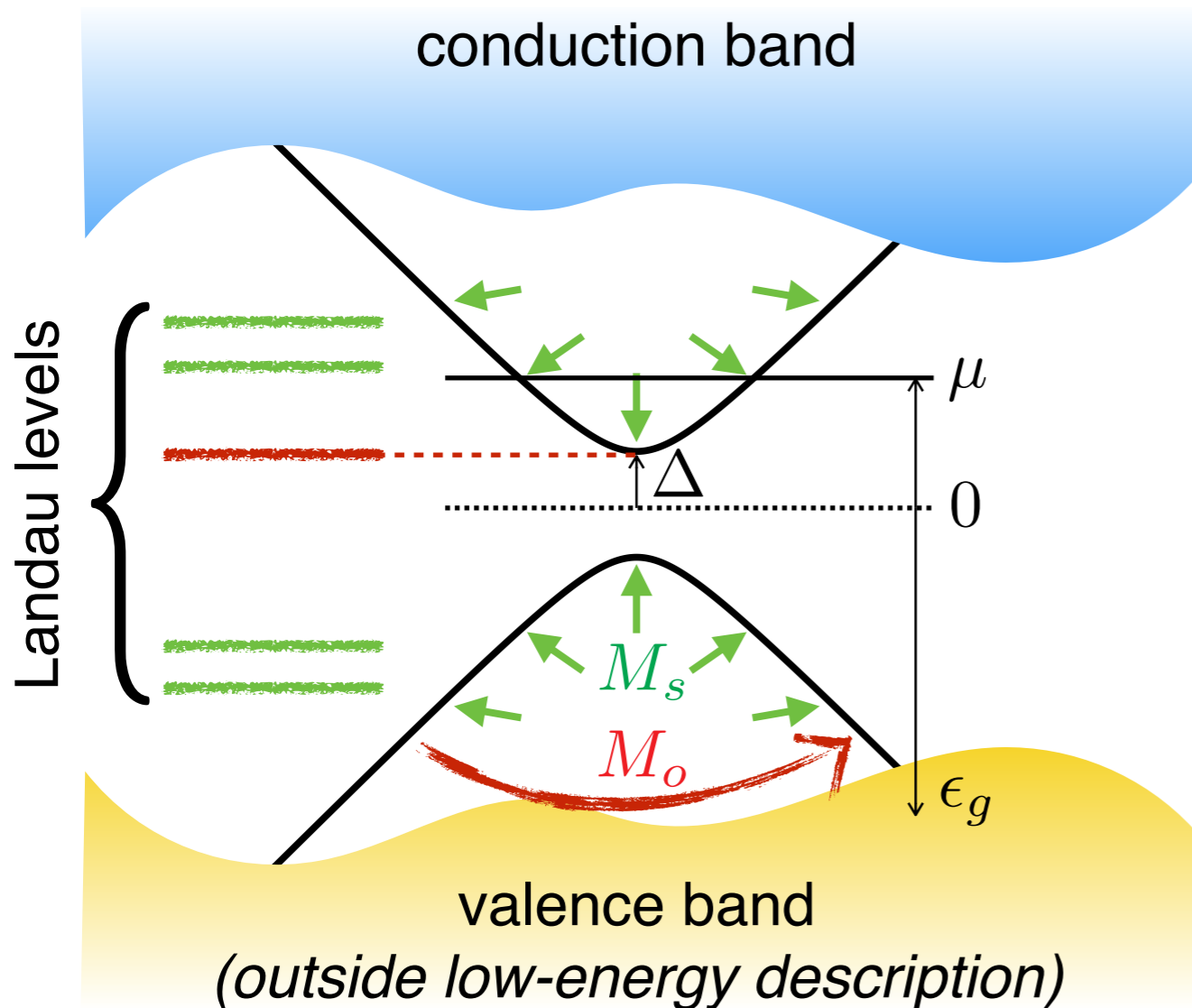
- ◆ TI in proximal coupling to a ferromagnetic texture
- ◆ In general, we have inhomogeneities in the electrical potential and magnetic exchange field
- ◆ What is the nature of the magnetic (spin and orbital) response of the TI surface?
- ◆ What is the feedback to (and coupling with) the magnetic layer?



Effective theory for magnetic response

- ◆ Peierls-substituted Dirac equation with Zeeman coupling:

$$H_0 = v \left(\mathbf{p} + \frac{e}{c} \mathbf{A} \right) \cdot \mathbf{z} \times \hat{\boldsymbol{\sigma}} - e\varphi + \frac{g}{2} \mu_B \mathbf{B} \cdot \hat{\boldsymbol{\sigma}}$$



Landau levels

$$\epsilon_0 = -\text{sgn}(B)\Delta$$

$$\epsilon_n = \text{sgn}(n) \sqrt{2(\hbar v/l)^2 |n| + \Delta^2}$$

Pankratov, *PLA* (1987)

$$M = -\partial_B \Omega(\mu, T, B)$$

the thermodynamic potential Ω is dominated by the 0th LL at low fields

(orbital) persistent current:

$$\mathbf{j} = -c\mathbf{z} \times \nabla M_o$$

(helical) planar spin density:

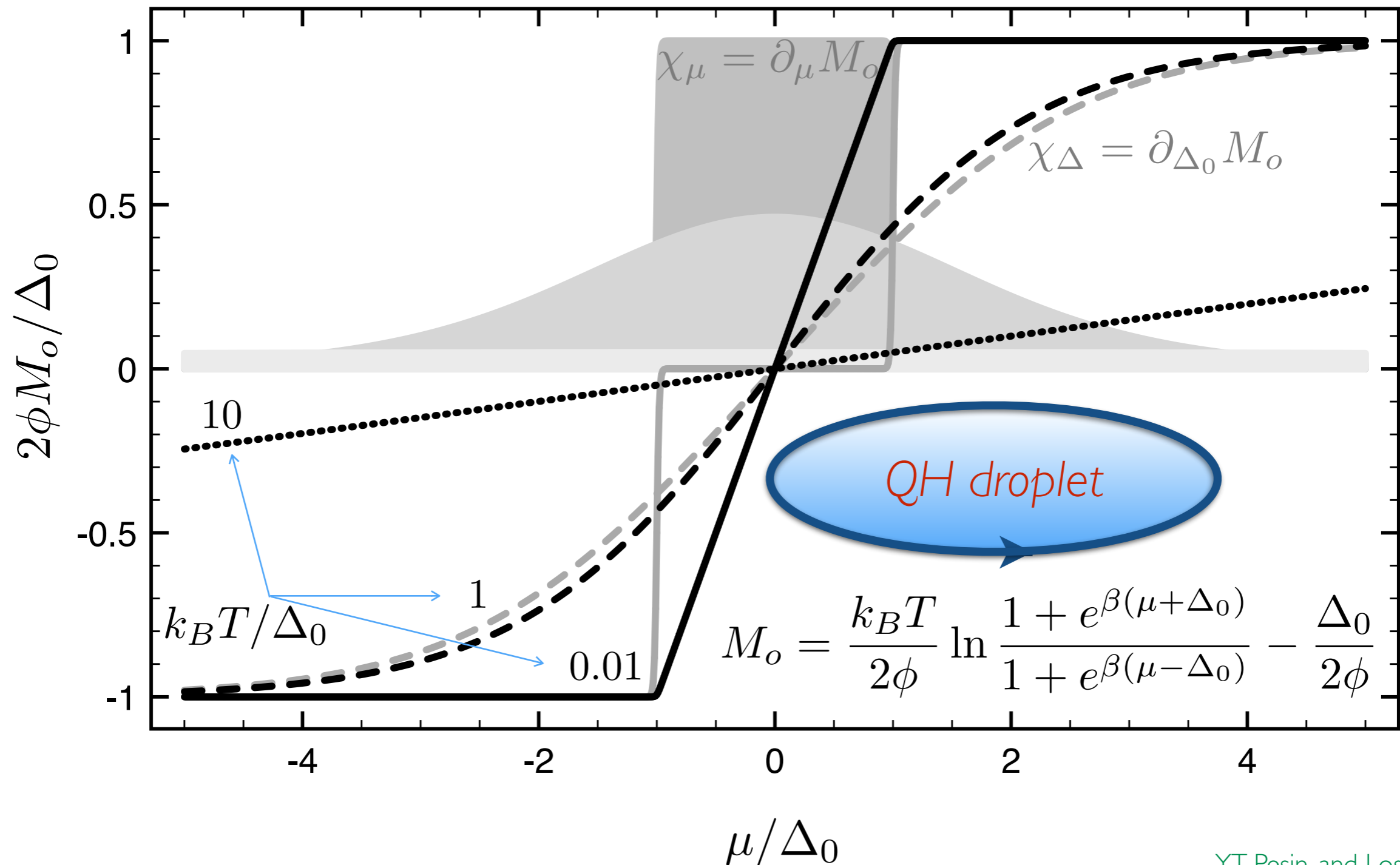
$$\boldsymbol{\rho} \times \mathbf{z} = -\frac{c}{ev} \delta_{\mathbf{A}} H = \frac{\mathbf{j}}{ev}$$

Magnetic response

- Spin/orbital contributions to the (local) out-of-plane magnetization:

$$M = M_s + M_o$$

$$M_s = \Delta_0 \frac{g}{4m\phi} \frac{\epsilon_g}{v^2}$$



Exchange coupling and local spin density

- ◆ The local exchange interaction has the general form

$$H' = J(n_x \hat{\sigma}_x + n_y \hat{\sigma}_y) + J_{\perp} n_z \hat{\sigma}_z$$

\mathbf{n} - magnetic order (direction)

$\hat{\sigma}$ - electron spin

- ◆ We are thus tasked with calculating the out-of-plane and planar components of the spin density
- ◆ Due to the electronic helicity, the out-of-plane orbital magnetization has the full information about the planar spin density $\boldsymbol{\rho}_{\parallel}$:

$$\boldsymbol{\rho}_{\parallel} \equiv \mathbf{z} \times \boldsymbol{\rho} \times \mathbf{z} = \frac{\mathbf{z} \times \mathbf{j}}{ev} = \frac{J_{\perp} \chi_{\Delta} \nabla n_z - e \chi_{\mu} \mathbf{E}}{4\pi \hbar v}$$

$$\chi_{\Delta} \equiv 2\phi \partial_{\Delta_0} M_o = \frac{\sinh(\beta\mu)}{\cosh(\beta\Delta_0) + \cosh(\beta\mu)} \quad \chi_{\mu} \equiv 2\phi \partial_{\mu} M_o = \frac{\sinh(\beta\Delta_0)}{\cosh(\beta\Delta_0) + \cosh(\beta\mu)}$$

- universal scaling functions

Feedback on the magnetization

- ◆ Landau-Lifshitz equation:

$$s(1 + \alpha \mathbf{n} \times) \partial_t \mathbf{n} = \mathbf{n} \times \left(\mathbf{H}_{\text{eff}} - \underbrace{J \boldsymbol{\rho}_{\parallel} - J_{\perp} \rho_z \mathbf{z}}_{\text{spin torque}} \right) = \mathbf{n} \times \mathbf{H}^*$$

$$\mathbf{H}^* \equiv -\delta_{\mathbf{n}}(\Omega_0 + \Omega')$$

(intrinsic) magnetic free energy

TI feedback (spin torque)

- ◆ The TI feedback is obtained by integrating electrons out:

$$\Omega' = \int d^2 \mathbf{r} \mathcal{F}'[\mathbf{n}] \quad \mathcal{F}'[\mathbf{n}] = -\frac{K n_z^2}{2} - \frac{\Gamma_{\text{DM}}}{2} \underbrace{(n_z \nabla \cdot \mathbf{n} - \mathbf{n} \cdot \nabla n_z)}_{\text{DMI}} - \Gamma_{\text{ME}} \underbrace{\mathbf{E} \cdot \mathbf{n}}_{\text{ME effect}}$$

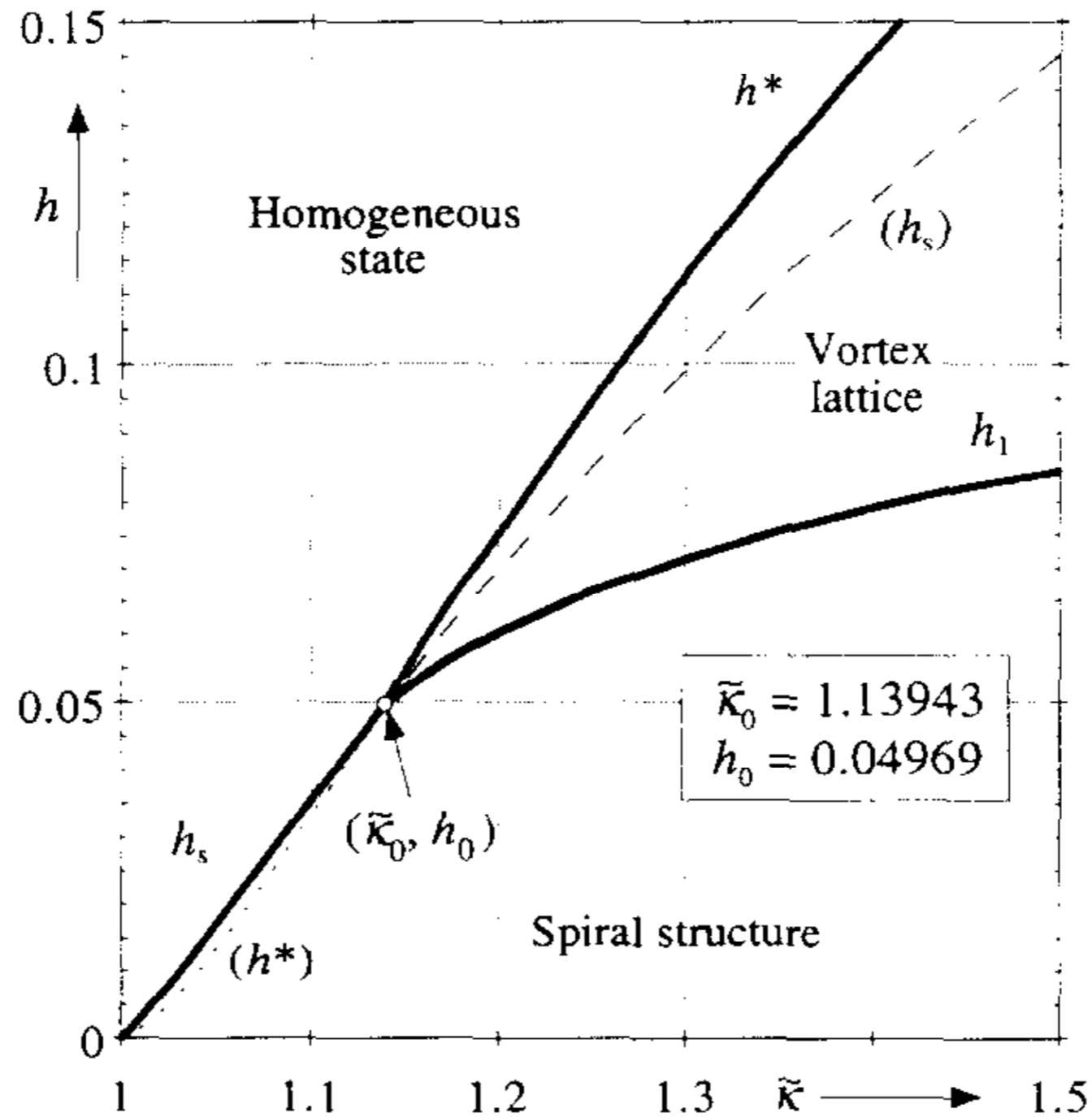
Magnetic free energy, with universal scaling of the DMI and magnetoelectric coefficients with the exchange-induced gap, temperature, and chemical potential:

$$\Gamma_{\text{DM}} = (J J_{\perp} / 4\pi \hbar v) \chi_{\Delta}$$

$$\Gamma_{\text{ME}} = (J e / 4\pi \hbar v) \chi_{\mu}$$

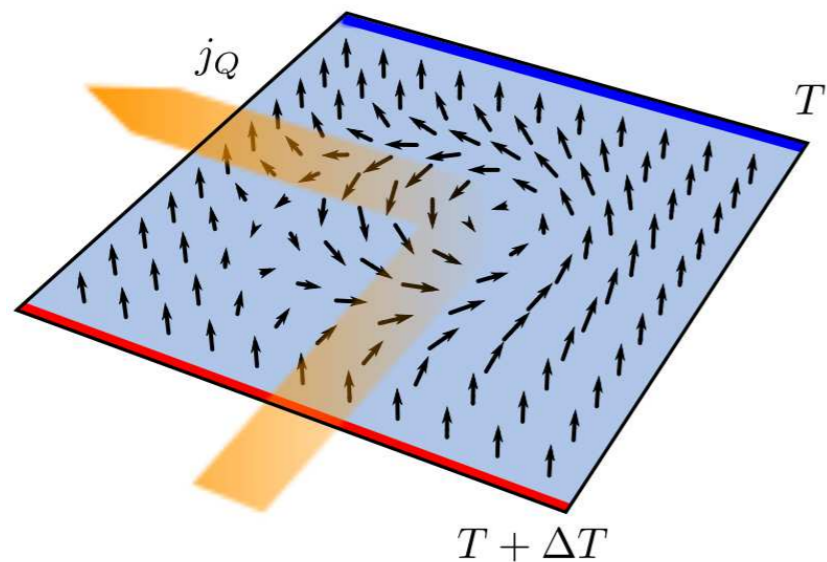
Skyrmionic lattice

- ◆ Phase diagram engendered by the DMI:



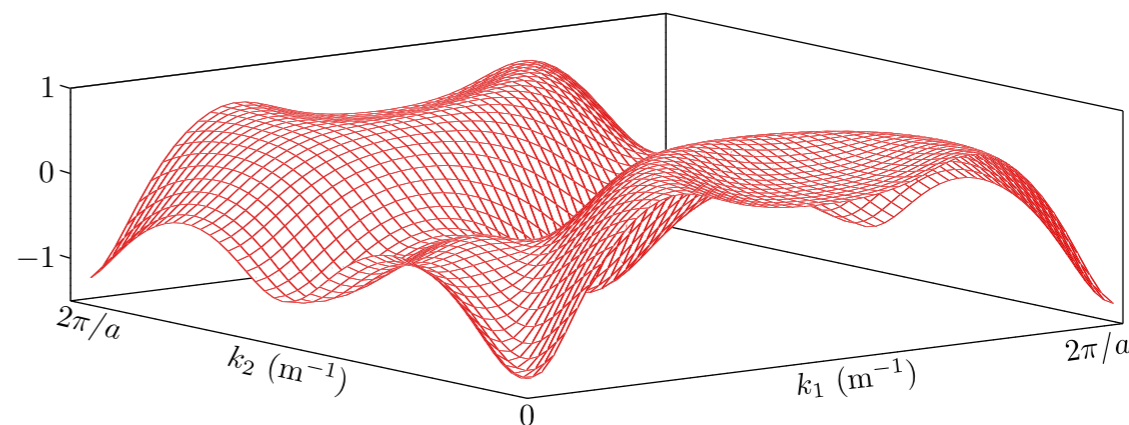
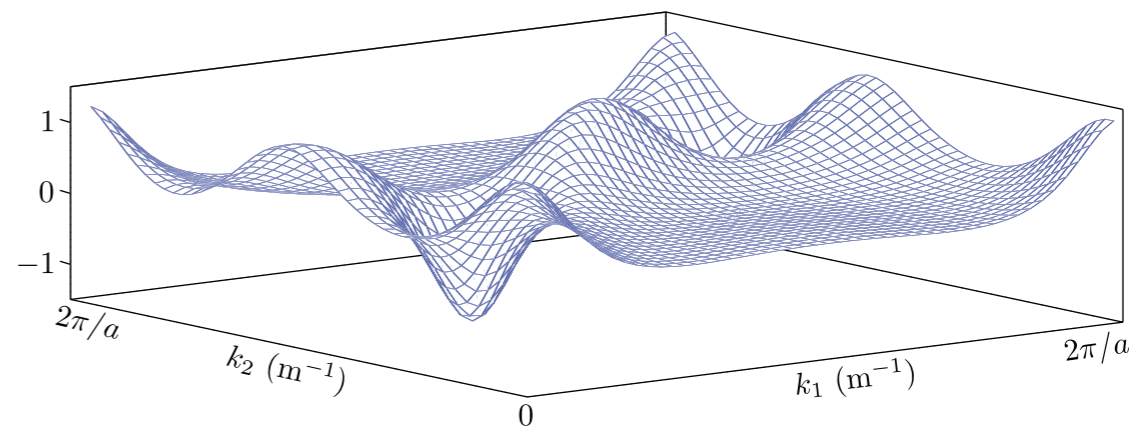
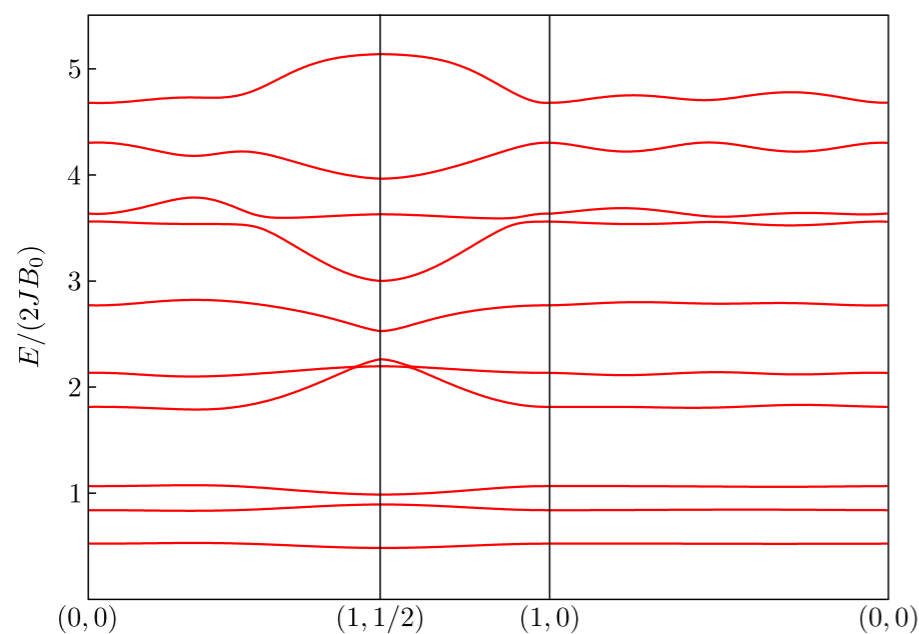
Thermal Hall effect

- The magnetic texture fomented by the DM interaction can produce Landau-level-like magnonic Chern bands, which harbor magnonic chiral edge states and induce a thermal Hall effect:



$$i\hbar\partial_t m_+ = A(\nabla/i + \alpha)^2 m_+$$

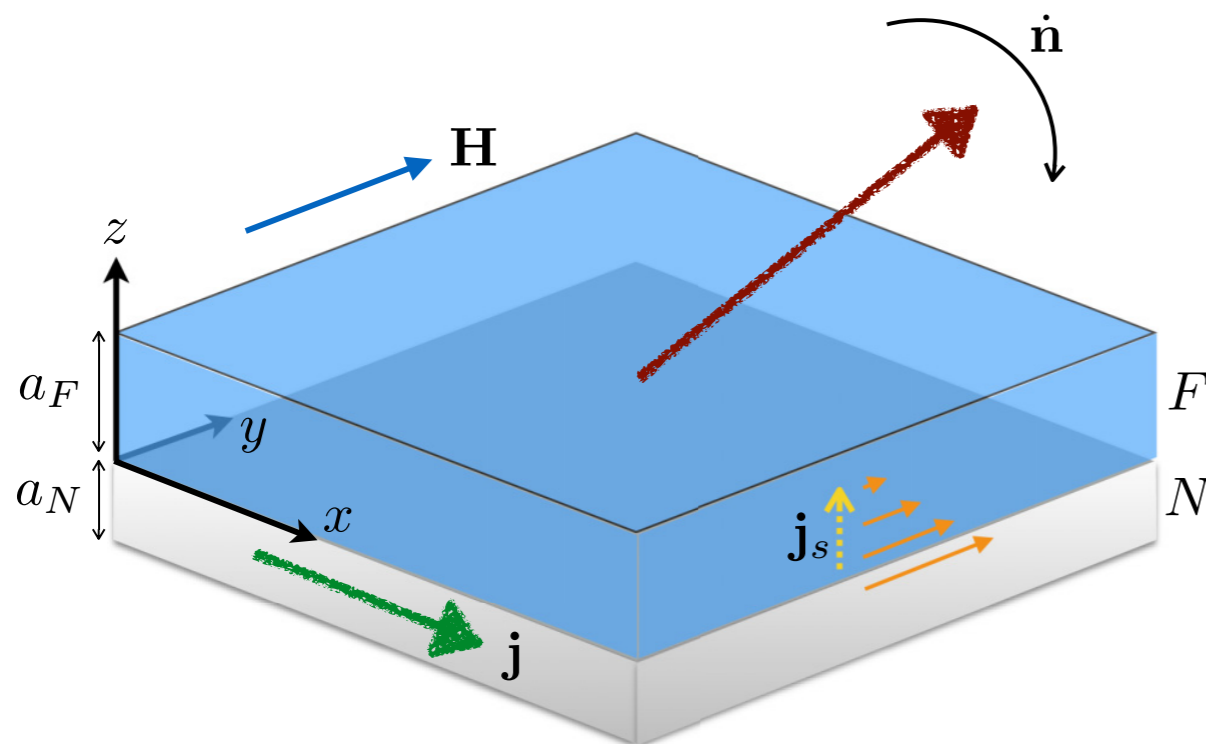
$$\alpha = \hat{R}^{-1} \left(\nabla - \frac{JJ_{\perp}}{vA} \hat{\mathbf{I}} \right) \hat{R} \Big|_{12}$$



Spin Hall phenomenology of magnetic dynamics

► Our understanding of the interaction between electric (transport) currents and magnetic dynamics at magnetic/nonmagnetic interfaces is based on the phenomenological theory according to:

- Structural/crystalline symmetries
- Onsager reciprocity
- Nonequilibrium thermodynamics



$$s(\dot{\mathbf{n}} + \mathbf{n} \times \hat{\alpha}\dot{\mathbf{n}}) = \mathbf{H}^* \times \mathbf{n} + \boldsymbol{\tau} \quad \text{torque}$$

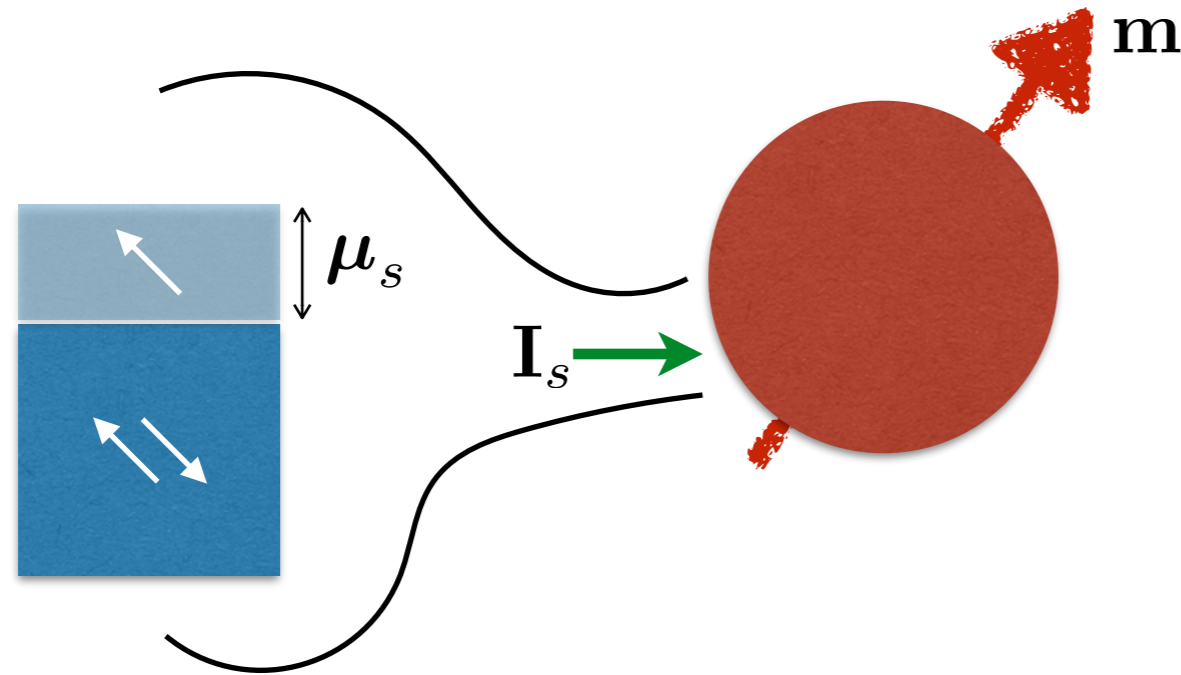
$$L\dot{\mathbf{j}} + \hat{\rho}\dot{\mathbf{j}} = \mathbf{E} + \boldsymbol{\epsilon} \quad \text{motive force}$$

$$\boldsymbol{\tau} = (\eta + \vartheta\mathbf{n} \times)(\mathbf{z} \times \mathbf{j}) \times \mathbf{n}$$

$$\boldsymbol{\epsilon} = [(\eta + \vartheta\mathbf{n} \times)\dot{\mathbf{n}}] \times \mathbf{z}$$

Towards magnetic insulators and extreme SOI

- ◆ For the ferromagnetic metals where the exchange interaction is strong and normal metals where spin-orbit coupling is weak:



$$-\frac{d\mu_s}{dt} \leftarrow \mathbf{I}_s \equiv \boldsymbol{\tau} \rightarrow \frac{d\mathbf{m}}{dt}$$

spin-transfer torque/pumping

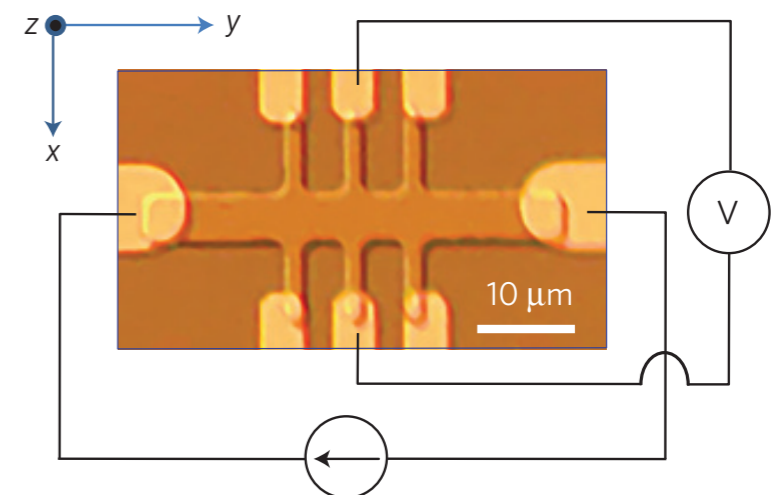
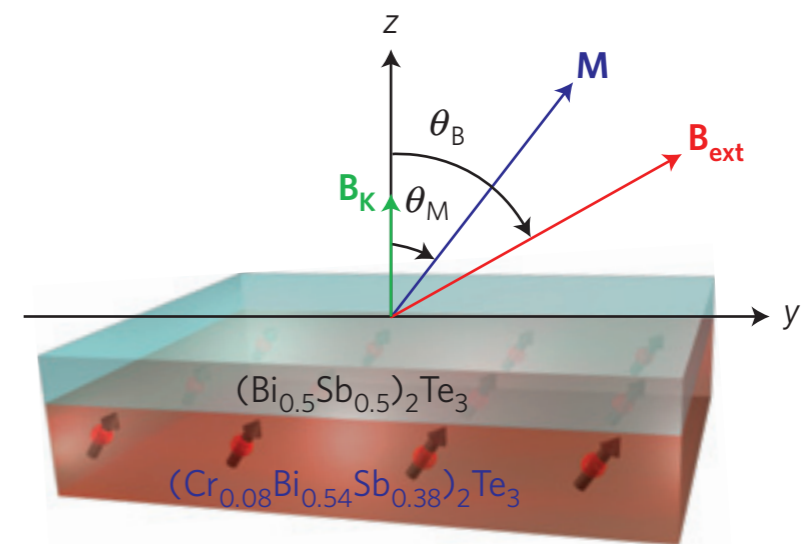
YT et al., RMP (2005)

- ◆ Recently: growing interest toward replacing ferromagnetic metals with insulators and normal metals with conductors exhibiting strong spin-orbit interactions

- Direct/inverse *spin Hall effects*
- Spin transport via magnetic dynamics (e.g., magnons, domain walls)

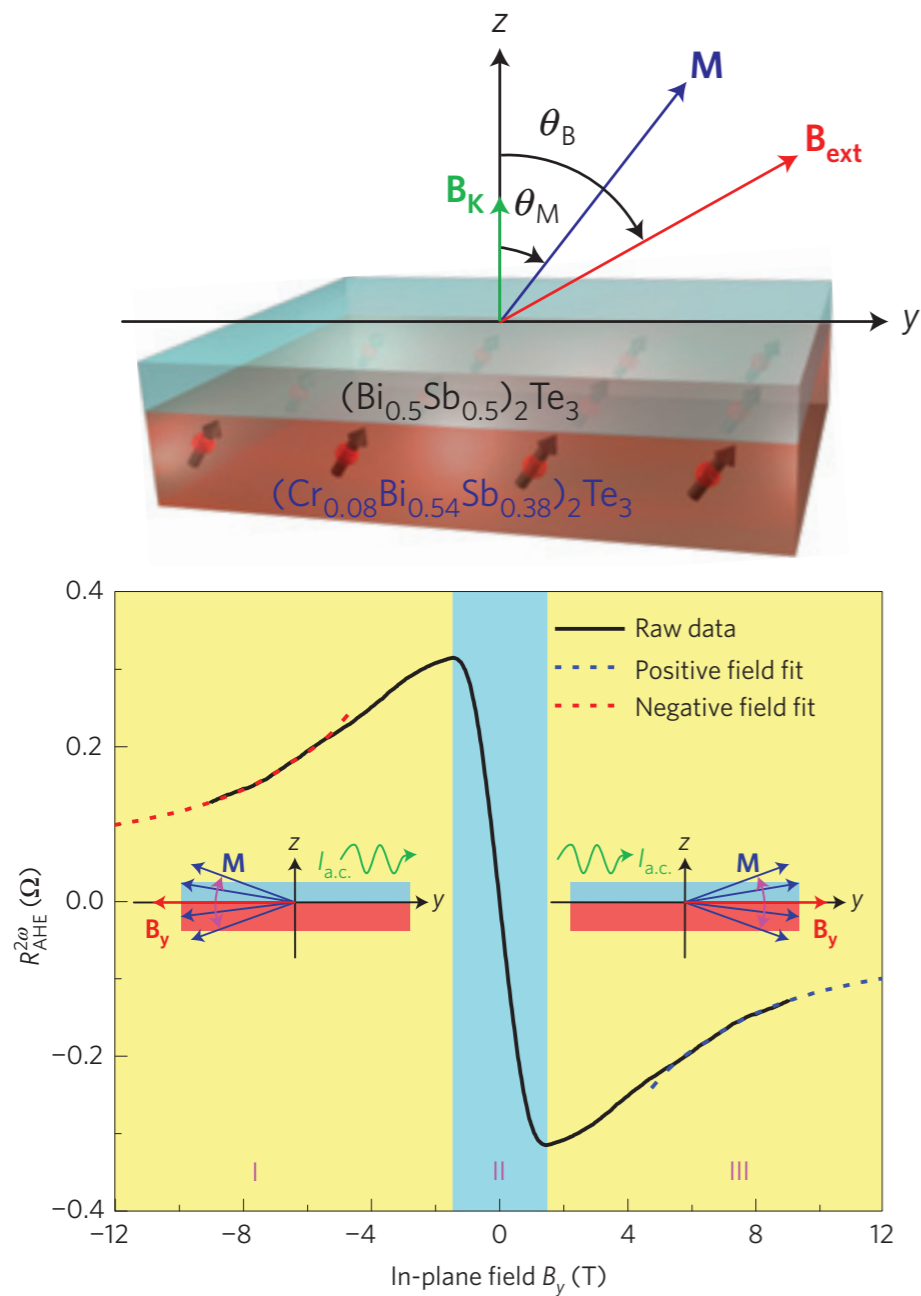
TI-based magnetic heterostructures

- ◆ Perhaps the most dramatic magnetoelectric phenomena can be expected at the interfaces between magnetic and topological insulators:
 - Electric current is localized at the interface, where the magnetic proximity effects are strongest
 - The spin-orbit interactions constitute the dominant contribution to the electronic Hamiltonian: This limit is opposite to the nonrelativistic regime
 - If the Dirac electrons are gapped by the ferromagnet, the bilayer provides an essentially nondissipative magnetoelectric compound

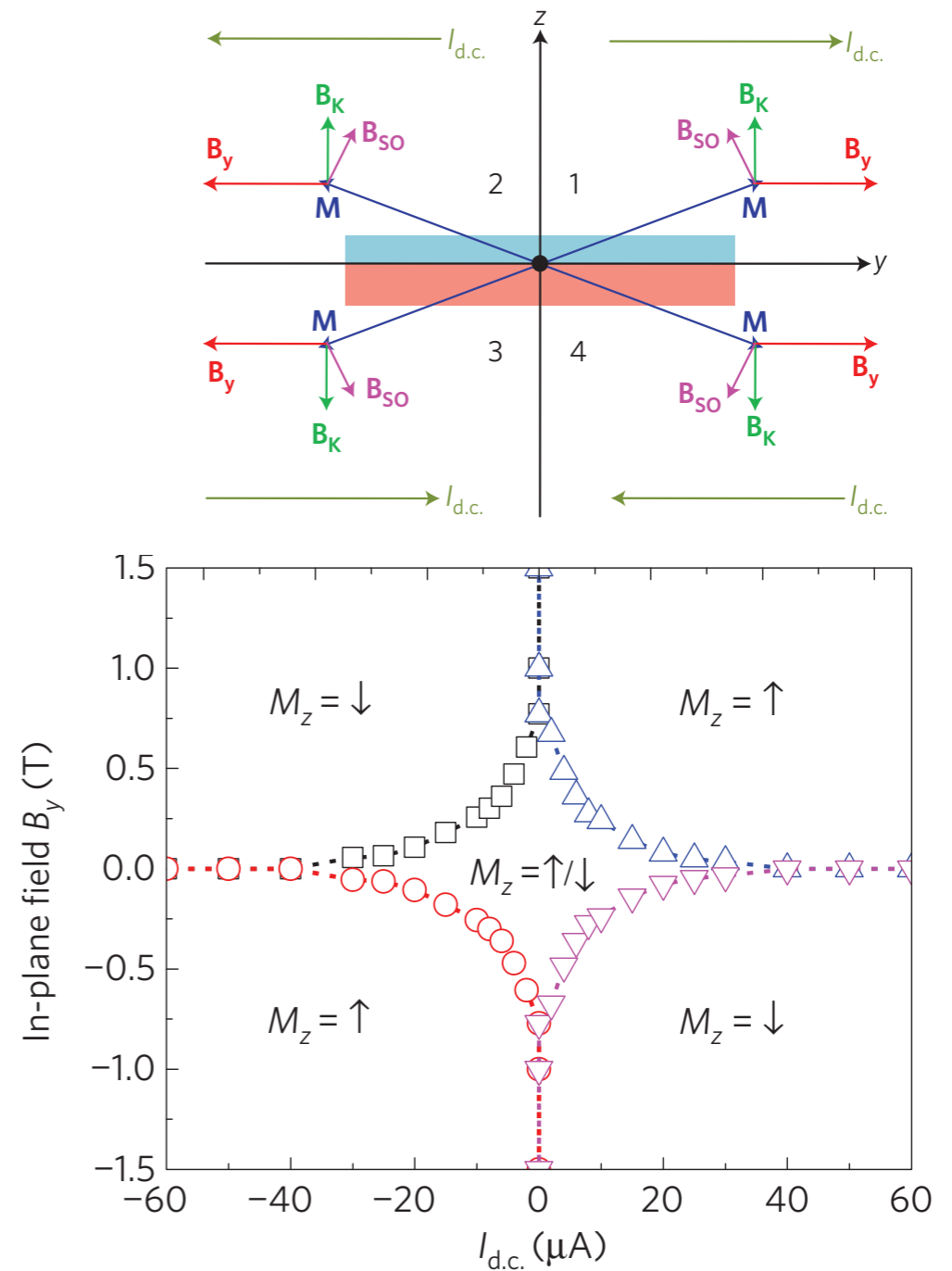


Magnetically-doped topological insulators

- Our phenomenology can be applied to analyze current-driven dynamics in magnetically-doped TI's:



2nd-harmonic AHE measurement of the effective spin Hall angle: $\tan \theta_{\text{SH}} > 10^2$



Deterministic current-induced switching

Electronic chiral mode bound to a magnetic DW

- Free-energy density

$$\mathcal{F}_0 = \frac{A}{2} [(\partial_x \mathbf{n})^2 + (\partial_y \mathbf{n})^2] - \frac{K}{2} n_z^2$$

- Domain-wall profile

$$\theta(\mathbf{r}) = 2 \tan^{-1} e^{-(x-x_{\text{dw}})/\lambda_{\text{dw}}}$$

$$\phi(\mathbf{r}) = \phi_{\text{dw}}$$

- Soft variables are provided by the zero-mode amplitudes x_{dw} and ϕ_{dw}

- Dirac Hamiltonian

$$H_0 = v (\mathbf{p} + e\mathbf{A}/c) \cdot \mathbf{z} \times \hat{\boldsymbol{\sigma}} - e\varphi + \Delta \hat{\sigma}_z$$

- Charge response

$$\rho = -\sigma_{xy} B_z, \quad \mathbf{j} = \sigma_{xy} \mathbf{z} \times \mathbf{E}$$

- Hall conductance

$$\sigma_{xy} = \text{sgn}(\Delta) \frac{e^2}{2h}$$

Redlich, *PRL* (1984); Jackiw, *PRD* (1984)

- ◆ Local exchange coupling:

$$H' = J(n_x \hat{\sigma}_x + n_y \hat{\sigma}_y) + J_{\perp} n_z \hat{\sigma}_z$$

- ◆ Charge pumping by magnetic dynamics:

$$\beta_z = \frac{J}{ev} \nabla \cdot \mathbf{n}, \quad \epsilon = -\frac{J}{ev} \mathbf{z} \times \partial_t \mathbf{n}$$

Garate and Franz, *PRL* (2010); Nomura, and Nagaosa, *PRB* (2010)

Magnetic torques

- Torque is governed by the helical locking of spin and current:

$$\boldsymbol{\tau} = (J\rho_{\parallel} + J_{\perp}\rho_z\mathbf{z}) \times \mathbf{n} \quad \text{where} \quad \rho_{\parallel} = \frac{\mathbf{z} \times \mathbf{j}}{ev}$$

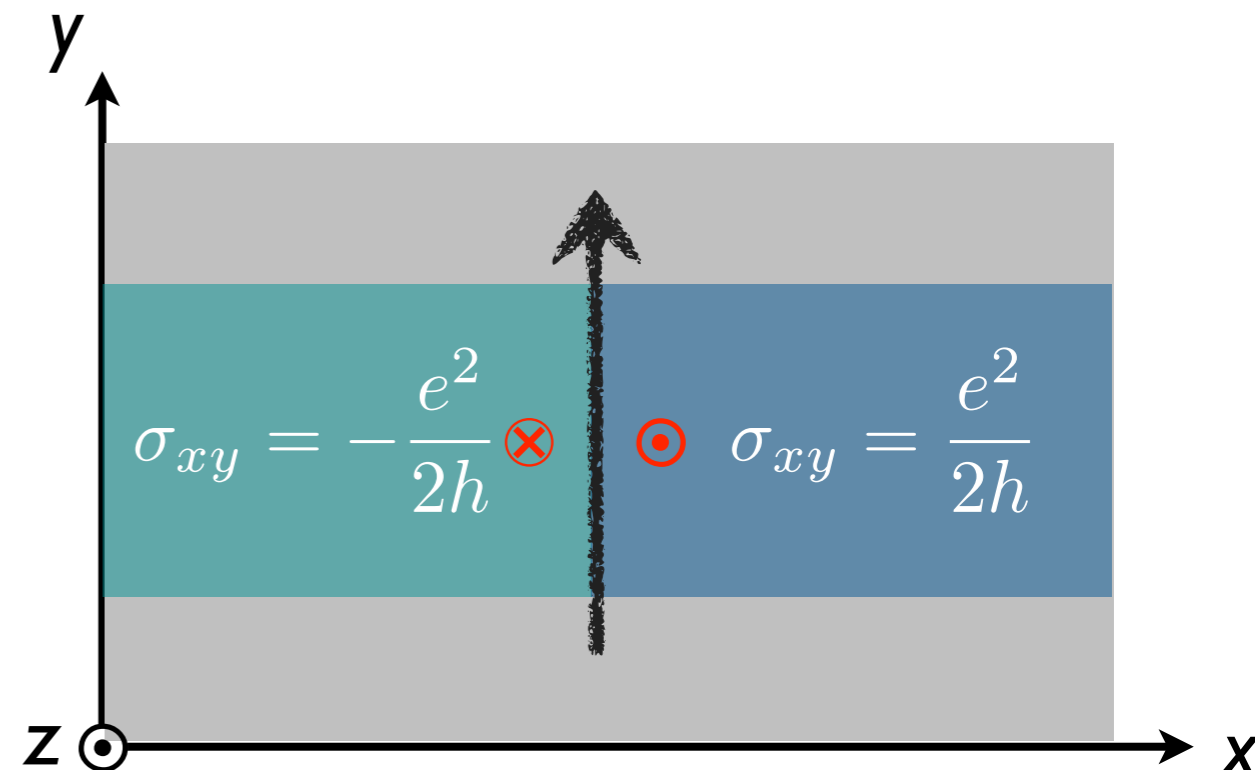
which precisely reproduces the nondissipative sector of the Onsager-reciprocal torque/pumping structure

- The total current \mathbf{j} consists of the dynamics-induced transport (Hall) and texture-induced equilibrium (persistent) contributions:

$$\mathbf{j} = \sigma_{xy} \frac{J}{ev} \mathbf{z} \times \partial_t \mathbf{n} \times \mathbf{z} - c\mathbf{z} \times \nabla M_o$$

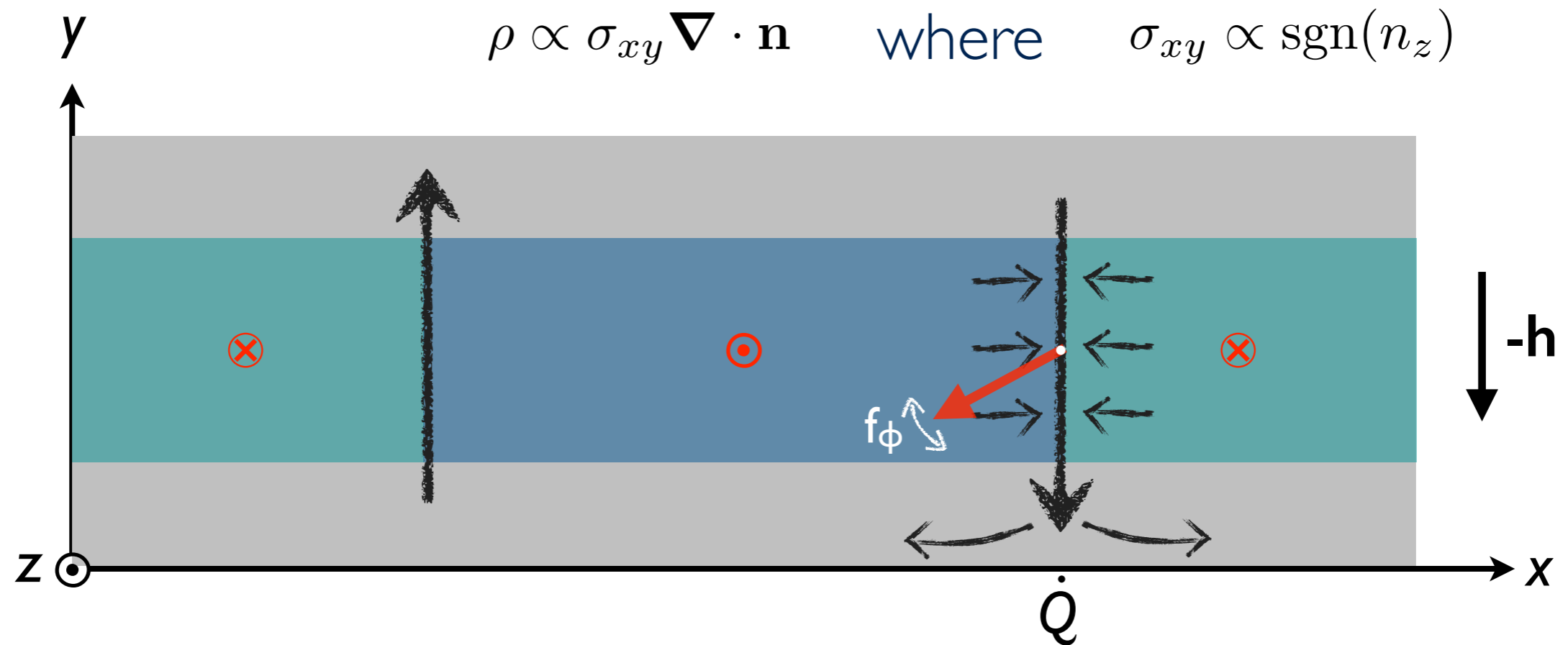
pumping
DM/ME

spin Hall phenomenology
0th Landau level



Charge pumping along a domain wall

- The magnetic dynamics redistributes charges between the gapless chiral modes and the adjacent gapped regions:

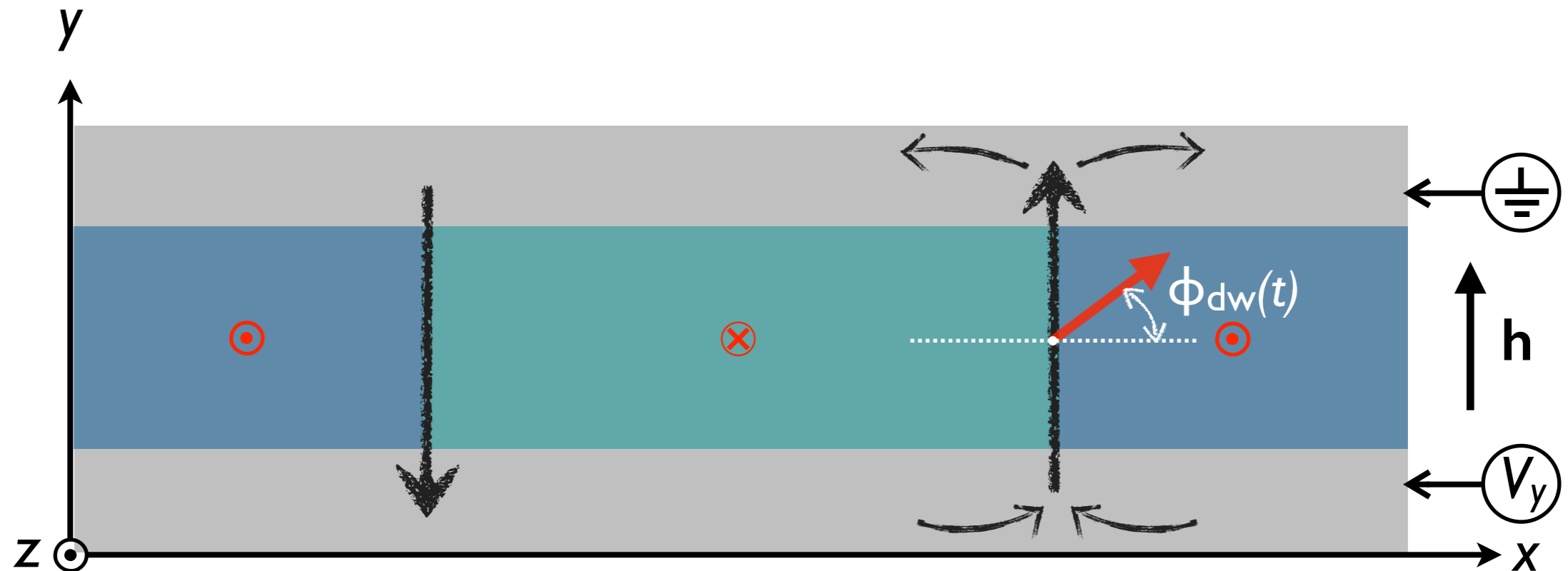


$$\epsilon \propto \mathbf{z} \times \partial_t \mathbf{n} \rightarrow \mathbf{y} \sin \phi_{dw} \dot{\phi}_{dw}$$

- The domain-wall dynamics thus pumps current along the chiral mode (“squeezing” it out of the gapped quantum-Hall regions)

Onsager-reciprocal torque

- The coupled dynamics of the canonically conjugate pair $(x_{\text{dw}}, \phi_{\text{dw}})$



realizes a damped analog of the Josephson junction, whose equilibrium phase can be arbitrarily controlled by the applied voltage and magnetic field:

$$\dot{x}_{\text{dw}} \propto (f_\phi + j \sin \phi_{\text{dw}}), \quad \dot{\phi}_{\text{dw}} \propto \frac{\alpha}{\lambda_{\text{dw}}} \dot{x}_{\text{dw}} \quad \text{Onsager} \quad \epsilon \propto \sin \phi_{\text{dw}} \dot{\phi}_{\text{dw}}$$

$$f_\phi \equiv -\partial_{\phi_{\text{dw}}} F \propto JJ_\perp \sin \phi_{\text{dw}} + h\lambda_{\text{dw}} \cos \phi_{\text{dw}}$$

Summary

- ◆ Universal scaling functions for the orbital magnetization (the three spin components are contained in the out-of-plane spin and orbital magnetizations), leading to DMI and $\mathbf{E} \cdot \mathbf{n}$ ME effect
- ◆ Zeroth Landau level and spin Hall phenomenology encapsulate the essential equilibrium and nonequilibrium magnetoelectric properties
- ◆ Magnetic domain walls bind electronic chiral modes, thus imprinting reconfigurable interconnects, which could be “loaded” by a voltage bias, microwaves, and thermal gradients

[YT and Loss, PRL \(2012\)](#)

[Pesin and MacDonald, PRL \(2013\)](#)

[YT and Bender, PRB \(2014\)](#)

[Fan et al., Nature Mat. \(2014\)](#)

[YT, Pesin, and Loss, PRB \(2015\)](#)

