Quasiclassical circuit theory of spin transport in superconducting heterostructures

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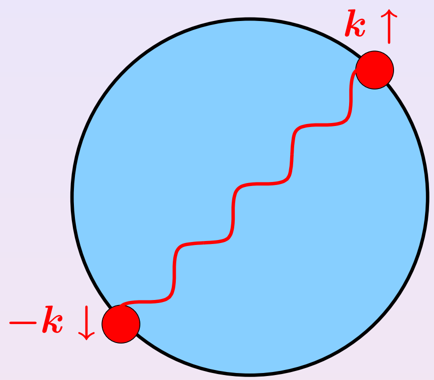
Outline

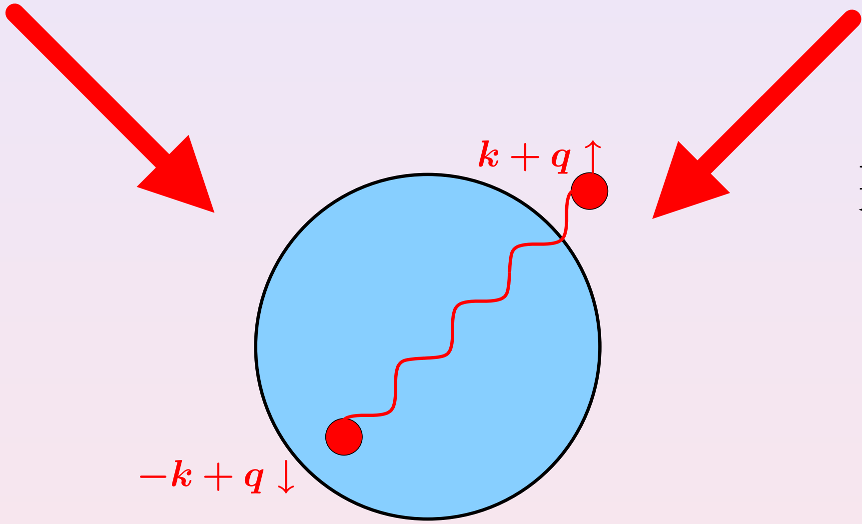
- Some history of superconductor-ferromagnet heterostructures
- Quasiclassical Greens functions and boundary conditions
- Quantum Circuit Theory
- Spectral properties of SF bilayers
- (Spin-)Thermoelectric effects in SF

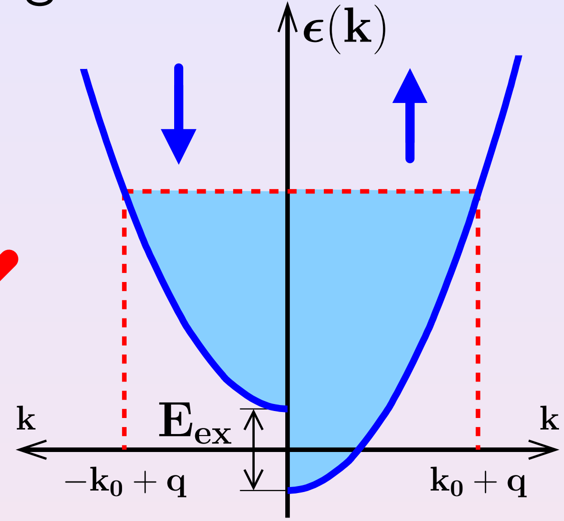
General properties of SF

Superconductor: attractive interaction through virtual phonons

Magnetism: Bandsplitting by exchange interaction







total momentum

$k_\uparrow - k_\downarrow = 0$

Spin splitting

$k_\uparrow \neq k_\downarrow$

[Fulde+Ferrel (1964); Larkin+Ovchinnikov (1965)]

Cooper pairs with momentum $2q = \frac{2E_{ex}}{\hbar v_F} = \frac{1}{\xi_F} \neq 0$

Macroscopic wave function: $\Delta\cos(2qr)$ or Δe^{i2qr}

spin dependent pair wave function/supercurrent!

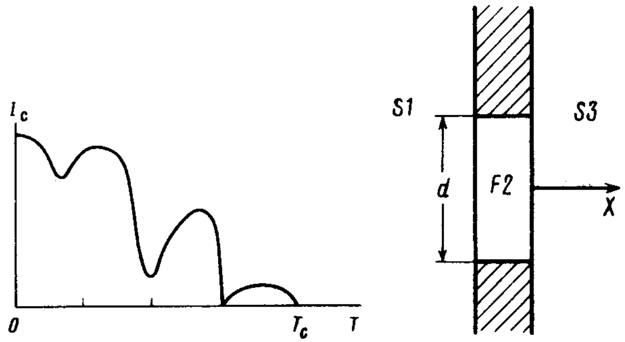
Critical current oscillations

- Fulde, Ferrell (1964), Larkin, Ovchinnikov (1964)
  Superconductor with exchange field H_{ex} → inhomogeneous order parameter

$$\Delta(x) = \Delta\cos(q_{ex}x)$$

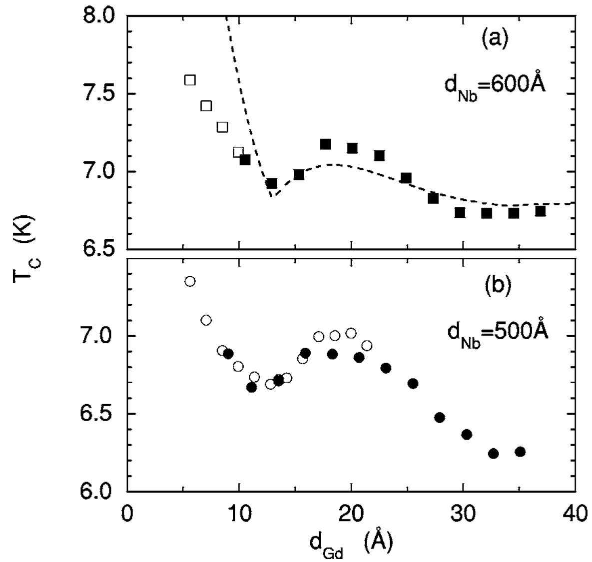
$$q_{ex} = \frac{\hbar v_F}{2H_{ex}}$$

- Buzdin, Bulaevskii, Panyukov (1982)
  Critical current of an SFS Josphson
  junction oscillates!



$$I_c = I_c^0 e^{-q_{ex}d_F}\cos(q_{ex}d_F)$$

- Buzdin and Kuprianov (1990); Radovic *et al.* (1991)
  Critical temperature of SFS oscillates
  as function F-thickness
  (Exp. observed by Jiang et al.
  and Mercaldo et al 1995,…)

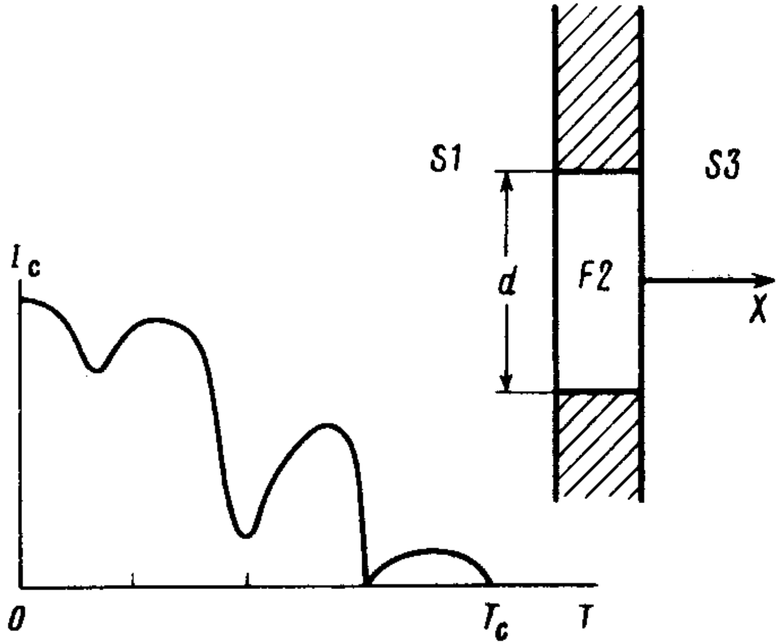


History of SF (some aspects...)

- Buzdin, Bulaevskii, Panyukov (1982)
  Critical current of an SFS Josephson
  junction oscillates!

$$I_c = I_c^0 e^{-q_{ex} d_F} \cos(q_{ex} d_F)$$

$$q_{ex} = \frac{\hbar v_F}{2H_{ex}}$$



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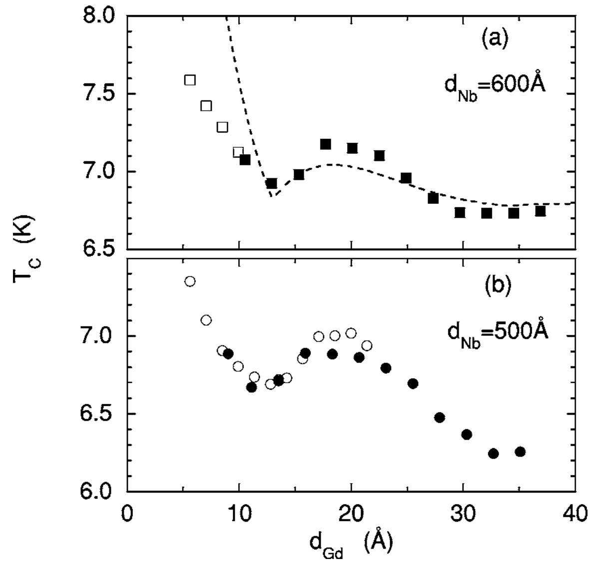
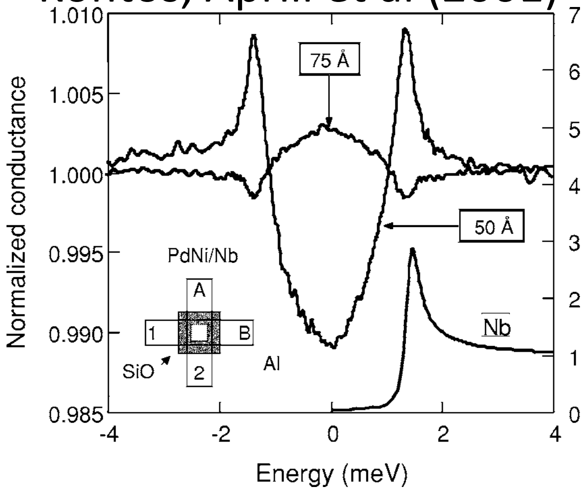
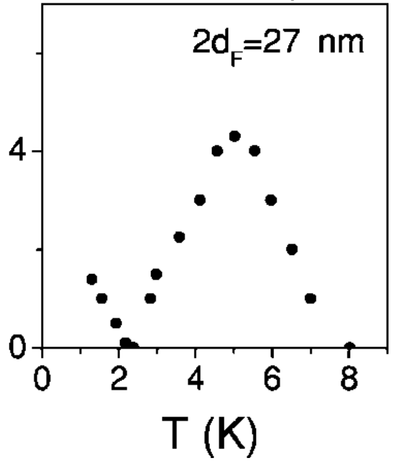


(Some) Experimental progress since 2000

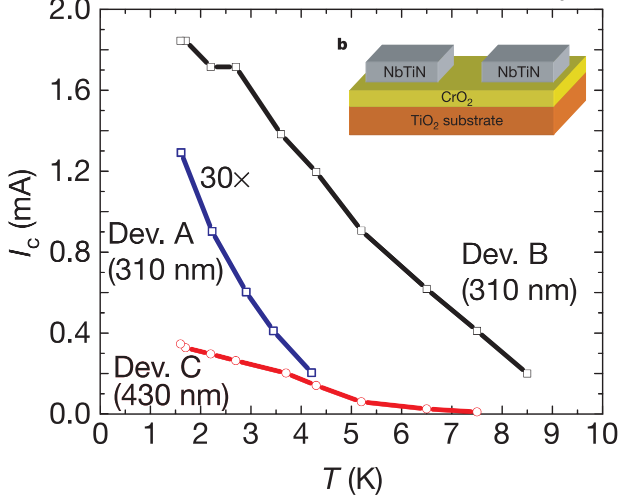
Density of states oscillation
Kontos, Aprili et al (2001)



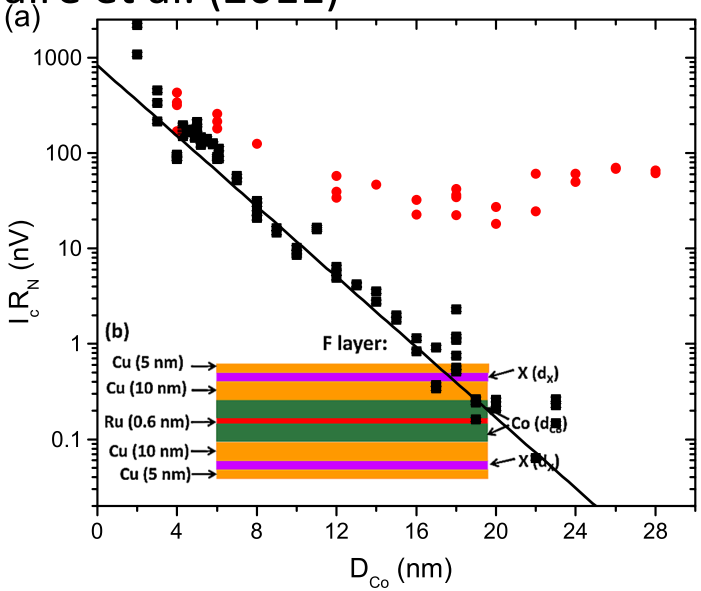
Critical current oscillations
Ryazanov et al. (2001)



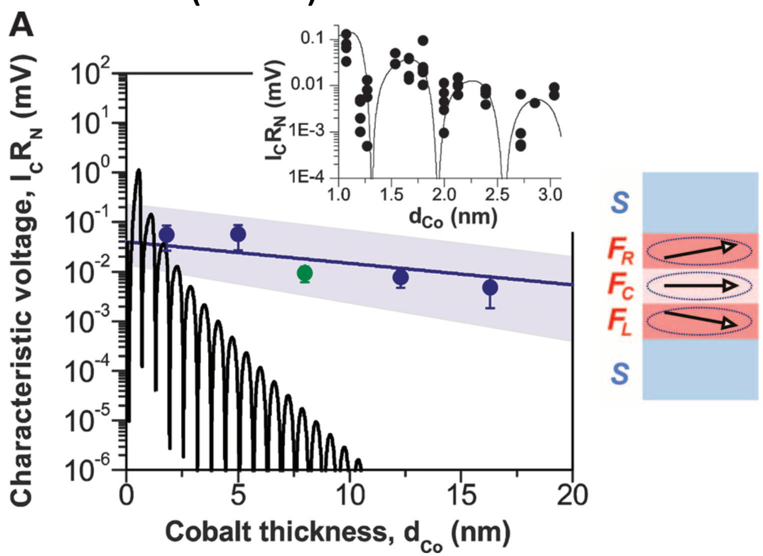
Half-metal-Josephson effect
Keizer, Gönnenwein, et al. (2006)



Long-range triplet supercurrent
Khaire et al. (2011)



Triplet supercurrent by conical ferromagnet
Robinson et al. (2011)



Quasiclassical Theory
&
Boundary Conditions

The quasiclassical theory of inhomogeneous superconductors

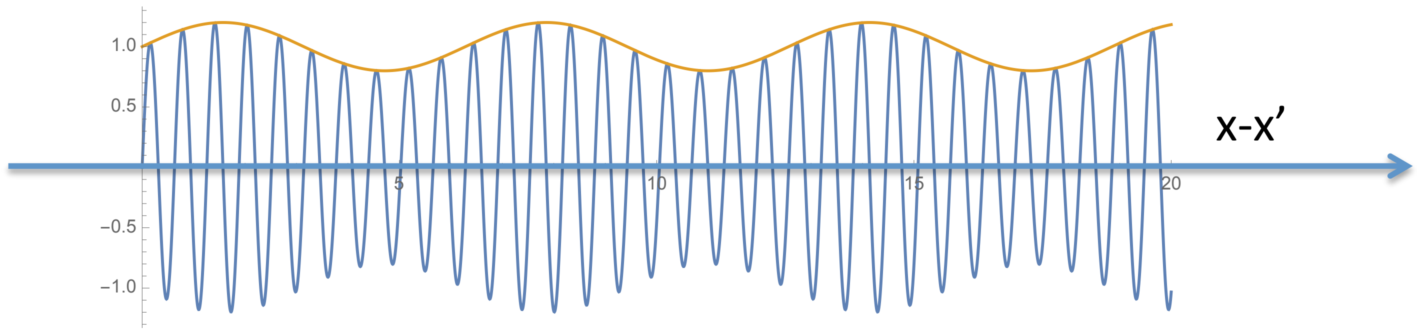
General superconductors: Gorkov equations for matrix Greens function
[Note: GF is in general a matrix in Nambu-Keldysh-Spin-….-Space]

$$\Psi = \left(\psi_{\uparrow}, \psi_{\downarrow}, \psi_{\downarrow}^{\dagger}, -\psi_{\uparrow}^{\dagger}\right)$$

[Gorkov (1957), Abrikosov (1958)]

$$\breve{G}(x,x') = -i\left\langle \mathcal{T}\,\Psi(x)\Psi^{\dagger}(x')\right\rangle \approx \breve{G}(x,x)e^{ik_F(x-x')}$$

Contains full microscopic oscillations on scale of Fermi wavelength



Usually unnecessary amount of information!

Realistic case: impurities, microscopic imperfections lead to random elastic scattering
→ all interference effects ~exp(ik$_F$x) are washed out

The quasiclassical theory of inhomogeneous superconductors

Transport-like equation for the qc Greensfunction

$$\frac{\vec{p}^2}{2m} - \epsilon_F \approx \frac{p_F^2}{2m} - i\frac{\vec{p}_F}{m}\vec{\nabla} - \epsilon_F = -i\vec{v}_F\vec{\nabla}$$

$$\left[\underbrace{-i\partial_t}_{\varepsilon} + i\vec{v}_F\vec{\nabla} + \breve{\Sigma}(\vec{x},\varepsilon), \breve{G}(\vec{v}_F,\vec{x},\varepsilon)\right] = 0$$

[Eilenberger (1968); Larkin, Ovchinnikov (1968)]

$$\vec{j} = e^2 N_0 \int d\varepsilon \left\langle \mathrm{tr}\hat{\tau}_3 \breve{G}_K(\vec{v}_F,\vec{x},\varepsilon)\right\rangle_{v_F}$$

current density

angular average

Normalization condition $\breve{G}(\vec{v}_F,\vec{x},t,t')^2 = \breve{1}$

Selfenergy contributions (phonons, impurities, Coulomb,...)

E.g. $\breve{\Sigma}_{ph}(\vec{x}) = \breve{\Delta}(\vec{x}) = \lambda\int d\varepsilon\left\langle \breve{G}(x,v_F,\varepsilon)\right\rangle_{off-diag}$ $\breve{\Sigma}_{imp}(\vec{x},\varepsilon) = \frac{1}{2\tau}\left\langle \breve{G}\right\rangle_{\vec{v}_F}$

Pairing potential

Elastic impurity scattering

Diffusive approximation [Usadel 1970]

$$l = v_F\tau \ll \underbrace{\frac{v_F}{\Delta}}_{\xi_S}, \underbrace{\frac{v_F}{kT}}_{\xi_T}$$

$$\breve{G}(\vec{v}_F,\vec{x}) = \breve{G}(\vec{x}) + \vec{v}_F\breve{\vec{G}}(\vec{x}) + \ldots$$

Usadel equation (quantum diffusion equation)

$$D = v_F^2\tau / 3$$

$$D\partial_x\breve{G}(x)\partial_x\breve{G}(x) = \left[-iE\hat{\tau}_3 + \hat{\Delta} + \hat{\Sigma}', \breve{G}(x)\right]$$

$$\vec{j} = e^2 N_0 D\int d\varepsilon \mathrm{tr}\hat{\tau}_3\breve{G}(\vec{x})\vec{\nabla}\breve{G}(\vec{x})_K$$

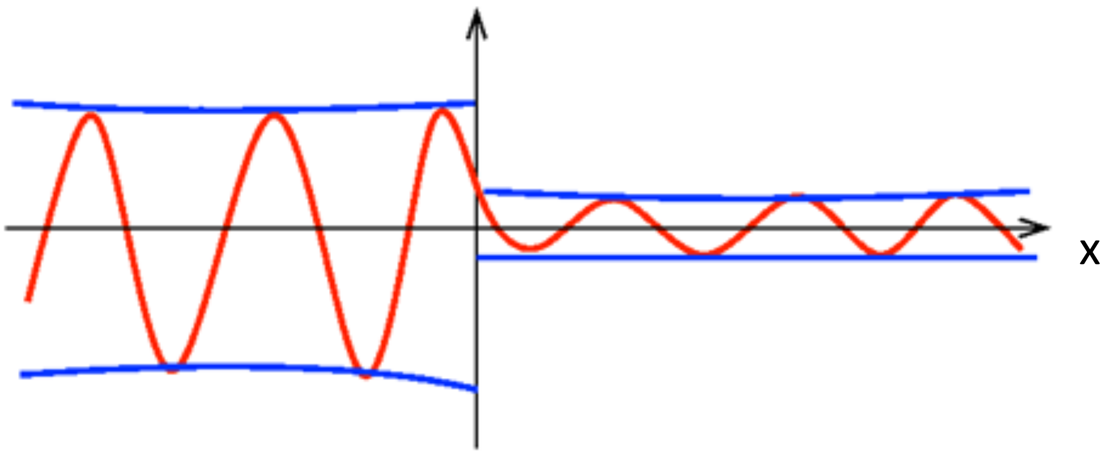
current density

The quasiclassical theory and the boundary condition problem

[Eilenberger (1968), Larkin & Ovchinnikov (1968)]

Principle of quasiclassics: integrate out "fast oscillations" $e^{ik_F(x-x')}$

Equation reduced to envelope functions: $\breve{G}(x,x') \approx \breve{G}(x,x) e^{ik_F(x-x')}$



Envelope functions (quasiclassical Green's functions) have jumps at atomically sharp interfaces, which cannot be derived within the quasiclassical theory

Need for separately derived boundary conditions!

HISTORY OF BOUNDARY CONDITIONS IN THE QUASICLASSICAL THEORY

- Zaitsev (1984): boundary conditions for ballistic interfaces
  - spin-degenerate interface, only transmission probabilities
  - complicated non-linear equations
- Rainer, Sauls, Millis (1988): spin-active interface, ballistic
  - general scattering matrix, complicated non-linear equations
- Kupriyanov, Lukichev (1988): diffusive case
  - Only small transmission
  - One parameter: tunnel conductance

$$G_T = 2G_Q \sum_n T_n$$

- Nazarov (1999): diffusive case
  - arbitrary transmission, spin-degenerate
- More aspects: Zaikin, Shelankov, Eschrig,....

Circuit theory formulation of the diffusive theory and the boundary condition

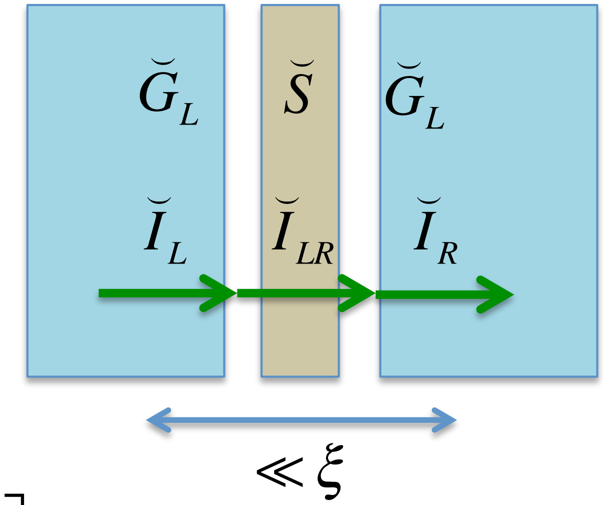
Matrix current (unit area) $\breve{I}(x) = -\sigma \breve{G}(x)\partial_x \breve{G}(x)$

Conductivity $\sigma = e^2 N_0 D$

Usadel equation $\partial_x \breve{I}(x) = \underbrace{-\frac{\sigma}{D}\left[-iE\hat{\tau}_3 + \hat{\Delta} + \hat{\Sigma}, \breve{G}(x)\right]}_{\text{leakage (or source) of coherence}}$

Characteristic length $\xi = \sqrt{\frac{\hbar D}{\mathrm{Max}(E,\Delta,\Sigma)}}$

Close to an interface: $\breve{I}_{L,R} = -\sigma \breve{G}\partial_x \breve{G}\big|_{L,R}$



Scattering region = non-quasiclassical description
(scattering matrix S)

Nazarov (1999): for a spin-independent scattering matrix S

$$\breve{I}_{LR} = \frac{2e^2}{h}\sum_n \frac{T_n\left[\breve{G}_L, \breve{G}_R\right]}{4 + T_n\left(\left\{\breve{G}_L, \breve{G}_R\right\} - 2\right)} \overset{T_n \ll 1}{\approx} \frac{G_T}{2}\left[\breve{G}_L, \breve{G}_R\right]$$

Boundary condition: $\breve{I}_L = \breve{I}_{LR} = \breve{I}_R$ depends on transmission eigenvalues $\{T_n\}$

SF-Heterostructures: need for a spin-dependent boundary conditions!

The problem of spin-dependence (or energy) of the boundary scattering:

Isotropic GF
(Usadel, diffusive regions)

$$\breve{G}(x,E)$$

= Matrix
in Keldysh-Nambu-Spin space

General scattering matrix
(across the interface)

$$\hat{S}(E)=\begin{pmatrix} r & t \\ t' & r' \end{pmatrix}$$

= Matrix
in Nambu-Spin space

If: $\hat{S}(E)\sim\bar{S}\otimes\sigma_0$ (spin- & energy-independent)

Then: $\left[\breve{G}(x,E),\hat{S}\right]=0$ → extreme simplification!

Consequence: e.g. only $T_n=\mathrm{Eigenvalues}[t^\dagger t]$ enter the BC (Nazarov `99)

The way to spin-dependent boundary conditions in the diffusive case:

Generalization of the Nazarov-BC to weak spin dependence:

$$\delta\varphi = \arg r_\uparrow - \arg r_\downarrow$$

Spin-polarization conductance

$$G_P = G_Q \sum_n \left(T_{n\uparrow} - T_{n\downarrow}\right)$$

➔ Spin-polarized current

Spin-dependent interfacial phase shifts

$$G_\phi = G_Q \sum_n \delta\varphi_n$$

➔ Induced exchange splitting

Huertas-Hernando, Nazarov, Belzig (2002)

Taking into account strongly polarized magnetic insulators

$$G_{\phi 2} = G_Q \sum_n \delta\varphi_n^2 \qquad G_{\phi 3} = G_Q \sum_n \delta\varphi_n^3 \qquad G_{\phi 4} = \cdots$$

➔ pair-breaking

Cottet, Huertas-Hernando, Belzig, Nazarov (2009)

Barriers with constant spin polarization (per channel)

$$\hat{S}(E) = S \otimes \sigma_0 + S' \otimes \vec{m}\vec{\sigma}$$

→ all details of S and m enter! Machon, Belzig (2015)

Fully general spin-dependent BC
(including arbitrary polarization, textures, and spin-mixing)

$$\hat{S}(E) = \begin{pmatrix} \bar{r}(\vec{m}_L) & \bar{t}(\vec{m}) \\ \bar{t}'(\vec{m}) & \bar{r}'(\vec{m}_R) \end{pmatrix}$$

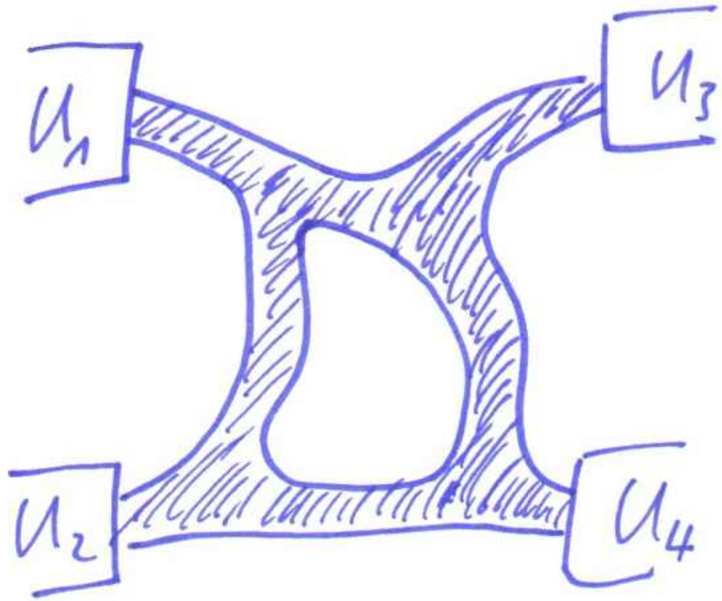
→ all details of S and all m’s enter!

Eschrig, Cottet, Belzig, Linder (2015)

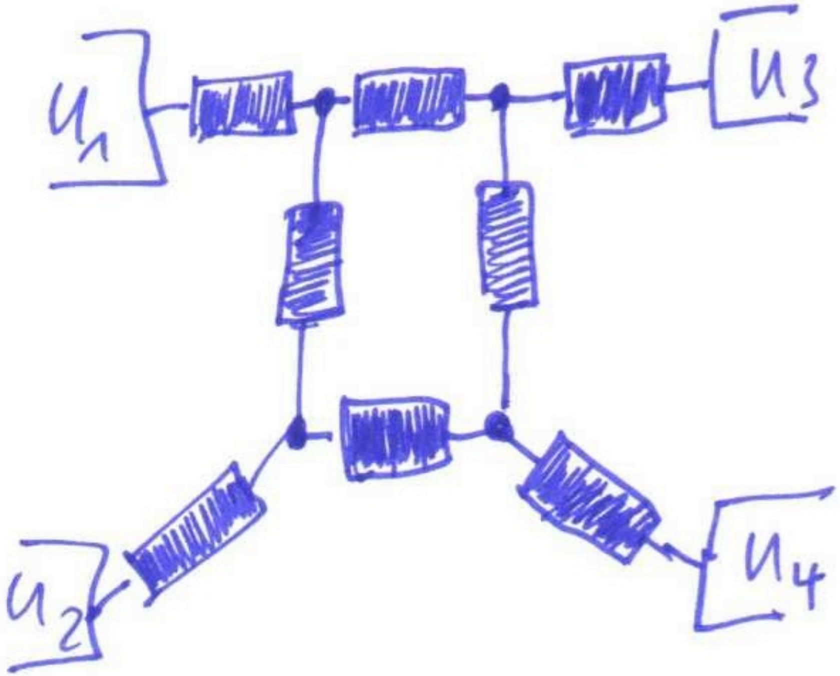
Quantum Circuit Theory

Classical circuit theory

- Description by full Poisson equations is very complicated and ineffective
- **Conserved** static currents flow between contacts with fixed voltages
- No current through boundaries



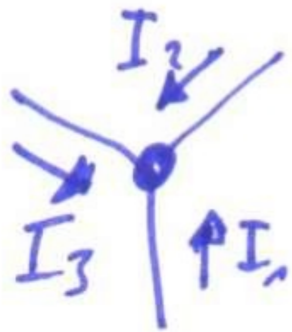
$$\vec{\nabla}\vec{j} = \Delta U = 0 \qquad \vec{n}\vec{j} = 0$$



The problem is drastically simplified by mapping onto a **discretized** structure with elements

- Network of **nodes and connectors** (resistors)
- Contacts to outer world: voltages in terminals are fixed
- The voltages on the nodes have to be determined by set of rules

Circuit theory and Kirchhoff rules



Conservations laws can be cast into two rules for an electric circuit composed of discrete elements

Rule 1: The current in each node is conserved (node rule)

Rule 2: The sum of all voltage differences around a closed loop is zero (loop rule)

At connections to the outer world, the voltages are fixed. The rules completely determine all **internal voltages and currents.**

In addition we need the microscopic description of the connector: **Ohms law**

$$I = \left(U_1 - U_2\right) / R$$

Remarkable consequence: all conductance properties of arbitrarily complicated networks are fully determined by the set of Kirchhoff Rules!

Nazarov 94-

Quantum Circuit theory

We use the matrix current: $$\hat{I}(x) = -\sigma \hat{g} \frac{\partial}{\partial x} \hat{g}$$

Electrical current: $$I = Tr \hat{\tau}_K \hat{I}$$

related to the gradient of a "matrix voltage" $\hat{g}$

The matrix current obeys a "conservation" law (up to decoherence)

$$D \frac{\partial}{\partial x} \hat{I}(x) = -iE\left[\hat{\tau}_3, \hat{g}\right] + \cdots$$

Other leakage/ source terms

Usadel equation

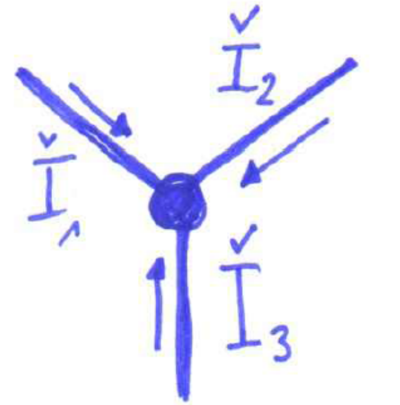
Decoherence/dephasing term (leakage of coherence)

$$\hat{\tau}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The matrix current conservation law is the basis of a circuit theory! We can construct a network of connectors (with matrix voltage drop) and nodes (with matrix voltages)

[Usadel 70]

Quantum Kirchhoff rules (without decoherence)



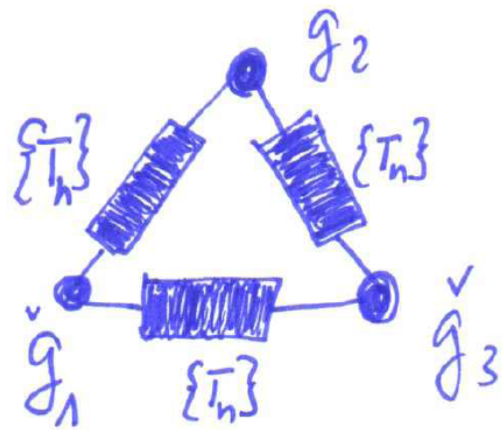
Principle: Consider a quantum electric circuit as composed of discrete elements with unknown matrix voltages

Rule 1: The **matrix** current in each node is conserved

$$\sum_i \hat{I}_i = 0$$

Rule 2: The **matrix** voltages obey the normalization

$$\hat{g}_i^2 = \hat{1}$$



In addition we need the description of the connector:
Quantum Ohms law

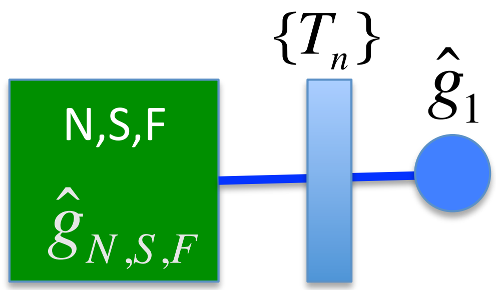
$$\hat{I}_{12} = \frac{e}{\hbar}\sum_n \frac{T_n\left[\hat{g}_1,\hat{g}_2\right]}{4+T_n\left(\left\{\hat{g}_1,\hat{g}_2\right\}-2\right)}$$

T_n: Transmission probabilities, determined by the microscopic details

Nonlinear functional relation between matrix voltages

Derivation requires a microscopic theory of the interface.

Leads: connected to circuit with some connector and a fixed Green's function (determine type of contact: normal metal, superconductor,….



[Nazarov 99]

Electron-hole (de)coherence

Decoherence can be taken into account analogously to a **leakage** current

$$-\frac{\partial}{\partial x}\hat{I}(x)=\left[-iE\hat{\tau}_3,\hat{g}(x)\right]$$

discretization

$$\hat{I}_1+\hat{I}_2+\hat{I}_{leakage}=0$$

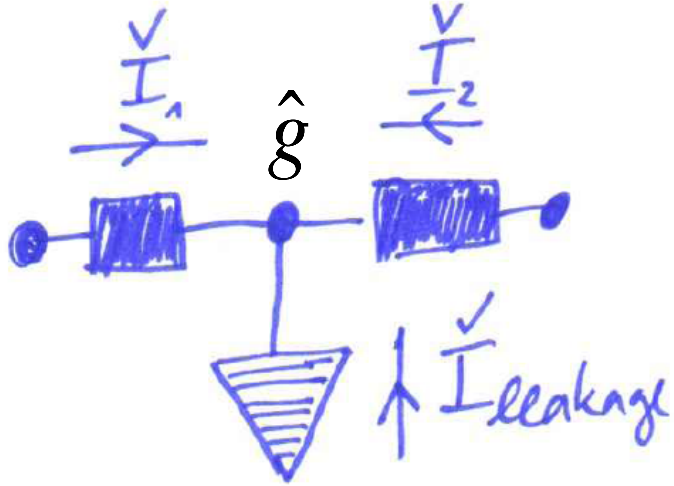
$$\hat{I}_{leakage}=\frac{G_Q}{\delta}\left[-iE\hat{\tau}_3,\hat{g}_c\right]$$

level spacing

Quantum mechanical decoherence has the same form as a **leakage current.**
No charge is lost, only coherence!

Other contributions (from self energies)

- superconductor (Δ) = source of coherence
- spin-flip scattering = loss of spin coherence
- Zeeman field/exchange = spin-dependent energy shift



Quantum circuit theory:

- Quantum Kirchhoff rules
- Matrix voltages and matrix currents
- Dephasing as leakage of coherence

Spin-dependent Quantum Circuit Theory (boundary condition)

Connectors (contacts)

$$\hat{I}_{1\to c}(\varepsilon) = \frac{G_T}{2}\left[\hat{G}_1, \hat{G}_c\right] + \frac{G_P}{2}\left[\left\{\hat{\kappa}, \hat{G}_1\right\}, \hat{G}_c\right] - i\frac{G_\phi}{2}\left[\hat{\kappa}, \hat{G}_c\right]$$

Nodes (finite dwell time):

Leakage of coherence

$$\hat{I}_L(\varepsilon) = -iG_Q\frac{\varepsilon}{\delta}\left[\hat{\tau}_3, \hat{G}_c\right]$$

Standard tunneling conductance

$$G_T = G_Q \sum_n \left(T_{n\uparrow} + T_{n\downarrow}\right)$$

➔ Usual charge current

Spin-polarization conductance

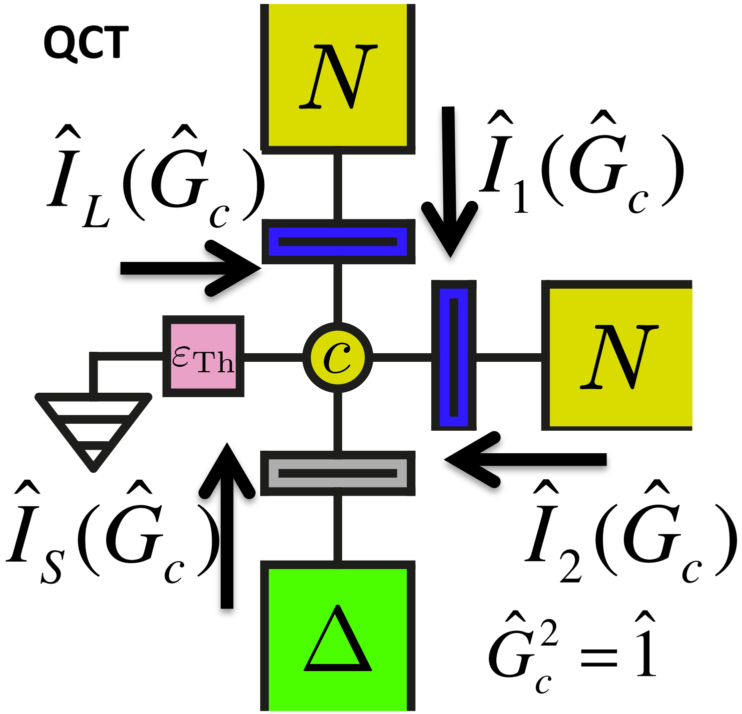
$$G_P = G_Q \sum_n \left(T_{n\uparrow} - T_{n\downarrow}\right)$$

➔ Spin-polarized current

Spin-dependent interfacial phase shifts

$$G_\phi = G_Q \sum_n \delta\phi_n$$

➔ Induced exchange splitting



Total matrix current conservation

$$\hat{I}_S + \hat{I}_1 + \hat{I}_2 + \hat{I}_L = 0$$

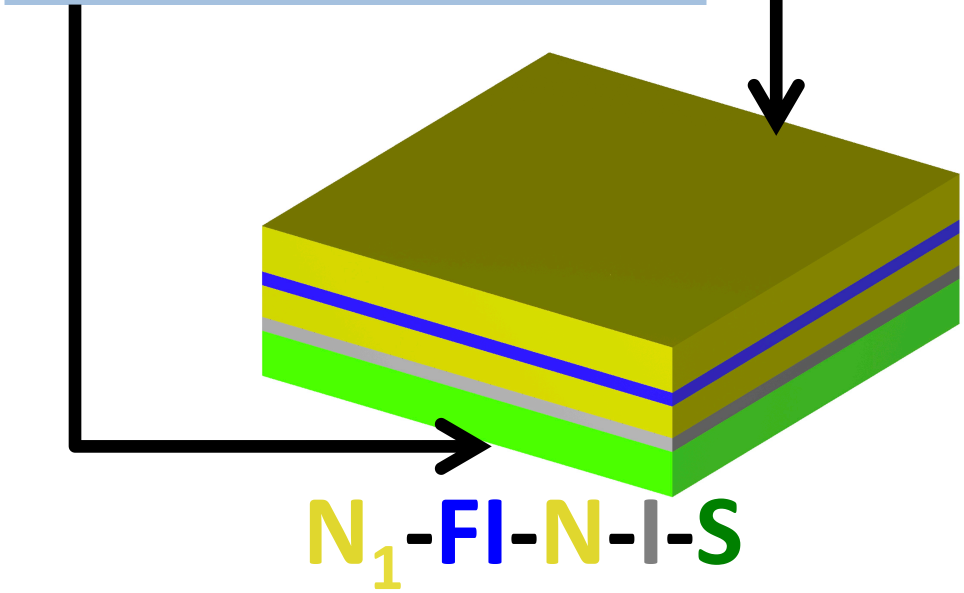
Huertas-Hernando, Belzig, Nazarov, PRL (2001)
Cottet, Huertas-Hernando, Belzig, Nazarov, PRB (2009)
Machon, Eschrig, Belzig, PRL (2013)

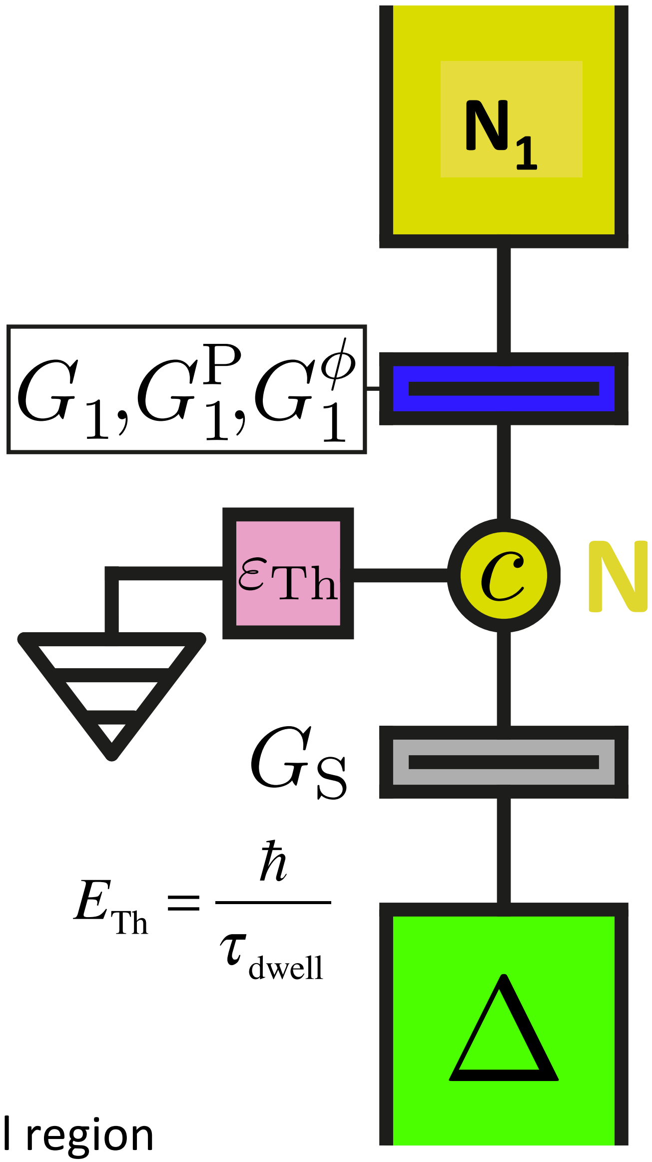
Some effects of the spin-dependent boundary conditions on the density of states

The density of states in a proximity metal with magnetic contacts

"stacked structure"

$$G(V) = \frac{dI}{dV} \sim N(eV)$$





$$E_{\mathrm{Th}} = \frac{\hbar}{\tau_{\mathrm{dwell}}}$$

Measure of coupling strength: **Thouless energy E_{th}**

- weak coupling = small E_{Th} = weak proximity
- strong coupling = large E_{Th} = strong proximity

Related to the inverse mean **dwell time** in the central region

[Machon, Eschrig, Belzig, Phys. Rev. Lett. **110**, 047002 (2013)]

How does the calculation look like in practice?

Spin-dependent the density of states in N

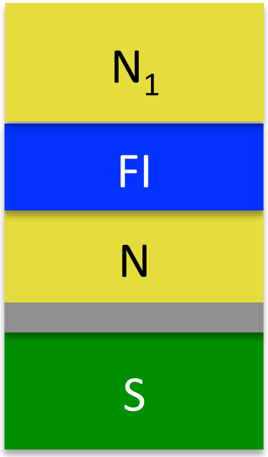
Total DOS: $N(\epsilon) = N_{\uparrow}(\epsilon) + N_{\downarrow}(\epsilon)$

[calculation from: Machon, Eschrig, Belzig, Phys. Rev. Lett. **110**, 047002 (2013)]

Spin polarization of the DOS $\dfrac{N_{\uparrow}(\epsilon) - N_{\downarrow}(\epsilon)}{N(\epsilon)}$

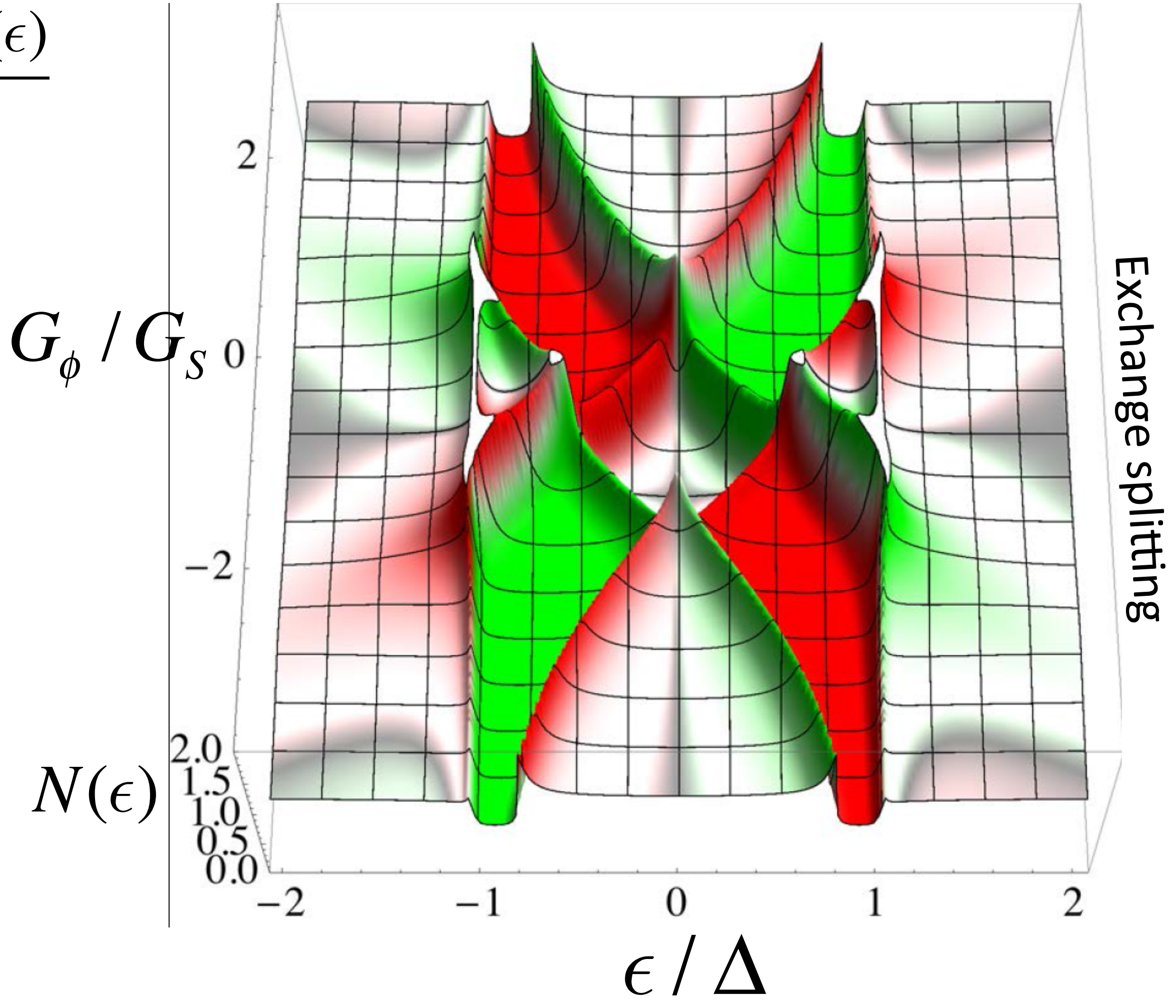
Red: spin up

Green: spin down



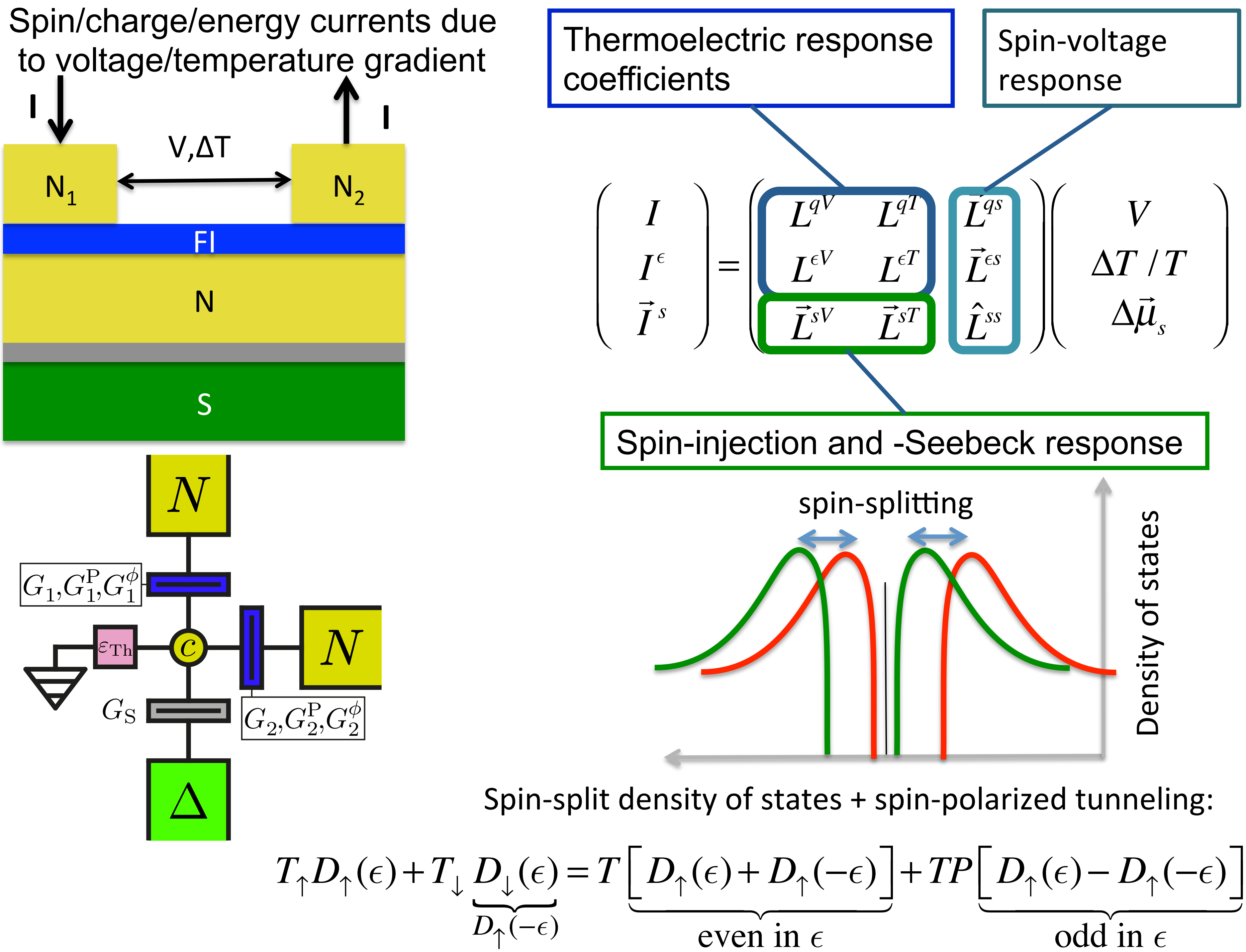
$$G_1 = \frac{G_S}{10}$$

$$E_{Th} = \Delta$$



100% spin-polarized energy bands*!

Spincaloritronics with an SF-heterostructure: Spinthermoelectric “transistor” structure



Spin/charge/energy currents due to voltage/temperature gradient
I
I
V,ΔT
N_1
N_2
FI
N
S
N
G_1, G_1^P, G_1^ϕ
ε_{Th}
c
N
G_S
G_2, G_2^P, G_2^ϕ
Δ
Thermoelectric response coefficients
Spin-voltage response
$\begin{pmatrix} I \\ I^\epsilon \\ \vec{I}^s \end{pmatrix} = \begin{pmatrix} L^{qV} & L^{qT} & \vec{L}^{qs} \\ L^{\epsilon V} & L^{\epsilon T} & \vec{L}^{\epsilon s} \\ \vec{L}^{sV} & \vec{L}^{sT} & \hat{L}^{ss} \end{pmatrix} \begin{pmatrix} V \\ \Delta T / T \\ \Delta \vec{\mu}_s \end{pmatrix}$
Spin-injection and -Seebeck response
spin-splitting
Density of states
Spin-split density of states + spin-polarized tunneling:
$T_\uparrow D_\uparrow(\epsilon) + T_\downarrow \underbrace{D_\downarrow(\epsilon)}_{D_\uparrow(-\epsilon)} = T \underbrace{\left[D_\uparrow(\epsilon) + D_\uparrow(-\epsilon) \right]}_{\text{even in } \epsilon} + TP \underbrace{\left[D_\uparrow(\epsilon) - D_\uparrow(-\epsilon) \right]}_{\text{odd in } \epsilon}$

Transistor thermopower

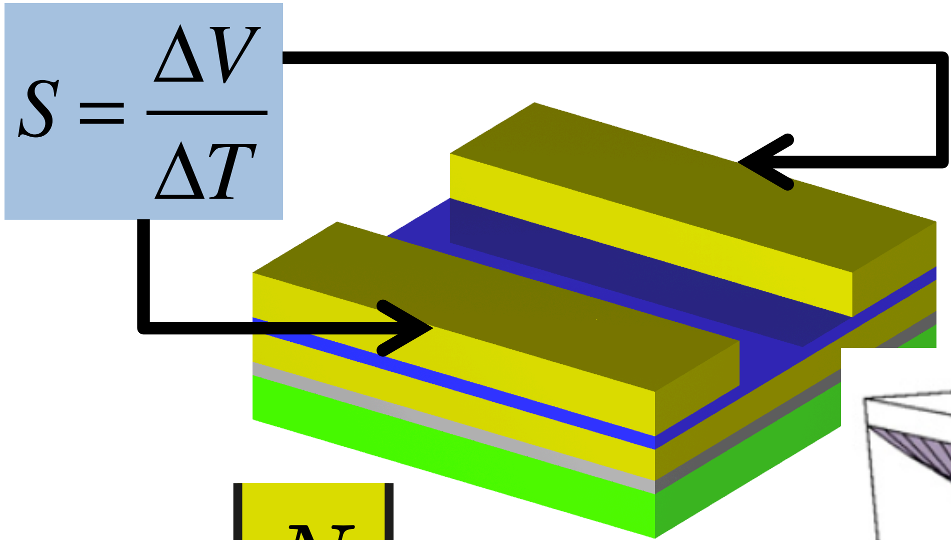
$$S = -\frac{1}{T}\frac{L^{qT}}{L^{qV}}$$

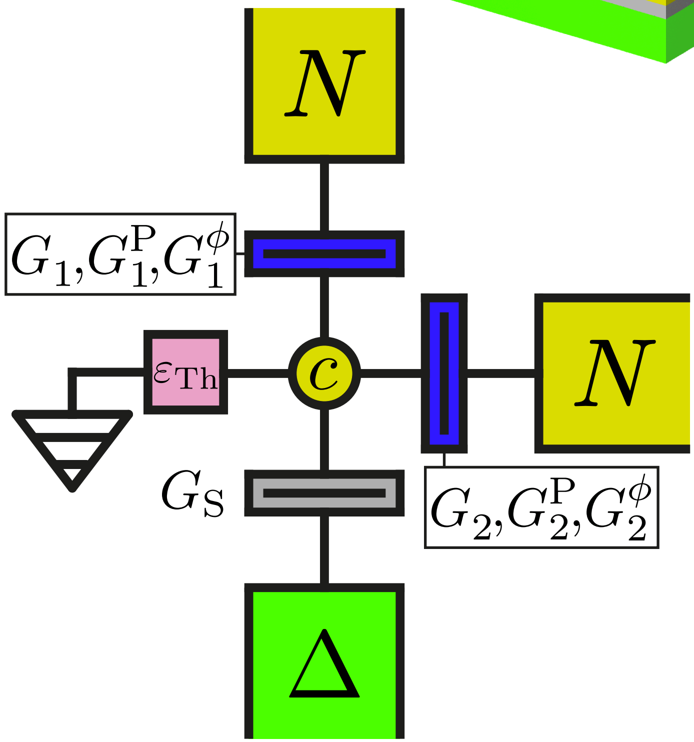
$$G_1 = G_2 = \frac{G_S}{100}$$

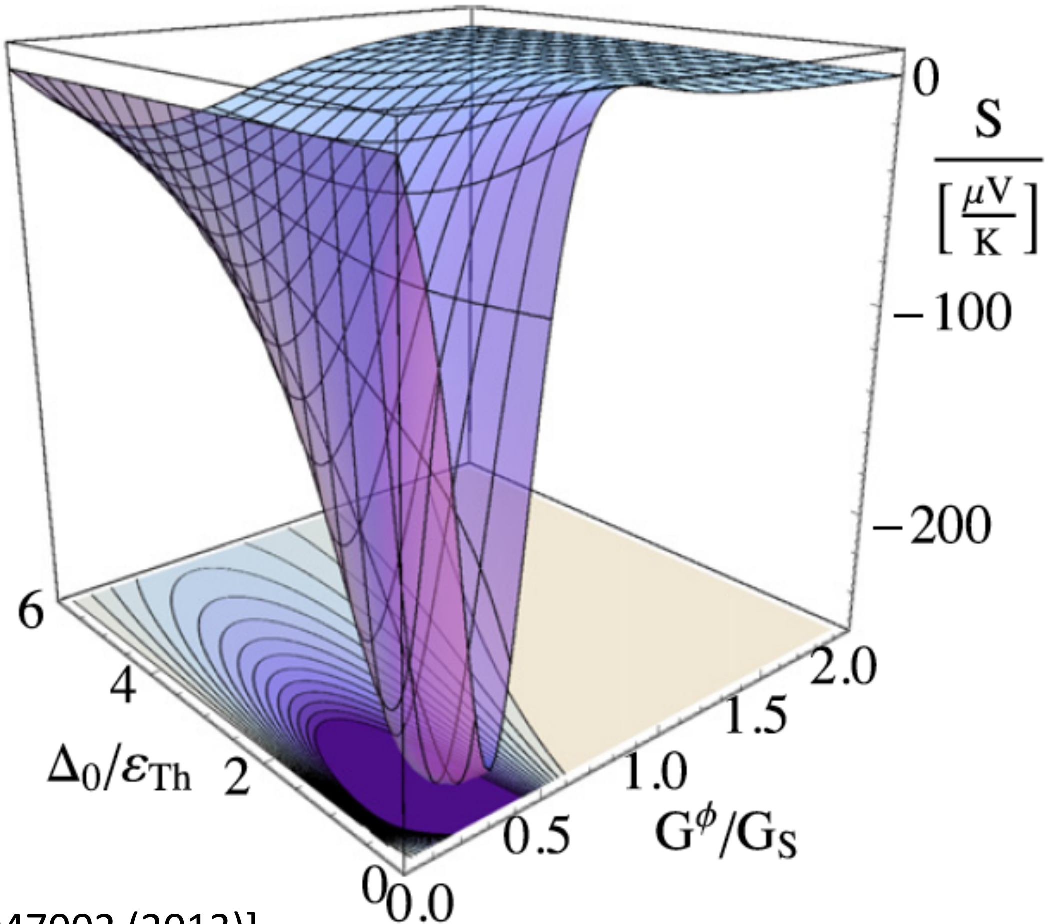
$$T = 0.1T_c$$

$$P_1 = P_2 = 90\%$$

Giant Seebeck coefficient!







[Machon, Eschrig, Belzig, Phys. Rev. Lett. **110**, 047002 (2013)]

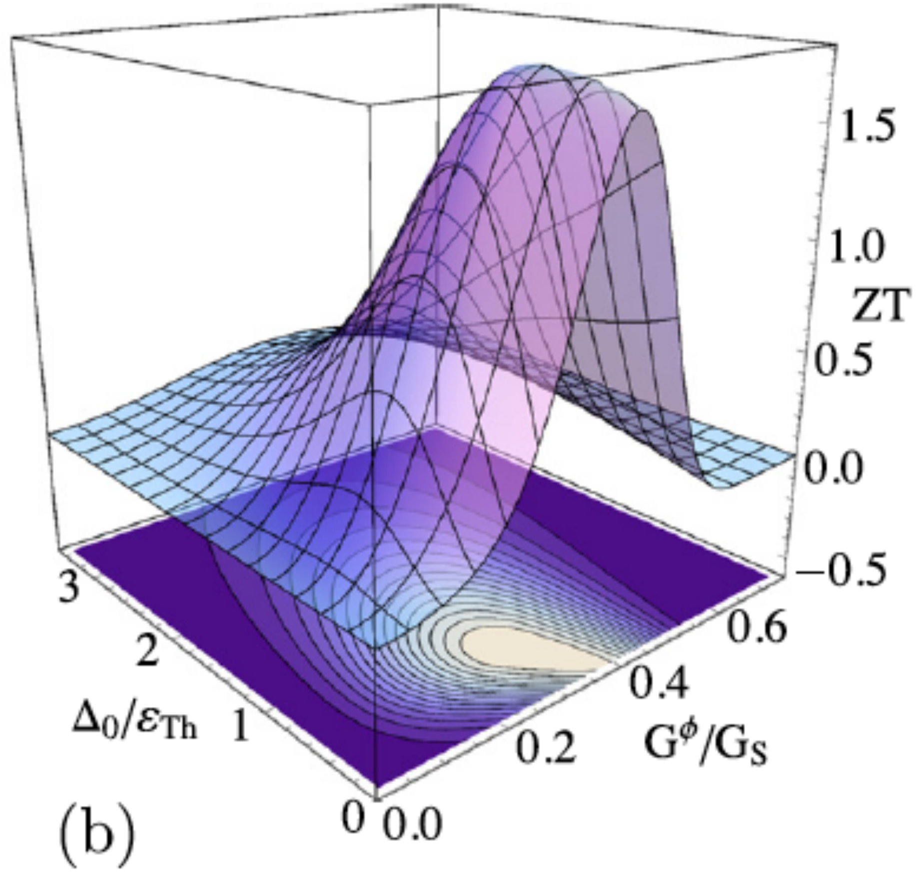
Thermoelectric figure of merit

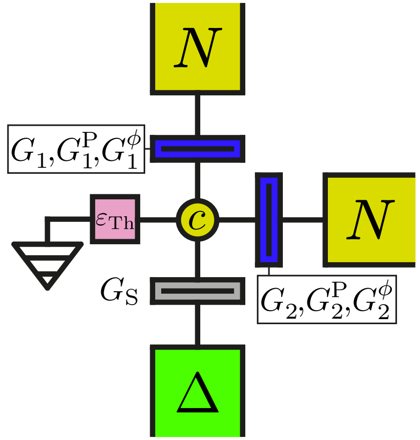
$$ZT = \frac{GS^2T}{\kappa} = \frac{(L^{qT})^2}{L^{qV}L^{\varepsilon T} - (L^{qT})^2}$$

$$G_1 = G_2 = G_S / 10$$

$$T = 0.1 T_c$$

$$P_1 = P_2 = 90\%$$

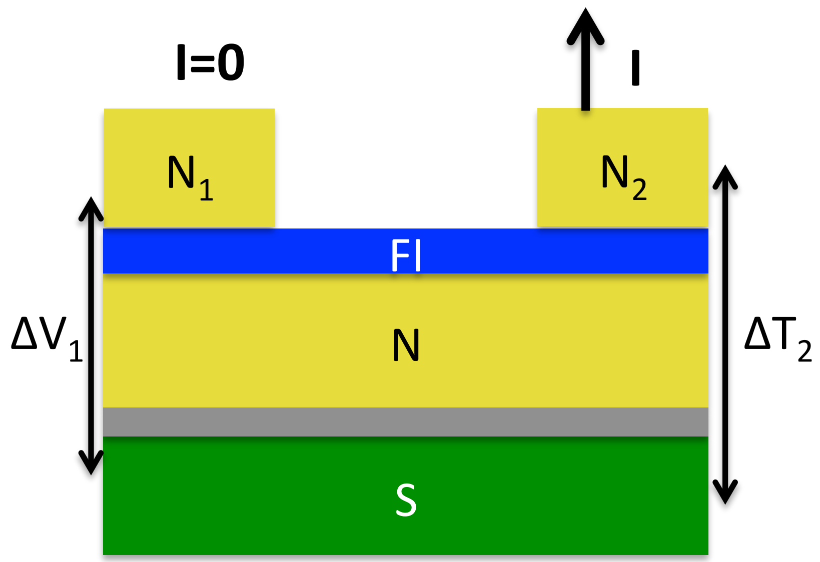


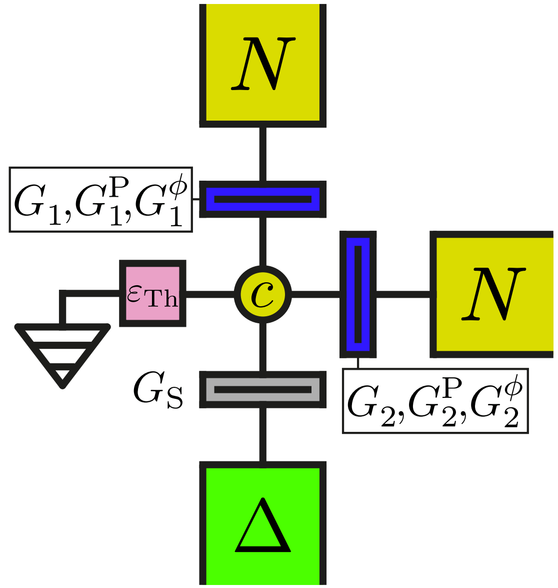


Huge (>1) figure of merit!

[Machon, Eschrig, Belzig, New J. Phys. **16**, 073002 (2014)]

Non-local caloritronics with an SF-heterostructure:



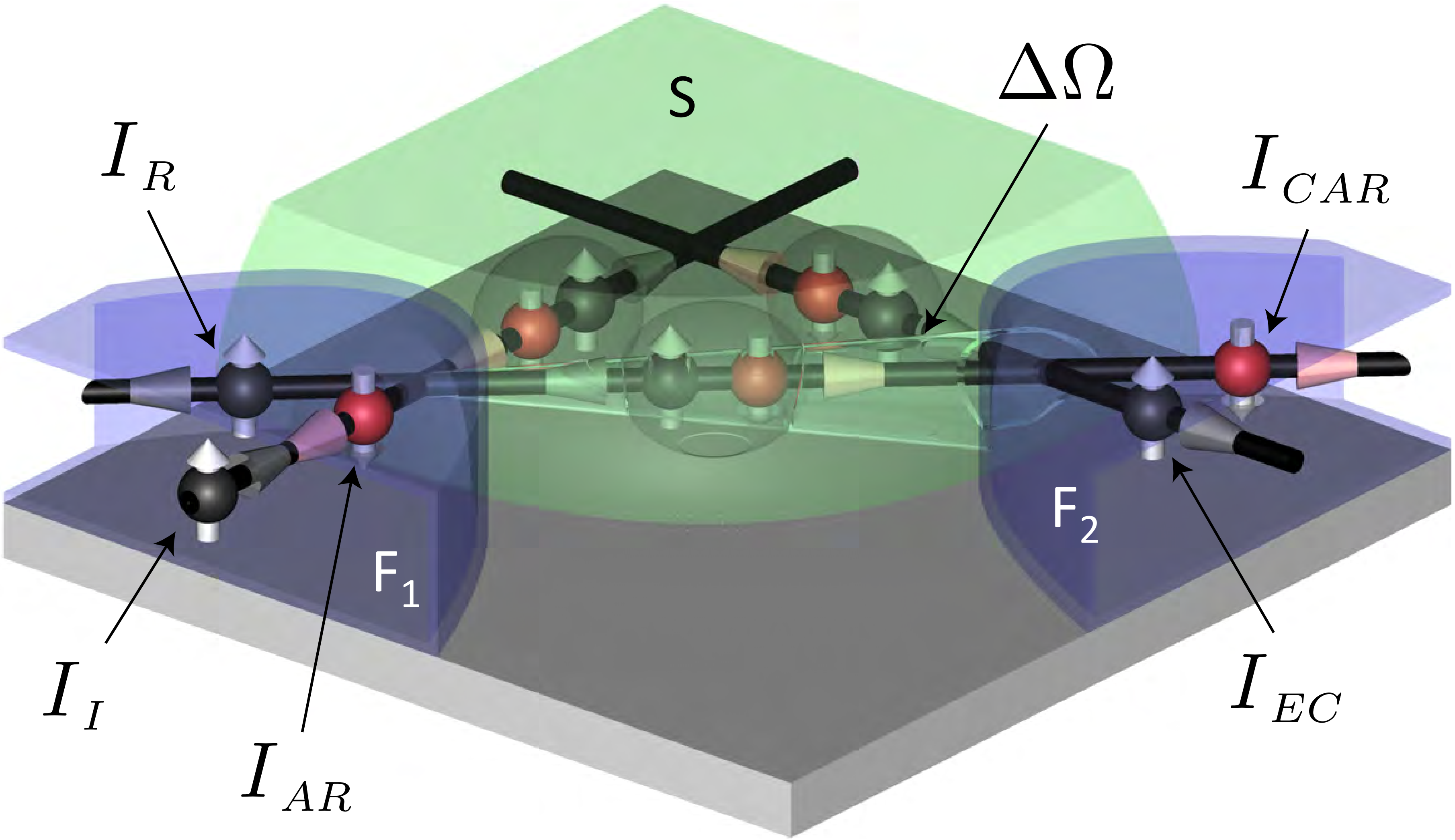


$$\begin{pmatrix} I_1^q \\ I_1^\epsilon \\ I_2^q \\ I_2^\epsilon \end{pmatrix} = \begin{pmatrix} L_{11}^{qV} & L_{11}^{qT} & L_{12}^{qV} & L_{12}^{qT} \\ L_{11}^{\epsilon V} & L_{11}^{\epsilon T} & L_{12}^{\epsilon V} & L_{12}^{\epsilon T} \\ L_{21}^{qV} & L_{21}^{qT} & L_{22}^{qV} & L_{22}^{qT} \\ L_{21}^{\epsilon V} & L_{21}^{\epsilon T} & L_{22}^{\epsilon V} & L_{22}^{\epsilon T} \end{pmatrix} \begin{pmatrix} \Delta V_1 \\ -\Delta T_1/T_S \\ \Delta V_2 \\ -\Delta T_2/T_S \end{pmatrix}$$

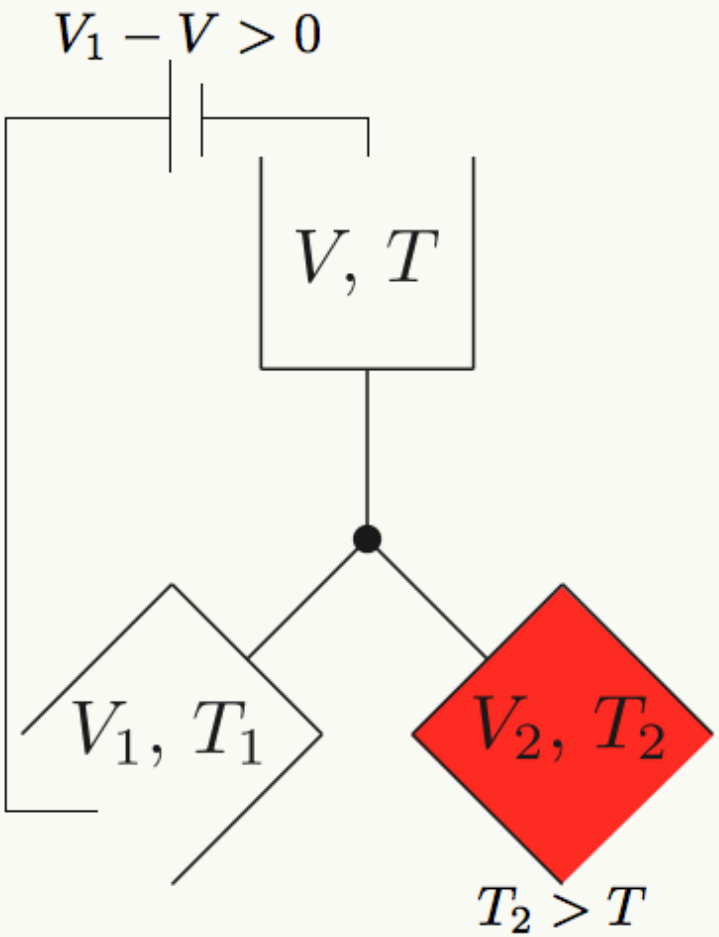
Local thermoelectric response

Nonlocal thermoelectric response

Ballistic limit: identifying the different processes



3-TERMINAL SYSTEM



local | nonlocal

$$\begin{pmatrix} I_1^q \\ I_1^\epsilon \\ I_2^q \\ I_2^\epsilon \end{pmatrix} = \begin{pmatrix} L_{11}^{qV} & L_{11}^{qT} & L_{12}^{qV} & L_{12}^{qT} \\ L_{11}^{\epsilon V} & L_{11}^{\epsilon T} & L_{12}^{\epsilon V} & L_{12}^{\epsilon T} \\ L_{21}^{qV} & L_{21}^{qT} & L_{22}^{qV} & L_{22}^{qT} \\ L_{21}^{\epsilon V} & L_{21}^{\epsilon T} & L_{22}^{\epsilon V} & L_{22}^{\epsilon T} \end{pmatrix} \begin{pmatrix} \Delta V_1 \\ -\Delta T_1/T_S \\ \Delta V_2 \\ -\Delta T_2/T_S \end{pmatrix}$$

$$\mathcal{S} = \Delta V_1/\Delta T_2$$

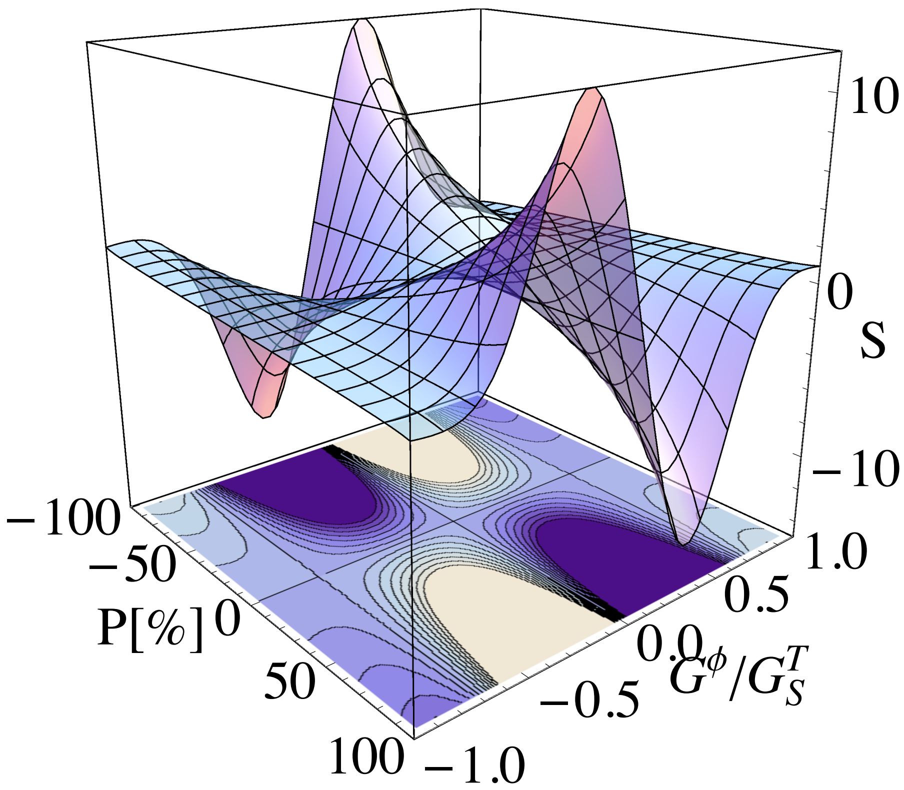
	I_1	I_2	ΔT_1	ΔV_2	$\mathcal{S}T_S$
I	free	0	0	0	L_{12}^{qT}/L_{11}^{qV}
II	0	free	0	0	L_{22}^{qT}/L_{21}^{qV}
III	$I_1 = I_2$		0	0	$\frac{L_{12}^{q,T}-L_{22}^{q,T}}{L_{11}^{q,V}-L_{21}^{q,V}}$
IV	0	0	free	0	$\frac{L_{12}^{qT}L_{21}^{qT}-L_{22}^{qT}L_{11}^{qT}}{L_{11}^{qV}L_{21}^{qT}-L_{21}^{qV}L_{11}^{qT}}$
V	0	0	0	free	$\frac{L_{12}^{qT}L_{22}^{qV}-L_{22}^{qT}L_{12}^{qV}}{L_{11}^{qV}L_{22}^{qV}-L_{21}^{qV}L_{12}^{qV}}$

Nonlocal thermopower

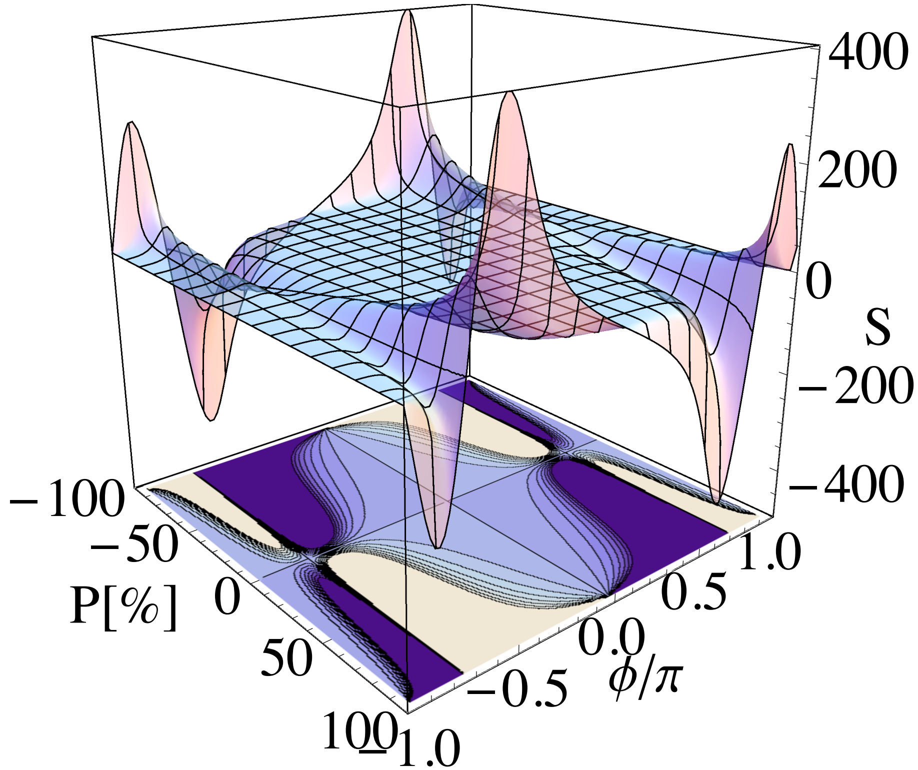
$$S = \frac{\Delta V_2}{\Delta T_1}$$

S in units of 8[μV/K]

Dirty case



Clean case

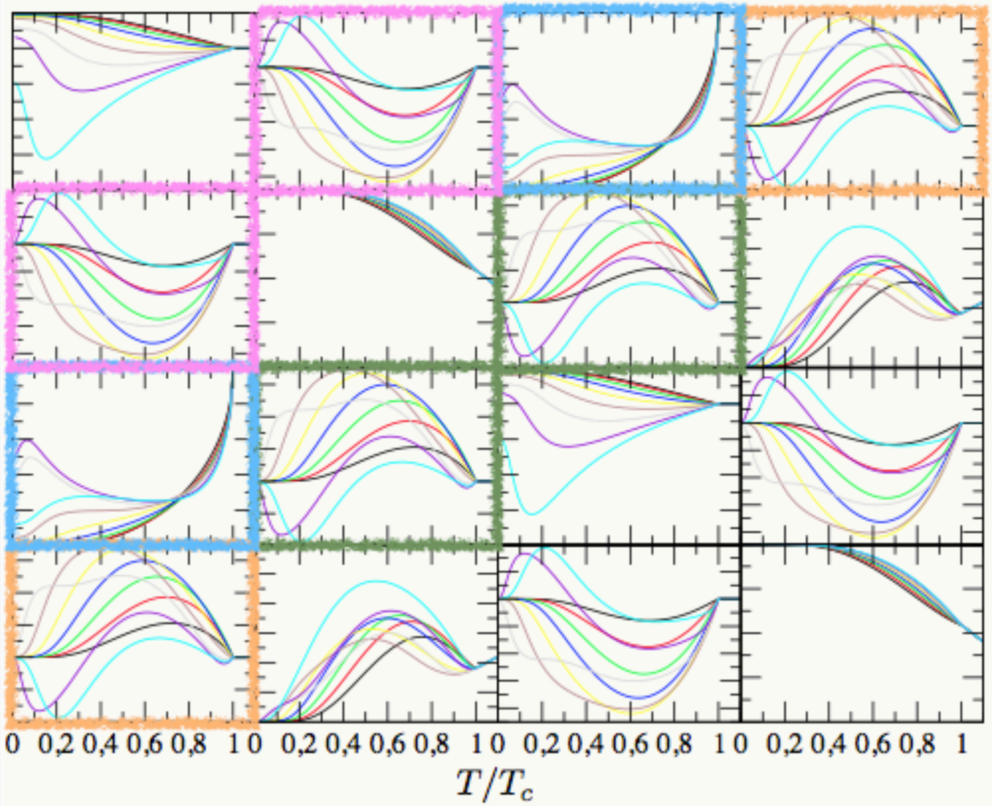


- Similar behaviour, larger magnitude in clean case
- Maximal nonlocal thermopower small exchange splitting (comparable to E_{th})
- Maximal for large polarization
- Sign change, more pronounced in clean limit

ONSAGER SYMMETRY

Temperatur dependence:

(colors=different spin-mixing)



Generalized Onsager-Kelvin relation:

$$\hat{L} = \begin{pmatrix} L_{11}^{qV} & L_{11}^{qT} & L_{12}^{qV} & L_{12}^{qT} \\ L_{11}^{\epsilon V} & L_{11}^{\epsilon T} & L_{12}^{\epsilon V} & L_{12}^{\epsilon T} \\ L_{21}^{qV} & L_{21}^{qT} & L_{22}^{qV} & L_{22}^{qT} \\ L_{21}^{\epsilon V} & L_{21}^{\epsilon T} & L_{22}^{\epsilon V} & L_{22}^{\epsilon T} \end{pmatrix}$$

$$\hat{L} = \hat{L}^T$$

Independent of:

- Transmission probabilities
- Spin-dependent parameters
- Relative magnetisation
- Clean or dirty limit

$G_1 = G_2 = G_S^T/10$ $\quad E_{Th} = \Delta$ $\quad P_1 = P_2 = 10\%$ $\quad G_1^{\phi} = G_2^{\phi} = 0.1 \cdots 0.6 \cdots 0.9\, G_S^T$

Conclusion/Summary

Quantum circuit theory of spin transport

- Circuit theory: nodes, connectors, leads
- Quantum Kirchhoff rules (Matrix current conservation)
- Spin-dependent boundary conditions/connector

Spin effects on proximity effect

- Spin-mixing in superconductors
- Superconducting proximity spin-split density of states due to a ferromagnetic insulator

Thermoelectricity:

- Spin-splitting + spin-polarized tunneling →large Seebeck effect
- Maximizing thermoelectric efficiency in a N-SFI-N transistor heterostructures
- Non-local Seebeck effect

General reference for QCT: Yu.V. Nazarov, arxiv:1999
Further references:
Machon, Eschrig, Belzig, Phys. Rev. Lett. **110**, 047002 (2013)
Machon, Eschrig, Belzig, New J. Phys. **16**, 073002 (2014)
Machon, Belzig , arxiv 02/2015
Eschrig, Cottet, Belzig, Linder arxiv 04/2015

The End