Thermal vector potential theory of transport induced by temperature gradient

多々良源　Gen TATARA

RIKEN Center for Emergent Matter Science (CEMS)

2015/6/1

Thermally-driven transport

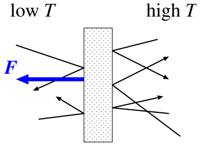
- Landauer-Büttiker formulation

$$j = -\kappa \sum_{\boldsymbol{k}} \nabla f(\epsilon_{\boldsymbol{k}}) = \kappa \int d\epsilon \nu(\epsilon) \left(-\frac{df}{d\epsilon} \right) \left(-\nabla \mu - \beta \epsilon \frac{\nabla T}{T} \right)$$
$$= -\sigma_E \nabla \mu - \sigma_T \nabla T$$

$$\boxed{\begin{aligned} \sigma_R &= \kappa \nu(\epsilon_F) \\ \sigma_T &= \kappa \frac{\pi^2}{3} k_B \nu'(\epsilon_F) \end{aligned}}$$

Wiedemann-Franz law

- Linear response thoery (Kubo formula) ?
  - Interaction Hamiltonian necessary
  - Quantum mechanical Hamiltonian for statistical force ??



Thermal vector potential theory of thermally-driven transport

Thermal effects: essential in insulator spintronics

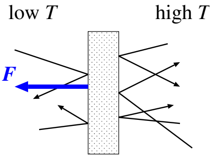
- Insulator magnets
  - Low damping (1/100 of metals)

    Fast magnetization switching, efficient signal transmission
  - Heat : Main driving mechanism of spin transport
- Waste heat
  - Spin Seebeck effect *Saitoh,2010-*

    Waste heat ⇒ Spin current ⇒ Electric current

Quantum theory including temperature gradient ?

- Temperature gradient $\nabla T \Rightarrow$ Force



$\nabla T \simeq$ Electric field

- No solution (?) *Nonequilibrium quantum statistical physics (Sasa-Tasaki)*

Luttinger's method

Luttinger, Phys. Rev. (1964)

- Assume that there is a scalar field (gravitational field) Ψ coupling to energy density $\mathcal{E}$

$$\boxed{H_{\mathrm{L}} = \int d\boldsymbol{r}\mathcal{E}\Psi}$$

- Write linear response expression for current, energy current

$$j = L_1 \left(E - T\nabla \frac{\mu}{T} \right) + L_2 T\nabla \frac{1}{T} - L_3 \nabla\Psi$$

$$j_{\mathcal{E}} = L_4 \left(E - T\nabla \frac{\mu}{T} \right) + L_5 T\nabla \frac{1}{T} - L_6 \nabla\Psi$$

$$L_1 = \langle j\, j \rangle \,, L_2 = \langle j\, j_{\mathcal{E}} \rangle \,, \ldots$$

- Determine Ψ from condition of equilibrium (no current)

$$\boxed{\nabla\Psi = -\frac{\nabla T}{T}} \,, \quad L_2 = L_3$$

Einstein's relation

- Transport coefficients are calculated by linear response theory (Kubo formula) with respect to Ψ

Luttinger's method

Luttinger, Phys. Rev. (1964)

$$H_{\mathrm{L}} = \int d\boldsymbol{r}\,\mathcal{E}\Psi$$

- Force $= -\nabla\Psi$
- Inhomogeneous Boltzmann weight   Local equilibrium

$$e^{-\frac{H}{k_B T}} \rightarrow e^{-\int d^3 r \frac{\mathcal{E}}{k_B T(\boldsymbol{r})}} = e^{-\int d^3 r \frac{\mathcal{E}}{k_B T}\left(1 - \frac{\boldsymbol{r}\cdot\nabla T}{T} + \cdots\right)} = e^{-\frac{1}{k_B T}\int d^3 r (\mathcal{E} + \Psi\mathcal{E})}$$

$$\nabla\Psi = -\frac{\nabla T}{T}$$

Luttinger's method *Luttinger, Phys. Rev. (1964)*

- Transport coefficients calculated from correlation functions

$$\sigma_T \leftarrow \langle j j_{\mathcal{E}} \rangle$$

  - Electrons *Smrcka'77, Cooper'97,Michaeli'09, Eich'14*
  - Magnons *Matsumoto'11*
  - Thermal torque *Kohno'14*

- Dangerous formalism

Naive application $\Rightarrow$ Divergence at $T \to 0$

  - Thermal Hall effect *Qin,Niu'11*
  - Thermal spin-transfer torque *Kohno'14*

- Equilibrium contribution needs to be subtracted carefully
  - Effects of diamagnetic current are not clear in Luttinger's formalism

Luttinger's method

Luttinger, Phys. Rev. (1964)

- Transport coefficients calculated from correlation functions

$$\sigma_T \leftarrow \langle j j_{\mathcal{E}} \rangle$$

  - Electrons *Smrcka'77, Cooper'97,Michaeli'09, Eich'14*
  - Magnons *Matsumoto'11*
  - Thermal torque *Kohno'14*

- Dangerous formalism

Naive application $\Rightarrow$ Divergence at $T \to 0$

  - Thermal Hall effect *Qin,Niu'11*
  - Thermal spin-transfer torque *Kohno'14*

- Equilibrium contribution needs to be subtracted carefully
  - Effects of diamagnetic current are not clear in Luttinger's formalism

Vector potential formulation would solve the problem

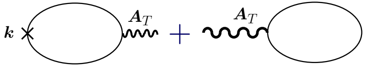
Diamagnetic current in electromagnetism

- Electric current

$$\boldsymbol{j} = \frac{e}{m}(\boldsymbol{k} - e\boldsymbol{A})c^\dagger c$$

  - $\boldsymbol{A}$: vector potential $\boldsymbol{E} = -\dot{\boldsymbol{A}}$
  - $\boldsymbol{j}^A = -\frac{e^2}{m}\boldsymbol{A}n$: Diamagnetic current Equilibrium contribution
- Linear response calculation

$\boldsymbol{j} =$



$$= \sigma_E \boldsymbol{E}$$

  - Diamagnetic current is essential to obtain physical current
- Scalar potential formulation

$$\boldsymbol{E} = -\nabla\phi$$

  - Role of diamagnetic current is not clear

Luttinger's method

Luttinger, Phys. Rev. (1964)

- Analogy with electromagnetism

	Electromagnetism	Thermal effects
Charge	Electric charge e	Energy density $\mathcal{E}$
Current	Electric current $\boldsymbol{j}$	Energy current density $\boldsymbol{j}_{\mathcal{E}}$
Potentials	ϕ, $\boldsymbol{A}$	Ψ
Field	$\boldsymbol{E} = -\nabla\phi - \dot{\boldsymbol{A}}$	$\boldsymbol{E}_T = -\nabla\Psi$

Luttinger's method

Luttinger, Phys. Rev. (1964)

- Analogy with electromagnetism

	Electromagnetism	Thermal effects
Charge	Electric charge e	Energy density $\mathcal{E}$
Current	Electric current $\boldsymbol{j}$	Energy current density $\boldsymbol{j}_{\mathcal{E}}$
Potentials	ϕ, $\boldsymbol{A}$	Ψ , $\boldsymbol{A}_T$
Field	$\boldsymbol{E} = -\nabla\phi - \dot{\boldsymbol{A}}$	$\boldsymbol{E}_T = -\nabla\Psi - \dot{\boldsymbol{A}}_T$

Luttinger's method

Luttinger, Phys. Rev. (1964)

- Analogy with electromagnetism

	Electromagnetism	Thermal effects
Charge	Electric charge e	Energy density $\mathcal{E}$
Current	Electric current $\boldsymbol{j}$	Energy current density $\boldsymbol{j}_{\mathcal{E}}$
Potentials	ϕ, $\boldsymbol{A}$	Ψ , $\boldsymbol{A}_T$
Field	$\boldsymbol{E} = -\nabla\phi - \dot{\boldsymbol{A}}$	$\boldsymbol{E}_T = -\nabla\Psi - \dot{\boldsymbol{A}}_T$

- No gauge symmetry for energy
  - Charge conservation = U(1) gauge invariance
    $\Rightarrow$ Scalar & vector potentials
  - Energy conservation = invariance under global time translation
    $t \rightarrow t + \alpha$

Luttinger's method

Luttinger, Phys. Rev. (1964)

- Analogy with electromagnetism

	Electromagnetism	Thermal effects
Charge	Electric charge e	Energy density $\mathcal{E}$
Current	Electric current $\boldsymbol{j}$	Energy current density $\boldsymbol{j}_{\mathcal{E}}$
Potentials	ϕ, $\boldsymbol{A}$	Ψ , $\boldsymbol{A}_T$
Field	$\boldsymbol{E} = -\nabla\phi - \dot{\boldsymbol{A}}$	$\boldsymbol{E}_T = -\nabla\Psi - \dot{\boldsymbol{A}}_T$

- No gauge symmetry for energy
  - Charge conservation = U(1) gauge invariance
    $\Rightarrow$ Scalar & vector potentials
  - Energy conservation = invariance under global time translation
    $t \to t + \alpha$
- Shitade *Prog. Theor. Exper. Phys. (2014)*
  - Assume invariance under local time translation $t \to t + \alpha(\boldsymbol{r}, t)$
    $\Rightarrow$ Gauge invariant theory of gravitational potential *Utiyama'56*

    $g_{\mu\nu} = \eta_{ab} h^a_\mu h^b_\nu$ h^a_μ: vielbein

    $h^0_0 = \Psi$: Scalar potential coupled to $\mathcal{E}$, $h^0_i = \boldsymbol{A}_T$: Vector potential coupled to $\boldsymbol{j}_{\mathcal{E}}$
  - Origin of local translational invariance ?

Thermal vector potential formalism

GT, Phys. Rev. Lett., 114, 196601 (2015)

- Luttinger's gravitational potential

$$H_{\mathrm{L}} = \int d\boldsymbol{r}\mathcal{E}\Psi$$

- Energy conservation law

$$\boxed{\dot{\mathcal{E}} + \nabla \cdot \boldsymbol{j}_{\mathcal{E}} = 0}$$

$$H_{\mathrm{L}} = \int d^3r \int_{-\infty}^{t} dt' \boldsymbol{j}_{\mathcal{E}}(t') \cdot \nabla\Psi(\boldsymbol{r}, t)$$

- Equivallent to

$$\boxed{H_{A_T} \equiv -\int d^3r \boldsymbol{j}_{\mathcal{E}}(\boldsymbol{r}, t) \cdot \boldsymbol{A}_T(t)}$$

$\int_{-\infty}^{\infty} dt H_{A_T} = \int_{-\infty}^{\infty} dt H_{\mathrm{L}}(t)$

$$\boxed{\partial_t \boldsymbol{A}_T(\boldsymbol{r}, t) = \nabla\Psi(\boldsymbol{r}, t) = \frac{\nabla T}{T}.}$$

Thermal vector potential formalism

GT, Phys. Rev. Lett., 114, 196601 (2015)

Another derivation

- Entropy change due to energy current $\boldsymbol{j}_{\mathcal{E}}$

$$\dot{\mathcal{S}} = -\int d^3 r \frac{1}{T} \nabla \cdot \boldsymbol{j}_{\mathcal{E}} = -\int d^3 r \boldsymbol{j}_{\mathcal{E}} \cdot \frac{\nabla T}{T^2}$$

Landau&Lifshitz

- Free energy change ($F = E - T\mathcal{S}$)

$$H_T \equiv \frac{1}{T} \int d^3 r \int_0^t \boldsymbol{j}_{\mathcal{E}}(t') dt' \cdot \nabla T$$

- H_T is equivalent to DC limit

$$H_{A_T} \equiv -\int d^3 r \boldsymbol{j}_{\mathcal{E}}(\boldsymbol{r}, t) \cdot \boldsymbol{A}_T(t)$$

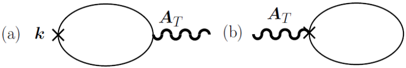
$$\boldsymbol{A}_T(t) \equiv \int_{-\infty}^{t} dt' \nabla \Psi(t'), \qquad \partial_t \boldsymbol{A}_T(\boldsymbol{r}, t) = \frac{\nabla T}{T}$$

Thermal vector potential formalism

GT, Phys. Rev. Lett., 114, 196601 (2015)

- Example 1 : Longituginal thermal conductivity

$$\boldsymbol{j} = \frac{e}{m}\boldsymbol{k} - \frac{e}{m}\epsilon \boldsymbol{A}_T =$$



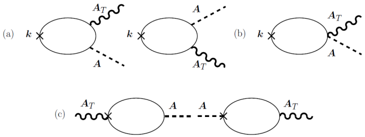
$$= \sigma_T \boldsymbol{E}_T, \qquad \boldsymbol{E}_T = \frac{\nabla T}{T}$$

$$\sigma_T \equiv \frac{e\hbar^5}{6m^2}\frac{1}{V}\sum_{\boldsymbol{k}} k^2 \epsilon_{\boldsymbol{k}} \int \frac{d\omega}{2\pi} f'(\omega) \frac{1}{\tau^2} |g^{\rm r}_{\boldsymbol{k}\omega}|^4$$

No divergence

- Example 2 : Thermal Hall conductivity

$$\boldsymbol{j} =$$



$$= \Theta_H(\boldsymbol{E}_T \times \boldsymbol{B})$$

$$\Theta_H = -i\frac{e^2\hbar^4}{6m^3 V}\sum_{\boldsymbol{k}}\int\frac{d\omega}{2\pi}k^2\epsilon_{\boldsymbol{k}}\left[f'(\omega)[(g^{\rm r}_{\boldsymbol{k},\omega})^2 g^{\rm a}_{\boldsymbol{k},\omega} - g^{\rm r}_{\boldsymbol{k},\omega}(g^{\rm a}_{\boldsymbol{k},\omega})^2] + \hbar f(\omega)[(g^{\rm a}_{\boldsymbol{k},\omega})^4 - (g^{\rm r}_{\boldsymbol{k},\omega})^4]\right]$$

No divergence

Thermal vector potential formalism

GT, Phys. Rev. Lett., 114, 196601 (2015) Why thermal vector potential works so well?

- Currents

$$j_i = \frac{e\hbar}{m}\frac{1}{V}\sum_{\boldsymbol{k}}\left[k_i - eA_i - \gamma^{ij}_k A_{T,j}\right] c^\dagger_{\boldsymbol{k}} c_{\boldsymbol{k}} \qquad \text{Electric current}$$

$$j_{\mathcal{E},i} = \frac{\hbar}{m}\frac{1}{V}\sum_{\boldsymbol{k}}\left[k_i\epsilon_{\boldsymbol{k}} - e\gamma^{ij}_k A_j - \gamma^{ij}_{T,k} A_{T,j}\right] c^\dagger_{\boldsymbol{k}} c_{\boldsymbol{k}} \qquad \text{Energy current}$$

$$\gamma^{ij}_{T,k} \equiv \epsilon_{\boldsymbol{k}}\left(\epsilon_{\boldsymbol{k}}\delta_{ij} + \frac{2\hbar^2}{m}k_i k_j\right)$$

Derived from energy conservation law $\dot{\mathcal{E}} = -\nabla\cdot\boldsymbol{j}_{\mathcal{E}}$

- $\Rightarrow$ Minimal coupling

$$H = \frac{\hbar^2}{2m}\sum_{\boldsymbol{k}}\left(\boldsymbol{k} - e\boldsymbol{A} - \epsilon_{k-eA}\boldsymbol{A}_T\right)^2 c^\dagger{}_{\boldsymbol{k}} c_{\boldsymbol{k}}.$$

Thermal vector potential is a ‘gauge field’

Thermal vector potential formalism

GT, Phys. Rev. Lett., 114, 196601 (2015) Why thermal vector potential works so well?

- Currents

$$j_i = \frac{e\hbar}{m}\frac{1}{V}\sum_{\boldsymbol{k}}\left[k_i - eA_i - \gamma_k^{ij}A_{T,j}\right]c_{\boldsymbol{k}}^\dagger c_{\boldsymbol{k}} \qquad \text{Electric current}$$

$$j_{\mathcal{E},i} = \frac{\hbar}{m}\frac{1}{V}\sum_{\boldsymbol{k}}\left[k_i\epsilon_{\boldsymbol{k}} - e\gamma_k^{ij}A_j - \gamma_{T,k}^{ij}A_{T,j}\right]c_{\boldsymbol{k}}^\dagger c_{\boldsymbol{k}} \qquad \text{Energy current}$$

$$\gamma_{T,k}^{ij} \equiv \epsilon_{\boldsymbol{k}}\left(\epsilon_{\boldsymbol{k}}\delta_{ij} + \frac{2\hbar^2}{m}k_ik_j\right)$$

Derived from energy conservation law $\quad \dot{\mathcal{E}} = -\nabla\cdot\boldsymbol{j}_{\mathcal{E}}$

- $\Rightarrow$ Minimal coupling

$$H = \frac{\hbar^2}{2m}\sum_{\boldsymbol{k}}\left(\boldsymbol{k} - e\boldsymbol{A} - \epsilon_{k-eA}\boldsymbol{A}_T\right)^2 c^\dagger{}_{\boldsymbol{k}}c_{\boldsymbol{k}}.$$

Thermal vector potential is a ‘gauge field’

- Gauge invariance in thermal effects? $\quad \nabla T = \text{gravity ?}$

Magnetization dynamics due to thermally-driven magnon

GT, Cond-mat:1505.01908

- Ferromagnet

$$\mathcal{E} = \frac{J}{2}(\nabla \boldsymbol{S})^2 - \frac{K}{2}(S_z)^2 + \frac{K_\perp}{2}(S_y)^2$$

- Energy current

$$\boldsymbol{j}_{\mathcal{E},i} = -\frac{J}{a^3}\nabla_i \boldsymbol{S} \cdot \dot{\boldsymbol{S}}$$

- Separation of variables

$$\boldsymbol{S} = U(\boldsymbol{r}, t)\widetilde{\boldsymbol{S}} \equiv U(\boldsymbol{r}, t)(S\hat{\boldsymbol{z}} + \delta \boldsymbol{s}),$$

$$U = \begin{pmatrix} \cos\theta\cos\phi & -\sin\phi & \sin\theta\cos\phi \\ \cos\theta\sin\phi & \cos\phi & \sin\theta\sin\phi \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}, \qquad \delta\boldsymbol{s} = \begin{pmatrix} \gamma(b^\dagger + b) \\ i\gamma(b^\dagger - b) \\ -b^\dagger b \end{pmatrix}$$

- Spin gauge field

$$\partial_\mu \boldsymbol{S} = U(\partial_\mu + iA_U)\widetilde{\boldsymbol{S}}, \qquad A_{U,\mu} \equiv -iU^{-1}\nabla_\mu U$$

Magnetization dynamics due to thermally-driven magnon

GT, Cond-mat:1505.01908

- Magnon spin transfer effect

$$H_{\rm st} = 4JS \int d^3 r \boldsymbol{A}_{\rm s} \cdot \boldsymbol{j}_{\rm m}$$

$$\boldsymbol{j}_{\rm m} \equiv -\frac{i}{2a^3}(b^\dagger \overset{\leftrightarrow}{\nabla} b) - \frac{i}{2a^3}\boldsymbol{A}_T(b^\dagger \overset{\leftrightarrow}{\partial}_t b)$$

'Diamagnetic'magnon current

- Thermal force on magnon

$$H_{A_T} \equiv -\int d^3 r \boldsymbol{j}_{\mathcal{E}} \cdot \boldsymbol{A}_T$$

$$j^{\rm (m)}_{\mathcal{E},i} \equiv -\frac{JS}{a^3}\left[\dot{b}^\dagger(\nabla_i b) + (\nabla_i b^\dagger)\dot{b}\right]$$

Magnetization dynamics due to thermally-driven magnon

GT, Cond-mat:1505.01908

- Thermally-induced magnon current

$$j_{\mathrm{m},i} = -\kappa \nabla_i T$$

$$\kappa = -\frac{4JS}{3\hbar}\alpha^2 \frac{1}{T}\frac{1}{V}\sum_{\boldsymbol{q}} q^2 \int \frac{d\omega}{2\pi} n'(\omega) \frac{\omega^3}{[(\omega-\omega_{\boldsymbol{q}})^2+(\alpha\omega)^2]^2}$$

$$= \frac{1}{2\pi^2 (JS)^{3/2}} \frac{1}{\alpha} k_B (k_B T)^{1/2} F(\beta\Delta_{\mathrm{sw}})$$

$$F(\beta\Delta_{\mathrm{sw}}) \equiv \int_{\beta\Delta_{\mathrm{sw}}}^{\infty} dx \frac{(x-\beta\Delta_{\mathrm{sw}})^{1/2}}{e^x - 1}$$

Magnetization dynamics due to thermally-driven magnon

GT, Cond-mat:1505.01908

- Equation of motion of a domain wall with magnon spin transfer

$$\dot{\phi} + \alpha \frac{\dot{X}}{\lambda} = \beta_T \frac{u_T}{\lambda}$$
$$\dot{X} - \alpha\lambda\dot{\phi} = v_c \sin 2\phi - P_T u_T$$

$$v_c \equiv \frac{K_\perp \lambda S}{2\hbar},$$

β_T : phenomenological force

$$u_T \equiv -\frac{k_B a^2}{\hbar} \nabla_z T$$

$$P_T \equiv \frac{F}{\pi^2 \sqrt{S}} \frac{1}{\alpha} \sqrt{k_B T \frac{a^2}{J S^2}}$$

Magnon spin transfer efficienty

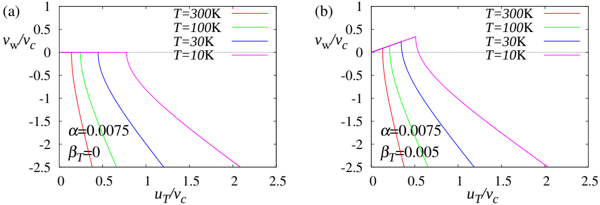
- $P_T = 6.7$ at $T = 300$K Large compared to current driven case ($P < 1$)
- Strongly temperature dependent

Magnetization dynamics due to thermally-driven magnon

GT, Cond-mat:1505.01908

- Domain wall speed

$$v_{\rm w} = \frac{\beta_T}{\alpha} u_T - \frac{v_{\rm c}}{1+\alpha^2} \sqrt{\left(P_T + \frac{\beta_T}{\alpha}\right)^2 \left(\frac{u_T}{v_{\rm c}}\right)^2 - 1}$$



- YIG

$$\alpha = 0.0075,\ S = 14,\ \frac{JS^2}{a^2} = 5.8 \times 10^{-21}\ \text{J},\ P_T = 0.40 \times \sqrt{T(\mathrm{K})}$$

  - $v_{\rm w} = -0.89 \times 10^{-2}$ m/s for $\nabla T = 20$ K/mm
  - Experiment -1.8×10^{-4} m/s *Jiang'13*

Pinning?

Summary

- Vector potential representation of thermal effect
  - Extention of Luttinger's formulation
  - Entropy change due to energy current
- Electron transport (Lingitudinal, Hall)
- Magnon-induced torque on domain wall

References

- *GT, Phys. Rev. Lett. 114, 196601 (2015).*
- *GT, Cond-mat:1505.01908.*

Grants *Kakenhi*

Japan Society for the Promotion of Science and The Ministry of Education, Culture, Sports, Science and Technology (MEXT), Japan