Magnetization pumping and dynamics in a Dzyaloshinskii-Moriya magnet

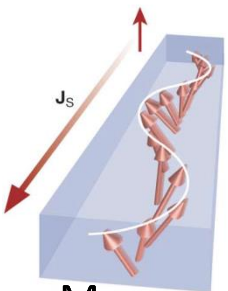
- Alexey A. Kovalev

Phys. Rev. B 89, 241101(R) (2014)
EPL (Europhys. Letters) 109, 67008 (2015)

Outline

- Describe magnon spin torques within LLG equation with stochastic magnetic fields
- Identify dissipative and non-dissipative torque contributions
- Discuss chiral derivative and generalizations to arbitrary form of Dzyaloshinskii-Moriya interactions
- Describe linear response theory of magnonic torques
- Discuss manipulation of skyrmions by temperature gradients, magnon pumping, and magnetization switching

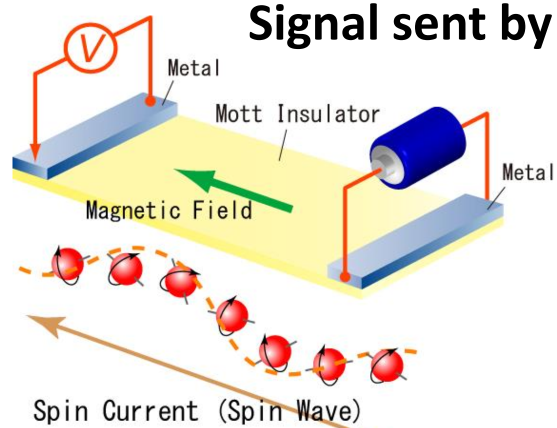
Magnon currents



Magnon

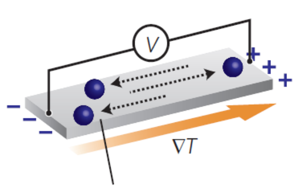
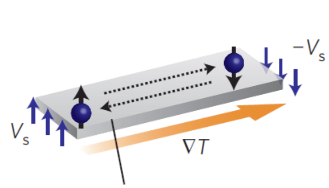
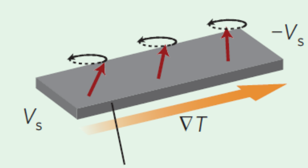
$= \uparrow +$ No charge

Signal sent by magnons



Y. Kajiwara, K. Harii, S. Takahashi, J. Ohe, K. Uchida, M. Mizuguchi, H. Umezawa, H. Kawai, K. Ando, K. Takanashi, S. Maekawa & E. Saitoh Nature **464**, 262 (2010)

Spin-Seebeck effect

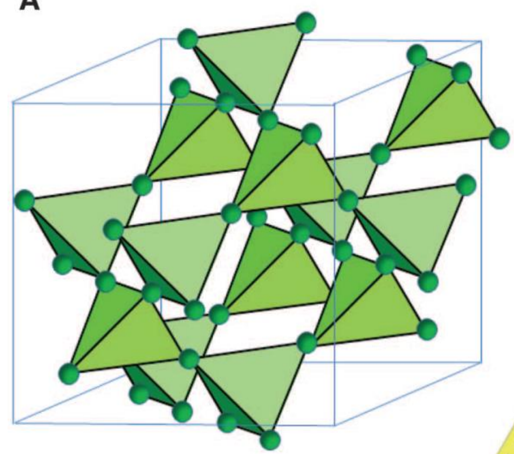
Material \ Output	Electricity	Magnetism
Conductor	**a** Seebeck effect  Metal or semiconductor	**b** Spin Seebeck effect  Ferromagnetic metal
Insulator	×	**c** Spin Seebeck effect  Magnetic insulator

K.Uchida,J.Xiao,H.Adachi, J.Ohe, S.Takahashi,J.Ieda, T. Ota,Y.Kajiwara,H.Umezawa,H.Kawai, G.E.W.Bauer, S.Maekawa& E.Saitoh,Nature Mat. **9**, 894 (2010)

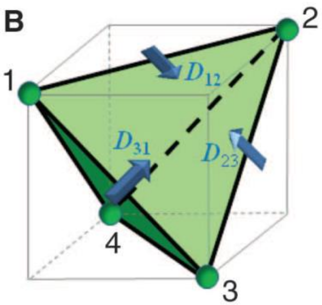
C. M. Jaworski, J. Yang,S. Mack, D. D. Awschalom, J. P. Heremans & R. C. Myers, Nature Mat. **9**, 898 (2010)

Magnon Hall effect

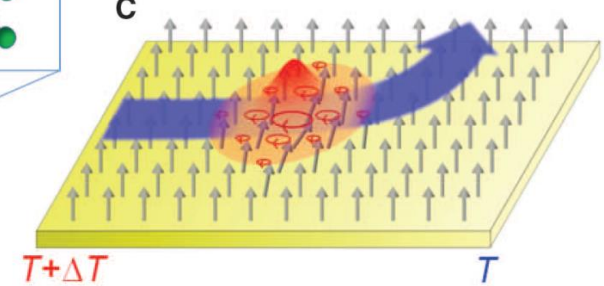
A



B



C



Y. Onose, T. Ideue, H. Katsura, Y. Shiomi, N.Nagaosa& Y. Tokura, Science **329**, 297 (2010)

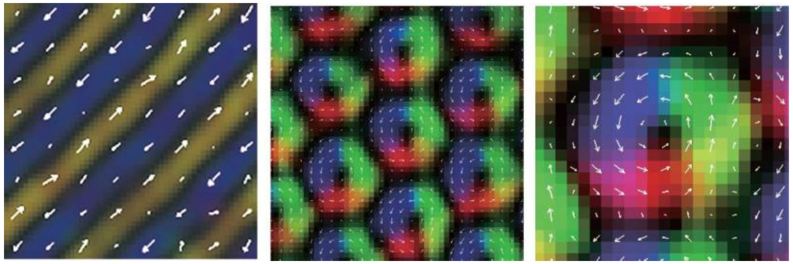
Spin-orbit interaction – source of interesting physics

Relativistic effects: $$H_{so} = \frac{\hbar}{2m^2c^2}\boldsymbol{\sigma}\cdot\left(\boldsymbol{\nabla}V(r)\times\mathbf{p}\right)$$

For conductors: leads to spin-orbit torques, anomalous Hall effect, spin Hall effects, quantum spin Hall effect etc..

For magnetic insulators:

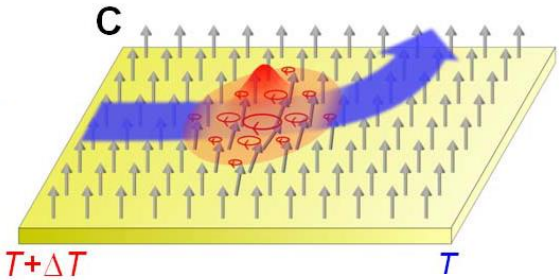
1. Formation of magnetic textures:



Tokura group, Nature 465, 901–904 (2010)

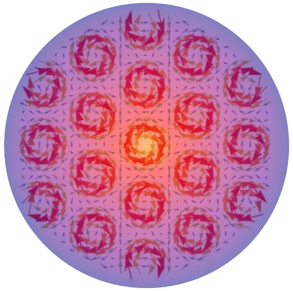
2. Magnon Hall effect:

Tokura group,
Science **329**, 297 (2010)



3. Proposals of edge effects:

R. Shindou, R. Matsumoto,
S. Murakami, and J-I Ohe,
Phys. Rev. B **87**, 174427 (2013)



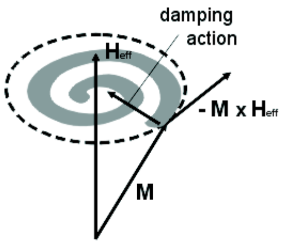
4. Analog of spin-orbit torque for homogeneous magnet: A. Manchon, P. B. Ndiaye, J-H Moon, H-W Lee, K-J Lee, arXiv:1401.0883

5. Manipulation of skyrmions:
M. Mochizuki et al.,
Nature Materials 13, 241–246 (2014)

Magnetic textures

Ferromagnet is well below Curie temperature: $\mathbf{M}(r,t) = M_s n(r,t) \quad M_s = const$

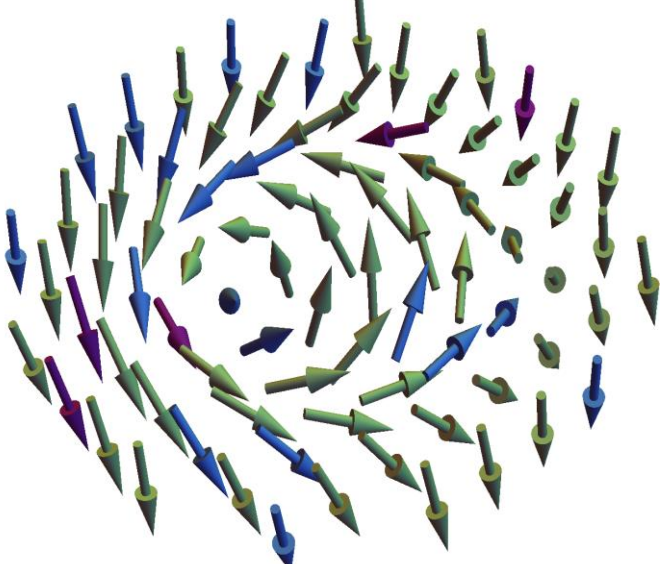
Landau-Lifshitz-Gilbert equation:



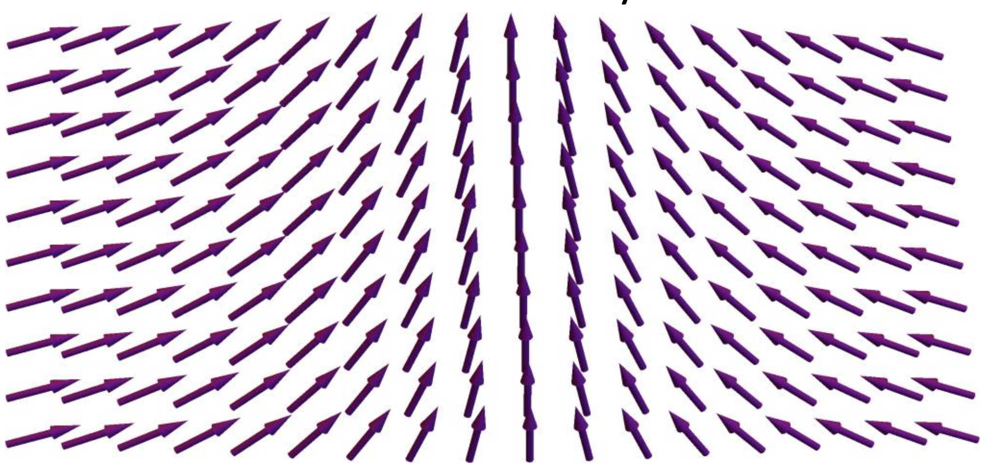
$$\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} + \alpha M \times \frac{\mathbf{M}}{dt}$$

Free energy defines the effective field: $\mathbf{H}_{\text{eff}} = -\frac{\delta F}{d\mathbf{M}}$

$$F = \mathbf{H} \cdot \mathbf{M} - \frac{A}{2}(\nabla \mathbf{M})^2 + F_{an}(\alpha, \beta, \gamma)...$$



Skyrmion



Domain wall

Magnetic textures can play the role of electro-magnetic field for magnons.

Local coordinate transformation

1. By introducing a coordinate-dependent rotation at each point we can write the free energy in the frame associated with local magnetization direction.

$$F = (A/2)(\mathcal{D}_\alpha \tilde{\mathbf{m}})^2 - \mathbf{m} \cdot \mathbf{H}$$

$$\tilde{\mathbf{m}} = \hat{R}(\mathbf{r})\mathbf{m} \qquad \mathcal{D}_\alpha = \partial_\alpha - (\partial_\alpha \hat{R})\hat{R}^{-1} = \partial_\alpha - \boldsymbol{\mathcal{A}}_\alpha \times$$

2. Ignoring quadratic terms, we obtained a free energy with spatially dependent Dzyaloshinskii-Moriya interaction.

3. Compare to the most general form of Dzyaloshinskii-Moriya interaction:

$$F_{\text{DMI}} = D_{\alpha\beta}\varepsilon_{\gamma\beta\nu} m_\nu \partial_\alpha m_\gamma$$
$$D_{\alpha\beta} = \mathbf{D}_\alpha \cdot \mathbf{e}_\beta$$

$$\Rightarrow \boxed{\begin{aligned} F &= (A/2)(\mathcal{D}_\alpha \mathbf{m})^2 \\ \mathcal{D}_\alpha &= \partial_\alpha + (\mathbf{D}_\alpha / A)\times \end{aligned}}$$

4. We will treat both effect on equal footing when appropriate.

$$\mathbf{D}'_\alpha / A = \mathbf{D}_\alpha / A - \boldsymbol{\mathcal{A}}_\alpha$$

Chiral derivative

1. By introducing a coordinate-dependent rotation at each point we can write the free energy with Dzyaloshinskii-Moriya interaction via rotated magnetizations, up to some added anisotropies.

$$F = (A/2)(\partial_\alpha \cdot \tilde{\mathbf{m}})^2 - \mathbf{m} \cdot \mathbf{H}$$

$$\mathbf{m} = \hat{R}(\mathbf{r})\tilde{\mathbf{m}}$$

2. To return to un-rotated magnetization we use the chiral derivative.

$$\mathcal{D}_\alpha = \partial_\alpha + \hat{R}\partial_\alpha \hat{R}^{-1}$$

Kim K.-W., Lee H.-W., Lee K.-J. and Stiles M. D., Phys. Rev. Lett., 111 (2013) 216601.

3. Chiral derivative for the most general form of Dzyaloshinskii-Moriya interaction:

$$F_{\mathrm{DMI}} = D_{\alpha\beta}\varepsilon_{\gamma\beta\nu} m_\nu \partial_\alpha m_\gamma$$
$$D_{\alpha\beta} = \mathbf{D}_\alpha \cdot \mathbf{e}_\beta$$

$$\Rightarrow \quad \boxed{\mathcal{D}_\alpha = \partial_\alpha + (\mathbf{D}_\alpha / A)\times}$$

4. We separate Dzyaloshinskii-Moriya tensor into symmetric and antisymmetric parts:

$$D_{\alpha\beta} = D_{\alpha\beta}^{sym} + \varepsilon_{\alpha\beta\gamma} D_\gamma^{ant}$$

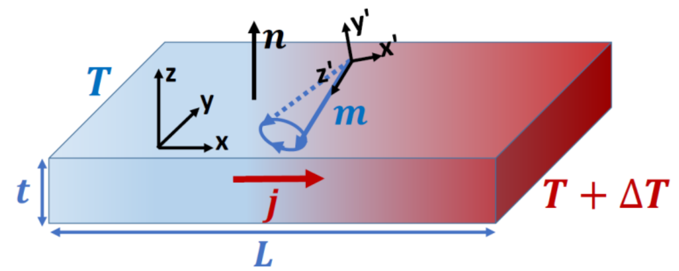
5. High symmetry cases:

$\mathbf{D}^{ant} = D\mathbf{n}$ --- Structural asymmetry

$D_{\alpha\beta}^{sym} = D\delta_{\alpha\beta}$ --- Noncentrosymmetric systems

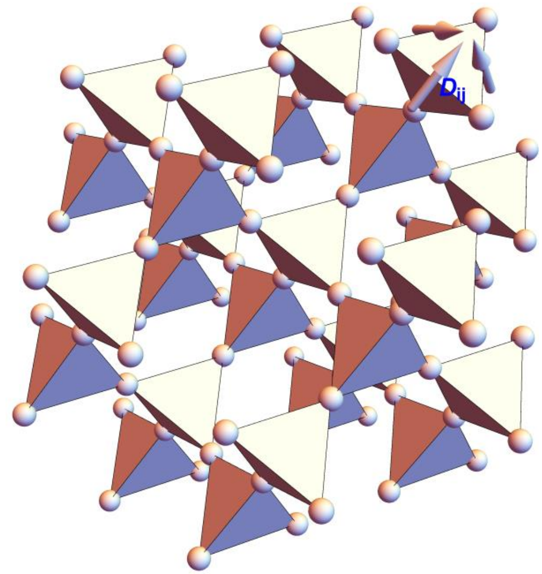
Dzyaloshinskii-Moriya Magnets

1. Consider thin-film Pt/Co(0.6 nm)/AlOx and similar structures where we naturally obtain structural asymmetry.
This case is considered in this talk with Dzyaloshinskii-Moriya interaction of the form:



$$F = (A/2)(\partial_\alpha \cdot \mathbf{m})^2 + D\mathbf{m} \cdot ([\mathbf{n} \times \boldsymbol{\partial}] \times \mathbf{m}) - \mathbf{m} \cdot \mathbf{H}$$

2. A general Dzyaloshinskii-Moriya magnets is described by a microscopic Hamiltonian, e.g. Lu2V2O7 with pyrochlore lattice.

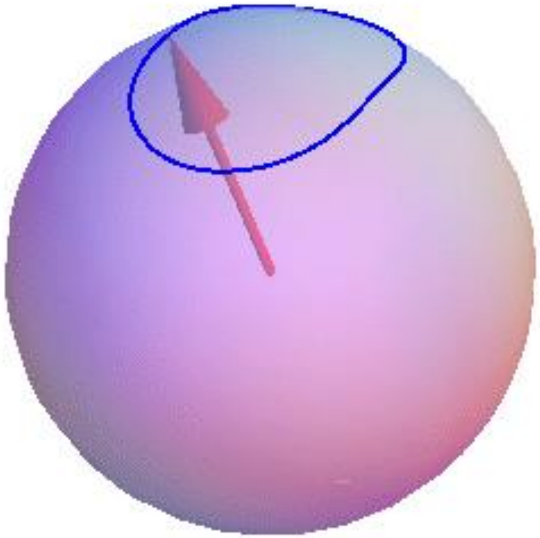


$$H_{\mathrm{DMI}} = \sum \mathbf{D}_{ij} \cdot (\mathbf{S}_i \times \mathbf{S}_j)$$

3. In the latter case additional effects related to accumulation of Berry phase have to be considered.
A. Mook, J. Henk, I. Mertig, Phys. Rev. B 89, 134409 (2014)

Geometric phase of spin

$$\Phi = A * S$$



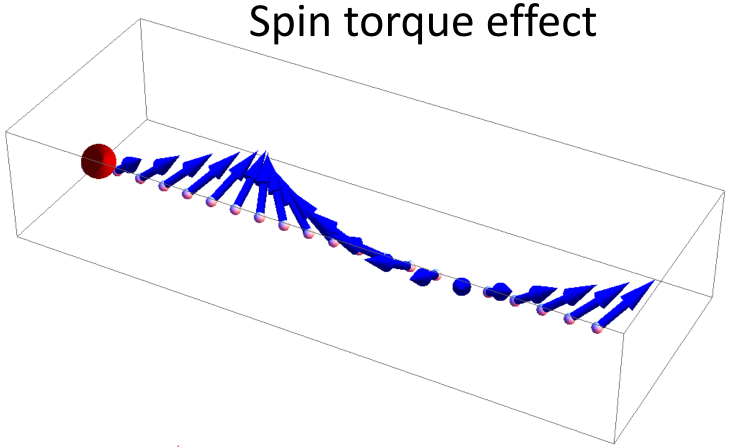
- Spin lives on a surface of a sphere.
- Spin returns to initial position and accumulates a Geometric phase – a product of encircled area and spin.

$|\boldsymbol{S}\rangle = e^{-i\hat{S}_z\phi}e^{-i\hat{S}_y\theta}e^{-i\hat{S}_z\chi}\,|\uparrow\rangle$ -- spin direction is defined by three Euler angles.

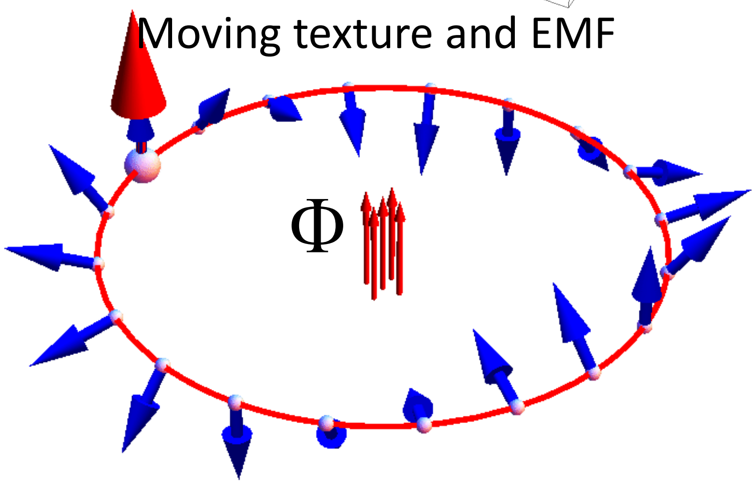
Geometric phase of spin -> $\boxed{\Phi = i\int_{R(0)}^{R(t)} \langle \boldsymbol{S}(R)|\,\nabla_R\,|\boldsymbol{S}(R)\rangle\, dR}$

Magnetic texture induced geometric phases

Spin torque effect

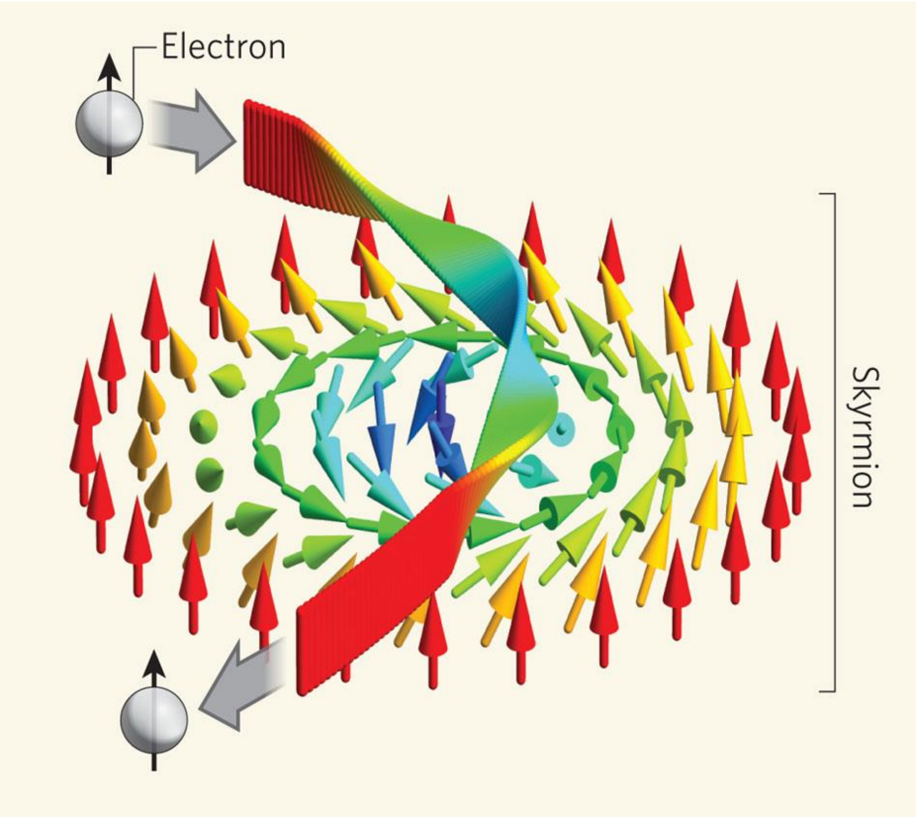


Moving texture and EMF



1. If magnetic field changes then by Faraday's law there is electro-motive force: $\mathcal{E} = -\partial_t \Phi$
2. Electron also accumulates additional phase Φ' due to magnetic texture. If the magnetic texture changes in time, by analogy to Faraday's law we have additional electromotive force: $\mathcal{E}' = -\partial_t \Phi'$

Fictitious magnetic field



Magnetic texture induced fictitious magnetic field deflects the trajectory.

C. Pfleiderer, A. Rosch, Nature 465, 880 (2010)
X. Z. Yu, Y. Onose, N. Kanazawa, J. H. Park, J. H. Han, Y. Matsui, N. Nagaosa & Y. Tokura Nature 465, 901 (2010)

Magnonic torques from the LLG equation

$$s(1+\alpha\mathbf{m}\times)\dot{\mathbf{m}}+s\mathbf{m}\times\mathbf{H}_{\text{eff}}=0$$

1. Separate magnetization dynamics into slow and fast components:

$$\mathbf{m}=(1-\mathbf{m}_f^2)^{1/2}\mathbf{m}_s+\mathbf{m}_f$$

2. Fast dynamics is found in the linear approximation.

3. When linear solutions are plugged back into the LLG equation they give spin torque from the second order terms:

$$\begin{aligned}\boldsymbol{\mathcal{T}}&=\mathbf{m}_s\times\mathbf{H}_{\text{eff}}^s-\langle\mathbf{m}\times\mathbf{H}_{\text{eff}}\rangle=A\mathbf{m}_s\times\boldsymbol{\mathcal{S}}\\&\approx A\langle\mathbf{m}_f\times\boldsymbol{\nabla}^2\mathbf{m}_f\rangle+2A\langle\mathbf{m}\times\partial_\alpha\mathbf{m}_s\partial_\alpha(\mathbf{m}_s\cdot\mathbf{m})\rangle\end{aligned}$$

4. We introduced transversal spin accumulation $\boldsymbol{\mathcal{S}}$.

5. Just like for electrons we recover two orthogonal directions for spin torque – corresponding to dissipative and non-dissipative contributions.

Transverse spin accumulation

1. We perform transformation aligning the axis z with local magnetization and use complex notation for all vectors, $a = a_x + ia_y$.

$$\boldsymbol{\mathcal{S}} = 2\langle \mathbf{m}_f(\partial_\alpha \mathbf{m}_s \cdot \partial_\alpha \mathbf{m}_f)\rangle - 2\langle \partial_\alpha \mathbf{m}_s(\mathbf{m}_f \cdot \partial_\alpha \mathbf{m}_f)\rangle$$

$$\Downarrow$$

$$S = \langle m_f(\partial_\alpha m_s \partial_\alpha m_f^* + \partial_\alpha m_s^* \partial_\alpha m_f)\rangle$$
$$- \langle \partial_\alpha m_s(m_f \partial_\alpha m_f^* + m_f^* \partial_\alpha m_f)\rangle = -\partial_\alpha m_s \langle m_f^* \partial_\alpha m_f\rangle$$

2. Spin accumulation can be found by calculating the average of the real and imaginary parts of the following operator:

$$\boxed{\hat{S} = -\frac{\mathbf{D}'_\alpha \times \mathbf{m}_s}{A}\partial_\alpha = -(\mathcal{D}_\alpha \mathbf{m}_s)\partial_\alpha}$$

Thermal magnons

We perform transformation using 3x3 matrix R, aligning the axis z with local magnetization:

$$\mathbf{m} \to \mathbf{m}' = \hat{R}\mathbf{m},\ \partial_\mu \to (\partial_\mu - \hat{A}_\mu),\ \hat{A}_\mu = \partial_\mu \hat{R}\hat{R}^{-1} \qquad \hat{A}\mathbf{m} = \mathbf{A}_\mu \times \mathbf{m}$$

$$\boxed{is[\partial_t(1 - i\alpha) - i\mathcal{A}_0^z]m_f = A\left(\partial_\alpha / i - \mathcal{A}_\alpha^z\right)^2 m_f + Hm_f}$$

$$m_f = m_x(\mathbf{r}, t) + im_y(\mathbf{r}, t)$$

Magnon current: $j_\alpha = A\,\mathrm{Im}\langle m_f^* \partial_\alpha m_f \rangle$

1. Thermal magnons have parabolic band like electrons.

2. Due to fictitious vector potential magnons are subject to electric and magnetic fields:

$$\boxed{\begin{aligned} \mathcal{E}_\alpha &= \hbar \widetilde{\mathbf{m}}_s \cdot (\partial_t \widetilde{\mathbf{m}}_s \times \partial_\alpha \widetilde{\mathbf{m}}_s) \\ \mathcal{B}_i &= (\hbar/2)\epsilon^{ijk} \widetilde{\mathbf{m}}_s \cdot (\partial_k \widetilde{\mathbf{m}}_s \times \partial_j \widetilde{\mathbf{m}}_s) \end{aligned}}$$

Stochastic LLG equation

1. Thermal effects are included via the stochastic LLG equation:

$$s(1+\alpha\mathbf{m}\times)\dot{\mathbf{m}} + \mathbf{m}\times(\mathbf{H}_{\text{eff}} + \mathbf{h}) = 0$$

$$\langle h_i(\mathbf{r},t)h_j(\mathbf{r}',t')\rangle = 2\alpha s k_B T(\mathbf{r})\delta_{ij}\delta(\mathbf{r}-\mathbf{r}')\delta(t-t')$$

W. F. Brown, Phys. Rev. 130, 1677 (1963)

2. Transverse Fourier transform to reduce to one dimensional equation:

$$m_f(\mathbf{q},\omega,x) = \int \frac{d^{d-1}\boldsymbol{\rho}\, d\omega}{(2\pi)^d} e^{i(\omega t - \mathbf{q}\boldsymbol{\rho})} m_f(\mathbf{r},t)$$

2. Correlator is found after solving the Helmholtz equation:

$$A(\partial_x^2 + k^2)m_f(x,\mathbf{q},\omega) = h(x,\mathbf{q},\omega)$$ --- Linearized LLG equation

3. In the rotated reference frame $S = S_x + iS_y$ for $\partial T(x)/\partial x = \text{constant}$

$$S = -\partial_x m_s \int \frac{d^{d-1}\mathbf{q}\, d\omega}{(2\pi)^d} \frac{\langle m_f(\mathbf{q},\omega,x)^* \partial_x m_f(\mathbf{q}',\omega',x)\rangle}{(2\pi)^d \delta(\mathbf{q}-\mathbf{q}')\delta(\omega-\omega')}$$

Results of analytic calculation

1. Torque is expanded in the Gilbert damping parameter α

$$\boxed{\mathcal{T} = -\hbar \partial_x m_s j_x (1 + i\beta)}$$ here $\partial T(x)/\partial x = \text{constant}$

$$j_x = (\partial_x T/T) \int d^d\mathbf{k}/(2\pi)^d \tau(\varepsilon)\varepsilon v_x^2 \partial f_0/\partial\varepsilon$$

$$\tau(\varepsilon) = (2\alpha\omega)^{-1}$$

2. We reproduce the result of relaxation time approximation to Boltzmann equation under assumption of quantum fluctuation dissipation theorem

$$2k_B T \to \hbar\omega \coth(\hbar\omega/2k_B T)$$

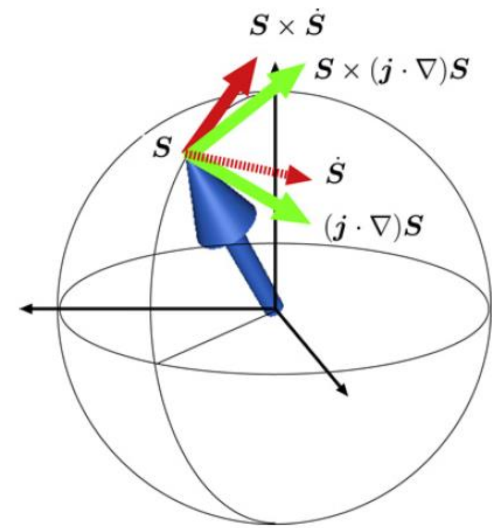
3. We also obtain the dissipative term with

$$\boxed{\beta/\alpha = (d/2)F_1(x)/F_0(x) \overset{x\to 0}{=} d/2}$$

$$F_0(x) = \int_0^\infty d\epsilon \epsilon^{d/2-1} \epsilon e^{\epsilon+x}/(e^{\epsilon+x}-1)^2$$

$$F_1(x) = \int_0^\infty d\epsilon (\epsilon + x) \epsilon^{d/2-1} e^{\epsilon+x}/(e^{\epsilon+x}-1)^2$$

$x = \hbar\omega_0/k_B T$ is calculated at magnon gap



Analogy to electron spin torque: G. Tatara, H. Kohno, J. Shibata, Physics Reports 468, 213 (2008)

LLG equation with magnonic torques

1. Amended LLG equation for slow component and magnonic torques

$$\boxed{s(1+\alpha\mathbf{m}_s\times)\dot{\mathbf{m}}_\mathbf{s}+\mathbf{m}_s\times\mathbf{H}^s_{\text{eff}}=-\left[1+\beta\mathbf{m}_s\times\right](\mathbf{j}^s_m\boldsymbol{\partial})\mathbf{m}_s}$$

$$\mathbf{j}^s_m=-\hbar\mathbf{j}$$

A. Kovalev, Phys. Rev. B 89, 241101(R) (2014)
Se Kwon Kim, Yaroslav Tserkovnyak, arXiv:1505.00818

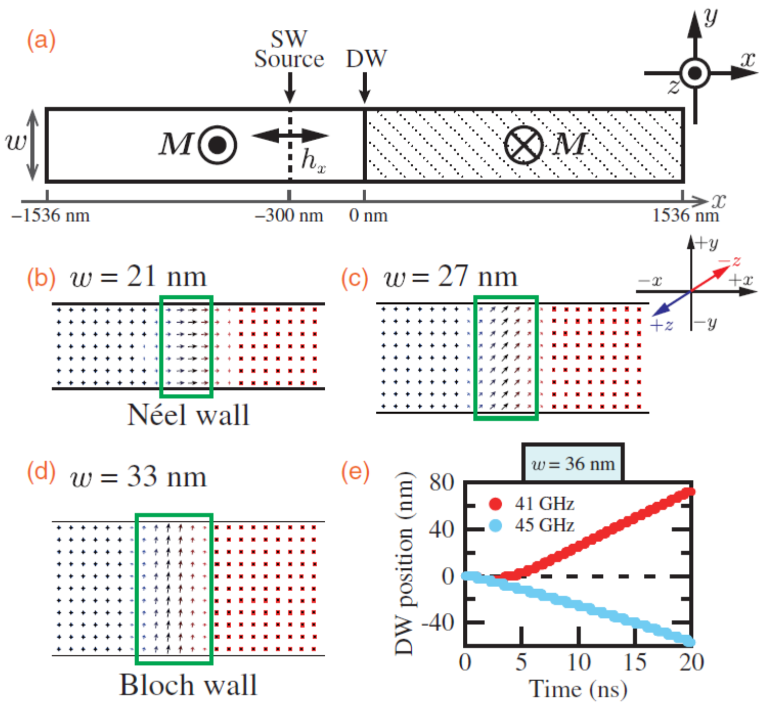
2. This approach should also apply to spin waves generated by external fields as long as the wavelength is sufficiently small. Domain wall will move towards the source.

3. Micromagnetic simulations also observe opposite direction of motion at some resonant frequencies.

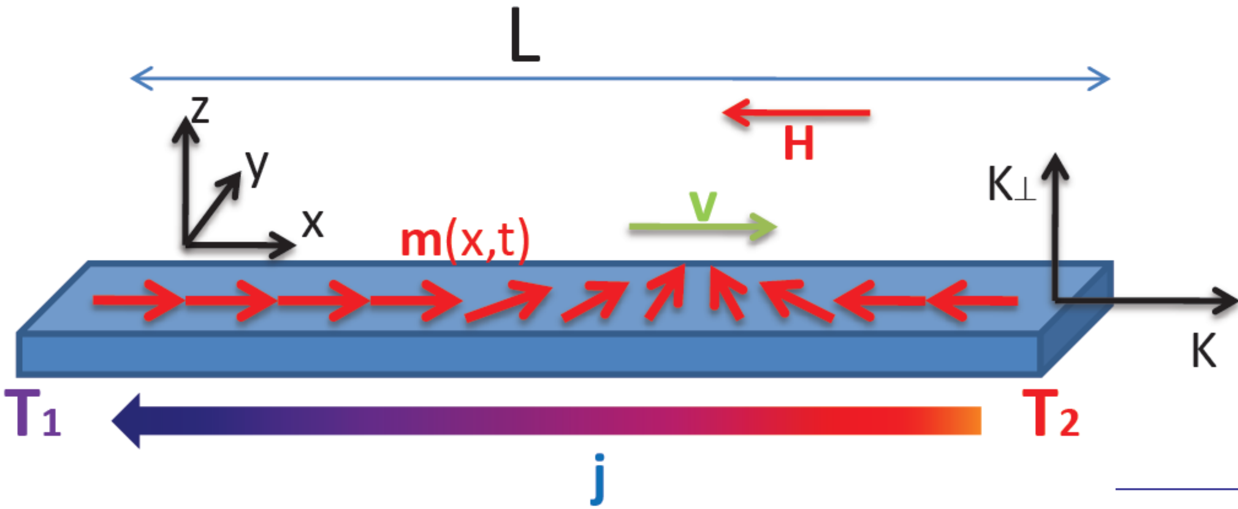
H. Hata, T. Taniguchi, H-W Lee, T. Moriyama and T. Ono, Appl. Phys. Express 7 033001 (2014)

4. Theory that includes linear momentum transfer:

P. Yan, A. Kamra, Y. Cao, and G. E.W. Bauer, Phys. Rev. B 144413 (2013)



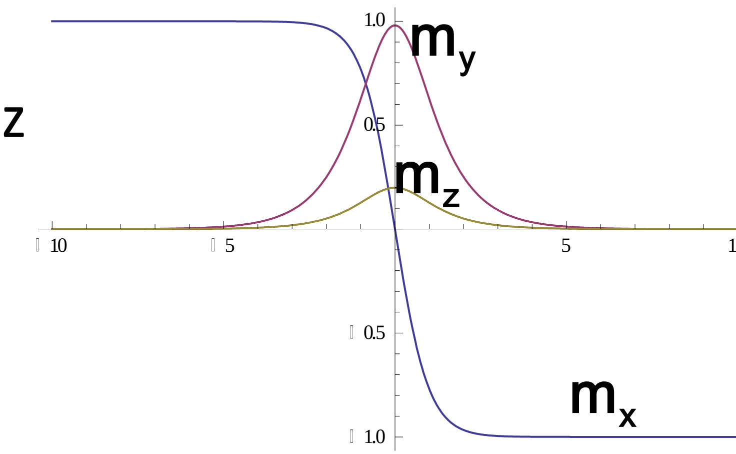
Domain wall propelled by magnons



Domain wall described by Walker ansatz

$$\varphi(\mathbf{r},t) \equiv \Phi(t), \quad \ln\tan\frac{\theta(\mathbf{r},t)}{2} = \frac{x - X(t)}{W(t)},$$

$$\mathbf{m} = (\cos\theta, \sin\theta\cos\varphi, \sin\theta\sin\varphi)$$



N.L. Schryer, L.R. Walker, J. Appl. Phys. 45, 5406 (1974)

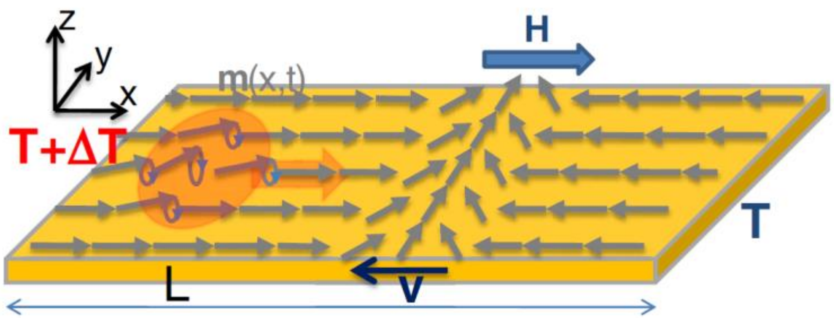
Domain wall velocity becomes:

$$\boxed{\dot{X} = \frac{\gamma H W - \hbar\beta j / s}{\alpha}}$$

A. A. Kovalev and Y. Tserkovnyak, EPL 97, 67002 (2012)

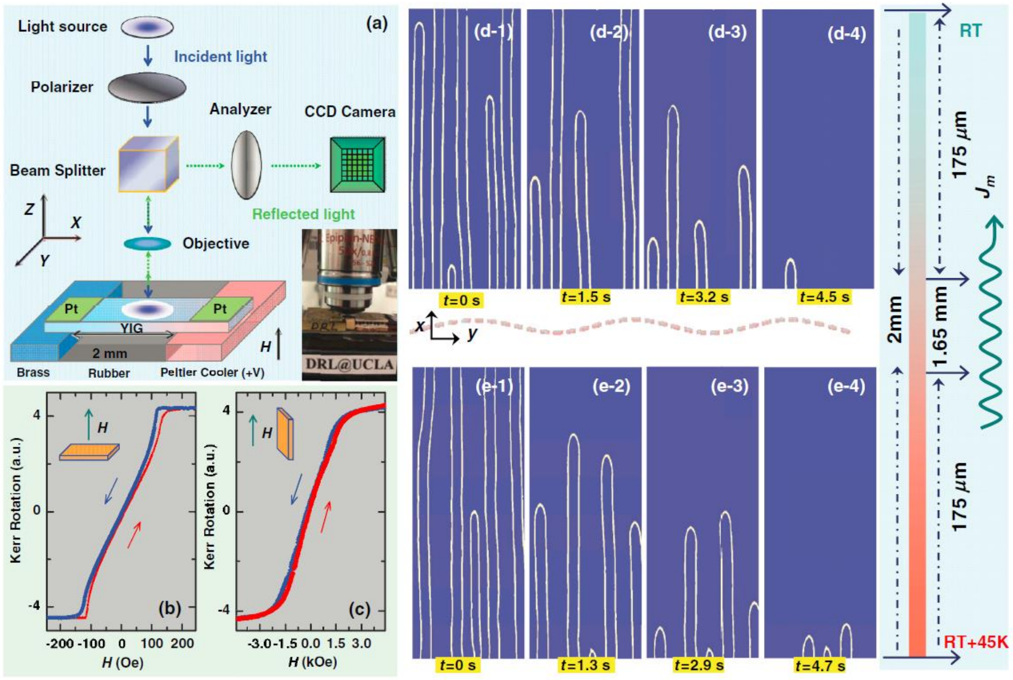
For thermal magnons: $j_\alpha = k_B \partial_\alpha T F_0 / (6\pi^2 \lambda \hbar \alpha)$

Domain wall moving by temperature gradients



A.A. Kovalev and Y. Tserkovnyak, EPL **97**, 67002 (2012)

Domain wall moves towards hot end.



W. Jiang et al., Phys. Rev. Lett. 110, 177202 (2013)

See also J. Torrejon, G. Malinowski, M. Pelloux, R. Weil, A. Thiaville, J. Curiale, D. Lacour, F. Montaigne, and M. Hehn, Phys. Rev. Lett. 109, 106601 (2012)

Magnonic manipulation of Skyrmions

1. Skyrmions are particularly stable in thin magnetic films
2. Temperature gradient will couple to motion of skyrmions via thermal magnon
3. We consider ferromagnet with Dzyaloshinskii-Moriya terms at sufficiently high temperatures

$$F = (A/2)(\partial_\alpha \cdot \mathbf{m})^2 - D_{\alpha\beta}\varepsilon_{\gamma\beta\nu} m_\nu \partial_\alpha m_\gamma$$

4. We use Thiele approach by introducing generalized coordinates q, i.e. $\dot{\mathbf{m}} = \sum_i \dot{q}_i \partial_{q_i} \mathbf{m}$
5. The skyrmion velocity becomes:

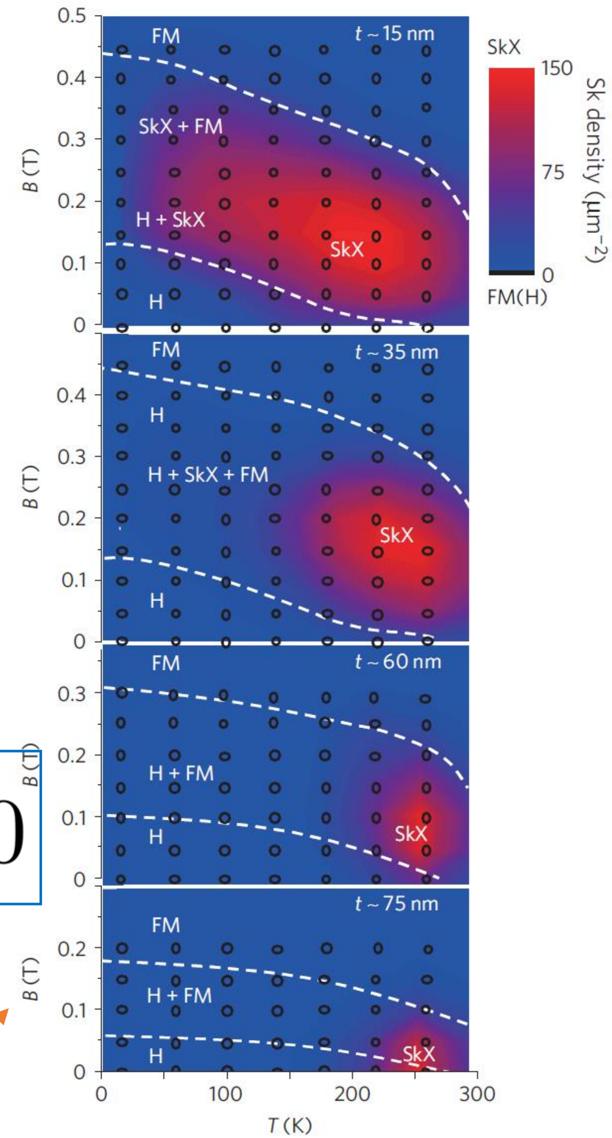
$$\boxed{\hat{\mathcal{G}}_1 \hbar \mathbf{j} + s\hat{\mathcal{G}}\mathbf{v} + \beta\hat{\eta}_1 \hbar \mathbf{j} + \alpha s \hat{\eta}\mathbf{v} = 0}$$

$$\mathcal{G}_{ij} = \int d^2\mathbf{r}(\partial_i \mathbf{m}_s \times \partial_j \mathbf{m}_s) \cdot \mathbf{m}_s/(4\pi)$$

$$\mathcal{G}_{1\,ij} = \int d^2\mathbf{r}(\partial_i \mathbf{m}_s \times \mathcal{D}_j \mathbf{m}_s) \cdot \mathbf{m}_s/(4\pi)$$

$$\eta_{ij} = \int d^2\mathbf{r}(\partial_i \mathbf{m}_s \cdot \partial_j \mathbf{m}_s)/(4\pi)$$

$$\eta_{1\,ij} = \int d^2\mathbf{r}(\partial_i \mathbf{m}_s \cdot \mathcal{D}_j \mathbf{m}_s)/(4\pi)$$



X. Z. Yu, N. Kanazawa, Y. Onose, K. Kimoto, W. Z. Zhang, S. Ishiwata, Y. Matsui and Y. Tokura, Nature Materials 10, 106–109 (2011) (FeGe)

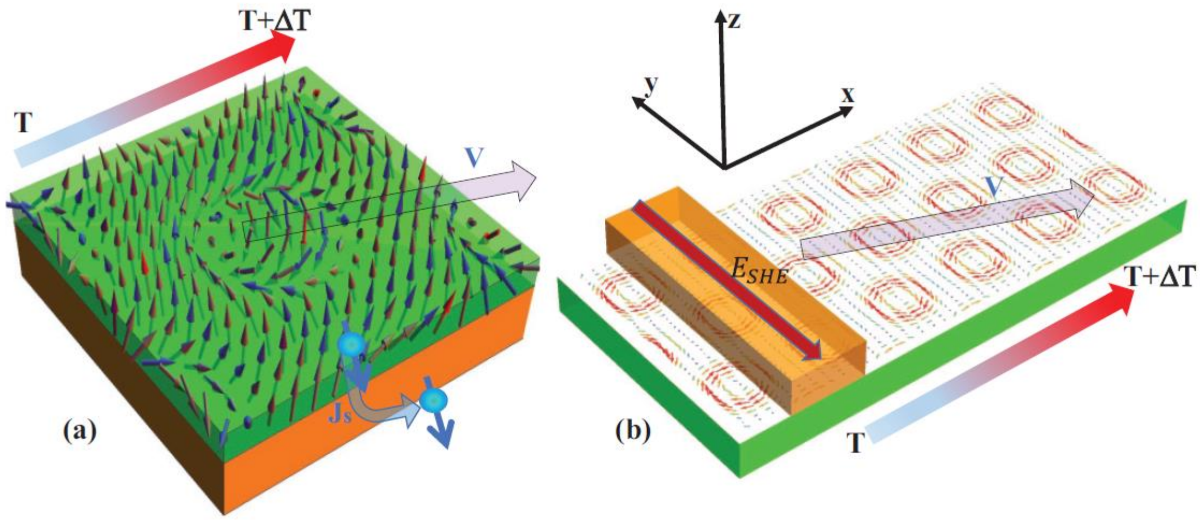
Skyrmionic spin Seebeck effect

1. Skyrmions will move in the direction of the hot region with additional side motion.

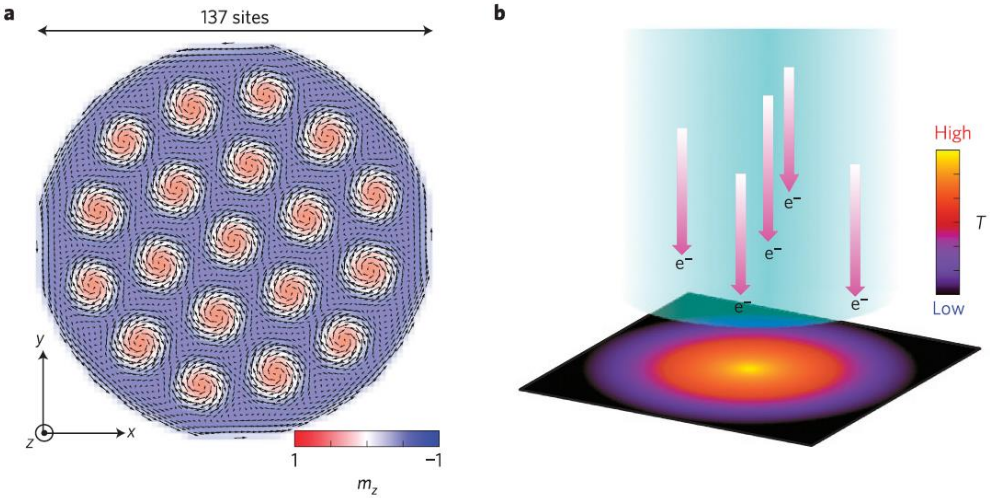
$$v_x = -\hbar j \frac{\mathcal{G}^2 + \alpha\eta\eta_1\beta}{s(\mathcal{G}^2 + \alpha^2\eta^2)}$$

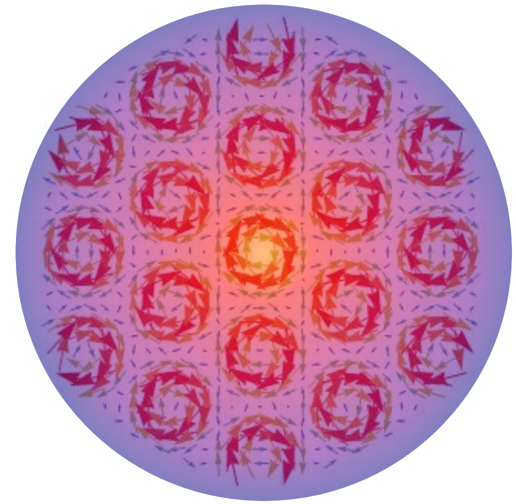
$$v_y = -\mathcal{G}\hbar j \frac{\alpha\eta - \beta\eta_1}{s(\mathcal{G}^2 + \alpha^2\eta^2)}$$

2. Two different regimes $\alpha\eta > \beta\eta_1$ and $\alpha\eta < \beta\eta_1$
3. Possible detection via spin pumping in the neighboring Pt layer.
4. For $\mathrm{Cu_2OSeO_3}$ the longitudinal velocity is estimated 0.1 m/s for 1K/µm



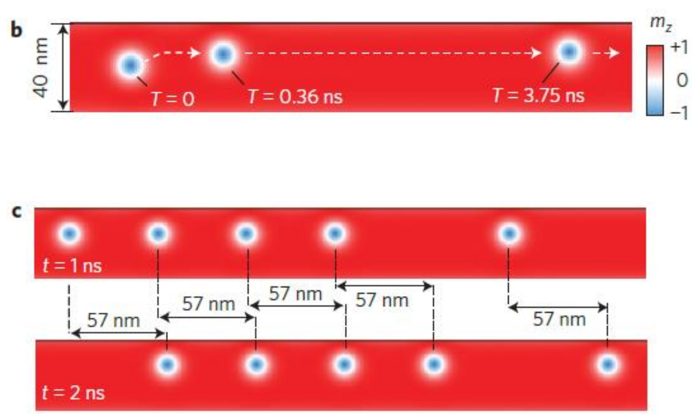
Current induced Skyrmion motion





M. Mochizuki, X. Z. Yu, S. Seki, N. Kanazawa, W. Koshibae, J. Zang, M. Mostovoy, Y. Tokura & N. Nagaosa, Nature Materials 13, 241–246 (2014)

No need for charge current as magnon current can induce skyrmion motion



In case of charge current we see very low thresholds which can be useful for memories.

J. Sampaio, V. Cros, S. Rohart, A. Thiaville & A. Fert, Nature Nanotechnology 8, 839–844 (2013)

Linear response theory for magnons

1. According to Luttinger, we can account for temperature gradient by introducing pseudo-gravitational potential:

$$e^{H/(k_B T)} \approx e^{(1+\chi)H/(k_B T_0)}$$

J. M. Luttinger, Phys. Rev. 135, 1505 (1964)

2. For an arbitrary multi-band Hamiltonian describing magnons we get

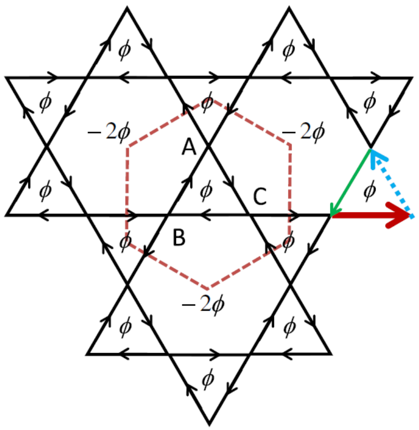
$$H_0 = \int d\mathbf{r}\Psi^\dagger(\mathbf{r}) H \Psi(\mathbf{r})\,; \qquad H_1 = \frac{1}{2}\int d\mathbf{r}\Psi^\dagger(\mathbf{r})(H\chi + \chi H)\Psi(\mathbf{r})$$

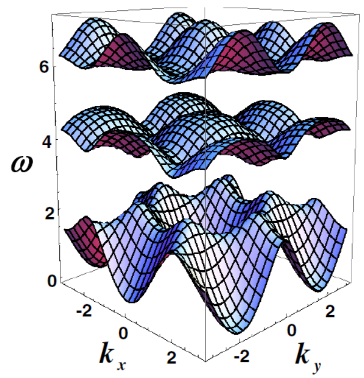
3. To calculate energy current response we need to find current-current correlator:

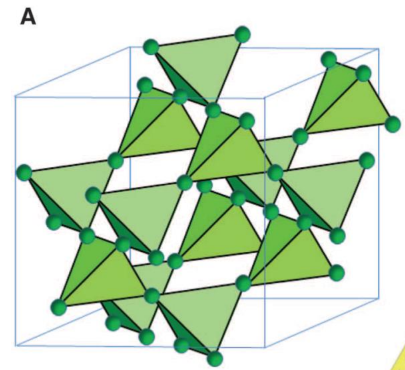
$$P_{\mu\nu} = \int_0^\beta d\tau e^{i\Omega t}\left\langle T_\tau \hat{J}^Q_\mu(\tau)\hat{J}^Q_\nu(0)\right\rangle$$

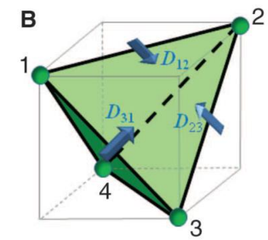
$$\left\langle \hat{J}^Q_\mu \right\rangle = \partial_\nu \chi \lim_{\Omega\to 0}\frac{P^R_{\mu\nu}(\Omega) - P^R_{\mu\nu}(0)}{i\Omega}$$

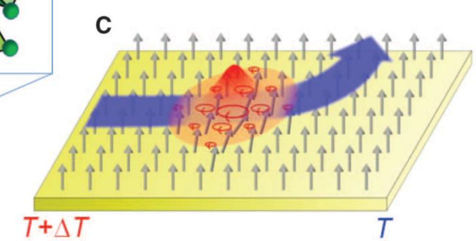
The magnon Hall effect











Y. Onose, T. Ideue, H. Katsura, Y. Shiomi, N.Nagaosa& Y. Tokura, Science 329, 297 (2010); R. Matsumoto and S. Murakami, Phys. Rev. Lett. 106(19), 197202 (2011);
A. Mook, J. Henk, I. Mertig, Phys. Rev. B 89, 134409 (2014)

Linear response for magnonic torques

1. We consider the Hamiltonian and disregard anisotropies at sufficiently high temperature:

$$H = H_{\text{ex}} + H_{\text{DMI}}$$

2. The response is considered with respect to the perturbation:

$$H_0 = \int d\mathbf{r}\Psi^\dagger(\mathbf{r})H\Psi(\mathbf{r})\,; \qquad H_1 = \frac{1}{2}\int d\mathbf{r}\Psi^\dagger(\mathbf{r})(H\chi + \chi H)\Psi(\mathbf{r})$$

4. To calculate the torque response we need to find the correlator:

$$\mathbf{P}_\nu = \int_0^\beta d\tau e^{i\Omega t}\left\langle T_\tau\left[\mathbf{m}\times\frac{\delta H_{\text{DMI}}}{\delta\mathbf{m}}(\tau)\right]\hat{J}_\nu^Q(0)\right\rangle$$

$$\langle\boldsymbol{\mathcal{T}}_\nu\rangle = \partial_\nu\chi\lim_{\Omega\to 0}\frac{\mathbf{P}_\nu^R(\Omega) - \mathbf{P}_\nu^R(0)}{i\Omega}$$

Comparison to theory of SO torques

1. The following correlator finds the anisotropy field in response to an electric field:

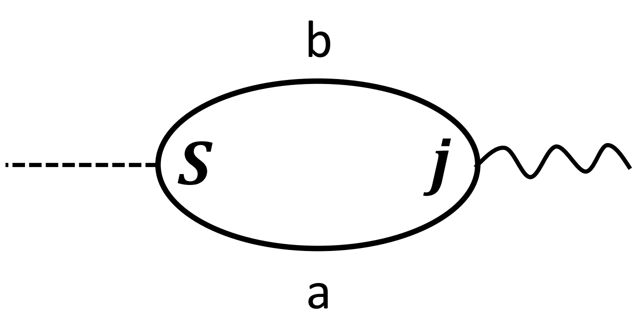
$$\mathbf{P}_\nu = \int_0^\beta d\tau e^{i\Omega t} \left\langle T_\tau \left[\frac{\delta H}{\delta \mathbf{m}}(\tau) \right] \hat{J}_\nu(0) \right\rangle$$

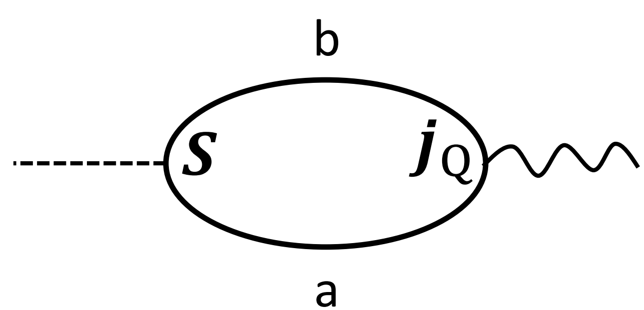
I. Garate and A. H. MacDonald, PRB 80, 134403 (2009); H. Kurebayashi,Jairo Sinova,D. Fang, A.C. Irvine,T. D. Skinner,J. Wunderlich, V. Novák,R. P. Campion,B. L. Gallagher,E. K. Vehstedt, L. P. Zarbo,K. Výborný, A. J. Ferguson& T. Jungwirth, Nature Nanotechnology 9, 211–217 (2014).

2. We essentially calculate the transverse spin accumulation.

3. Previously defined transverse magnon-spin accumulation is the proper operator to calculate in case of magnonic torque.

3. Diagrammatically we calculate the following diagrams.





Transverse magnon-spin accumulation

1. We arrive at the expression for the transverse spin accumulation:

$$\langle S_\nu \rangle = \frac{\hbar}{2\pi V} \int d\omega \frac{dn_B}{d\omega} \sum_{k,a\neq b} \left\langle \mathbf{k}, a | \hat{S} | \mathbf{k}, b \right\rangle \left\langle \mathbf{k}, b | J_\nu^Q | \mathbf{k}, a \right\rangle \left[G_{k,a}^R G_{k,b}^A - G_{k,a}^R G_{k,b}^R \right]$$

$$G_{k,a}^R = (G_{k,a}^A)^* = \frac{1}{\hbar\omega - \hbar\omega_k + i\Gamma}\,; \quad n_B = \{\exp[\hbar\omega/k_B T] - 1\}^{-1}$$

2. The spin accumulation operator can be found by expanding the torque up to the second order in the small fluctuations:

$$\hat{S} = -\frac{\mathbf{D}'_\alpha \times \mathbf{m}_s}{A} \partial_\alpha = -(\mathcal{D}_\alpha \mathbf{m}_s)\partial_\alpha$$

3. The corresponding torque is given by the expression:

$$\boldsymbol{\mathcal{T}} = A\mathbf{m}_s \times \boldsymbol{\mathcal{S}}$$

Application to single band LLG ferromagnet

1. We consider the following Hamiltonian:

$$H = (A/2)(\partial_\alpha \cdot \mathbf{m})^2 - D_{\alpha\beta}\varepsilon_{\gamma\beta\nu} m_\nu \partial_\alpha m_\gamma$$

$$D_{\alpha\beta} = \mathbf{D}_\alpha \cdot \mathbf{e}_\beta$$

2. The corresponding chiral derivative becomes:

$$\mathcal{D}_\alpha = \partial_\alpha + (\mathbf{D}_\alpha / A)\times$$

3. The spin accumulation operator becomes:

$$\hat{S} = -\frac{\mathbf{D}'_\alpha \times \mathbf{m}_s}{A}\partial_\alpha$$

4. We recover expression obtained before:

$$\begin{aligned}\mathcal{T} =& -2\hbar\partial_x m_s(\partial_x T/T)\int (d\omega/2\pi) G^R_{k,a} G^A_{k,b} \int d^d\mathbf{k}/(2\pi)^d \omega_k v_x^2 \partial n_B/\partial\omega \\ =& -\hbar\partial_x m_s(\partial_x T/T)\int d\omega_k \omega_k \frac{(\omega_k + i\Gamma)^{3/2}}{\Gamma}(A/s)^{3/2}(1/3\pi^2) n_B/\partial\omega_k\end{aligned}$$

$$\boxed{\mathcal{T} = -\hbar\partial_x m_s j_x (1 + i3\alpha/2)}$$

Agrees with the result from LLG equation.

Torques in Dzyaloshinskii-Moriya magnets

1. The general form of Dzyaloshinskii-Moriya interaction:

$$F_{\mathrm{DMI}} = D_{\alpha\beta}\varepsilon_{\gamma\beta\nu}m_\nu\partial_\alpha m_\gamma$$

$$D_{\alpha\beta} = \mathbf{D}_\alpha \cdot \mathbf{e}_\beta$$

2. We separate Dzyaloshinskii-Moriya tensor into symmetric and antisymmetric parts:

$$D_{\alpha\beta} = D^{sym}_{\alpha\beta} + \varepsilon_{\alpha\beta\gamma}D^{ant}_\gamma$$

3. High symmetry cases: $\eta = \hbar,\ \theta = \alpha\eta d/2$

$$\boldsymbol{\mathcal{T}} = (1/A)\left(\eta + \vartheta\mathbf{m}\times\right) j_\alpha \mathbf{D}_\alpha \times \mathbf{m}$$

$$\mathbf{D}^{ant} = D\mathbf{n} \qquad\qquad D^{sym}_{\alpha\beta} = D\delta_{\alpha\beta}$$

$$\boldsymbol{\mathcal{T}} = (D/A)\left(\eta + \vartheta\mathbf{m}\times\right)(\mathbf{n}\times\mathbf{j})\times\mathbf{m} \qquad \boldsymbol{\mathcal{T}} = (D/A)\left(\eta + \vartheta\mathbf{m}\times\right)\mathbf{j}\times\mathbf{m}$$

Thermodynamic argument

1. Write the rate of entropy production:

$$\dot{\mathbb{S}} = \int d^3\mathbf{r}\left(-\frac{\boldsymbol{\partial} T}{T^2}\cdot\mathbf{j}_q - \frac{\boldsymbol{\partial}\mu}{T}\cdot\mathbf{j} - \frac{\mathbf{H}_{\text{eff}}}{T}\cdot\dot{\mathbf{m}}\right)$$

$\mathbf{j}_q = \mathbf{j}_u - \mu\mathbf{j}$ - relation between energy and heat currents

2. We relate the currents to the thermodynamic conjugates via kinetic coefficients.

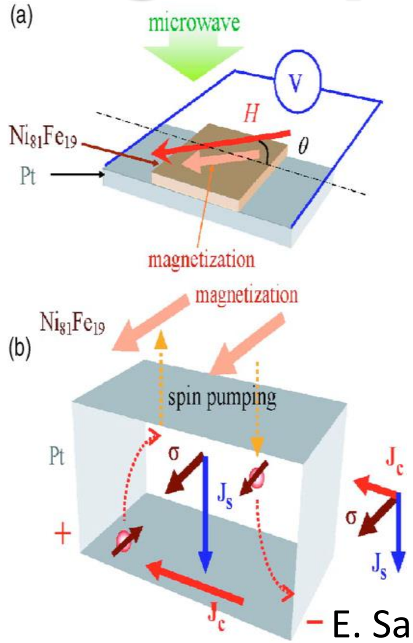
$$\boxed{\begin{aligned} -\partial_i\mu &= \hat{\varrho} j_i + \hat{\Pi}\partial_i T/T - (\eta\mathbf{m}\times\mathcal{D}_i\mathbf{m} + \vartheta\mathcal{D}_i\mathbf{m})\cdot\dot{\mathbf{m}} \\ j^h_{q,i} &= \hat{\Pi}^T j_i - \hat{\kappa}\partial_i T - (\eta_1\mathbf{m}\times\mathcal{D}_i\mathbf{m} + \vartheta_1\mathcal{D}_i\mathbf{m})\cdot\dot{\mathbf{m}} \end{aligned}}$$

3. We complete LLG equation by adding torque terms according to the Onsager principle.

$$\boxed{s(1+\alpha\mathbf{m}\times)\dot{\mathbf{m}} + \mathbf{m}\times\mathbf{H}_{\text{eff}} = (\eta + \vartheta\mathbf{m}\times)\, j_i\mathcal{D}_i\mathbf{m}}$$

$$\boxed{\mathcal{D}_\alpha = \partial_\alpha + (\mathbf{D}_\alpha/A')\times}$$

Magnon pumping and magnetization control



Pumped spin current is comparable to spin current in typical spin pumping experiments!

$$j_s = \sin^2\theta\hbar\omega g_{\uparrow\downarrow}/(4\pi)$$

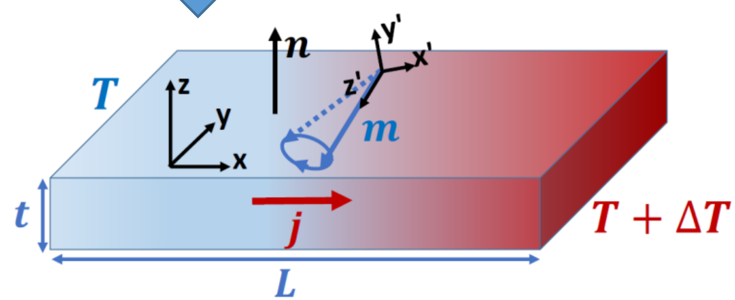
$$g_{\uparrow\downarrow} = D\vartheta/(At\eta\alpha)$$

Magnetization instability can develop at critical current given by:

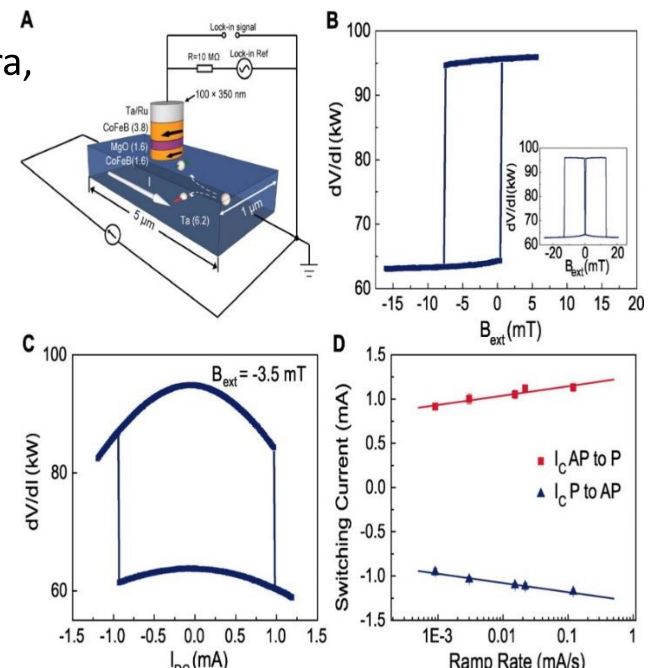
$$j_c = \frac{H + K/2}{|\vartheta/\alpha - \eta|}$$

Temperature gradient of $1\mathrm{K}/\mu\mathrm{m}$ should be sufficient for developing magnetization instability.

E. Saitoh, M. Ueda, H. Miyajima and G. Tatara, Appl. Phys. Lett. 88, 182509 (2006).



Single domain magnetization dynamics induced by microwave field pumps magnon and spin currents by virtue of Dzyaloshinskii-Moriya interactions. This can develop a temperature gradient along the sample. Alternatively, a temperature gradient can result in magnon current and torque on uniform magnetization according to the Onsager reciprocity principle.



L. Liu, Chi-F. Pai, Y. Li, H. W. Tseng, D. C. Ralph, R. A. Buhrman, Science 336 , 555-558 (2012).

Conclusions

- We described interplay between *fast and slow magnetization dynamics* in stochastic LLG equation in the presence of Dzyaloshinskii-Moriya interactions
- We formulated the *effective LLG equation* for slow dynamics where additional magnonic torques arise due to the fast dynamics
- The theory predicts *domain wall and skyrmion motion* by temperature gradients
- We describe *magnon pumping* by single domain magnetization dynamics and suggest a possibility to *reverse single domain magnetization* by temperature gradients in Dzyaloshinskii-Moriya magnets