

New Perspectives in Spintronic and Mesoscopic Physics  
at ISSP University of Tokyo 6/12/2015

# Coupled charge and magnetization in a Weyl semimetal



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in collaboration with  
Daichi Kurebayashi

# Coupled charge and magnetization in a Weyl semimetal

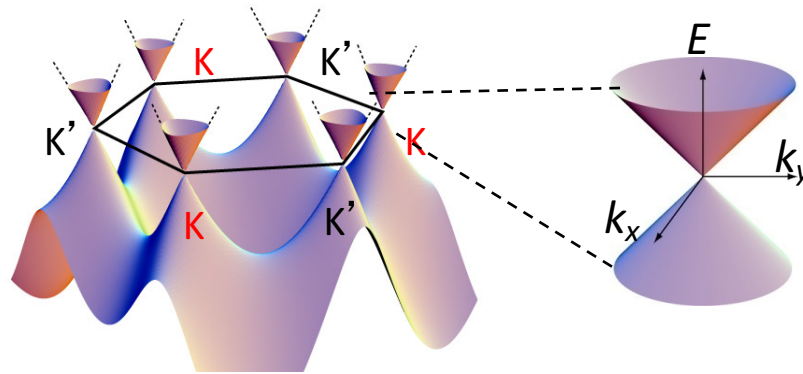
## outline

Introduction: What is a Weyl semimetal

1. Weyl semimetal in a magnetic Topological insulator
2. Magnetization-dynamics-induced charge pumping
3. Charge-induced spin torque

# What is a Weyl semimetal?

A **Weyl semimetal** is three-dimensional analogue of **graphene**



$$H^{2D} = v_F \begin{pmatrix} m_0 & p_x - ip_y \\ p_x + ip_y & -m_0 \end{pmatrix}$$

$m_0 = 0$  (for massless)

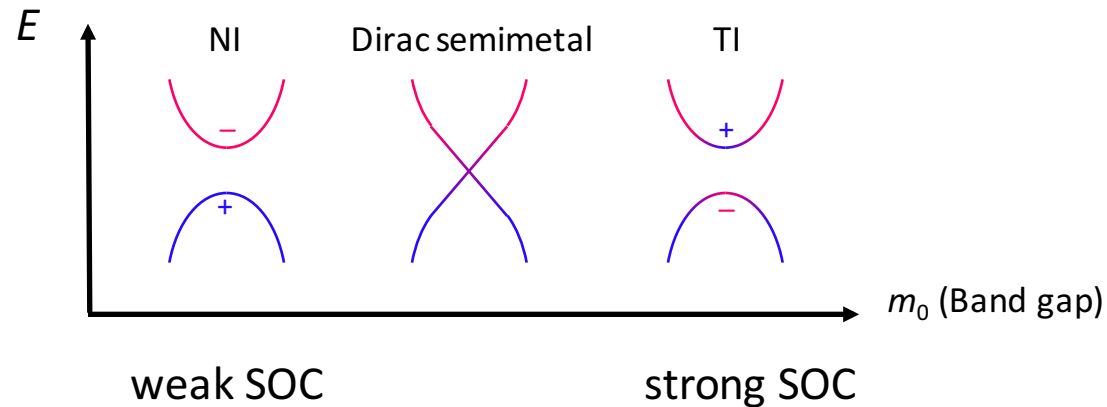
$$H^{3D} = v_F \begin{pmatrix} p_z & p_x - ip_y \\ p_x + ip_y & -p_z \end{pmatrix}$$

Dirac cone is stable in **3D**

# Topological states of matters

Spin-orbit coupling  $\longrightarrow$  Topologically nontrivial states

Kane & Mele (2005)



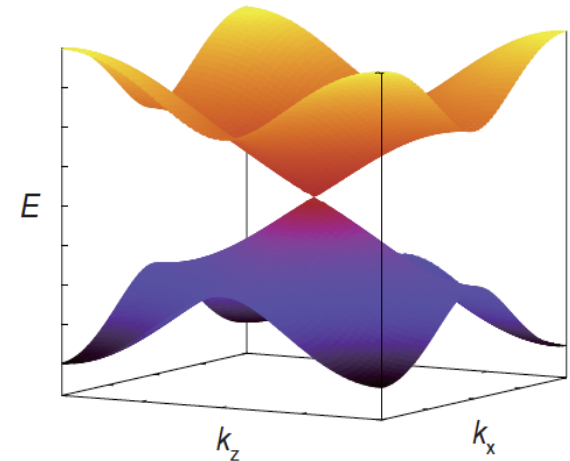
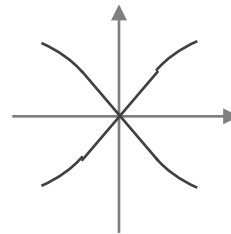
- Topological insulators (gapped)
- Topological semimetals (gapless)

# Dirac-Weyl semimetals

## Dirac semimetals

- 2D (graphene)  
Wallace (1947), ...
- 3D (accidental)  
Herring (1937), ...  
(symmetry protected)  
Wang et al., Young et al.(2012),...

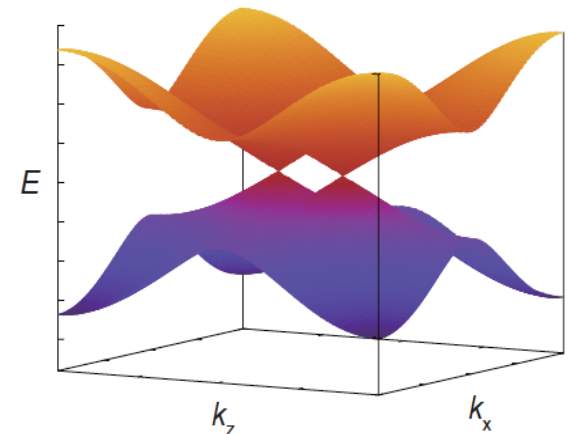
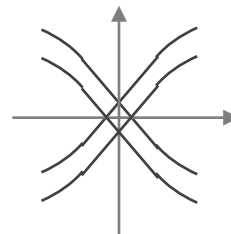
degenerate



## Weyl semimetals

- 3D (I-symmetry broken)  
Murakami (2007)  
Halasz&Balents (2012), ..
- 3D (T-symmetry broken)  
Wan et al. (2011)  
Burkov&Balents (2012), ..

non-degenerate



# Dirac-Weyl semimetals

## Dirac semimetals

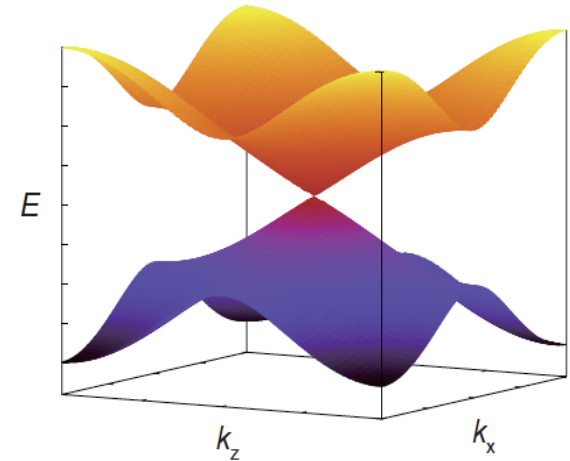
degenerate

$$H_{Dirac} = k_x \alpha_1 + k_y \alpha_2 + k_z \alpha_3 + m_0 \alpha_4$$

$m_0=0$  (massless)

$\alpha_i$ : 4x4 Dirac matrix

$$\{\alpha_i, \alpha_j\} = 2\delta_{ij}$$

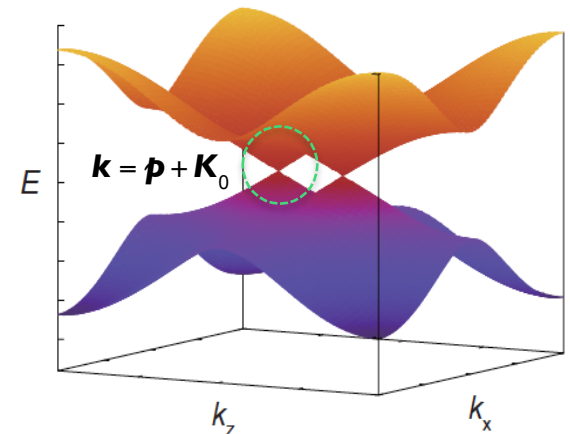


## Weyl semimetals

non-degenerate

$$H_{Weyl} = p_x \sigma_1 + p_y \sigma_2 + p_z \sigma_3$$

$s_i$ : 2x2 Pauli matrix



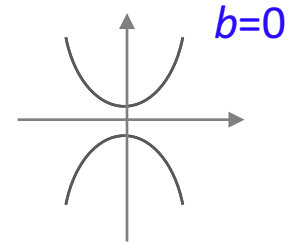
# Symmetry breaking

## Dirac hamiltonian

$$H_{Dirac} = k_x \alpha_1 + k_y \alpha_2 + k_z \alpha_3 + m_0 \alpha_4$$

$$\left\{ \begin{array}{l} \Theta^{-1} H_{Dirac}(\mathbf{k}) \Theta = H_{Dirac}(-\mathbf{k}) \quad \text{T-symmetry} \\ \Pi^{-1} H_{Dirac}(\mathbf{k}) \Pi = H_{Dirac}(-\mathbf{k}) \quad \text{I-symmetry} \end{array} \right.$$

degenerate



$$\alpha_i = \begin{bmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{bmatrix}, \quad \alpha_4 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$$

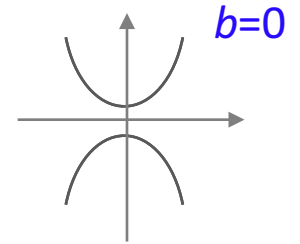
# Symmetry breaking

## Dirac hamiltonian

$$H_{Dirac} = k_x \alpha_1 + k_y \alpha_2 + k_z \alpha_3 + m_0 \alpha_4 + b S_z$$

$$\left\{ \begin{array}{l} \Theta^{-1} H_{Dirac}(\mathbf{k}) \Theta \neq H_{Dirac}(-\mathbf{k}) \quad \text{T-symmetry} \\ \Pi^{-1} H_{Dirac}(\mathbf{k}) \Pi = H_{Dirac}(-\mathbf{k}) \quad \text{I-symmetry} \end{array} \right.$$

degenerate



$$\alpha_i = \begin{bmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{bmatrix}, \quad \alpha_4 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$$

$$\Sigma_i = \begin{bmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{bmatrix}$$

spin of electrons



# Symmetry breaking

## Dirac hamiltonian

$$H_{Dirac} = k_x \alpha_1 + k_y \alpha_2 + k_z \alpha_3 + m_0 \alpha_4 + b S_z$$

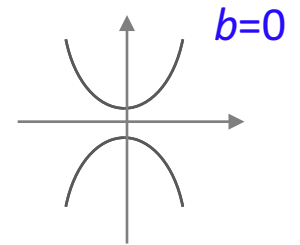
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$$\alpha_i = \begin{bmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{bmatrix}, \quad \alpha_4 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$$

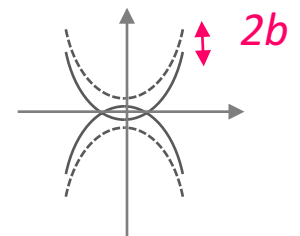
$$\Sigma_i = \begin{bmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{bmatrix}$$

spin of electrons

degenerate



non-degenerate



# Symmetry breaking

## Dirac hamiltonian

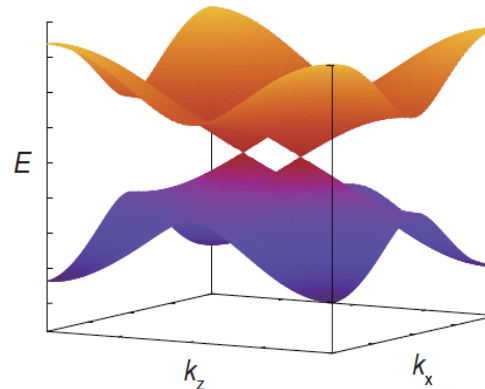
$$H_{Dirac} = k_x \alpha_1 + k_y \alpha_2 + k_z \alpha_3 + m_0 \alpha_4 + b S_z$$

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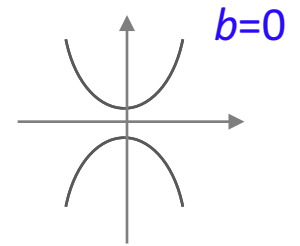
$$\alpha_i = \begin{bmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{bmatrix}, \quad \alpha_4 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$$

$$\Sigma_i = \begin{bmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{bmatrix}$$

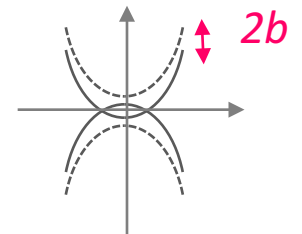
spin of electrons



degenerate



non-degenerate

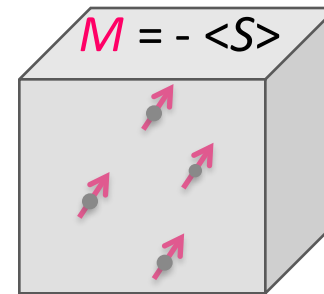


# Symmetry breaking

## Dirac hamiltonian

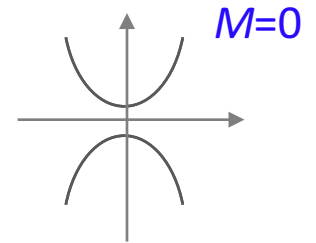
$$H_{Dirac} = k_x \alpha_1 + k_y \alpha_2 + k_z \alpha_3 + m_0 \alpha_4 + JM S_z$$

$$H_{sd} = J \sum_{i=1}^{N_{imp}} \mathbf{S}(\mathbf{R}_i) \cdot \Sigma$$

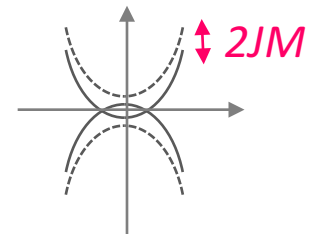


 local spins

degenerate



non-degenerate



# Symmetry breaking

## Dirac hamiltonian

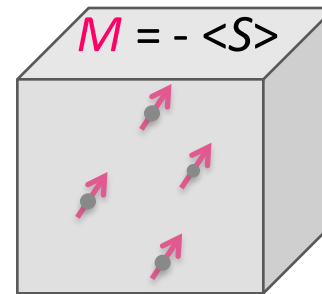
$$H_{Dirac} = k_x \alpha_1 + k_y \alpha_2 + k_z \alpha_3 + m_0 \alpha_4 + JM S_z$$

local spin     electron spin  
↓                    ↓

$$F = \frac{1}{2\chi_s} M^2 + \frac{1}{2\chi_e} m^2 - JMm$$

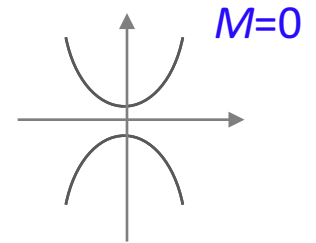
$$= \frac{1}{2\chi_s} (1 - J^2 \chi_e \chi_s) M^2 + \frac{1}{2\chi_e} (m - \chi_e JM)^2$$

$$H_{sd} = J \sum_{i=1}^{N_{imp}} \mathbf{S}(\mathbf{R}_i) \cdot \Sigma$$

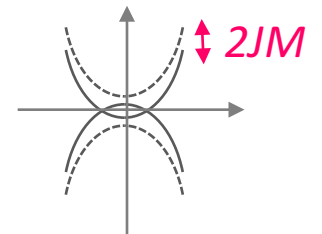


local spins

degenerate



non-degenerate

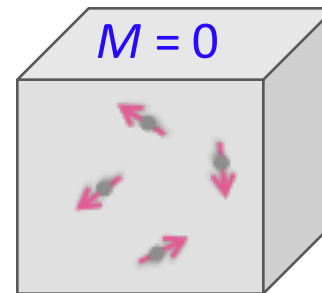


# Symmetry breaking

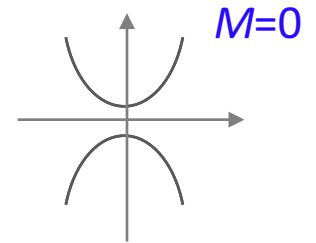
“Van Vleck ferromagnetism”

$$\begin{aligned}
 F &= \frac{1}{2\chi_s} M^2 + \frac{1}{2\chi_e} m^2 - JMm \\
 &= \frac{1}{2\chi_s} \underbrace{\left(1 - J^2 \chi_e \chi_s\right)}_{\text{---}} M^2 \\
 &\quad + \frac{1}{2\chi_e} \left(m - \chi_e JM\right)^2
 \end{aligned}$$

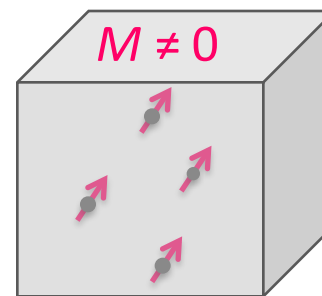
$$1 - J^2 \chi_s \chi_e > 0$$



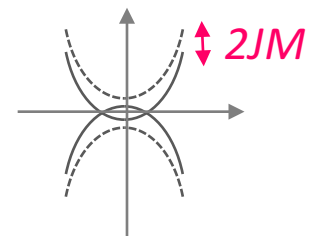
degenerate



$$1 - J^2 \chi_s \chi_e < 0$$



non-degenerate



 local spins

# Coupled charge and magnetization in a Weyl semimetal

## outline

Introduction: What is a Weyl semimetal

1. Weyl semimetal in a magnetic TI

2. Magnetization-dynamics-induced charge pumping

3. Charge-induced spin torque

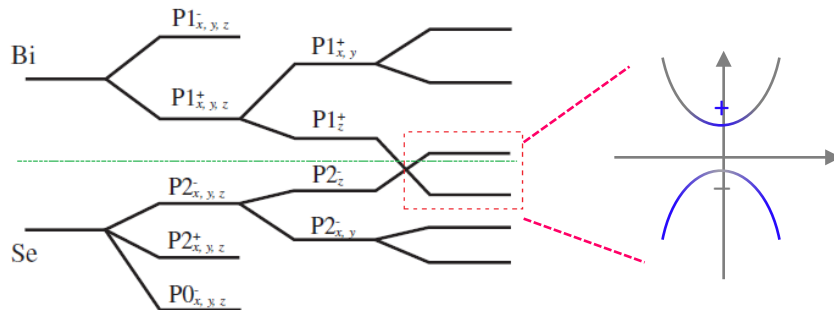
# Self-consistent theory

Kurebayashi, KN  
JPSJ 83 (2014) 063709.

$$H_{\text{total}} = H_e^{\text{MF}} + H_s^{\text{MF}} - N_{\text{imp}} J M m$$

Electrons

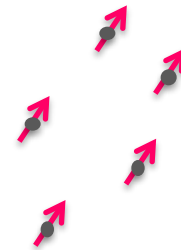
$$H_e^{\text{MF}} = \sum_{\mathbf{k}} c_{\mathbf{k}}^+ [H_0(\mathbf{k}) + x J M \Sigma_z] c_{\mathbf{k}}$$



$$H_0(\mathbf{k}) = \sum_{i=1}^3 R_i(\mathbf{k}) \alpha_i + m_0(\mathbf{k}) \alpha_4 + \varepsilon(\mathbf{k}) I$$

local spins

$$H_s^{\text{MF}} = J m \sum_{l=1}^{N_{\text{imp}}} S_z(\mathbf{r}_l)$$



local spins

Virtual crystal approximation

# Self-consistent theory

Kurebayashi, KN  
JPSJ 83 (2014) 063709.

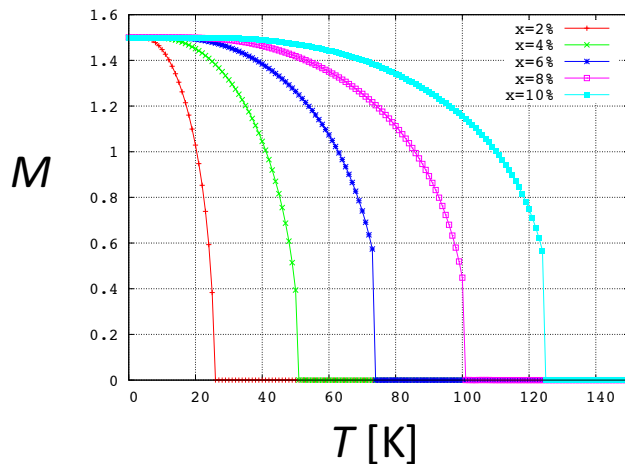
$$H_{\text{total}} = H_e^{\text{MF}} + H_s^{\text{MF}} - N_{\text{imp}} J M m$$

Electrons

local spins

$$H_e^{\text{MF}} = \sum_{\mathbf{k}} c_{\mathbf{k}}^{\dagger} [H_0(\mathbf{k}) + x J M \Sigma_z] c_{\mathbf{k}}$$

$$H_s^{\text{MF}} = J m \sum_{l=1}^{N_{\text{imp}}} S_z(\mathbf{r}_l)$$



$$m = \frac{1}{N} \sum_{i=1}^N \langle c_i^{\dagger} \Sigma_z c_i \rangle, \quad M = \frac{1}{N_i} \sum_{l=1}^{N_{\text{imp}}} \langle S_z(\vec{R}_l) \rangle$$

Virtual crystal approximation



# Self-consistent theory

Kurebayashi, KN  
JPSJ 83 (2014) 063709.

$$H_{\text{total}} = H_e^{\text{MF}} + H_s^{\text{MF}} - N_{\text{imp}} J M m$$

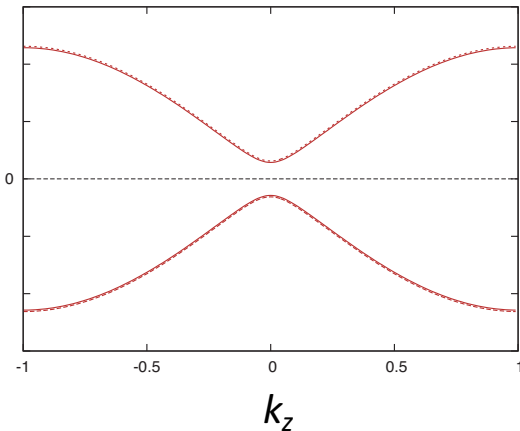
Electrons

local spins

$$H_e^{\text{MF}} = \sum_{\mathbf{k}} c_{\mathbf{k}}^+ [H_0(\mathbf{k}) + x J M \Sigma_z] c_{\mathbf{k}}$$

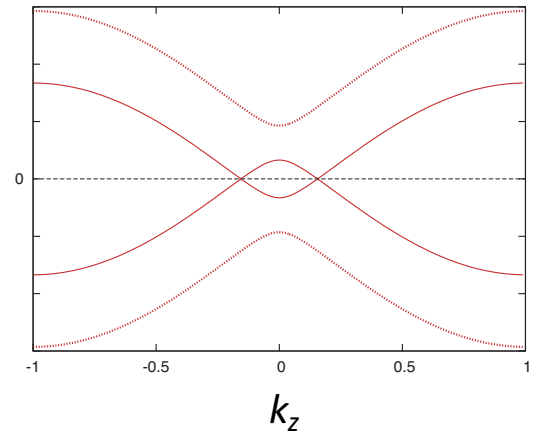
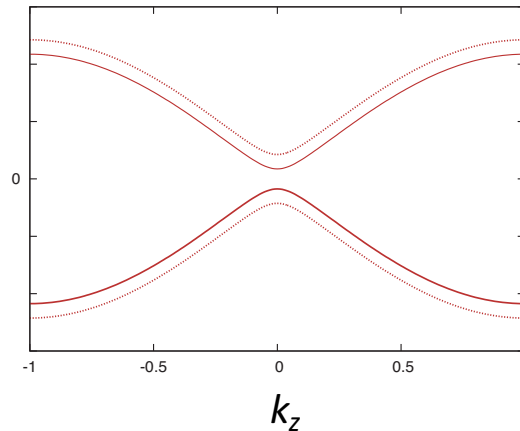
$$H_s^{\text{MF}} = J m \sum_{l=1}^{N_{\text{imp}}} S_z(\mathbf{r}_l)$$

$M = 0$



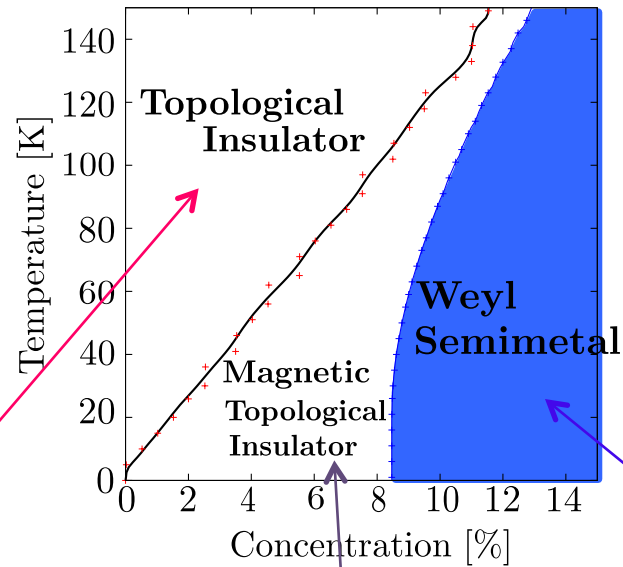
Magnetic transition

$M \neq 0$



# Weyl semimetal in a magnetic TI

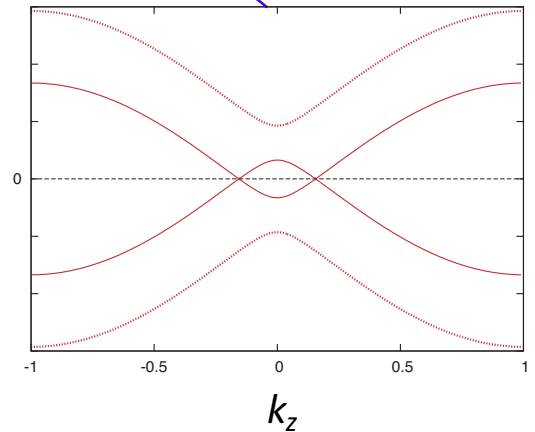
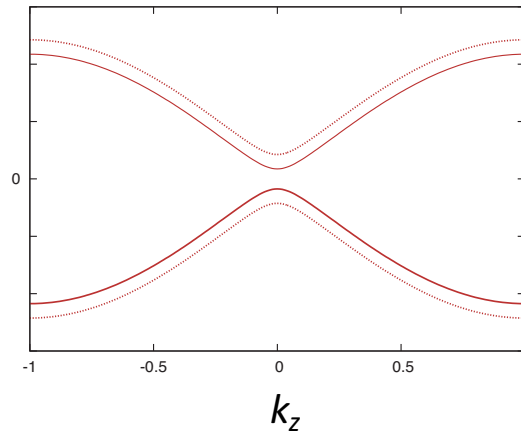
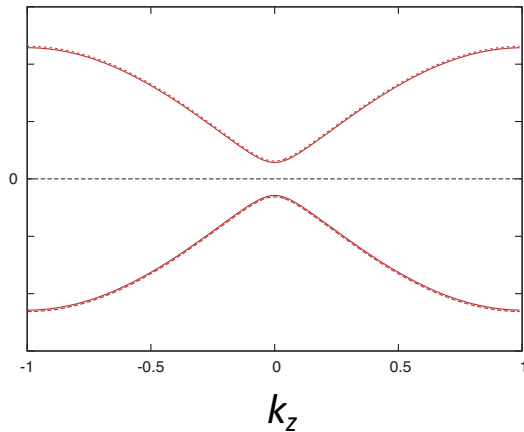
Kurebayashi, KN  
JPSJ 83 (2014) 063709.



$M = 0$

Magnetic transition

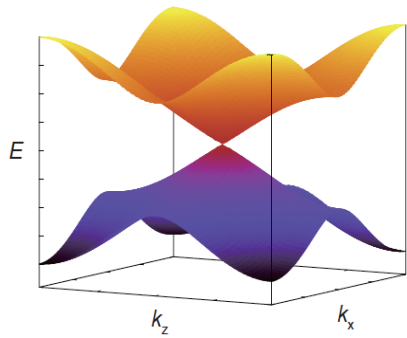
$M \neq 0$



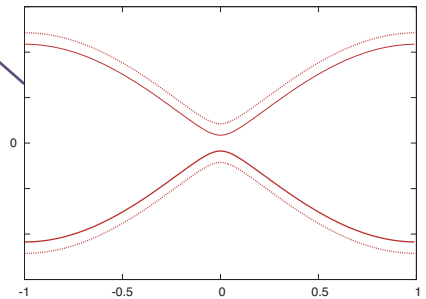
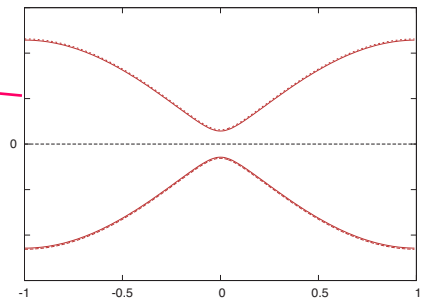
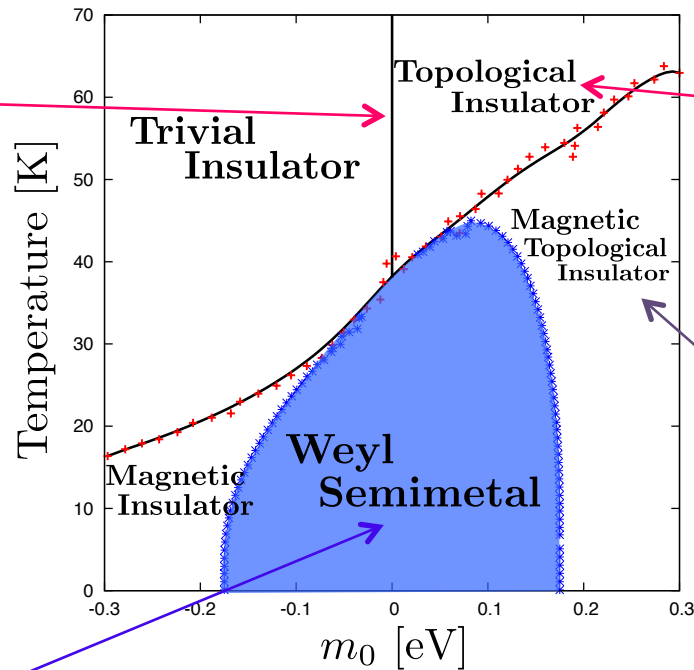
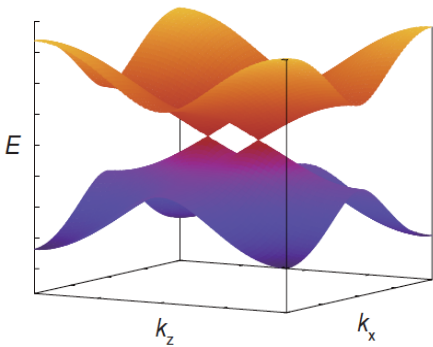
# Phase diagram

Kurebayashi, KN  
JPSJ 83 (2014) 063709.

Dirac semimetals



Weyl semimetals



weak SOC

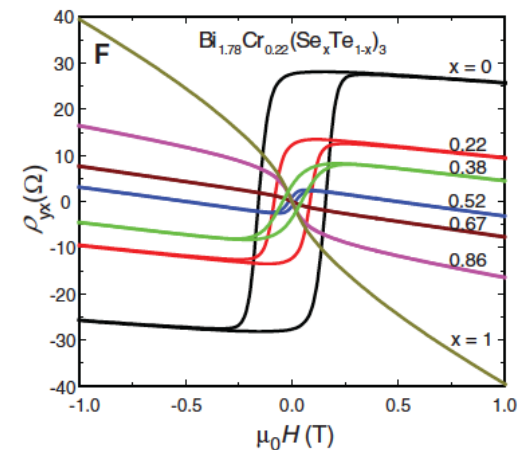
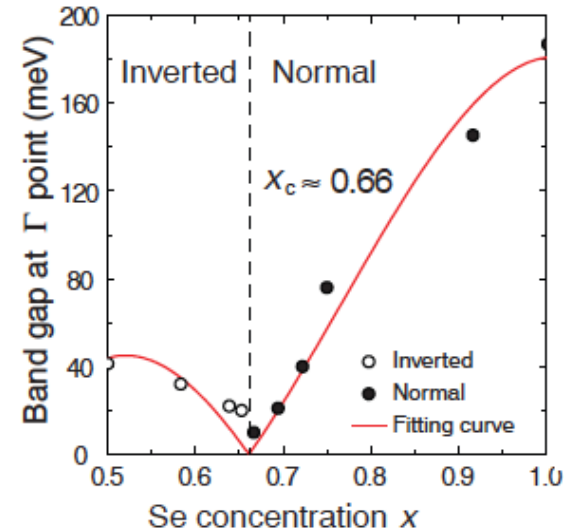
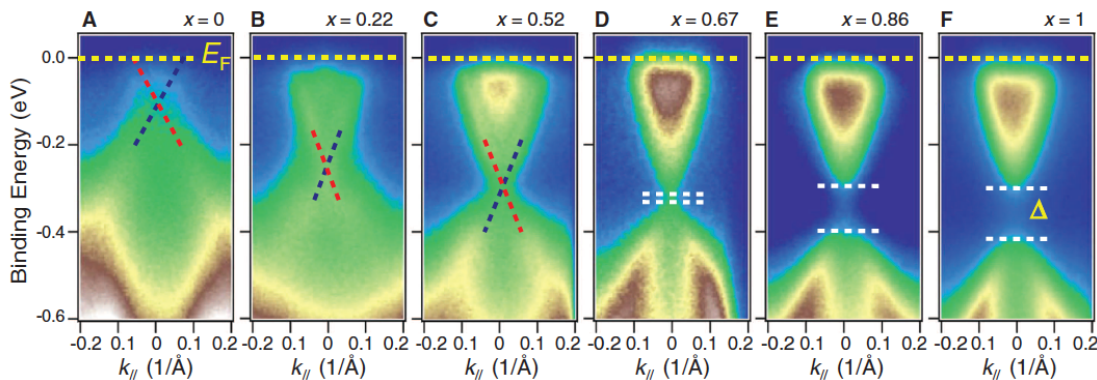
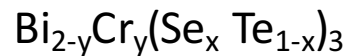
strong SOC

# Experiments of magnetic TIs

Science 339, 1582 (2013)

## Topology-Driven Magnetic Quantum Phase Transition in Topological Insulators

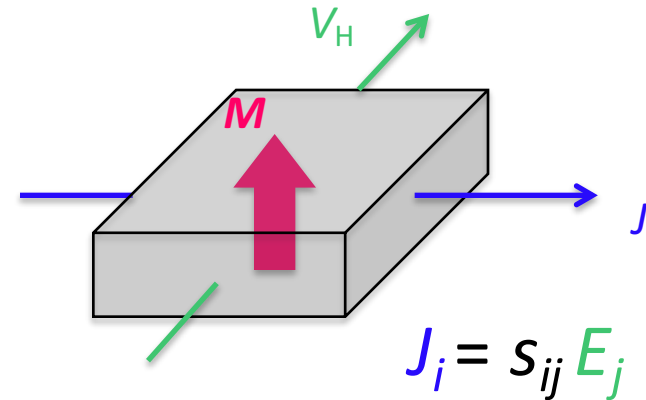
Jinsong Zhang,<sup>1\*</sup> Cui-Zu Chang,<sup>1,2\*</sup> Peizhe Tang,<sup>1\*</sup> Zuocheng Zhang,<sup>1</sup> Xiao Feng,<sup>2</sup> Kang Li,<sup>2</sup> Li-li Wang,<sup>2</sup> Xi Chen,<sup>1</sup> Chaoxing Liu,<sup>3</sup> Wenhui Duan,<sup>1</sup> Ke He,<sup>2†</sup> Qi-Kun Xue,<sup>1,2</sup> Xucun Ma,<sup>2</sup> Yayu Wang<sup>1†</sup>



# Anomalous Hall effect

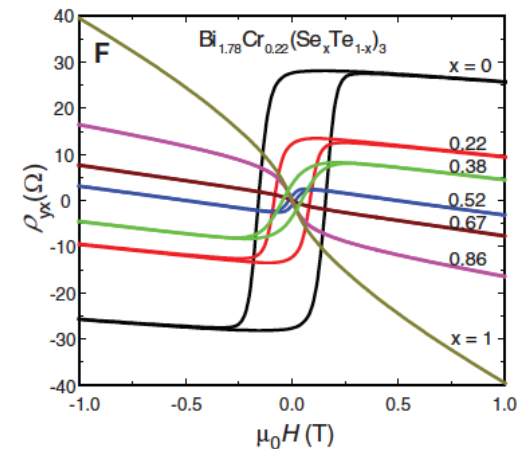
The Hall effect in materials with

- ferromagnetic order
- spin-orbit coupling



$$\rho_{xy} = R_0 B + R_s M$$

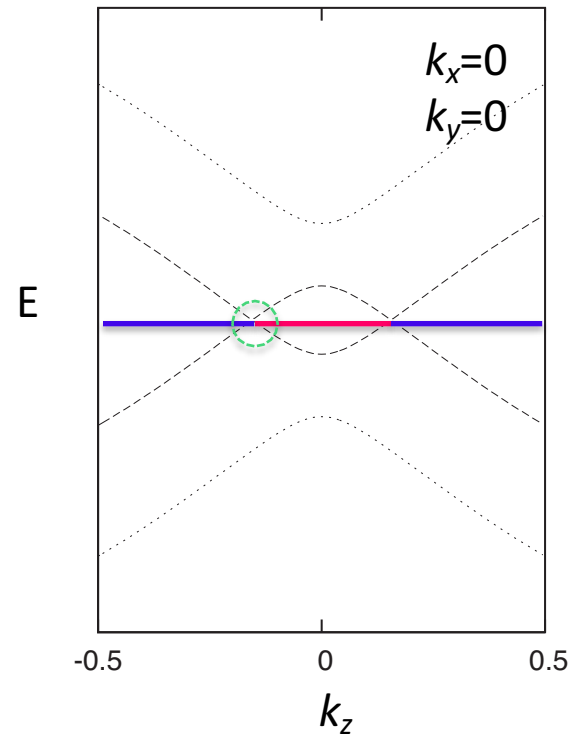
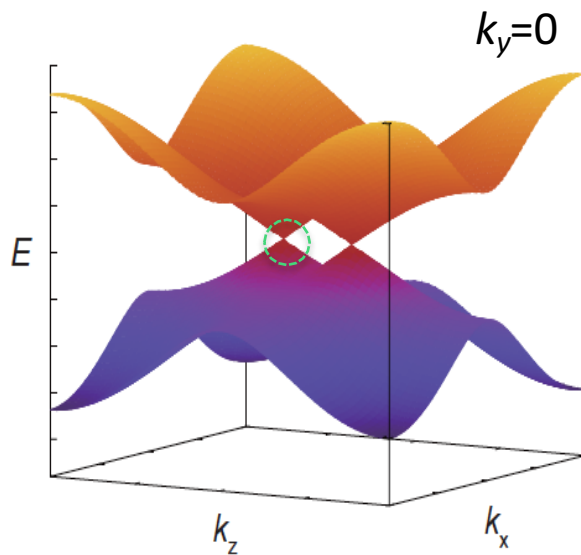
$$\rho_{xy} \propto M \quad (B \rightarrow 0)$$



# Anomalous Hall effect

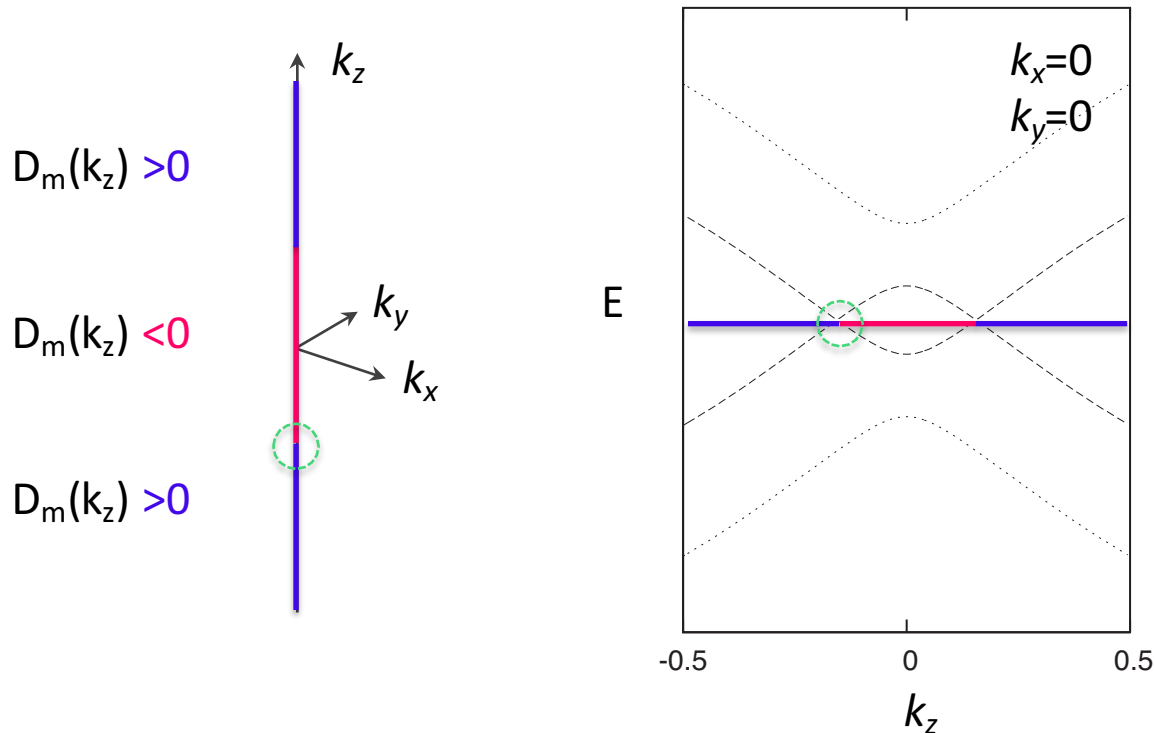
$$H_{\text{weyl}}(k_x, k_y, k_z) = k_x s_1 + k_y s_2 + D_m(k_z) s_3$$

3d Weyl SM



# Anomalous Hall effect

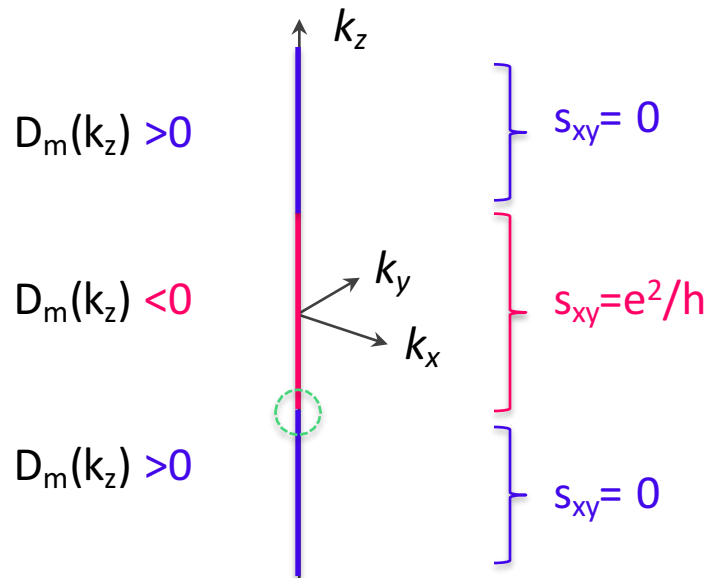
$$H_{\text{weyl}}(k_x, k_y, k_z) = k_x s_1 + k_y s_2 + D_m(k_z) s_3$$



# Anomalous Hall effect

$$H_{\text{weyl}}(k_x, k_y, k_z) = k_x s_1 + k_y s_2 + D_m(k_z) s_3$$

$$\sigma_{xy}^{2D}(k_z) = \frac{e^2}{2h} [1 - \text{sgn}(\Delta_m(k_z))]$$



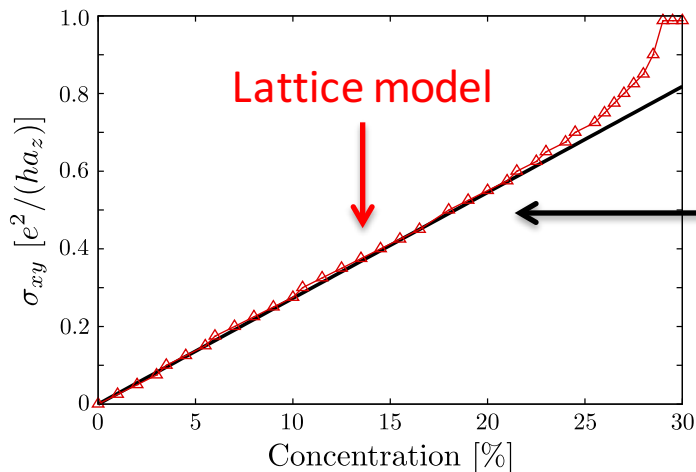
$$\begin{aligned} \sigma_{xy}^{3D} &= \int \frac{dk_z}{2\pi} \sigma_{xy}^{2D}(k_z) \\ &= \frac{e^2}{2\pi h} 2K_{\text{Weyl}} \end{aligned}$$



# Anomalous Hall effect

$$H_{\text{weyl}}(k_x, k_y, k_z) = k_x s_1 + k_y s_2 + D_m(k_z) s_3$$

$$\sigma_{xy}^{2D}(k_z) = \frac{e^2}{2h} [1 - \text{sgn}(\Delta_m(k_z))]$$



$$\begin{aligned} \sigma_{xy}^{3D} &= \int \frac{dk_z}{2\pi} \sigma_{xy}^{2D}(k_z) \\ &= \frac{e^2}{2\pi h} \underbrace{2K_{\text{Weyl}}}_{2JM} \end{aligned}$$

Kubo formula by D. Kurebayashi

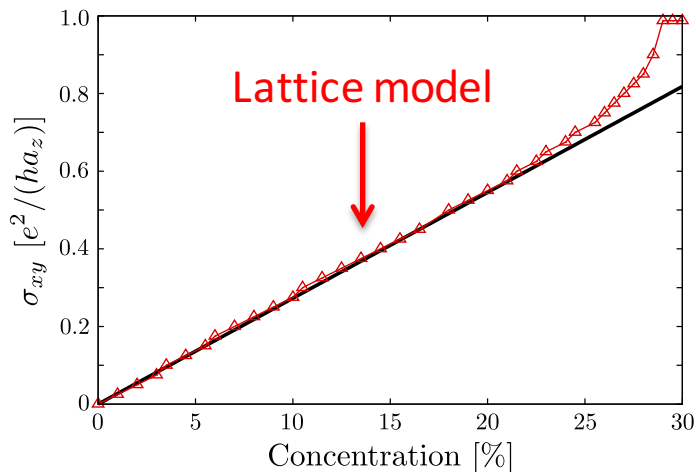
# Remarks

Magnetic order in TI as “Van Vleck ferromagnetism”.

Weyl semimetal can be realized in magnetic TIs.

(Cr-doped  $\text{Bi}_2(\text{Se}_x\text{Te}_{1-x})_3$ )

Anomalous Hall effect is a signature of the Weyl semimetal.



Kubo formula by D. Kurebayashi

