Waiting for rare entropic fluctuation in mesoscopic physics

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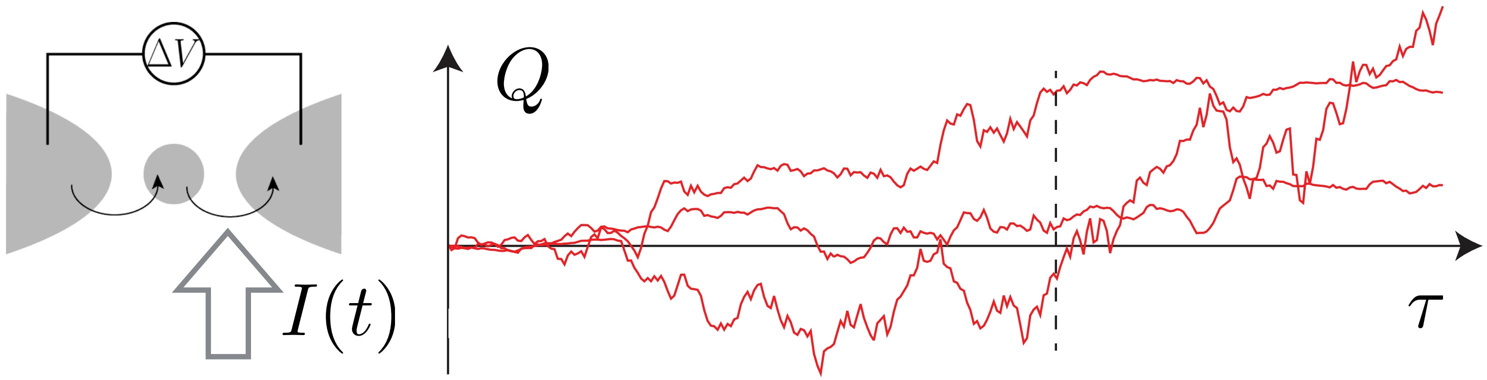
1. Counting statistics

◇ Measuring charge transfer



measure $I(t)$

accumurate $Q = \int_0^\tau dt I(t)$

◇ Statistics given at the "fixed" time

Probability $P(Q)$

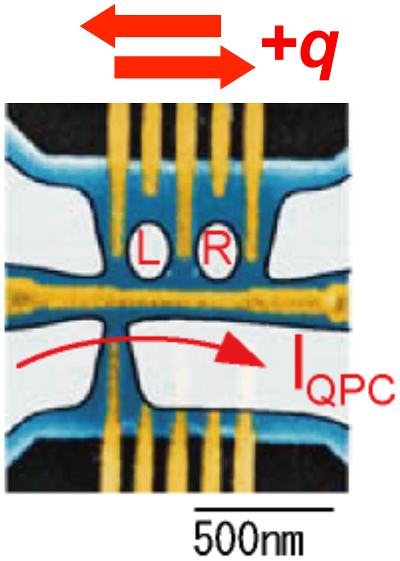
Cumulants $\langle Q^n \rangle_c$

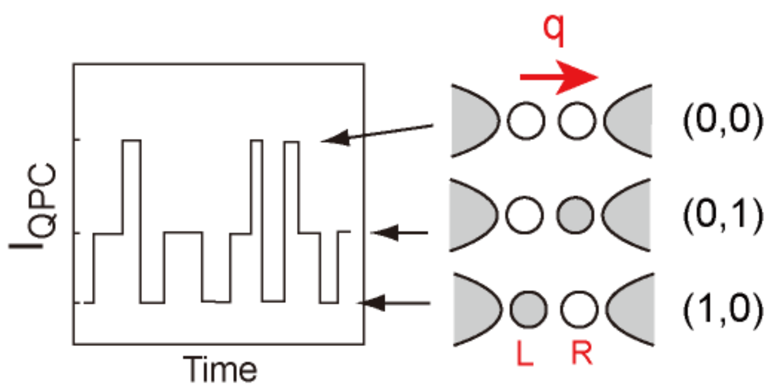
◇ One expects "information" from "fixed time statistics"

Examples of experiment

T. Fujisawa et al., Science (2006)

◇ Classical transport via coupled QDs

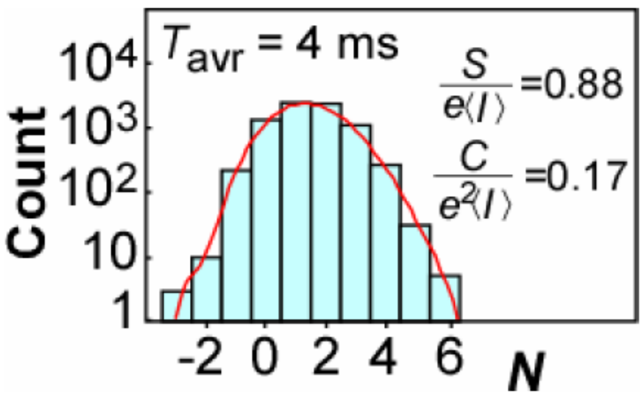




(1,0) → (0,1) forward tunneling

(0,1) → (1,0) backward tunneling

◇ Distribution of transmitted charge



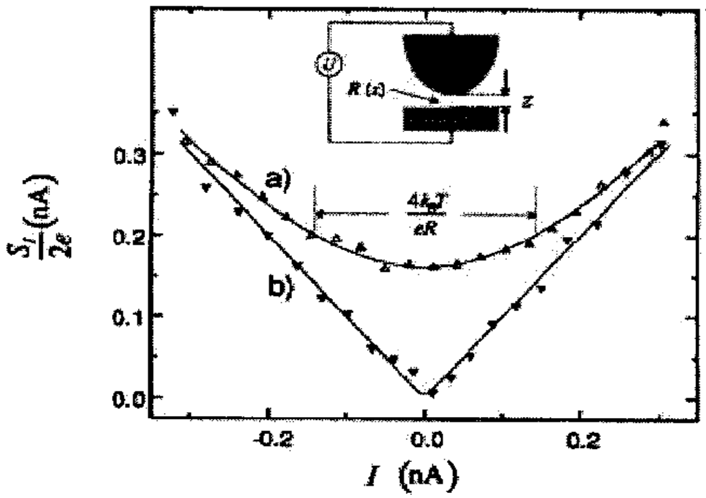
2. Information from "fixed time statistics"

◇ Zero temperature - Shot Noise -

Average current $I_1 \propto T$

Current noise $I_2 \propto T(1-T) \sim T \quad \text{if } T \ll 1$

Fano factor $F = \dfrac{I_2}{I_1} = 2e$ "Noise is the signal"



◇ Finite temperature ?

Fluctuation relation What is this ?

Fluctuation relation at the finite temperature

◇ Robust relation derived from time reversal symmetry

- Current context

$$P(-Q) \sim e^{-Q\beta\Delta V} P(Q)$$

- Entropy context (general)

$$P(-S) = e^{-S} P(S)$$

◇ This reproduces linear response results and predicts nonlinear response

Def. $\langle Q^n \rangle_c / \tau := \sum_k L_{n,k} (\beta \Delta V)^k / k!$

- FDT (Kubo formula) $L_{1,1} = L_{2,0}/2$
- Nonlinear response, e.g., $L_{1,2} = L_{2,1}$

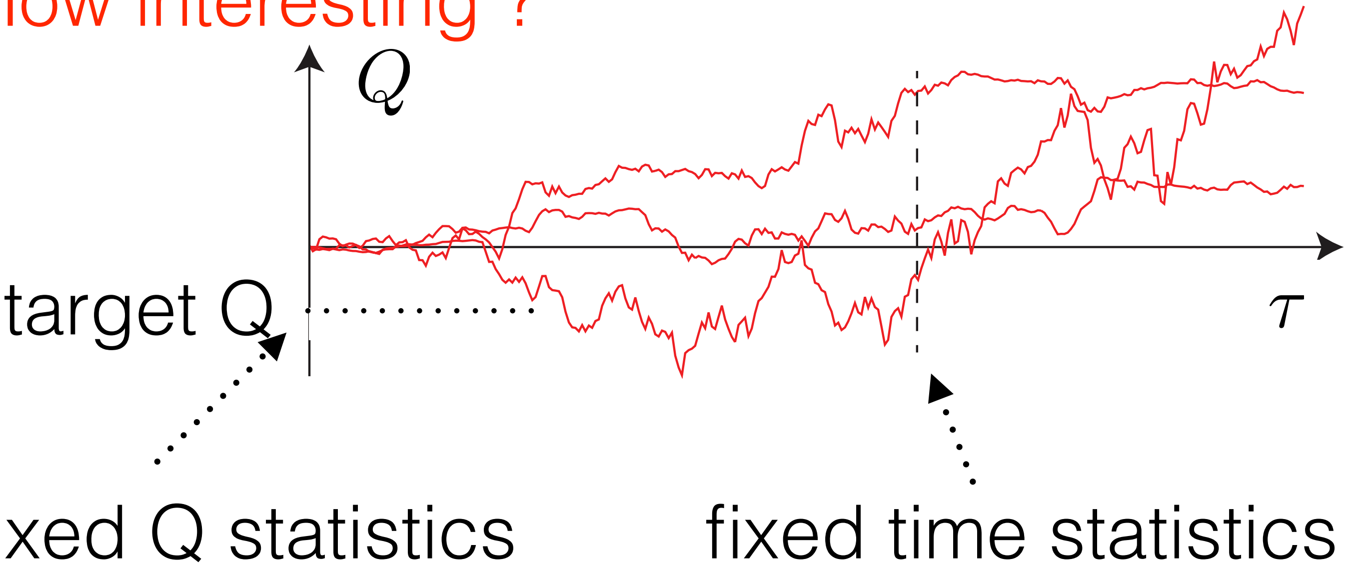
Today's talk

3. From fixed time to fixed Q statistics

◇ So far, statistics at the fixed time

- Questions -

What is fixed Q statistics ?
How formulated ?
Relation between fixed time and fixed Q physics ?
How interesting ?

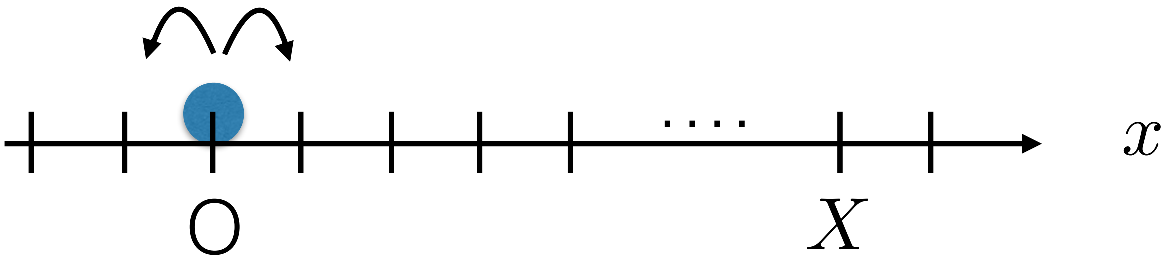


◇ Mathematically unambiguous statistics

: First passage time distribution (FPTD) to get Q

The simplest FPTD: random walk

◇ Biased random walk



◇ Distribution at large time

$$P(x,t) = \frac{1}{\sqrt{2\pi I_2 t}} e^{-\frac{(x-I_1 t)^2}{2I_2 t}} \qquad I_n = \left\langle \left(\int_0^\tau dt\, x(t) \right)^n \right\rangle_c \Big/ \tau \Big|_{\tau\to\infty}$$

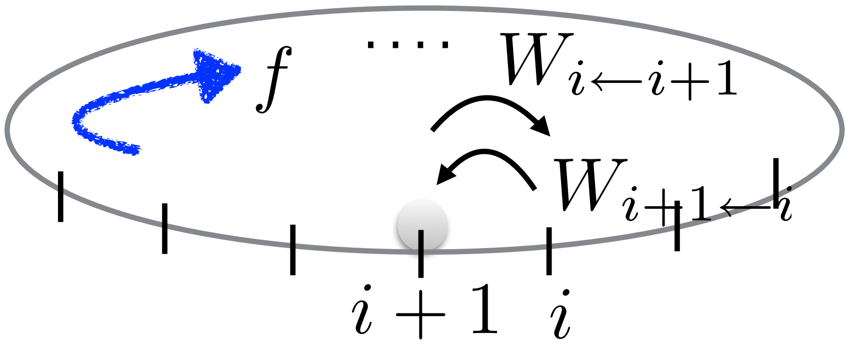
◇ First passage time distribution (FPTD) to reach X

$$\mathcal{F}_{rw}(X,t) = \frac{|X| e^{-\frac{(X-I_1 t)^2}{2I_2 t}}}{\sqrt{2\pi I_2 t^3}} \quad \longrightarrow e^{-\frac{I_1^2}{2I_2} t - \frac{3}{2}\log t}$$

The FPTD for entropic variables

Several models

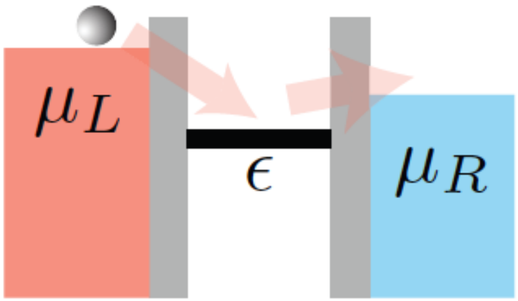
(a) Thermally hopping electronic systems



Experiments
S. Toyabe et al., Nature Physics(2010)
V. Blickle et al., PRL (2007)

FPTD for winding number

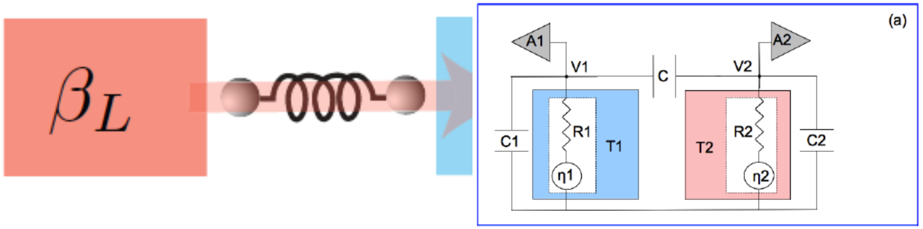
(b) Charge transfer via quantum dot (classical transport)



Experiments
T. Fujisawa et al., Science (2006)
B. Kung et al. ,Phys. Rev. X (2012)

FPTD for charge transfer

(c) Heat transfer via coupled spring system



Experiments
J. R. Gomez-Solano, Europhys Lett.(2010)
S. Ciliberto et al., PRL (2013)

FPTD for heat transfer

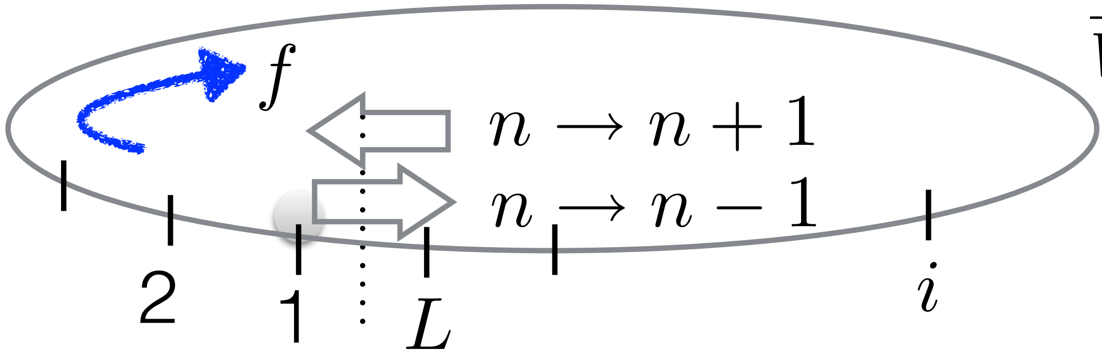
Model (a): thermally hopping electronic system
The FPTD for the winding number

◇ Master equation $\dot{P}_j(t) = \sum_{j'=j\pm 1} W_{j\leftarrow j'} P_{j'}(t) + W_{j,j} P_j(t)$

local detailed balance

$$\frac{W_{i+1\leftarrow i}}{W_{i\leftarrow i+1}} = e^{\beta(U_i - U_{i+1} + f)}$$

◇ Several definitions



Counting the number of passing the line: n

$T_{j\leftarrow i}(n,t)$: transition probability $(j,n) \leftarrow (i,0)$, $t \leftarrow t=0$

$F_{j\leftarrow i}(n,t)$: FPTD to get the pair $(j,n) \leftarrow (i,0)$

◇ FPTD for winding number $\mathcal{N}$

$$\mathcal{F}(\mathcal{N},t) = \sum_j F_{j\leftarrow j}(n=\mathcal{N},t) P_j^{SS}$$

Asymptotic behaviour of the FPTD

◇ Solving T and F with several basic equation like

$$T_{j\leftarrow i}(n,t) = \int_0^t du\, T_{j\leftarrow j}(0,t-u)F_{j\leftarrow i}(n,u)$$

◇ Asymptotic behaviour

Use of fluctuation relation symmetry yields

$$\mathcal{F}_{\mathrm{asym}}(t) = A(\mathcal{N})\exp\left(-\Gamma t - (3/2)\log t\right)$$

$$\Gamma = \frac{I_1^2}{2I_2} + \frac{I_3 I_1^3}{6I_2^3} + \frac{(3I_3^2 - I_2 I_4)I_1^4}{24 I_2^5} + \cdots$$

$$I_k = \langle \mathcal{N}^k \rangle_c / \tau \Big|_{\tau\to\infty}$$

Crucial observation on asymptotic behaviour

$$\mathcal{F}_{\mathrm{asym}}(\mathcal{N},t) = A(\mathcal{N}) \exp\left(-\Gamma t - (3/2)\log t\right)$$

$$\Gamma = \frac{I_1^2}{2I_2} + \frac{I_3 I_1^3}{6I_2^3} + \frac{(3I_3^2 - I_2 I_4)I_1^4}{24 I_2^5} + \cdots$$

$$I_k = \langle \mathcal{N}^k \rangle_c / \tau \Big|_{\tau\to\infty}$$

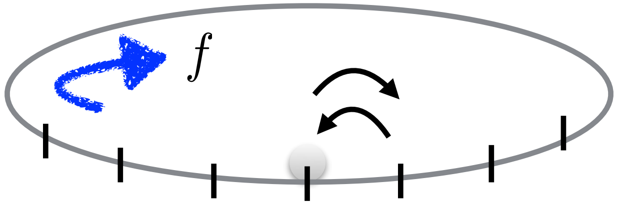
1. Asymptotic behaviour does not depend on $\mathcal{N}$
   (even negative winding follows the same form)

2. Relaxation rate is written with cumulants

3. First order reproduces random walk picture
   valid for linear response $(\text{small } I_1)$

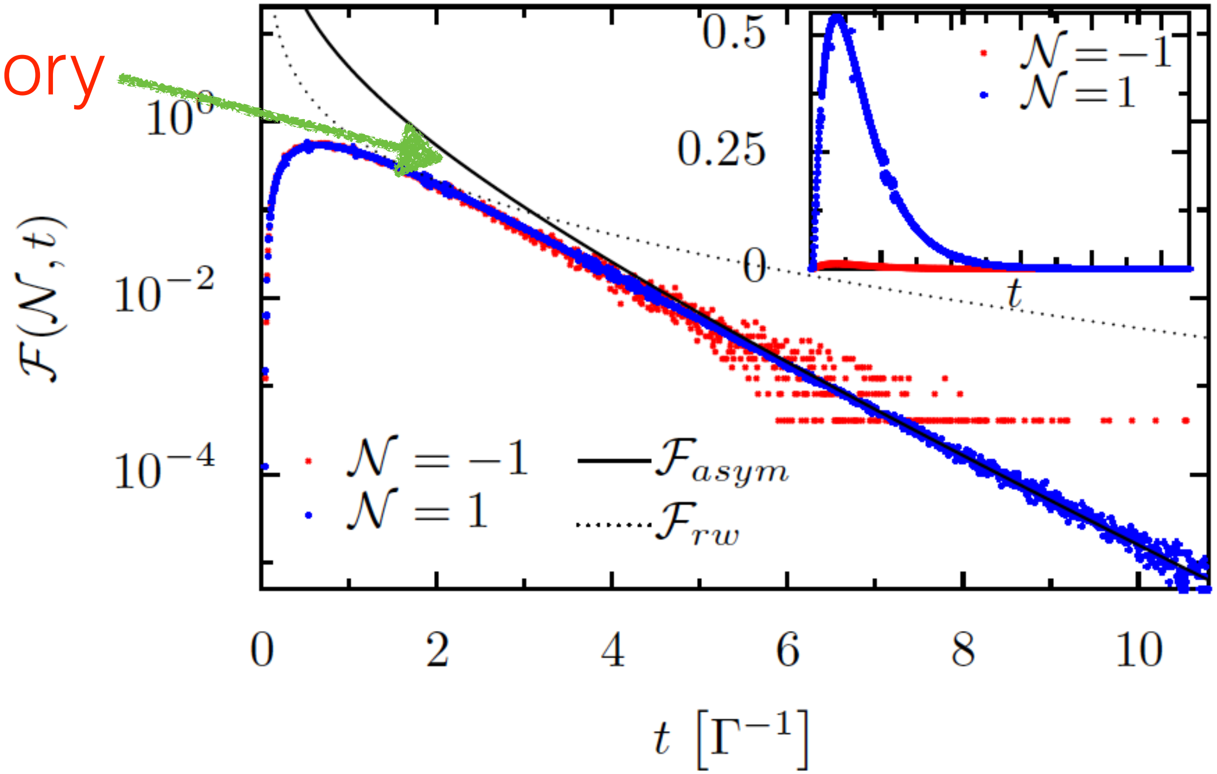
4. (3/2)log t correction

Numerical demonstration



◇ Normalized FPTD for winding number

Theory



This asymptotic expression is general

$$\mathcal{F}_{\rm asym}(\mathcal{X},t) = A(\mathcal{X}) \exp\left(-\Gamma t - (3/2)\log t\right)$$

$$\Gamma = \frac{I_1^2}{2I_2} + \frac{I_3 I_1^3}{6I_2^3} + \frac{(3I_3^2 - I_2 I_4)I_1^4}{24 I_2^5} + \cdots$$

$$I_k = \langle \mathcal{X}^k \rangle_c / \tau \Big|_{\tau\to\infty}$$

$\mathcal{X}$ is a physical quantity proportional to entropy production

(a) Thermally hopping electronic systems

$\mathcal{X} = \text{winding number}$

(b) Charge transfer via quantum dot

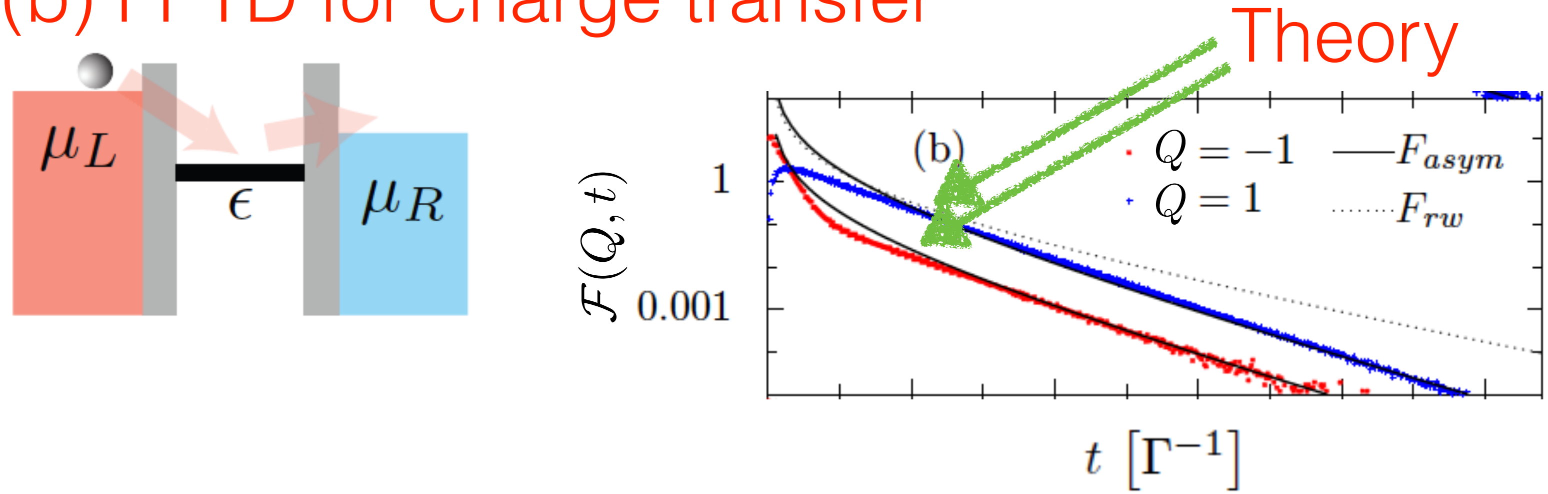
$\mathcal{X} = \text{charge transfer}$

(c) Heat transfer via coupled spring system

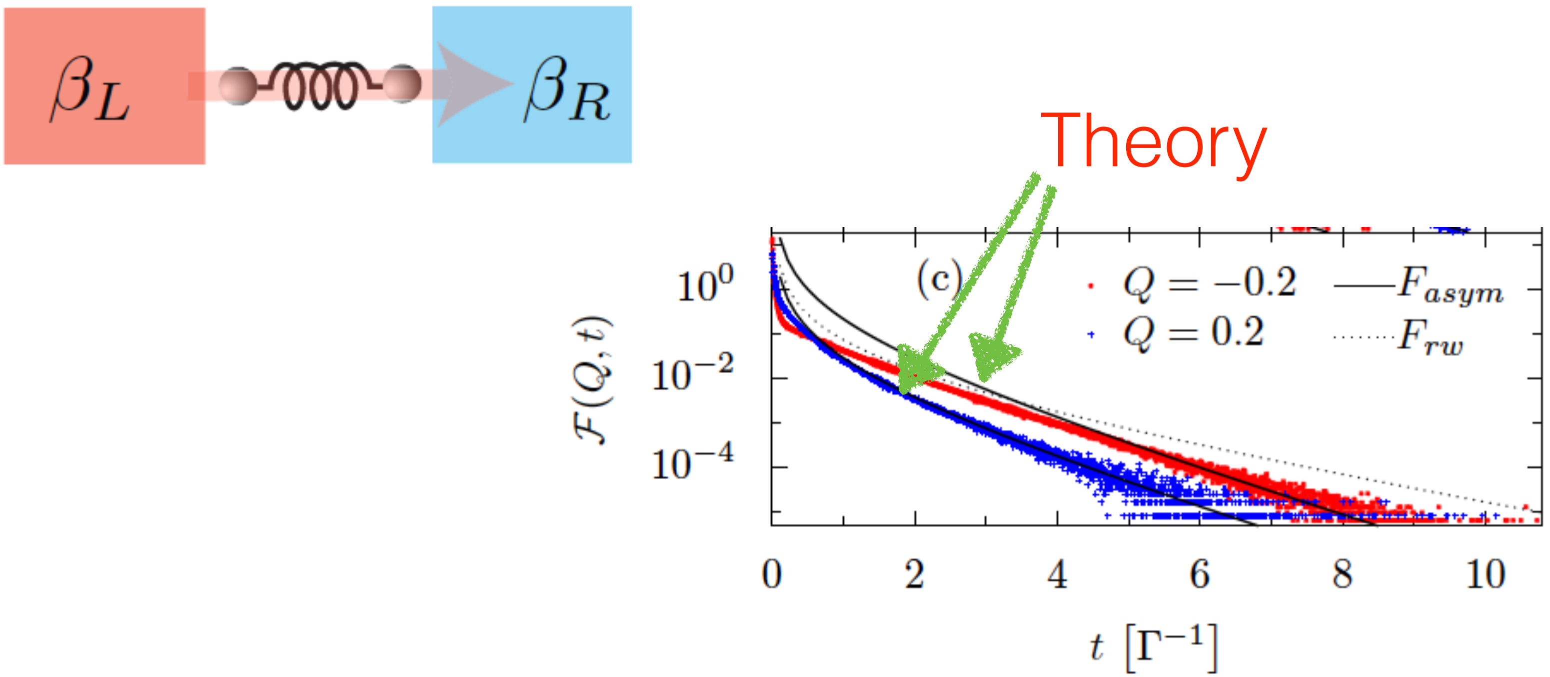
$\mathcal{X} = \text{heat transfer}$

Numerical demonstration

(b) FPTD for charge transfer



(c) FPTD for heat transfer



Summary

Ref.: arXiv:1504.02187

◇ We considered fixed target value statistics

◇ The first passage time distribution was studied (FPTD)

◇ The FPTD is connected to fixed time statistics in asymptotic behaviour

◇ Asymptotic behaviour has universal expression

Thank you for attention !