Spin-dependent thermoelectric effects and ~~spin-triplet-supercurrent~~ in mesoscopic superconductors

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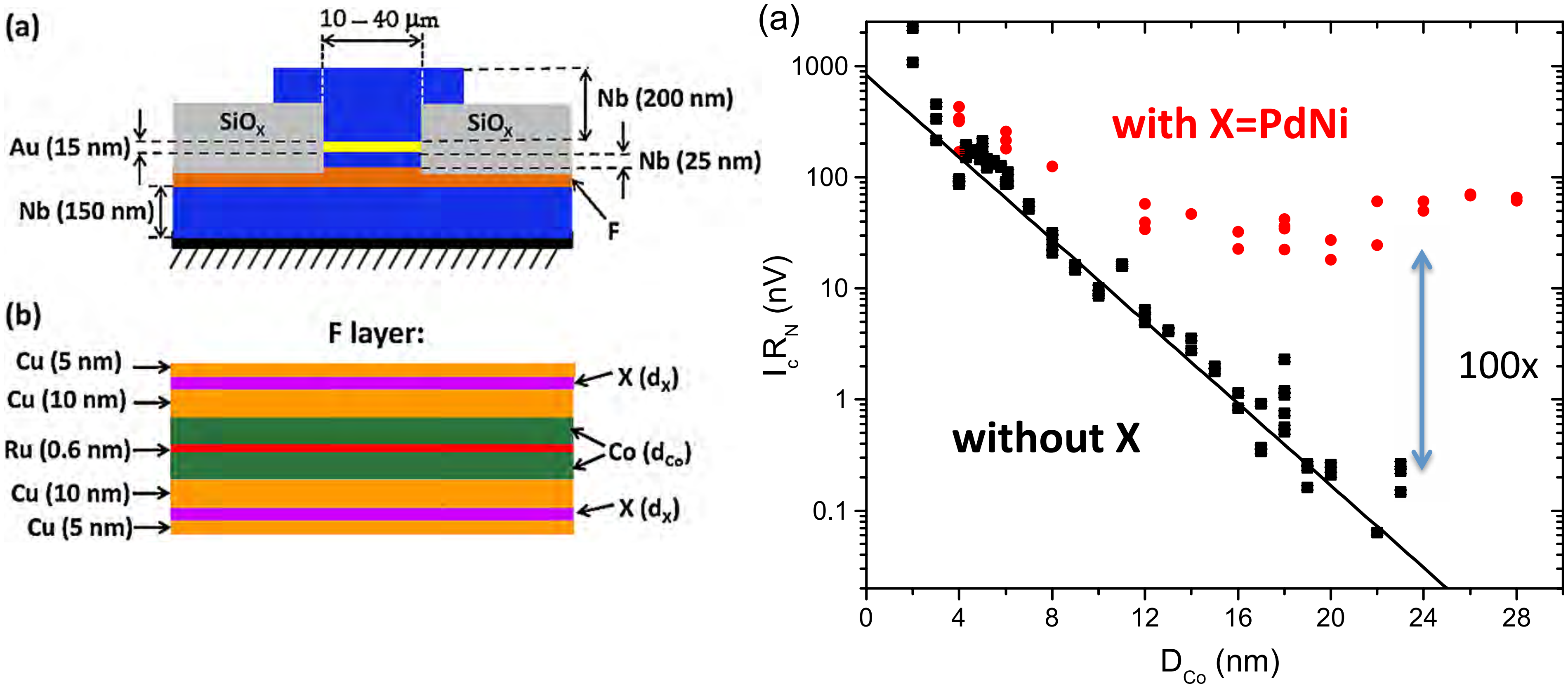
Phys. Rev. Lett. **110**, 047002 (2013)
New J. Phys. **16**, 073002 (2014)

Key experiments: Long-range triplet proximity effect in SXFFXS structures



- Josephson current through a ferromagnetic multilayer
- Adding non-collinear magnetic layers increase the critical current

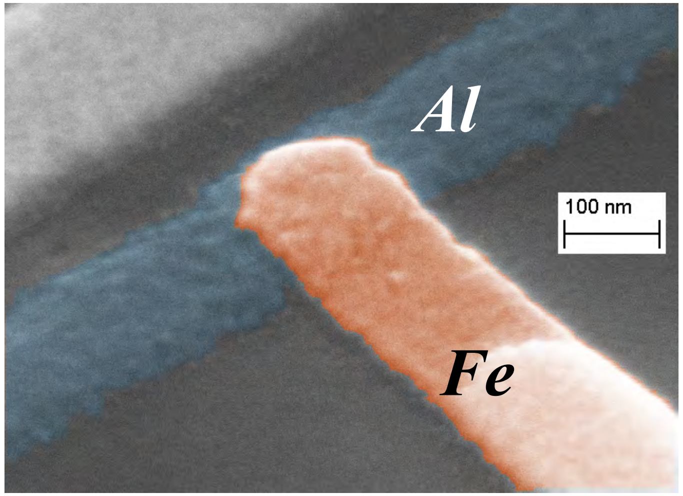
[Khaire, Khasawneh, Pratt, Birge, PRL (2010)]

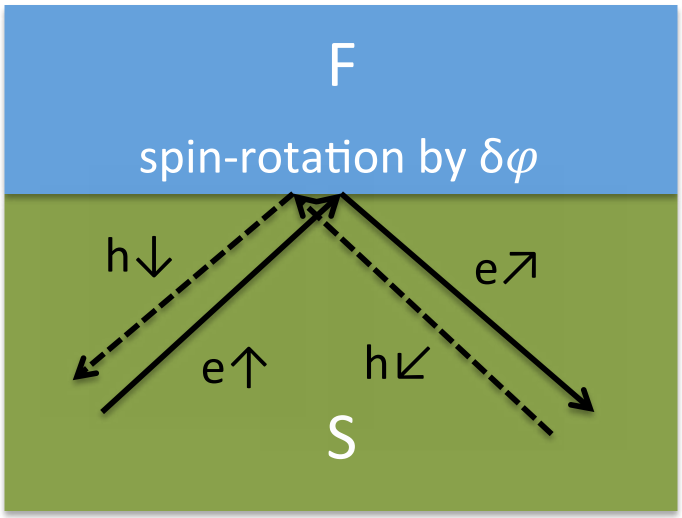
Similar experiment: adding spin-spiral ferromagnet (X=Ho)

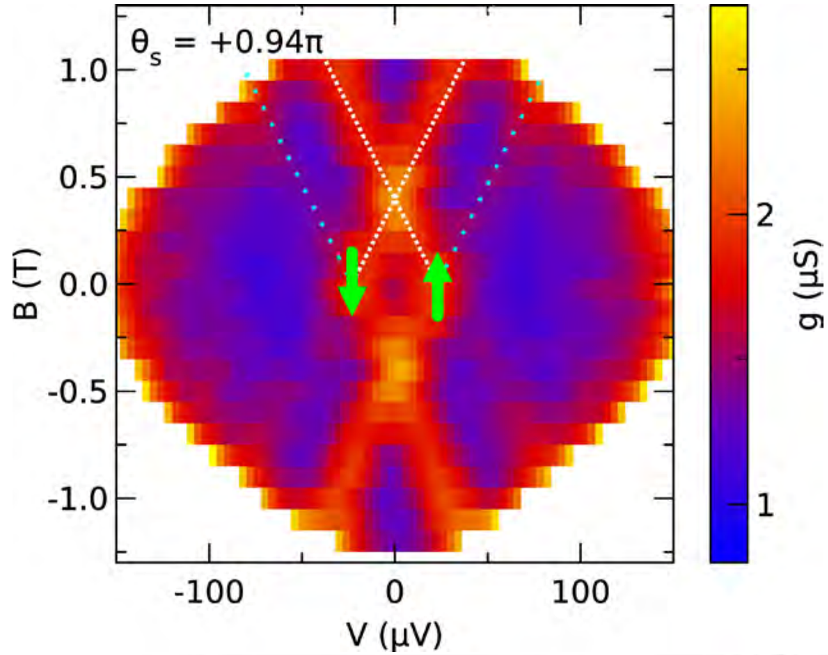
[Robinson, Witt, Blamire, Science (2010)]

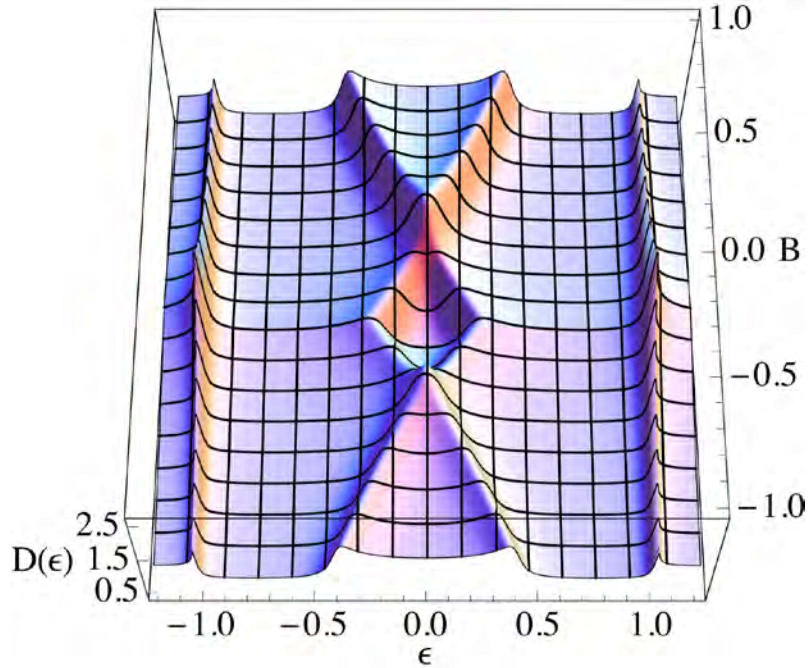
Experiment:

Andreev bound states at interfaces





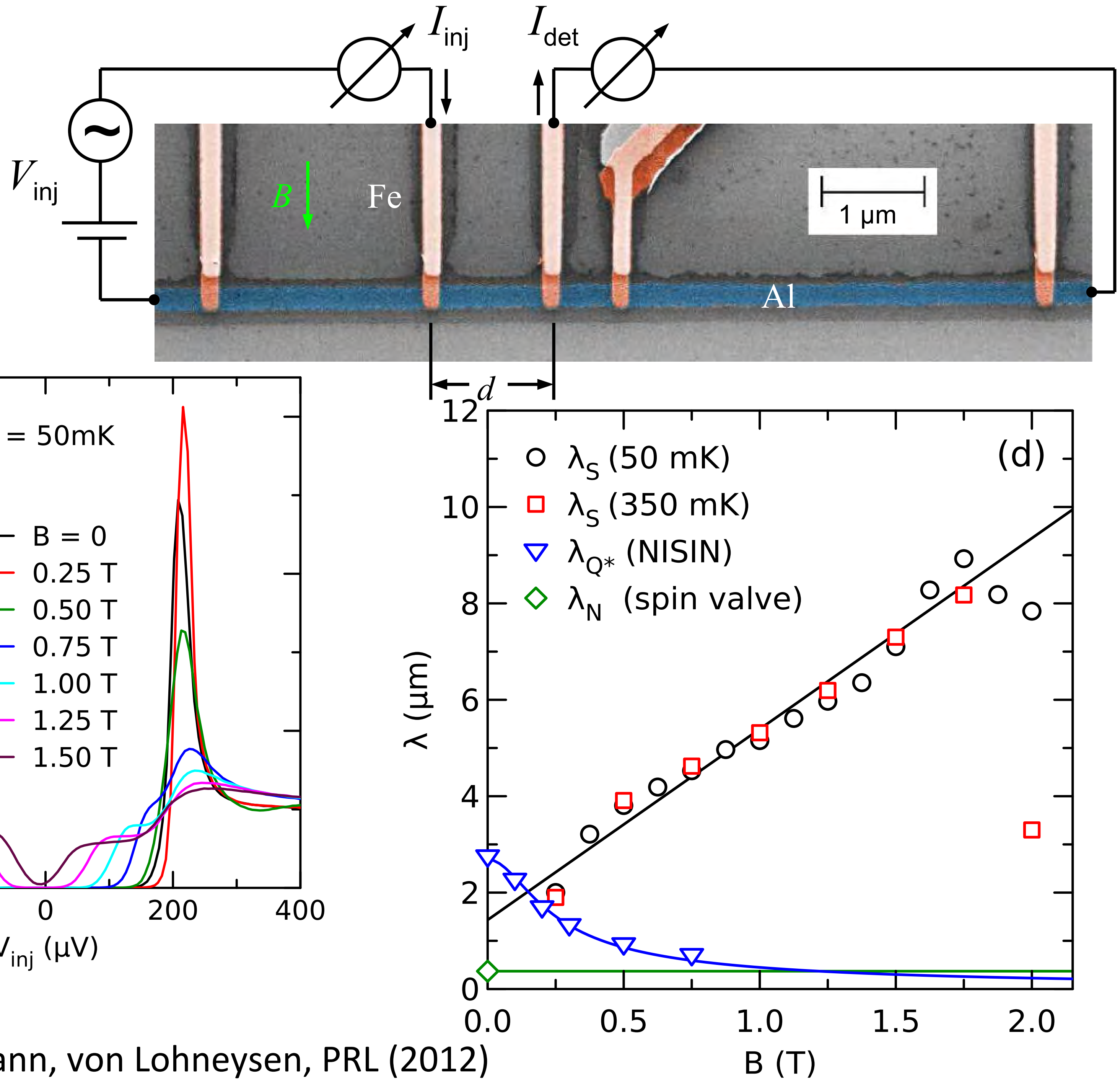




F. Hübler, M. J. Wolf, T. Scherer, D. Wang, D. Beckmann und H. v. Löhneysen,
Phys. Rev. Lett. **109**, 087004 (2012)

Theory: Machon, Belzig

Experiment:
Long-range
spin transport



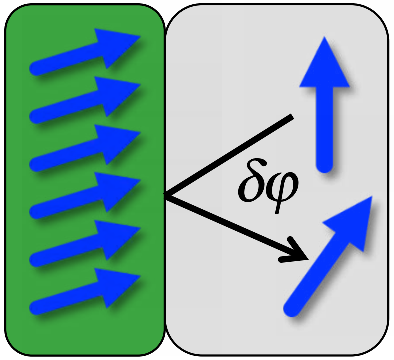
Hübler, Wolf, Beckmann, von Lohneysen, PRL (2012)
See also: Quay, Chevallier, Bena, Aprili, Nat. Phys. (2013).

Content:

- **Spin-mixing** in superconductors
- Seebeck effect and **thermoelectricity**
- Superconducting proximity spin-split density of states due to a **ferromagnetic insulator**
- **Spin-splitting + spin-polarized** tunneling as origin of a **large Seebeck effect**
- Maximizing **thermoelectric efficiency** in a superconductor-ferromagnet heterostructure
- **Spin Seebeck effect and spin injection**
- **Non-local Seebeck effect** in a multi-terminal structure

Spin mixing and interfacial phase shifts

Noncollinear FN transport



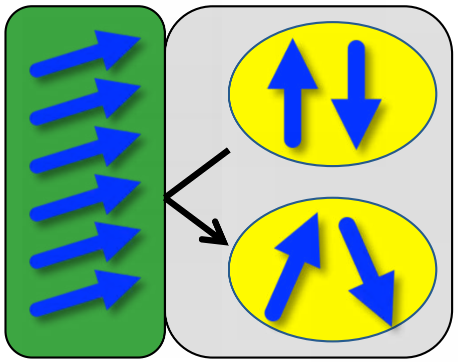
$$G_{\uparrow\downarrow} = \frac{e^2}{\hbar}\mathrm{Tr}\left[1 - r_\uparrow r_\downarrow^\dagger\right]$$

- Spin-pumping
- Gilbert damping
- Spin Seebeck

[Brataas, Nazarov, Bauer PRL 2000]

$T_{\uparrow\downarrow}, \delta\varphi \ll 1$

Spin-active interface FS transport



$$G^\phi = \frac{e^2}{\hbar}\sum_n \delta\varphi_n = \mathrm{Im}\, G_{\uparrow\downarrow}$$

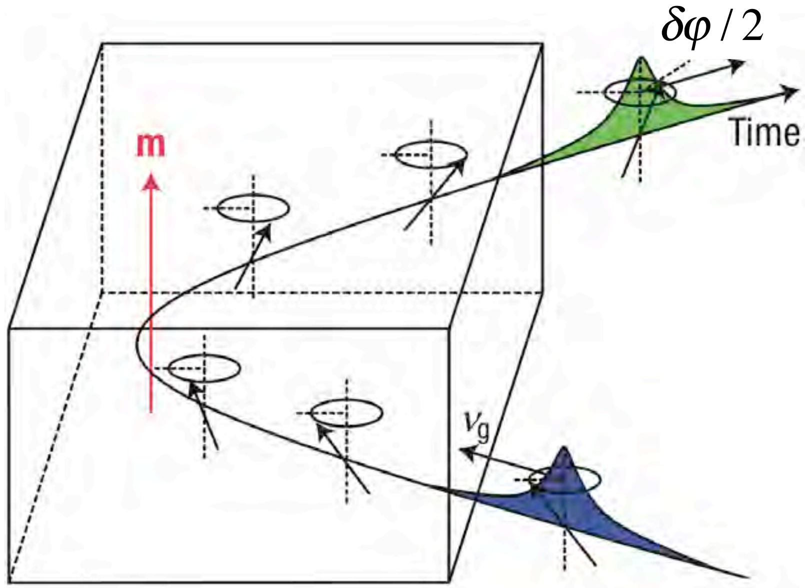
- Spin triplet pairing
- 100% spin valve
- Spin supercurrent

[Huertas, Nazarov, Belzig PRL 2002]

Phase shift of the reflection amplitudes

$$r_\uparrow = \left|r_\uparrow\right| e^{i\varphi_\uparrow} \qquad r_\downarrow = \left|r_\downarrow\right| e^{i\varphi_\downarrow}$$

$$\delta\varphi = \varphi_\uparrow - \varphi_\downarrow (= \vartheta)$$



Effective spin torque → acts on Cooper pair wave function

[Eschrig and Löfwander, Nat. Phys. 2003]

Triplet Cooper pair generation due to spin-dependent scattering

$$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

singlet

$F^z \uparrow$ $\quad S_\sigma^z = \begin{pmatrix} r_\sigma^z & t'^z_\sigma \\ t_\sigma^z & r'^z_\sigma \end{pmatrix}$

$$\frac{1}{\sqrt{2}}\left(t_\downarrow^z t_\uparrow^{z*}|\uparrow\downarrow\rangle - t_\uparrow^z t_\downarrow^{z*}|\downarrow\uparrow\rangle\right) = |t_\uparrow^z t_\downarrow^z|\left[\cos(\vartheta_s)\underbrace{\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}}_{|S=0\rangle} - i\sin(\vartheta_s)\underbrace{\frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}}_{|S=1,M=0\rangle}\right]$$

Spin-dependent interfacial phase shifts → **Effective magnetic field**

$$|t_\uparrow^z t_\downarrow^z|\frac{-i\sin(\vartheta_s)}{\sqrt{2}}\left(|\uparrow_x\uparrow_x\rangle + |\downarrow_x\downarrow_x\rangle\right)$$

Equal spin triplet!

$F^x \rightarrow$ $\quad S_\sigma^x = \begin{pmatrix} r_\sigma^x & t'^x_\sigma \\ t_\sigma^x & r'^x_\sigma \end{pmatrix}$

$t_\downarrow^x = 0$

$$T_\uparrow^x |t_\uparrow^z t_\downarrow^z|\frac{-i\sin(\vartheta_s)}{\sqrt{2}}|\uparrow_x\uparrow_x\rangle$$

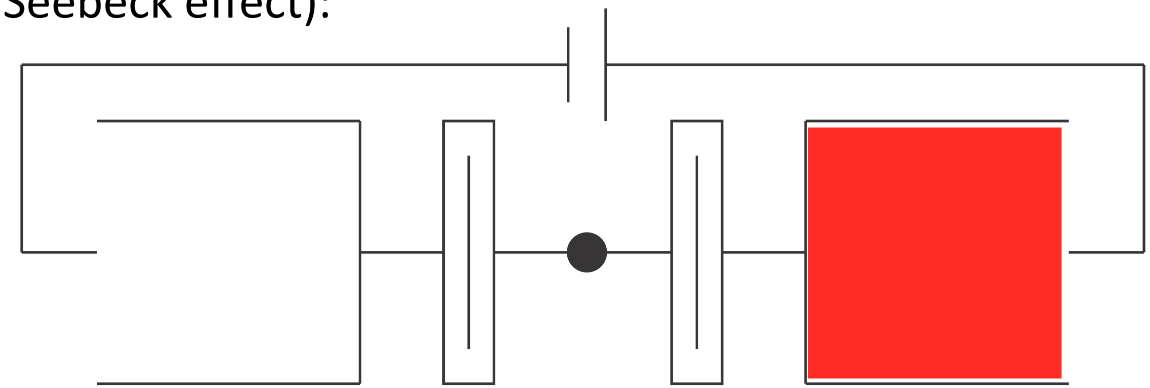
Theories: Bergeret, Volkov, Efetov (01); Eschrig, et al. (03); Nazarov; Tanaka, Golubov, Linder, ….

Thermoelectric response matrix:

Linear response for charge and energy current:

$$\begin{pmatrix} I^q \\ I^\epsilon \end{pmatrix} = \begin{pmatrix} L^{qV} & L^{qT} \\ L^{\epsilon V} & L^{\epsilon T} \end{pmatrix} \begin{pmatrix} \Delta V \\ -\Delta T/T \end{pmatrix}$$

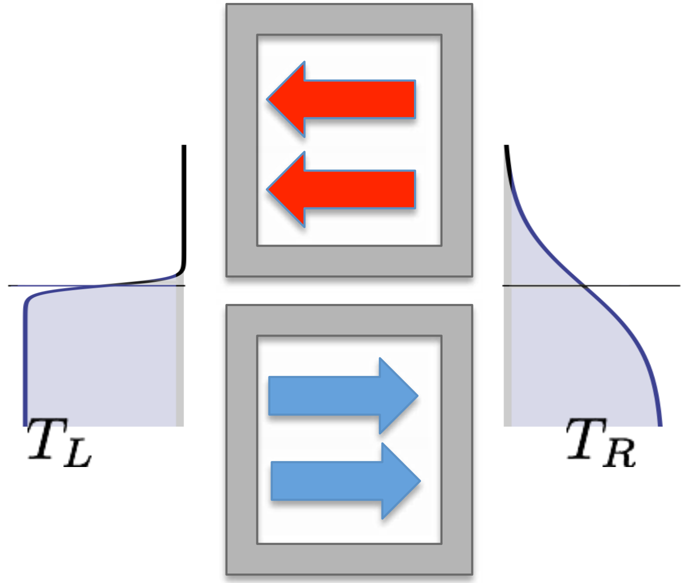
Thermopower (Seebeck effect):



$$I^q \stackrel{!}{=} 0 \;\Rightarrow\; \Delta V = \underbrace{\frac{1}{T}\frac{L^{qT}}{L^{qV}}}_{S} \Delta T$$

Relation of Seebeck and Peltier $L^{qT} = L^{\epsilon V}$ Onsager symmetry

Seebeck effect: the microscopic picture

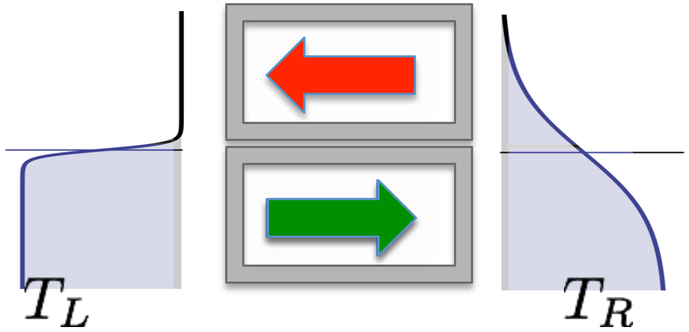


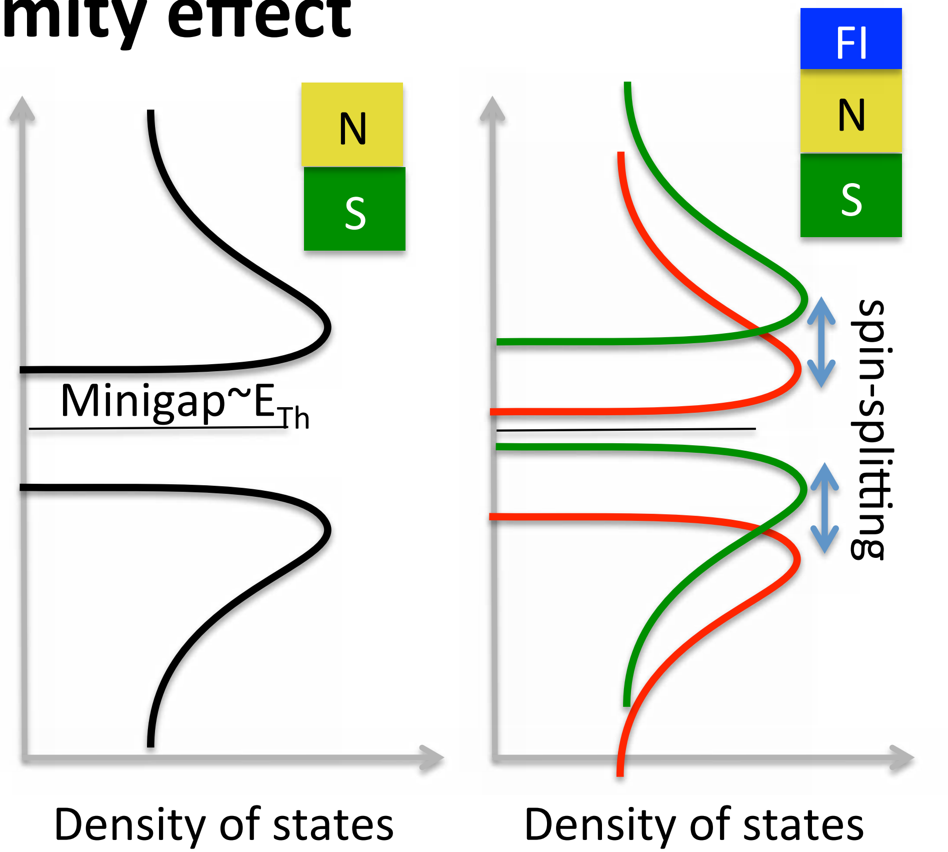
Thermoelectric coefficient related to energy asymmetry:

$$L^{qT} \sim \left|\epsilon\right| \left[D(\epsilon) - D(-\epsilon) \right]_{\epsilon = k_B T, eV} \underset{\text{(linearized)}}{\overset{\text{Mott}}{\approx}} \left(k_B T\right)^2 \left. \frac{dD(\epsilon)}{d\epsilon} \right|_{\epsilon = \epsilon_F}$$

[see e.g. Giazotto, Heikkila, Luukanen, Savin, and Pekola, Rev. Mod. Phys. **78**, 217 (2006)]

Tayloring the energy-dependent density of states by the superconducting proximity effect





Spin-split density of states combined with spin-polarized tunneling:

$$T = \left(T_\uparrow + T_\downarrow\right)/2 \qquad P = \left(T_\uparrow - T_\downarrow\right)/2T$$

$$T_\uparrow D_\uparrow(\epsilon) + T_\downarrow \underbrace{D_\downarrow(\epsilon)}_{D_\uparrow(-\epsilon)} = T\underbrace{\left[D_\uparrow(\epsilon) + D_\uparrow(-\epsilon)\right]}_{\text{even in }\epsilon} + TP\underbrace{\left[D_\uparrow(\epsilon) - D_\uparrow(-\epsilon)\right]}_{\text{odd in }\epsilon}$$

[Machon, Eschrig, Belzig, Phys. Rev. Lett. **110**, 047002 (2013)]

Quasiclassical Greens functions for superconductivity

Quantum Circuit Theory (Nazarov 1999)

$$\hat{G}_c = -i\left\langle \mathcal{T}_c \Psi(x,t)\Psi^\dagger(x,t')\right\rangle \quad \Psi = \left(\psi_\uparrow,\psi_\downarrow,\psi_\downarrow^\dagger,-\psi_\uparrow^\dagger\right)$$

Nodes (dwell time):

Leakage of coherence

$$\hat{I}_L(\varepsilon) = -iG_\Sigma \frac{\varepsilon}{\varepsilon_{Th}}\left[\hat{\tau}_3,\hat{G}_c\right]$$

Connectors (contacts)

Standard tunneling conductance

$$G_T = G_Q \sum_n \left(T_{n\uparrow} + T_{n\downarrow}\right)$$

➔ Usual charge current

Spin-polarization conductance

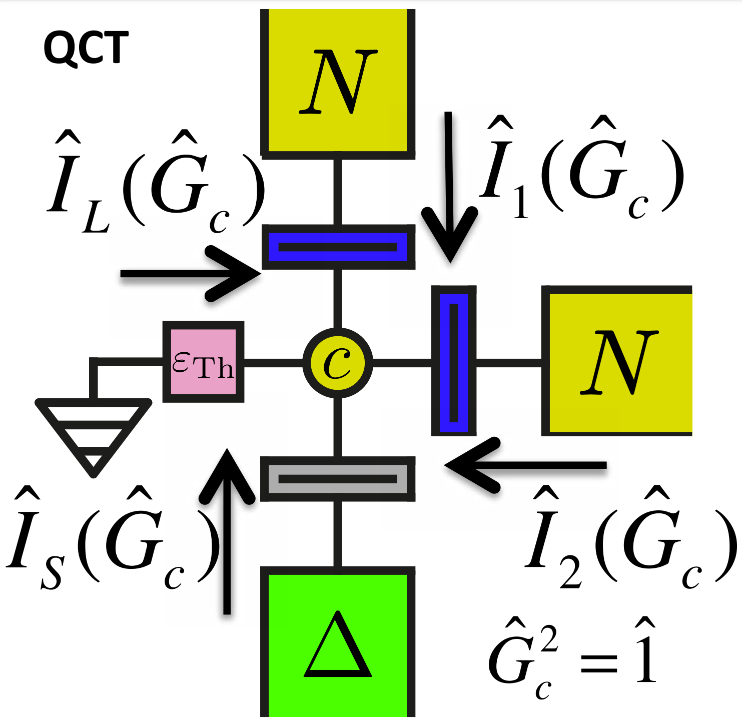
$$G_P = G_Q \sum_n \left(T_{n\uparrow} - T_{n\downarrow}\right)$$

➔ Spin-polarized current

Spin-dependent interfacial phase shifts

$$G_\phi = G_Q \sum_n \delta\phi_n$$

➔ Induced exchange splitting



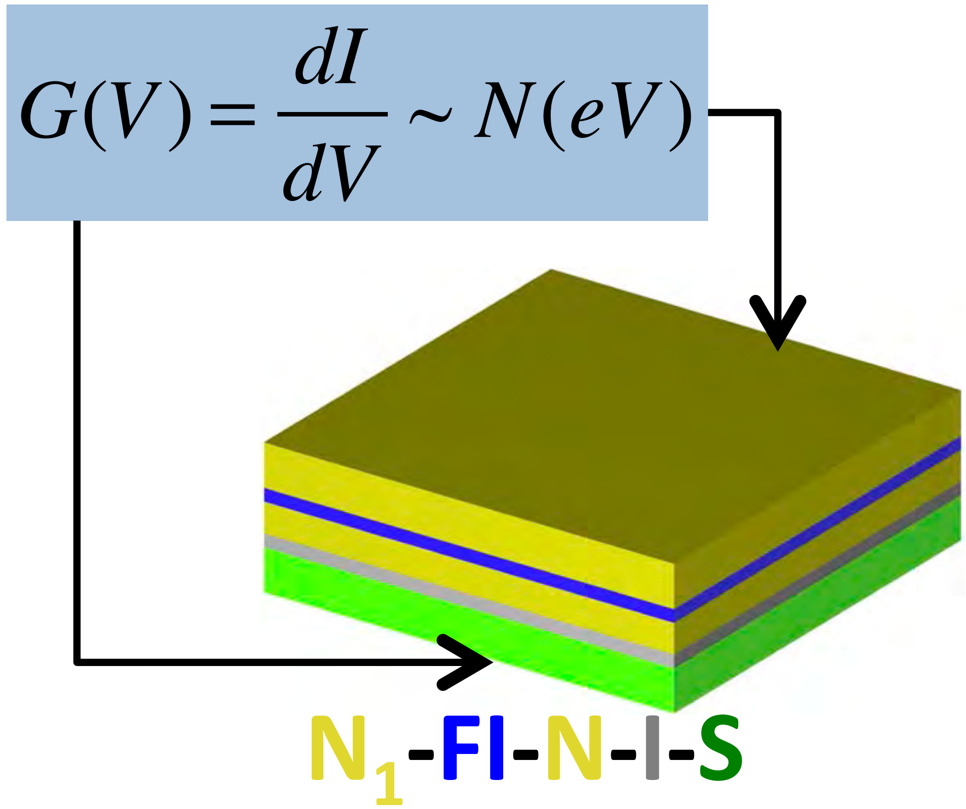
Total matrix current conservation

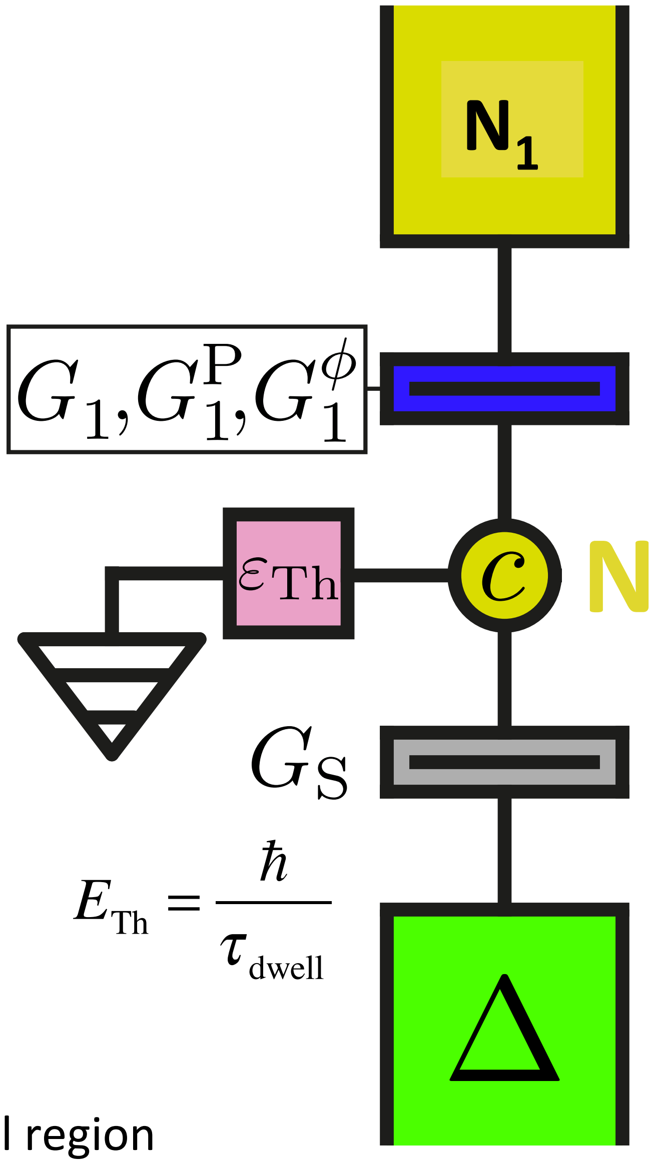
$$\hat{I}_S + \hat{I}_1 + \hat{I}_2 + \hat{I}_L = 0$$

Huertas-Hernando, Belzig, Nazarov, PRL (2001)
Cottet, Huertas-Hernando, Belzig, Nazarov, PRB (2009)
Machon, Eschrig, Belzig, PRL (2013)

The density of states in a proximity metal with magnetic contacts

"stacked structure"





Measure of coupling strength: **Thouless energy E_{th}**

- weak coupling = small E_{Th} = weak proximity
- strong coupling = large E_{Th} = strong proximity

Related to the inverse mean **dwell time** in the central region

[Machon, Eschrig, Belzig, Phys. Rev. Lett. **110**, 047002 (2013)]

Spin-dependent the density of states in N

Spin polarization

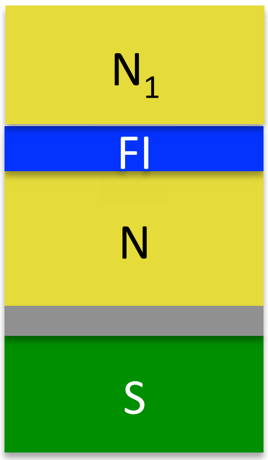
$$\frac{D_{\uparrow}(\epsilon)-D_{\downarrow}(\epsilon)}{D(\epsilon)}$$

Red: spin up

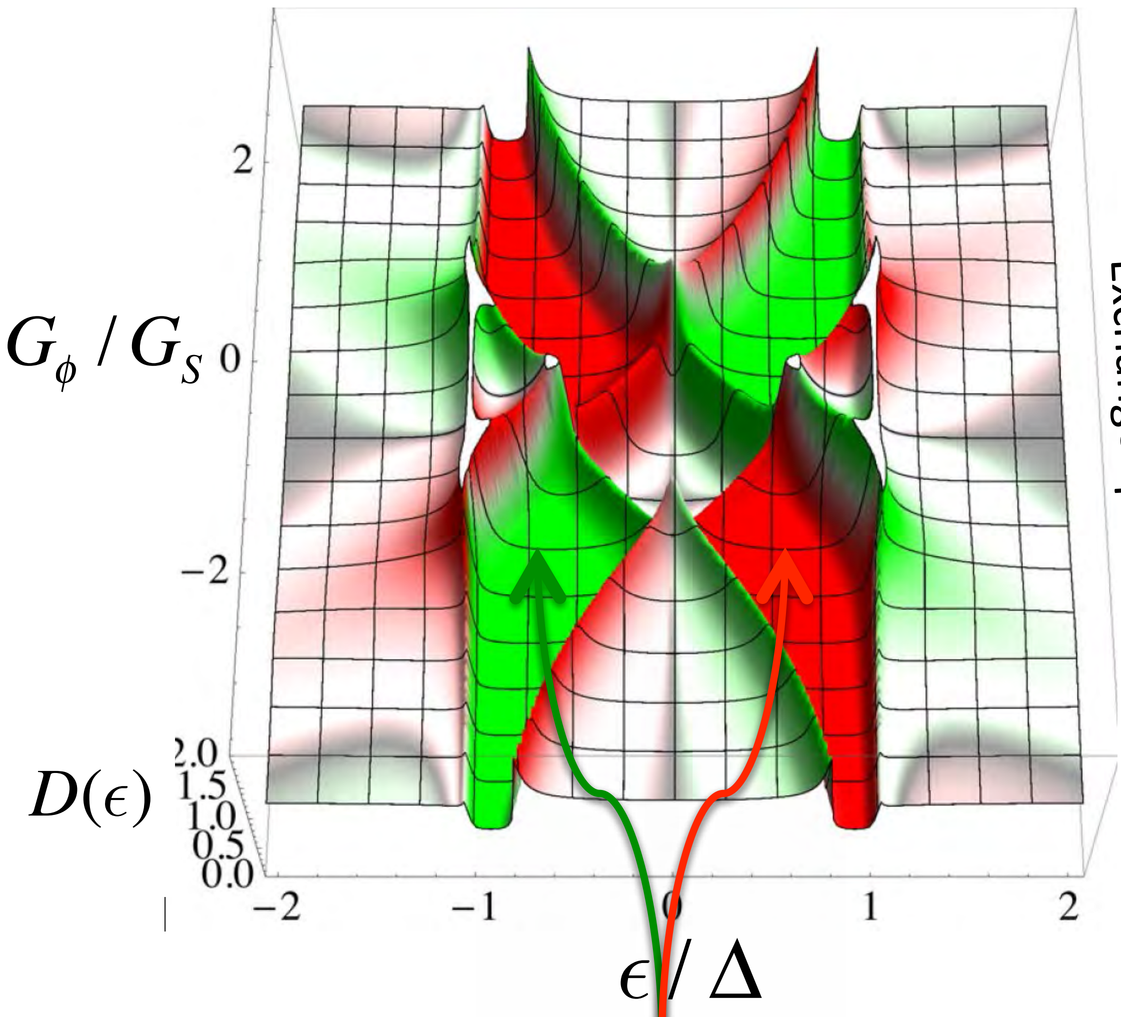
Green: spin down

$$G_1=\frac{G_S}{10}$$

$$E_{Th}=\Delta$$

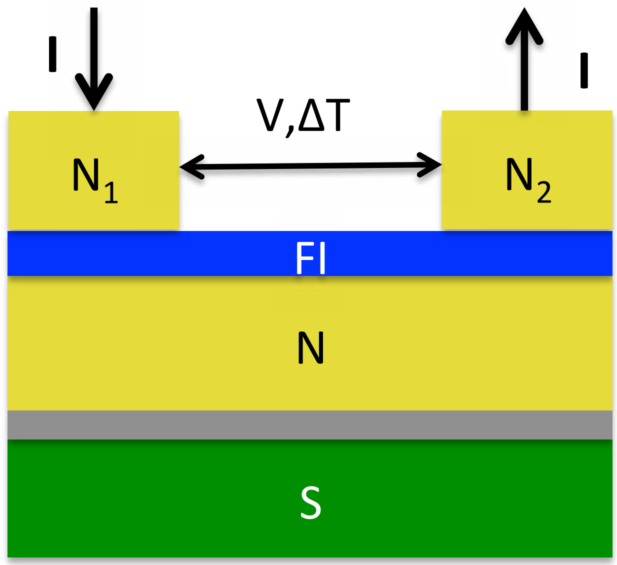


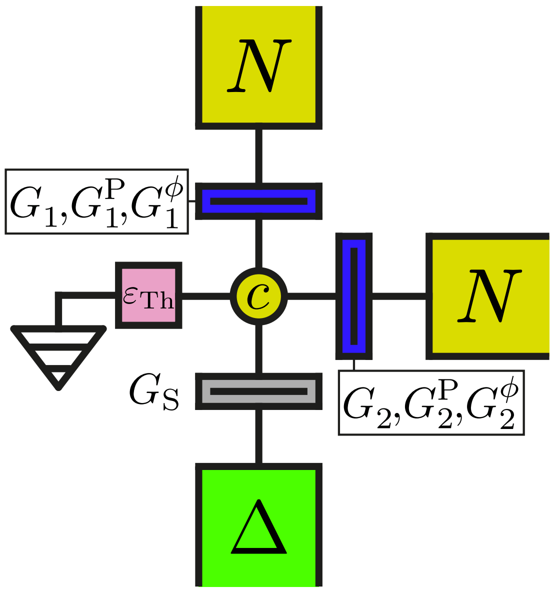
Total DOS: $D(\epsilon)=D_{\uparrow}(\epsilon)+D_{\downarrow}(\epsilon)$



100% spin-polarized energy bands!

Spincaloritronics with an SF-heterostructure: Spinthermoelectric “transistor” structure





Spin/charge/energy currents due to voltage/temperature gradient

Thermoelectric response coefficients

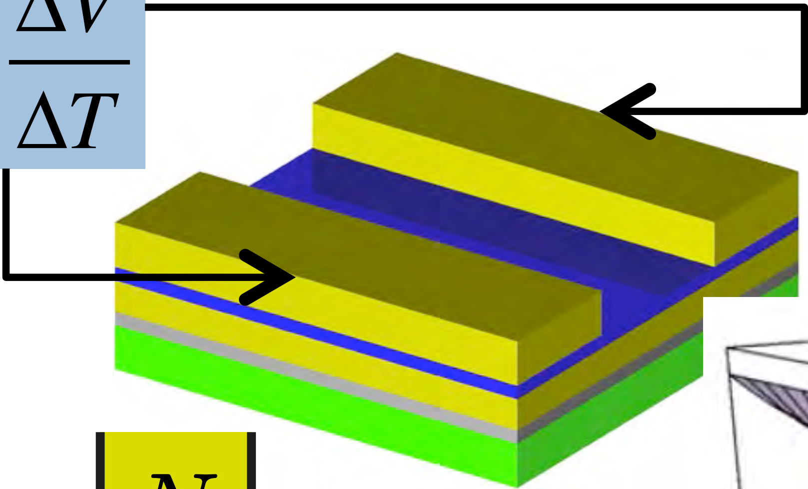
Spin-voltage response

$$\begin{pmatrix} I \\ I^{\epsilon} \\ \vec{I}^{s} \end{pmatrix} = \begin{pmatrix} L^{qV} & L^{qT} & \vec{L}^{qs} \\ L^{\epsilon V} & L^{\epsilon T} & \vec{L}^{\epsilon s} \\ \vec{L}^{sV} & \vec{L}^{sT} & \hat{L}^{ss} \end{pmatrix} \begin{pmatrix} V \\ \Delta T / T \\ \Delta \vec{\mu}_{s} \end{pmatrix}$$

Spin-injection and -Seebeck response

Transistor thermopower

$$S = \frac{\Delta V}{\Delta T}$$



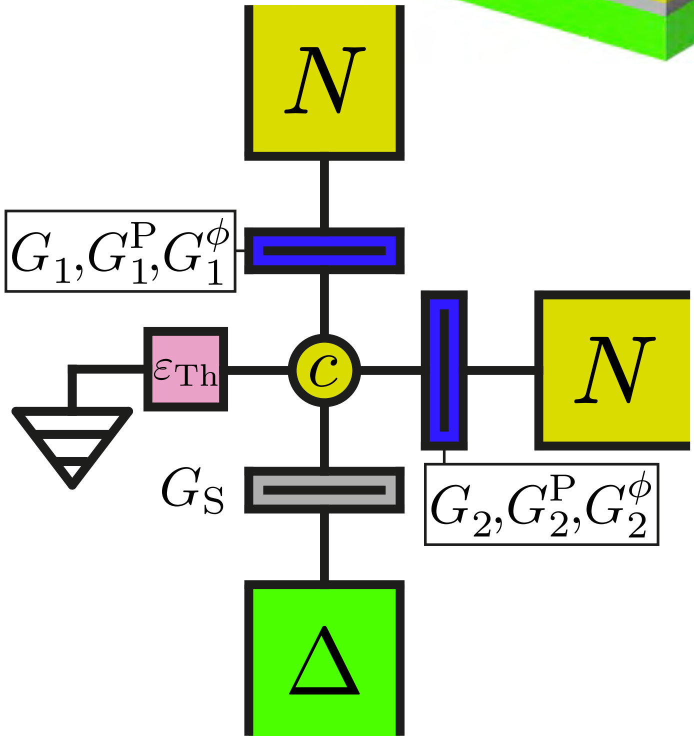
$$S = -\frac{1}{T}\frac{L^{qT}}{L^{qV}}$$

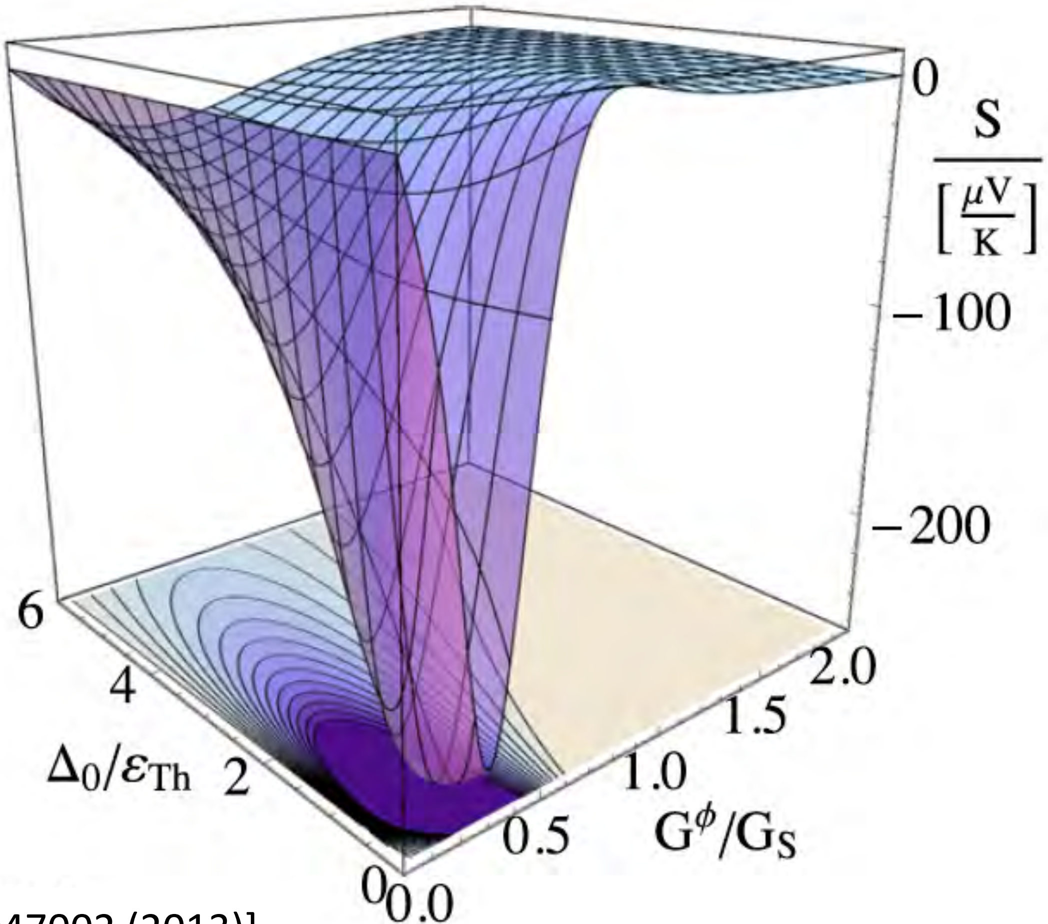
$$G_1 = G_2 = \frac{G_S}{100}$$

$$T = 0.1T_c$$

$$P_1 = P_2 = 90\%$$

Giant Seebeck coefficient!





[Machon, Eschrig, Belzig, Phys. Rev. Lett. **110**, 047002 (2013)]

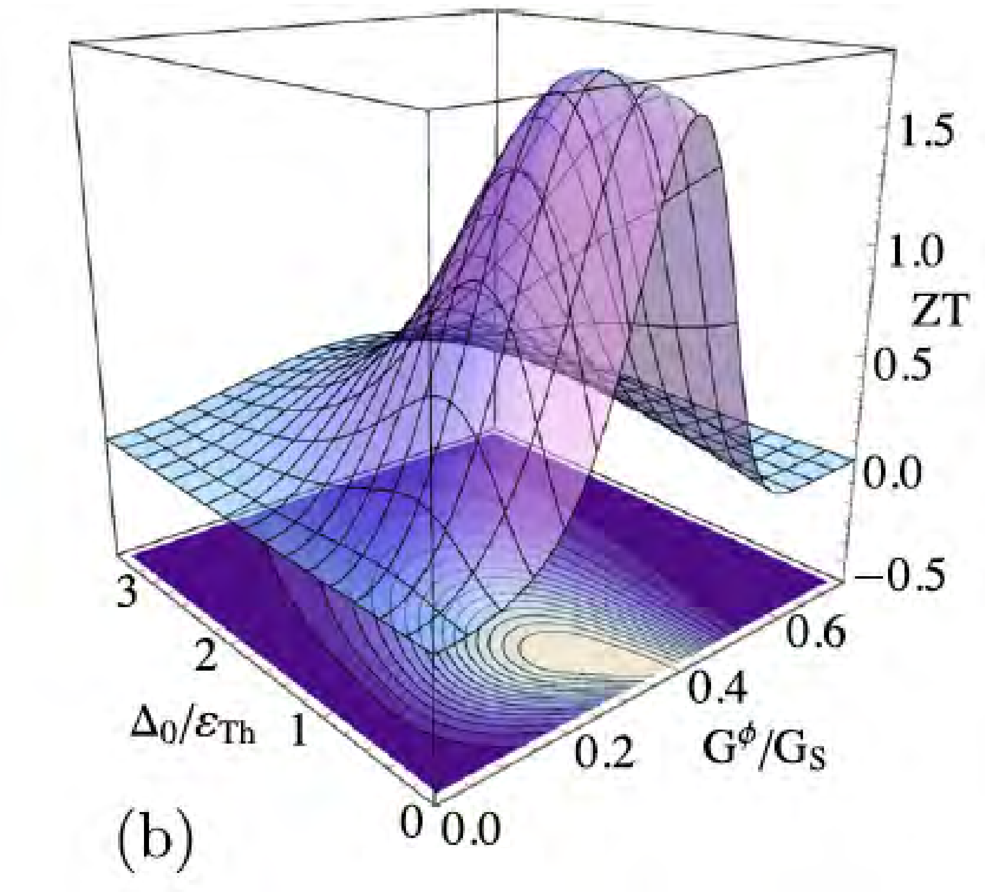
Thermoelectric figure of merit

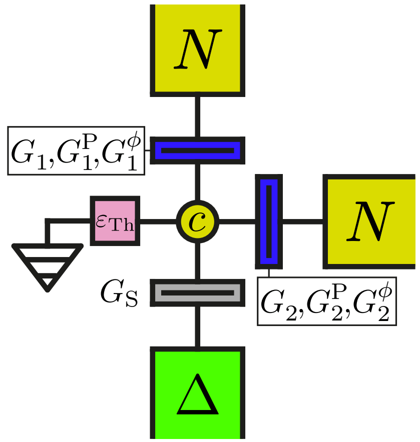
$$ZT = \frac{GS^2T}{\kappa} = \frac{(L^{qT})^2}{L^{qV}L^{\varepsilon T} - (L^{qT})^2}$$

$$G_1 = G_2 = G_S / 10$$

$$T = 0.1T_c$$

$$P_1 = P_2 = 90\%$$

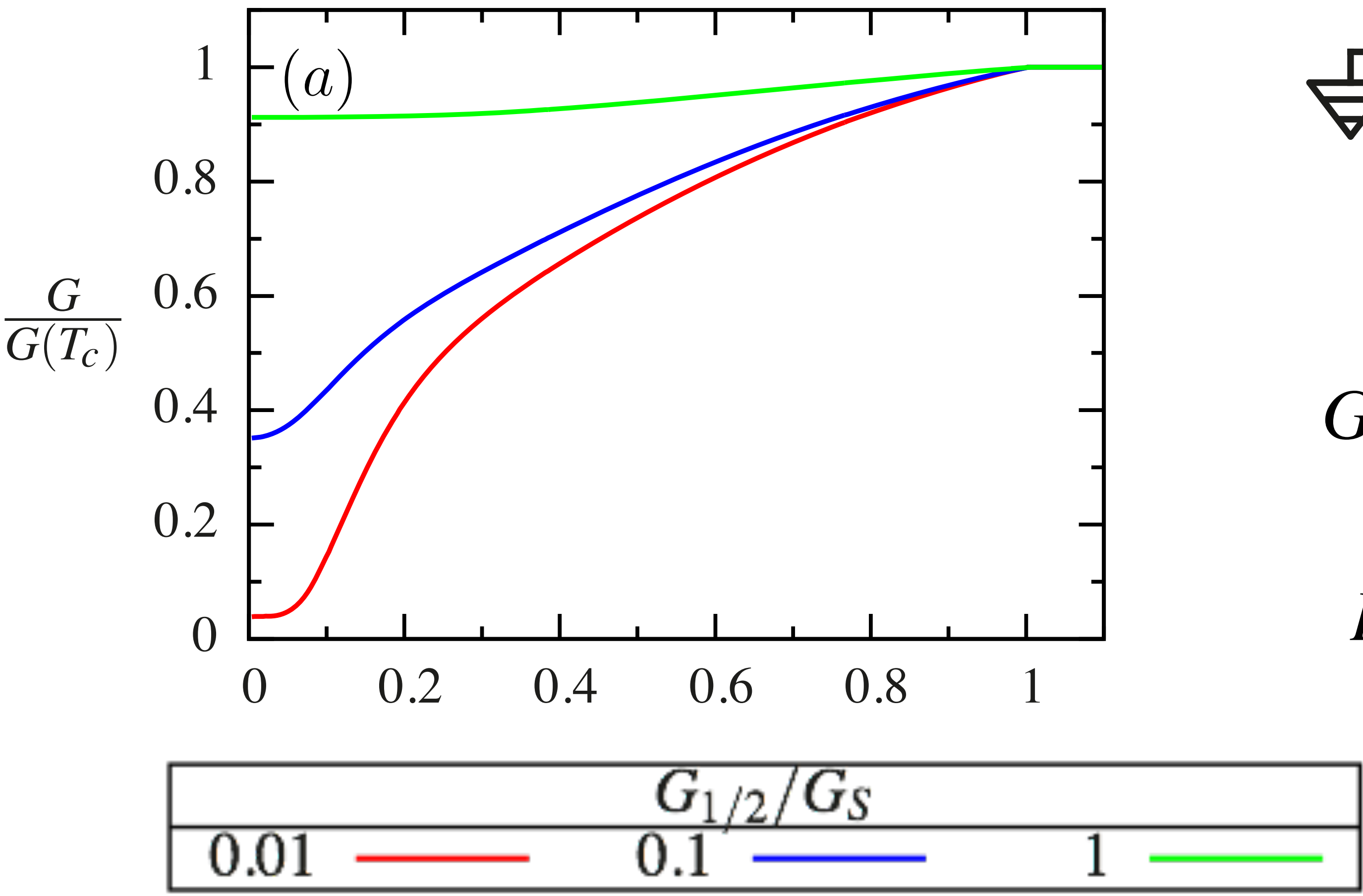


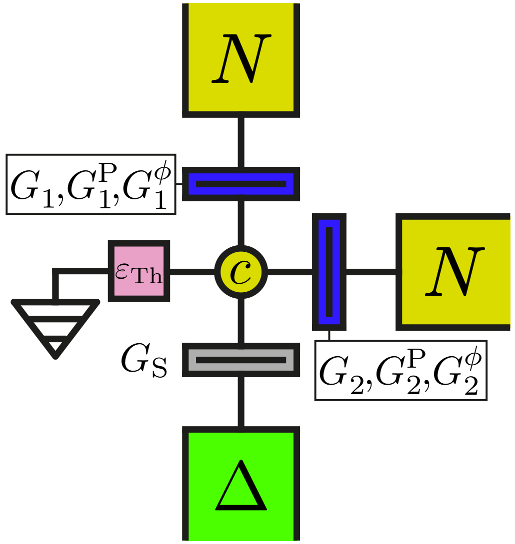


Huge (>1) figure of merit!

[Machon, Eschrig, Belzig, New J. Phys. **16**, 073002 (2014)]

Temperature dependence: Conductance





$$G_1 = G_2 = G_S / 10$$

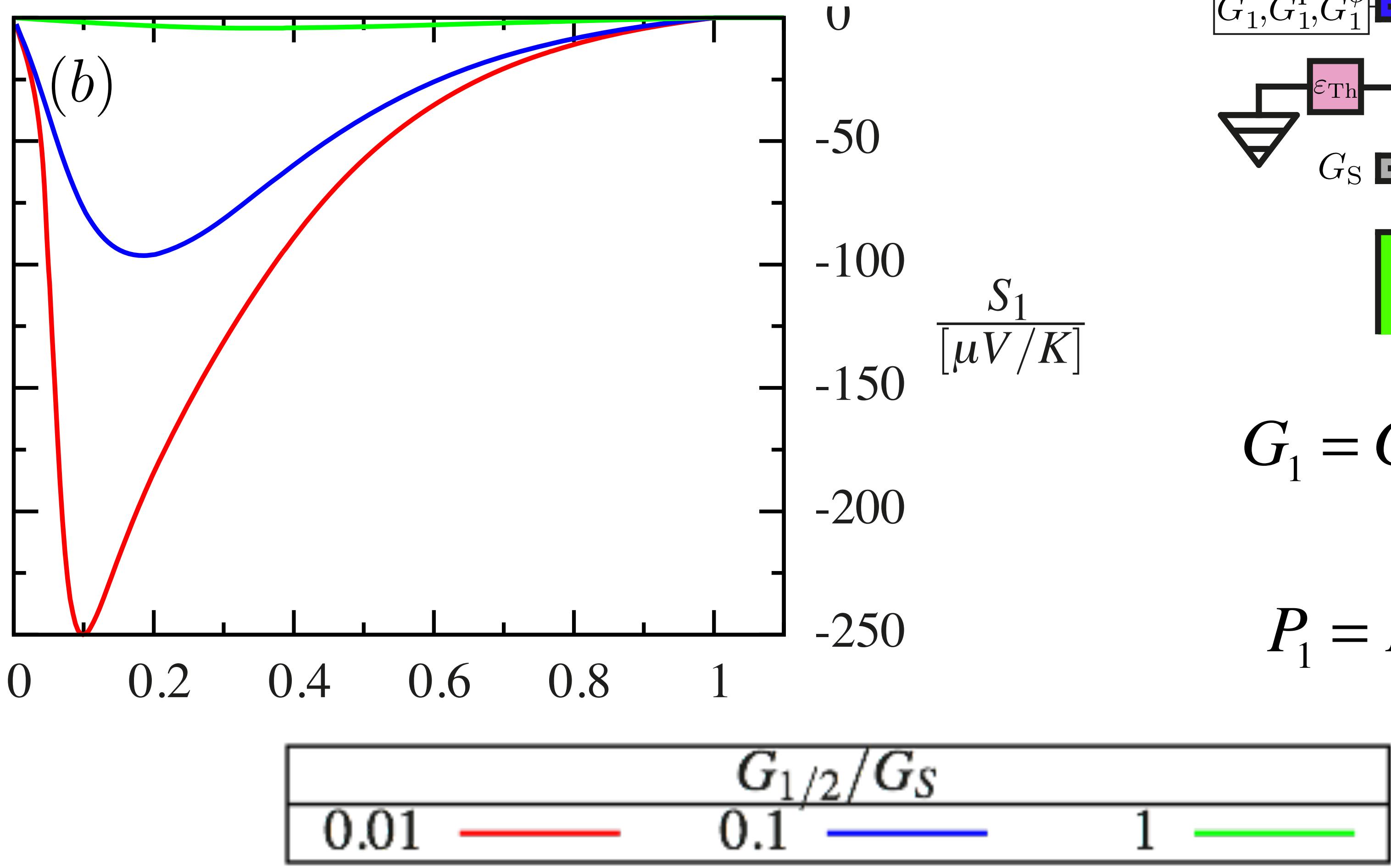
$$T = 0.1T_c$$

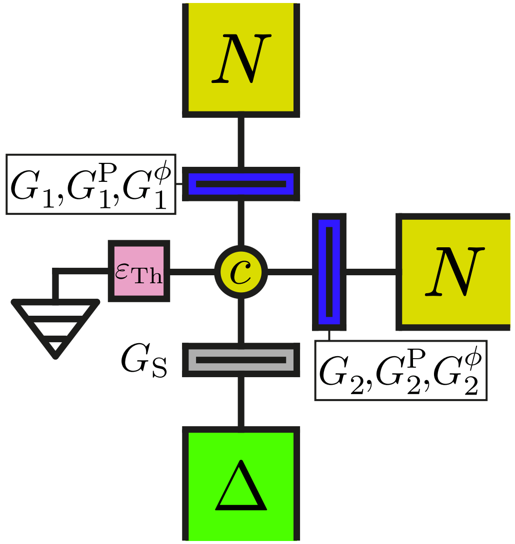
$$P_1 = P_2 = 90\%$$

- Strongly reduced below $k_B T \sim E_{th}$ (for weak coupling $G_{1/2} \ll G_S$)
- Only weak temperature dependence for strongly coupled normal leads

[Peter Machon, Matthias Eschrig, Wolfgang Belzig, New J. Phys. **16**, 073002 (2014)]

Temperature dependence: Thermopower





$$G_1 = G_2 = G_S / 10$$

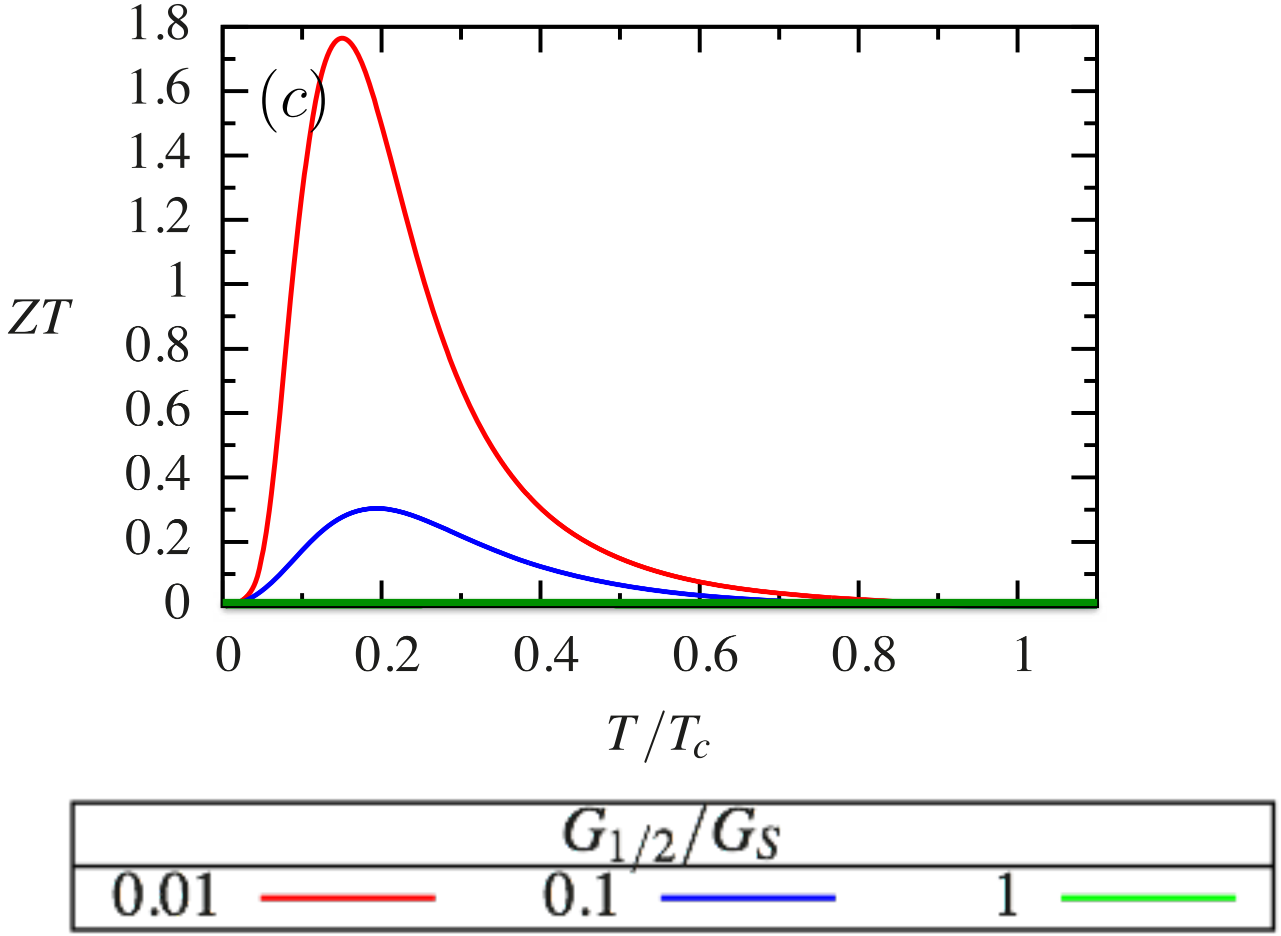
$$T = 0.1 T_c$$

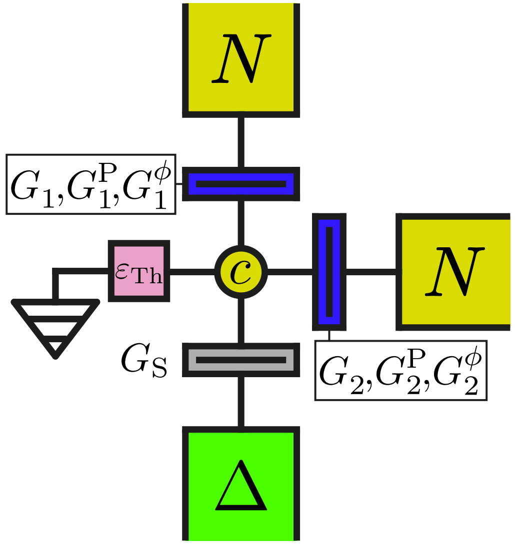
$$P_1 = P_2 = 90\%$$

- Pronounced maximum at $k_B T \sim E_{Th}$
- Strongly coupling the normal leads diminishes ZT due to smearing of the DOS

[Peter Machon, Matthias Eschrig, Wolfgang Belzig, New J. Phys. **16**, 073002 (2014)]

Temperature dependence: Figure of merit





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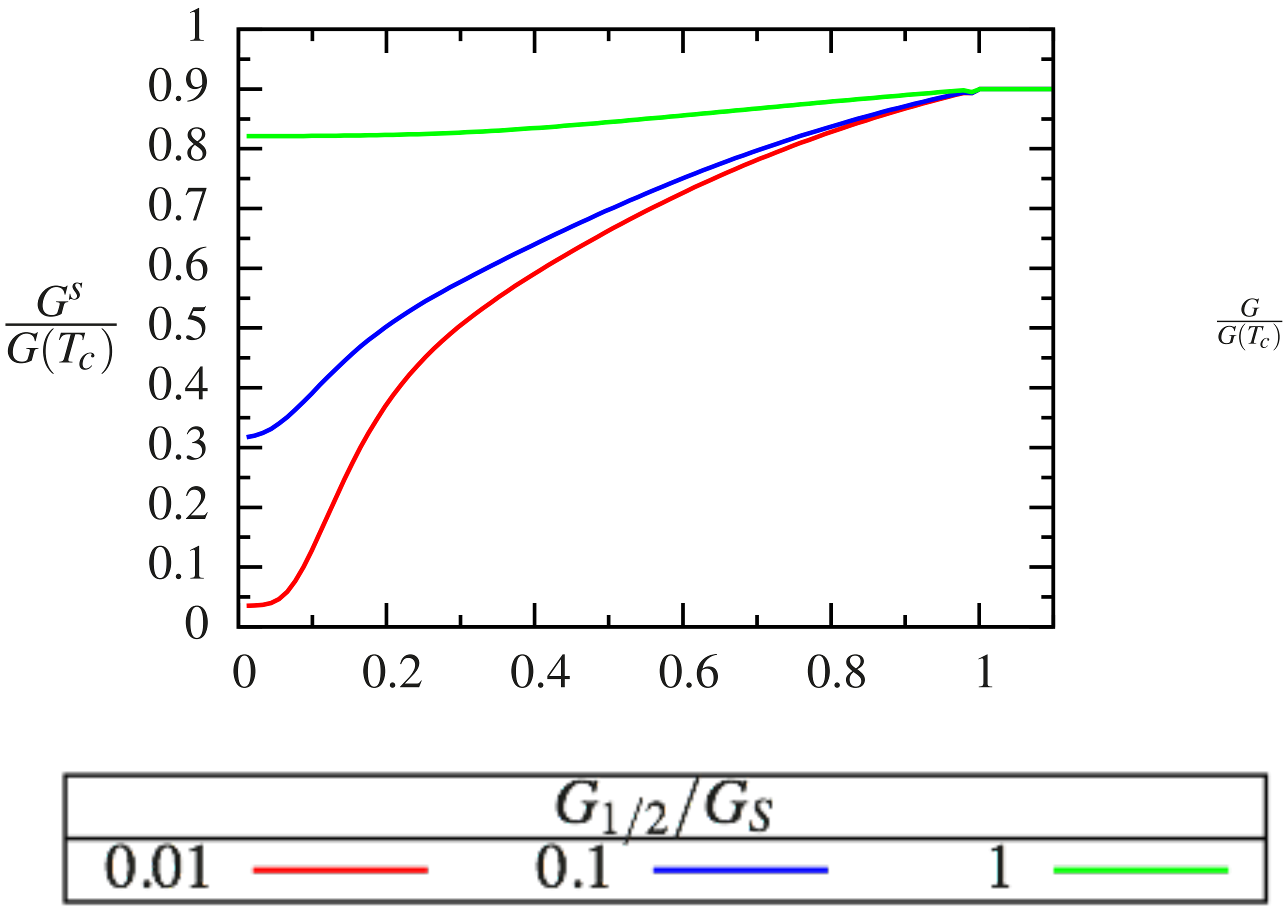
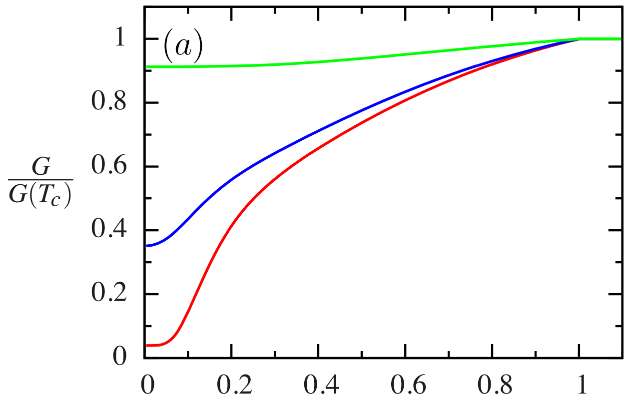
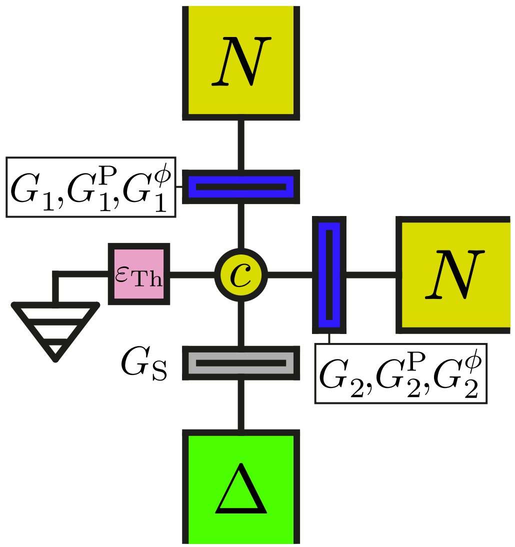
[Peter Machon, Matthias Eschrig, Wolfgang Belzig, New J. Phys. **16**, 073002 (2014)]

Temperature dependence: Spin conductance



c.f. charge conductance.

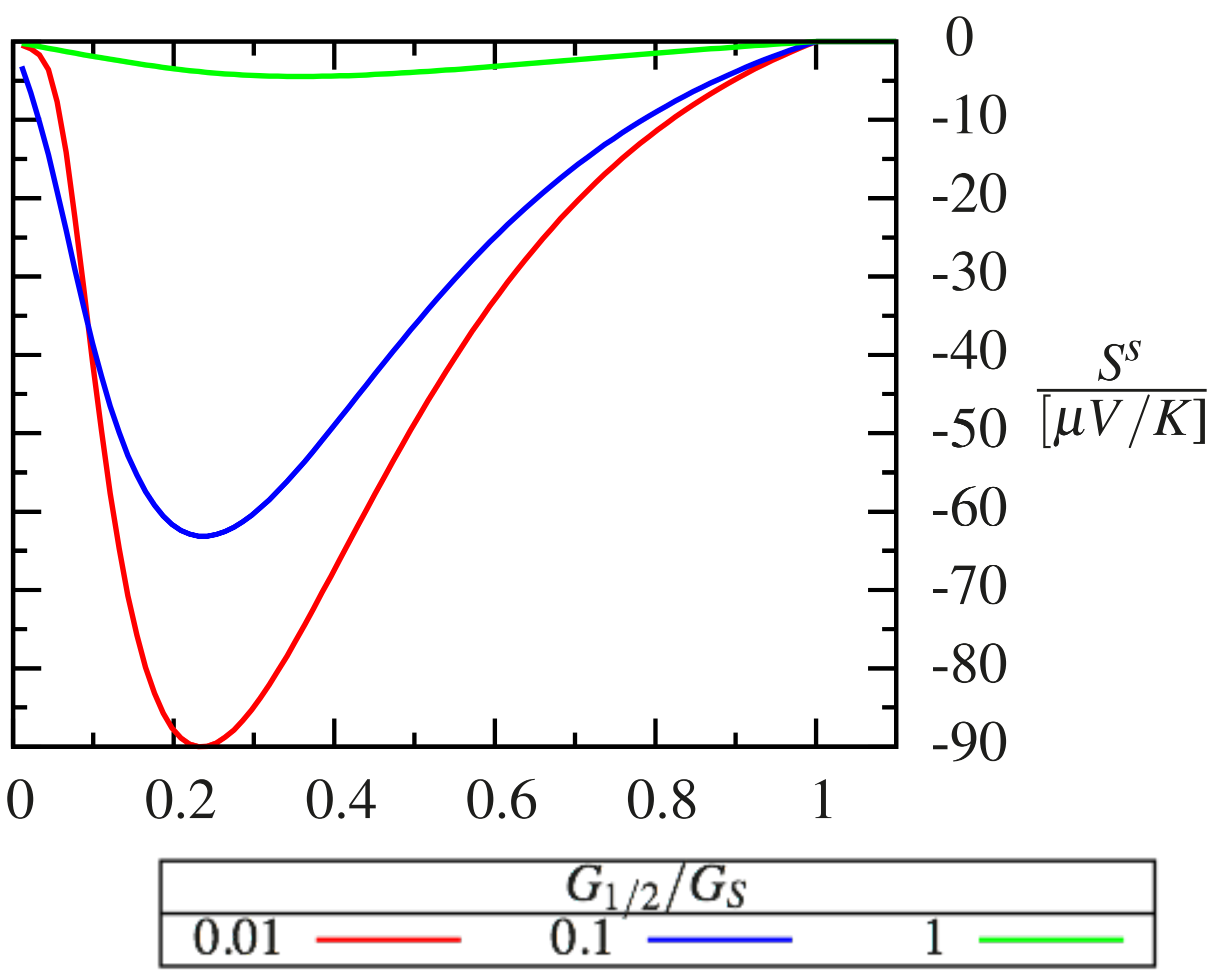




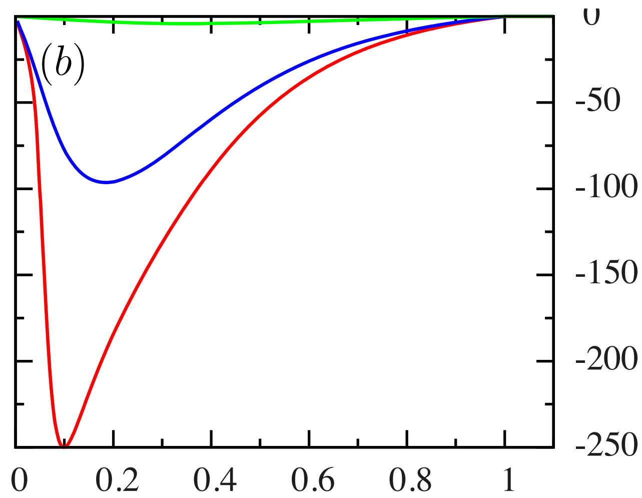
- Spin conductance = polarization x charge conductance

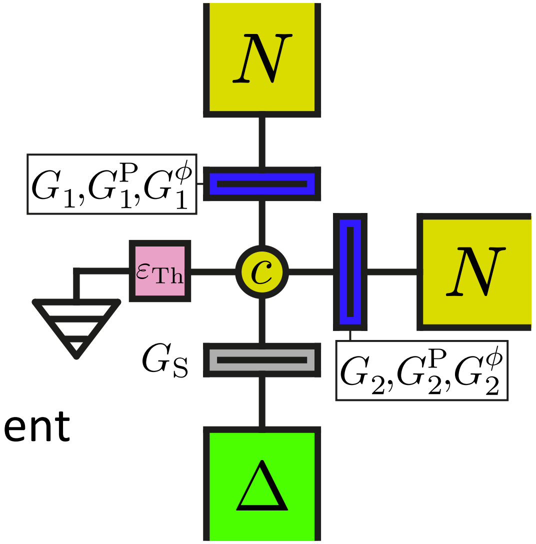
[Peter Machon, Wolfgang Belzig, unpublished]

Temperature dependence: Spin Seebeck



c.f. charge Seebeck S

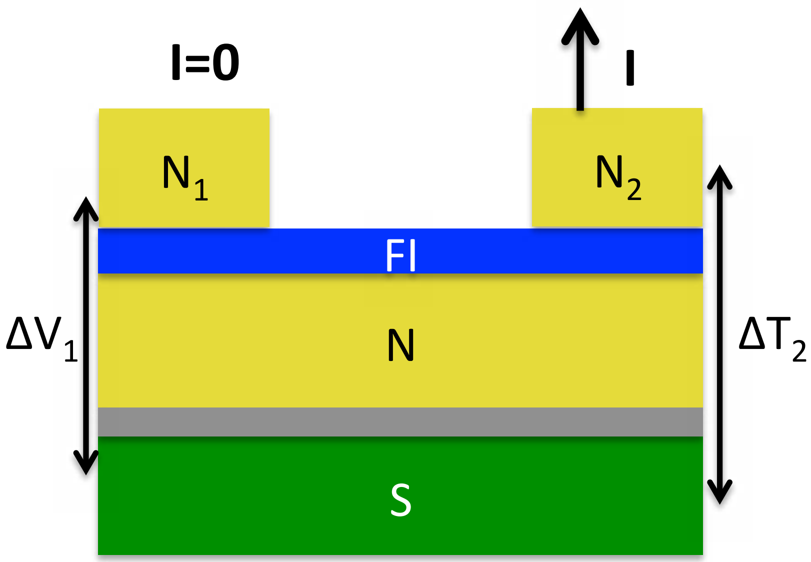


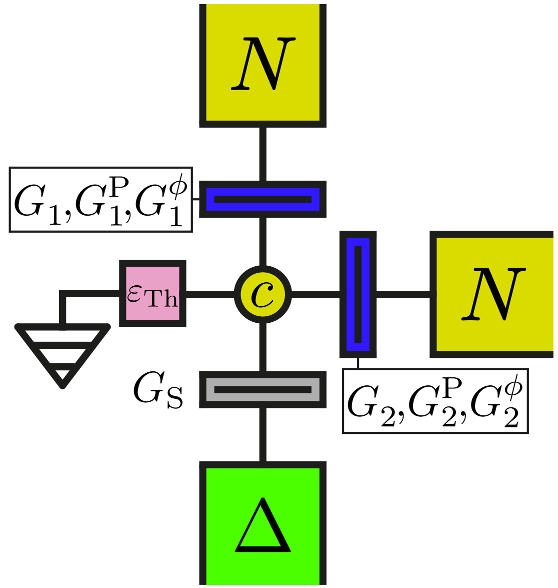


- Pronounced maximum at $k_B T \sim E_{Th}$
- **Different temperature** dependence than charge Seebeck coefficient

[Peter Machon, Wolfgang Belzig, unpublished]

Non-local caloritronics with an SF-heterostructure:





$$\begin{pmatrix} I_1^q \\ I_1^\epsilon \\ I_2^q \\ I_2^\epsilon \end{pmatrix} = \begin{pmatrix} L_{11}^{qV} & L_{11}^{qT} & L_{12}^{qV} & L_{12}^{qT} \\ L_{11}^{\epsilon V} & L_{11}^{\epsilon T} & L_{12}^{\epsilon V} & L_{12}^{\epsilon T} \\ L_{21}^{qV} & L_{21}^{qT} & L_{22}^{qV} & L_{22}^{qT} \\ L_{21}^{\epsilon V} & L_{21}^{\epsilon T} & L_{22}^{\epsilon V} & L_{22}^{\epsilon T} \end{pmatrix} \begin{pmatrix} \Delta V_1 \\ -\Delta T_1/T_S \\ \Delta V_2 \\ -\Delta T_2/T_S \end{pmatrix}$$

Local thermoelectric response

Nonlocal thermoelectric response

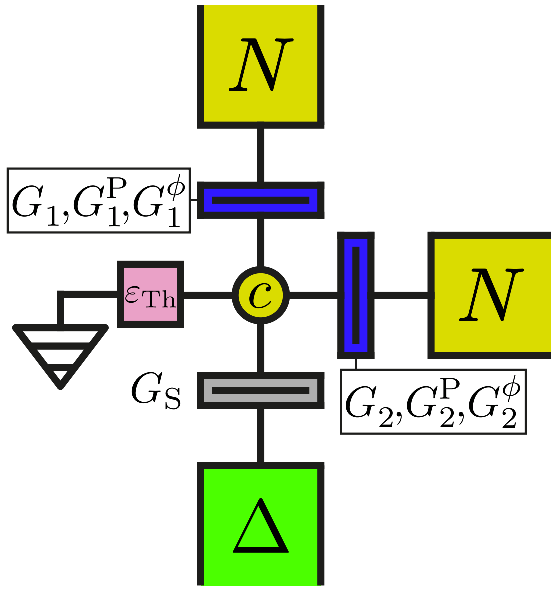
Nonlocal thermopower

$$S = \frac{\Delta V_1}{\Delta T_2} = \frac{1}{T}\frac{L_{12}^{qT}}{L_{11}^{qV}}\bigg|_{I_1^q=0}$$

80

[μV/K]

-80



$$G_1 = G_2 = \frac{G_S}{100}$$

$$T = 0.1T_c$$

$$P_1 = P_2 = 90\%$$

- Maximal nonlocal thermopower for exchange splitting (comparable to E_{th})
- Maximal for large polarization
- Sign change for larger mixing conductance

Conclusions

- Superconductor-Ferromagnet heterostructures are interesting for low-temperature energy current control
- Spin-dependent interfacial phase shifts at a ferromagnetic insulator are useful for spin-dependent proximity effect
- The combination of spin-splitting and spin-polarized transport enables giant thermoelectric effects
- Large thermoelectric figure of merit, interesting for applications in low-temperature energy control
- Large Spin Seebeck coefficients (~100μV/K) can be obtained using proximity structures
- Prediction of nonlocal thermoelectric effects in multi-terminal structures

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