Fluctuation Theorem for a Small Engine and Magnetization Switching by Spin Torque

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Outline

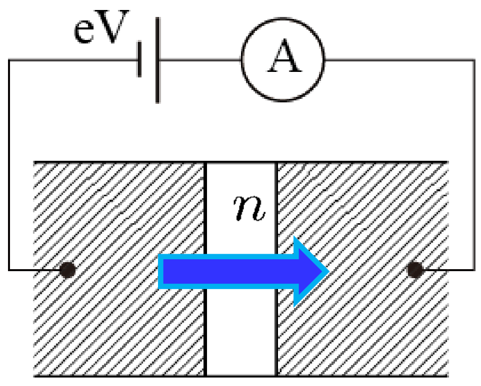
- ☐ Introduction
  - Full-counting statistics
  - Fluctuation theorem & Small heat engine
  - Current induced spin transfer torque & Magnetization switching

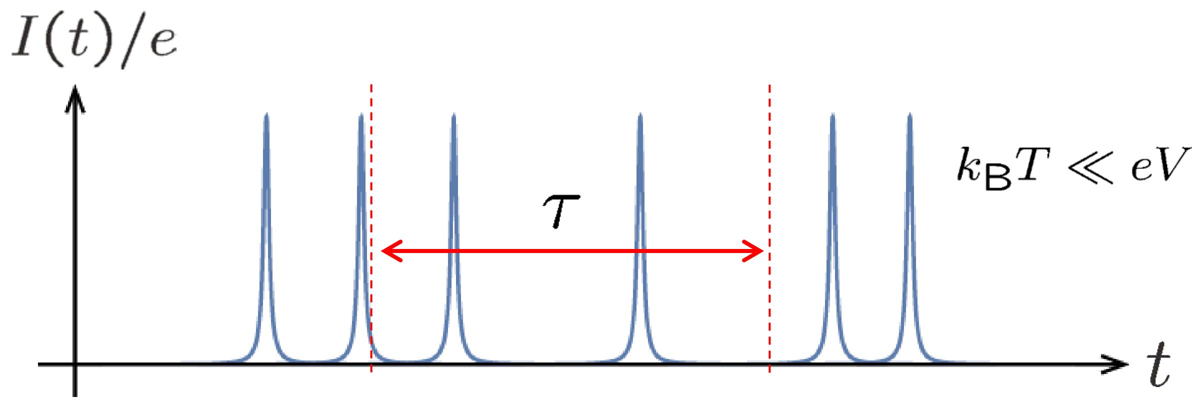
- ☐ Magnetization switching by fluctuating spin torque
  - Langevin equation in energy space
  - Full-counting statistics under adiabatic spin-pumping
  - Switching exponent & Threshold bias voltage:
    Escape of a particle from a meta-stable state induced by fluctuating non-equilibrium & non-conservative force

- ☐ Summary

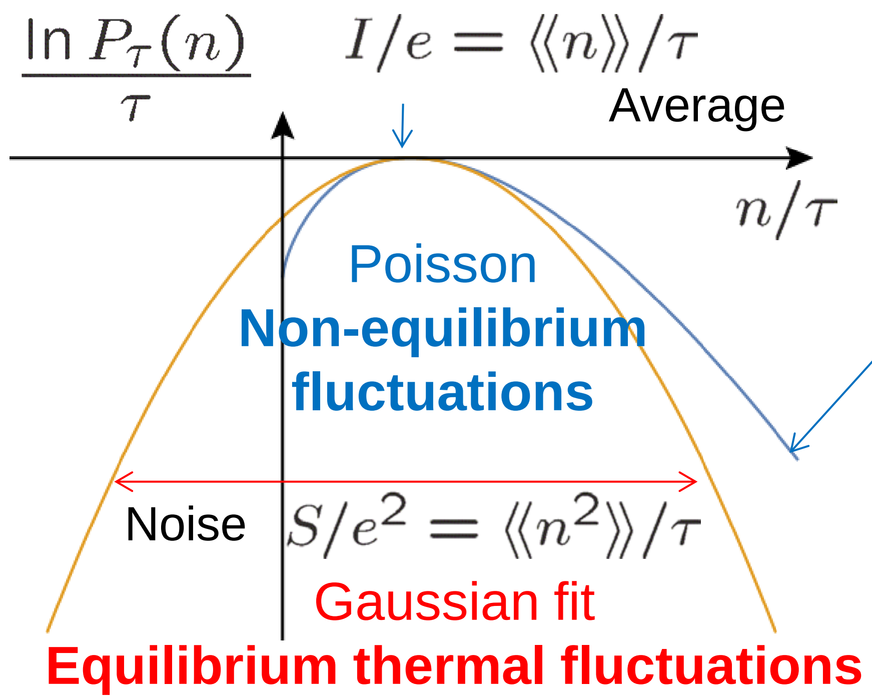
Full counting statistics

Levitov, H.-W. Lee, Lesovik, J. Math. Phys., 1996





Number of transmitted electrons $n = \int_t^{t+\tau} dt' \, I(t')/e$



Skewness

$\langle\langle n^3 \rangle\rangle/\tau$

Large deviation from Gauss distribution

Cumulant generating function

$$\mathcal{F}_G(\lambda) = \ln \sum_n P_\tau(n) e^{in\lambda}$$

Counting field

$$\langle\langle n^m \rangle\rangle = \left. \frac{\partial^m \mathcal{F}_G(\lambda)}{\partial (i\lambda)^m} \right|_{\lambda=0}$$

Fluctuation theorem

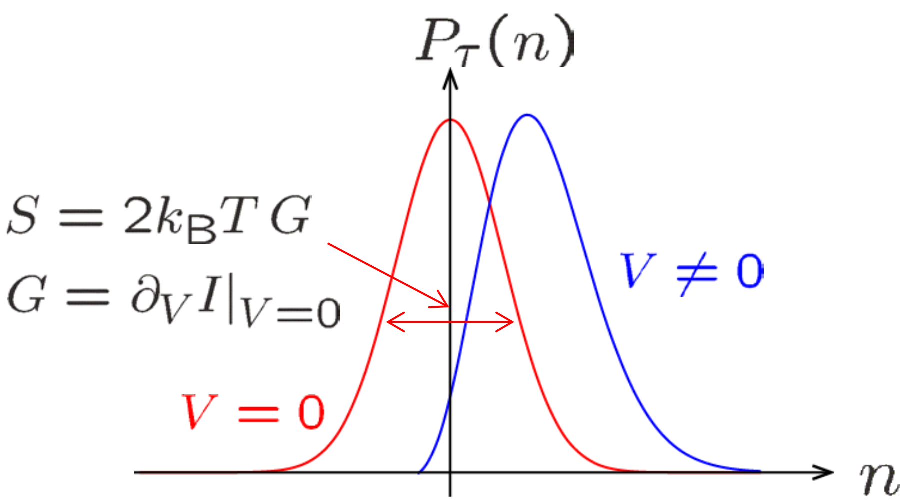
Evans, Cohen, Morris, 1993, Saito, Dhar, PRB 2007, Bochkov, Kuzovlev, 1977, etc.

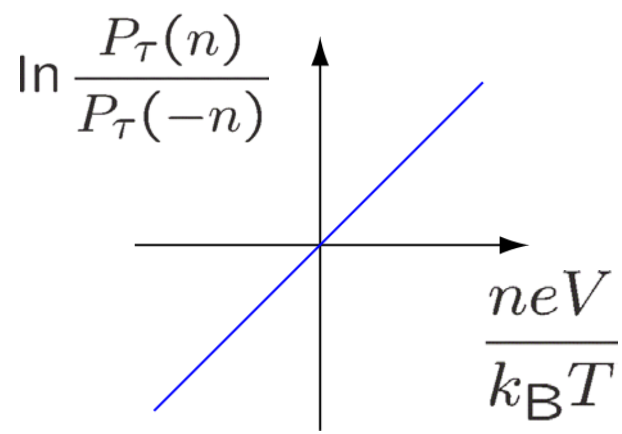
$$\frac{P_\tau(-\Delta S)}{P_\tau(+\Delta S)} = \exp(-\Delta S)$$

ΔS entropy production

- ✓ Micro-reversibility → FT is valid in far from equilibrium regime
- ✓ Second law of thermodynamics [Jarzynski PRL 1997, Crooks PRE 1999, etc.]
- ✓ Fluctuation-dissipation theorem & Onsager relations [Gallavotti PRL 1996]
- ✓ Extension to non-linear response regime [Tobiska, Nazarov, PRB 2005; Saito, YU PRB 2008]

$$\Delta S = \frac{neV}{k_B T}$$ Joule heat ⇒ $$\frac{P_\tau(-n)}{P_\tau(n)} = \exp\left(-\frac{neV}{k_B T}\right)$$

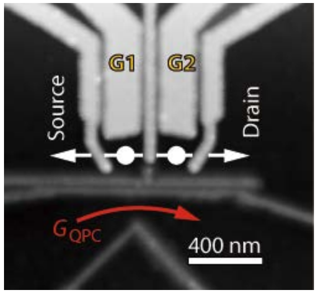


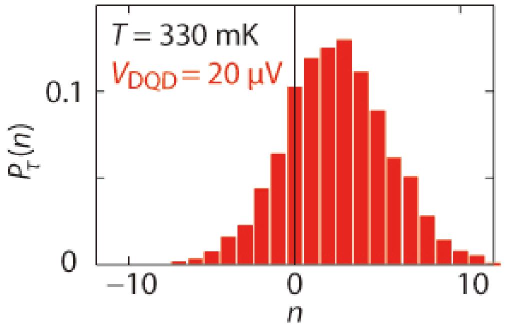


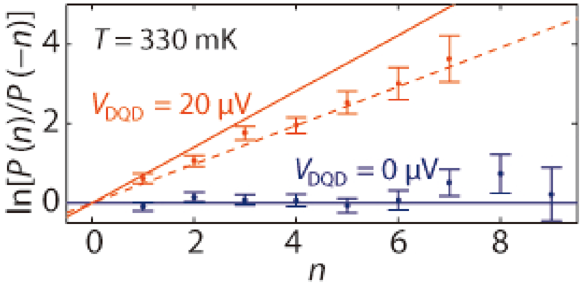
Unfolding-refolding cycles

Liphardt, et al., Nature 2002, Collin, et al., Nature 2005

Single-electron transport



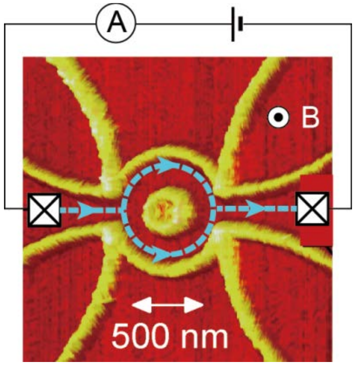


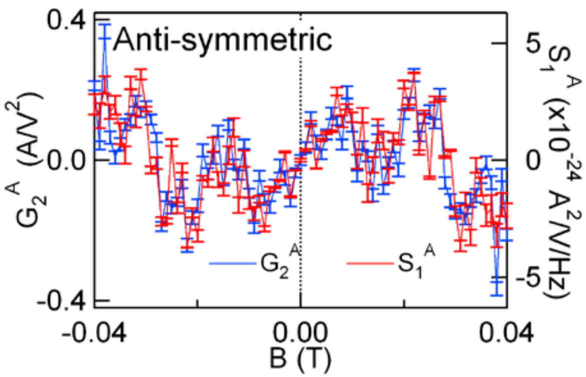


YU, Golubev, Marthaler, Saito, Fujisawa, Gerd Schön, PRB 2010, Küng, Rossler, Beck, Marthaler, Golubev, YU, Ihn, Ensslin, PRX 2012

Nonlinear conductance & Linear response of noise

Nakamura, Yamauchi, Hashisaka, Chida, Kobayashi, Ono, Leturcq, Ensslin, Saito, YU, Gossard, PRL 2010, PRB 2011





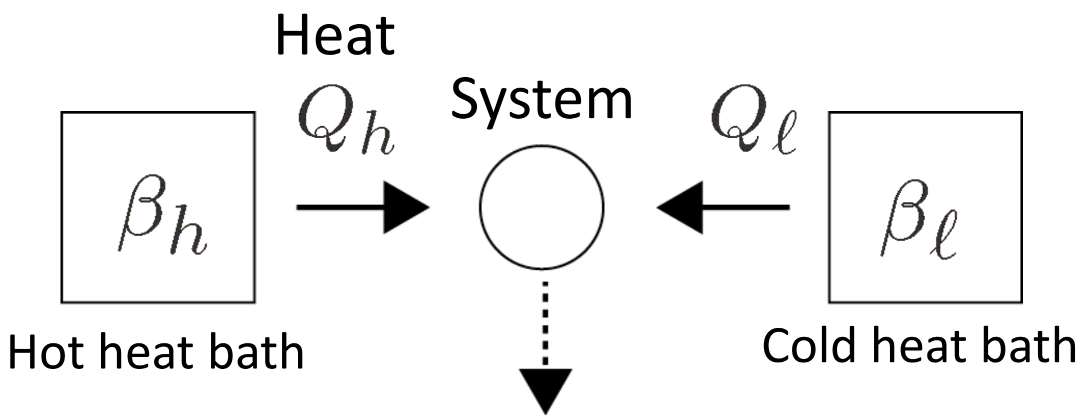
$$\frac{\partial S^A(B)}{\partial V} = 3k_BT\frac{\partial G^A(B)}{\partial V}$$

$$S^A(B) = S(B) - S(-B)$$

→ Nonlinear Onsager relations

Fluctuation theorem for a small heat engine

[Sinitsyn, J. Phys. A 2011, Campisi, J. Phys. A 2014, Verley, Willaert, Van den Broeck, Esposito, Nat Commun 2014]



$$\beta = \frac{1}{k_{\mathrm{B}}T}$$

Work $W = Q_h + Q_\ell$

$$\frac{P_{R,\tau}(-Q_h, -W)}{P_\tau(Q_h, W)} = \exp\left[-Q_h(\beta_h - \beta_\ell) - \beta_\ell W\right]$$

Jensen's inequality

$$\langle \mathrm{e}^{-\Delta S}\rangle \geq \mathrm{e}^{-\langle \Delta S\rangle}$$

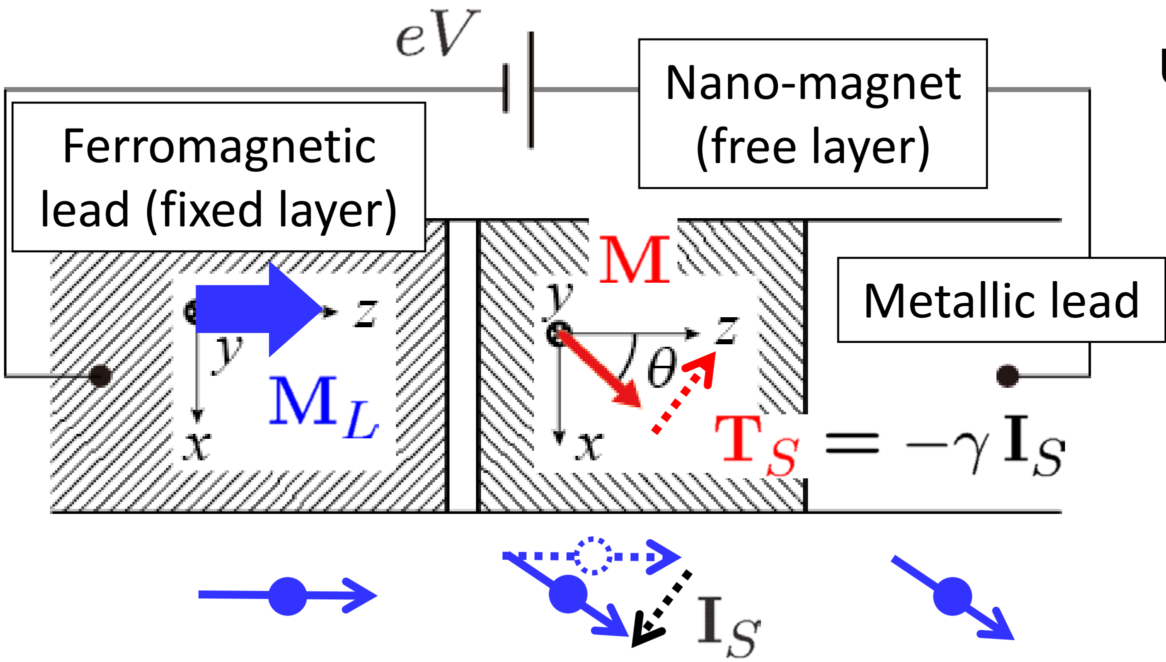
$$\eta = \frac{\langle W\rangle}{\langle Q_h\rangle} \leq 1 - \frac{\beta_h}{\beta_\ell}$$

Carnot theorem

✓ FT for a small engine is applicable to various nano-systems

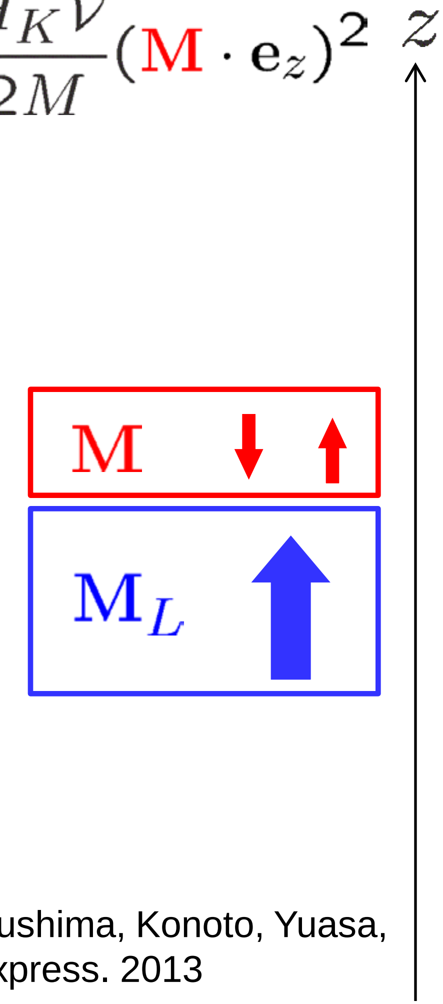
Spin-transfer torque in a ferromagnetic tunnel junction

[Slonczewski, J. Magn. Magn. Mater. 1996]

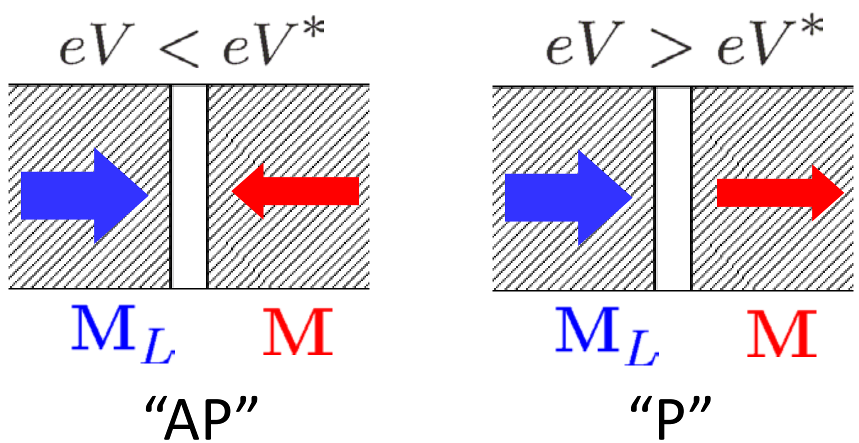


Uniaxial anisotropic energy

$$E = -\frac{H_K \mathcal{V}}{2M}(\mathbf{M}\cdot\mathbf{e}_z)^2$$



Magnetization switching above the critical voltage



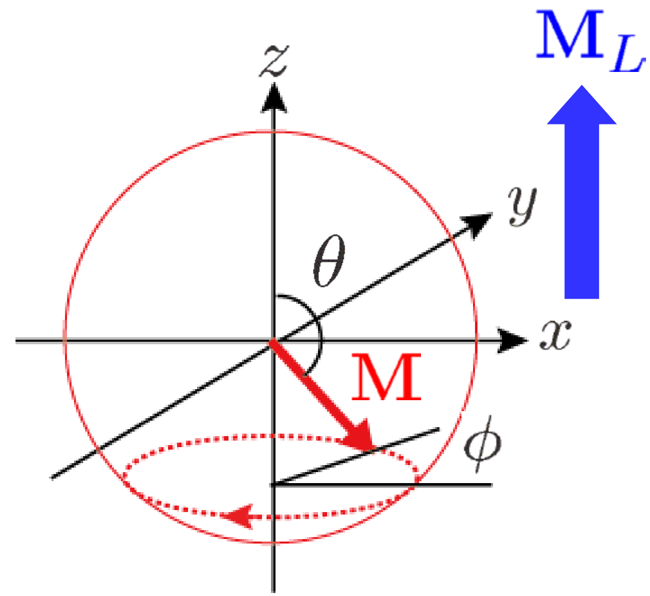
Yakushiji, Fukushima, Konoto, Yuasa, Appl. Phys. Express. 2013

Fluctuation induced magnetization switching

Stochastic LLG equation [Apalkov, Visscher PRB 2005]

$$\dot{\mathbf{M}} = -\gamma \mathbf{M} \times (\mathbf{H}_{\text{eff}} + \mathbf{h}) + \frac{\alpha\, \mathbf{M} \times \dot{\mathbf{M}}}{\mathcal{V}} - \frac{\gamma \mathbf{I}_S}{\mathcal{V}}$$

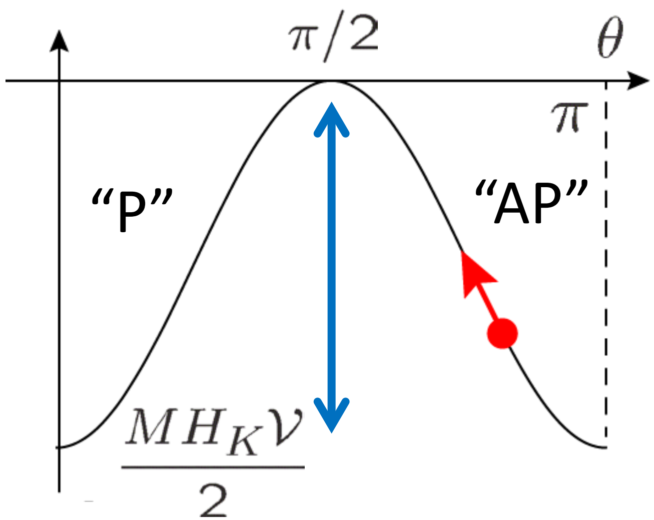
$$\mathbf{H}_{\text{eff}} = H_K \cos\theta\, \mathbf{e}_z$$



- ➢ Fluctuation-dissipation theorem → Gauss

$$\langle h_j(t) h_k(t') \rangle = 2\alpha k_{\text{B}} T \delta(t - t') \delta_{jk}$$

$$E = -\frac{M H_K \mathcal{V}}{2} (\cos\theta)^2$$



Switching rate $P \approx e^{-\Delta}$

$V = 0$ $\quad \Delta = \frac{M H_K \mathcal{V}}{2 k_{\text{B}} T}$ Arrhenius law [Brown PR 1963]

$0 < eV < eV^*$ [Taniguchi, Imamura, PRB 2011] $\quad \Delta = \frac{M H_K \mathcal{V}}{2 k_{\text{B}} T} \left(1 - \frac{V}{V^*}\right)^2$

- ➢ **What happens when the spin-torque fluctuates?**

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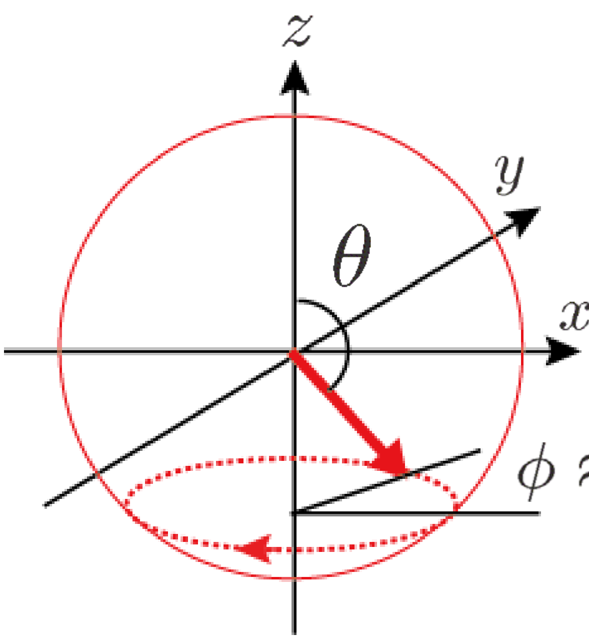
Langevin equation in the energy space

$$\dot{\mathbf{M}} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} - \frac{\gamma \mathbf{I}_S}{\mathcal{V}}$$ ← Weak spin torque

$$\mathbf{H}_{\text{eff}} = H_K \cos\theta \, \mathbf{e}_z$$

Energy variation is small after the single precession

$$\overline{\dot{E}(t)} = -\mathcal{V}\, \overline{\dot{\mathbf{M}} \cdot \mathbf{H}_{\text{eff}}} \approx \overline{\mathbf{I}_{Sz}(t)}\, \Omega(\theta(t))$$



$$\phi \approx \Omega t$$

$$\Omega = \gamma H_K \cos\theta$$

$$\boxed{\overline{E}(t+\Delta t) - \overline{E}(t) = w \equiv s\hbar\Omega}$$

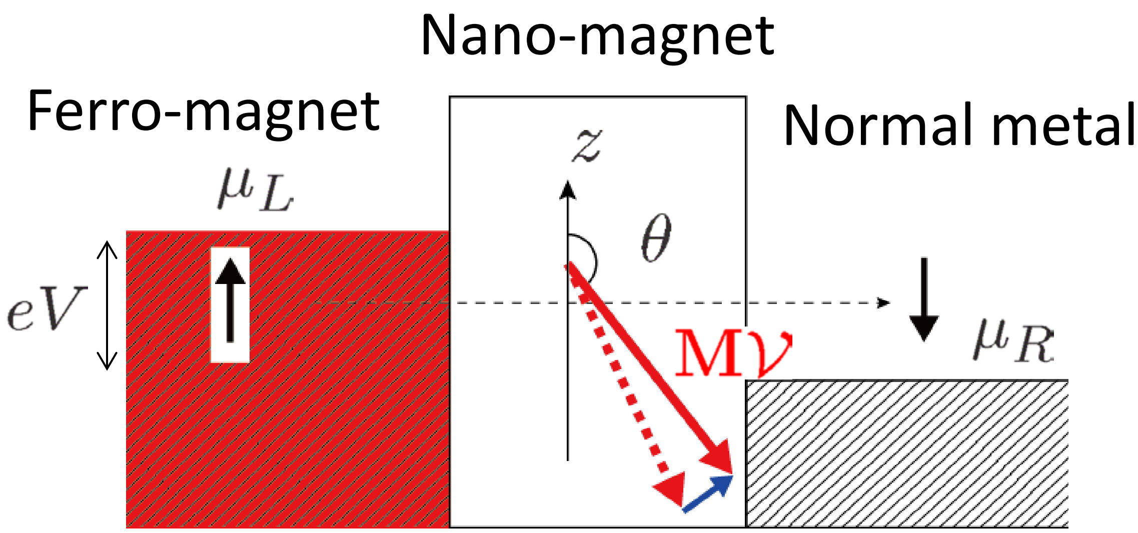
Fluctuating work done by spin-torque

$$s = \int_t^{t+\Delta t} dt' \overline{\mathbf{I}_{Sz}(t')}/\hbar$$

Number of flipped spins

Full-counting statistics under adiabatic spin-pumping

[Andreev, Kamenev, PRL 2000]



$4\times\ 4$ S-matrix → Scattering by the nano-magnet

$$\mathbf{S} = \begin{pmatrix} \mathbf{r} & \mathbf{t}' \\ \mathbf{t} & \mathbf{r}' \end{pmatrix}$$

$$\mathbf{r} = \begin{pmatrix} r_{\uparrow\uparrow} & r_{\uparrow\downarrow} \\ r_{\downarrow\uparrow} & r_{\downarrow\downarrow} \end{pmatrix}$$

$$\mathbf{S}(\phi,\theta) = e^{-i\Omega t\sigma_z/2}\mathbf{S}(\theta)e^{i\Omega t\sigma_z/2}$$

→ Adiabatic spin pumping by precession motion

[Brataas, Tserkovnyak, Bauer, PRL 2008]

$$\ln \sum_{n,s} P_{\Delta t}(n,s;\Omega)e^{i\lambda n+i\chi s} = \mathcal{F}_G(\lambda,\chi;\Omega)\Delta t$$

$$s = \int_t^{t+\Delta t} dt' \overline{\mathrm{I}_{S_z}(t')}/\hbar \qquad n = \int_t^{t+\Delta t} dt\, I(t)/e$$

Number of flipped spins Number of transmitted electrons

Ferromagnetic insulating nano-magnet

$$\left.\begin{aligned}\mathcal{F}_G(\lambda,\chi;\Omega) = &\sum_{\nu=\pm} \sin^2\theta\, G_{L\uparrow,R\downarrow} \frac{\nu(eV-\hbar\Omega)}{1-e^{-\nu\beta(eV-\hbar\Omega)}}(e^{i\nu(\lambda+\chi)}-1) \\ &+\sum_{\nu=\pm} \sin^2\theta\, G_{L\downarrow,R\uparrow} \frac{\nu(eV+\hbar\Omega)}{1-e^{-\nu\beta(eV+\hbar\Omega)}}(e^{i\nu(\lambda-\chi)}-1)\end{aligned}\right\} \text{Spin mixing tunneling}$$

$$+\sum_{\pm}\Gamma_{\pm}(e^{\pm i\lambda}-1) \qquad \text{Tunneling}$$

→ Spintronic fluctuation theorem [YU, Imamura, J. Phys. Conf. Ser. 2010; López, Lim, Sánchez, PRL 2012, Wang, Feldman, arXiv:1502.06572]

$$P_{R,\Delta t}(-n,s;\Omega) = P_{\Delta t}(n,s;-\Omega)e^{-\beta(\underbrace{neV}_{q}+\underbrace{s\hbar\Omega}_{w})}$$

Joule heat q $\qquad$ w Work

$$\frac{P_{R,\Delta t}(-q,-w)}{P_{\Delta t}(q,w)} = \exp\left[-\beta q - \beta w\right] \quad \Rightarrow \quad \frac{\langle w\rangle}{\langle q\rangle} \le 1 \text{ “Carnot theorem”}$$

→ FT for a small engine

Critical bias voltage

$$\frac{\langle\langle w\rangle\rangle}{\Delta t} = \hbar\Omega \frac{\partial \mathcal{F}_G(\lambda=0,\chi;\Omega)}{\partial(i\chi)}\Big|_{\chi=0} = \Omega\, \mathrm{I}_{Sz}^{\Omega=0} - p_{\mathrm{pump}} = 0$$

Power gain by spin-torque Power dissipation by spin-pumping

$$\begin{cases} \mathrm{I}_{Sz}^{\Omega=0} = (G_{L\uparrow,R\downarrow} - G_{L\downarrow,R\uparrow})\,\hbar \sin^2\theta\, eV \\ p_{\mathrm{pump}} = (G_{L\downarrow,R\uparrow} + G_{L\uparrow,R\downarrow}) \sin^2\theta\, (\hbar\Omega)^2 \end{cases}$$

[Slonczewski, JMMM 1996]

$$|\Omega^*| > \max[\gamma H_K \cos\theta]$$

Critical voltage

$$\hbar\Omega^* = \frac{G_{L\uparrow,R\downarrow} - G_{L\downarrow,R\uparrow}}{G_{L\uparrow,R\downarrow} + G_{L\downarrow,R\uparrow}} eV \quad \Longrightarrow \quad \frac{eV^*}{2\mu_{\mathrm{B}} H_K} = \frac{G_{L\downarrow,R\uparrow} + G_{L\uparrow,R\downarrow}}{|G_{L\downarrow,R\uparrow} - G_{L\uparrow,R\downarrow}|}$$

Spin-torque shot noise induces probabilistic switching below the critical voltage

→ Path-integral & Optimal-path approximation [Sukhorukov, Jordan PRL 2007]

$$P = \int \mathcal{D}\xi \int_{E(0)}^{E(\tau)} \mathcal{D}E\, e^{iS} \approx e^{-\Delta}$$

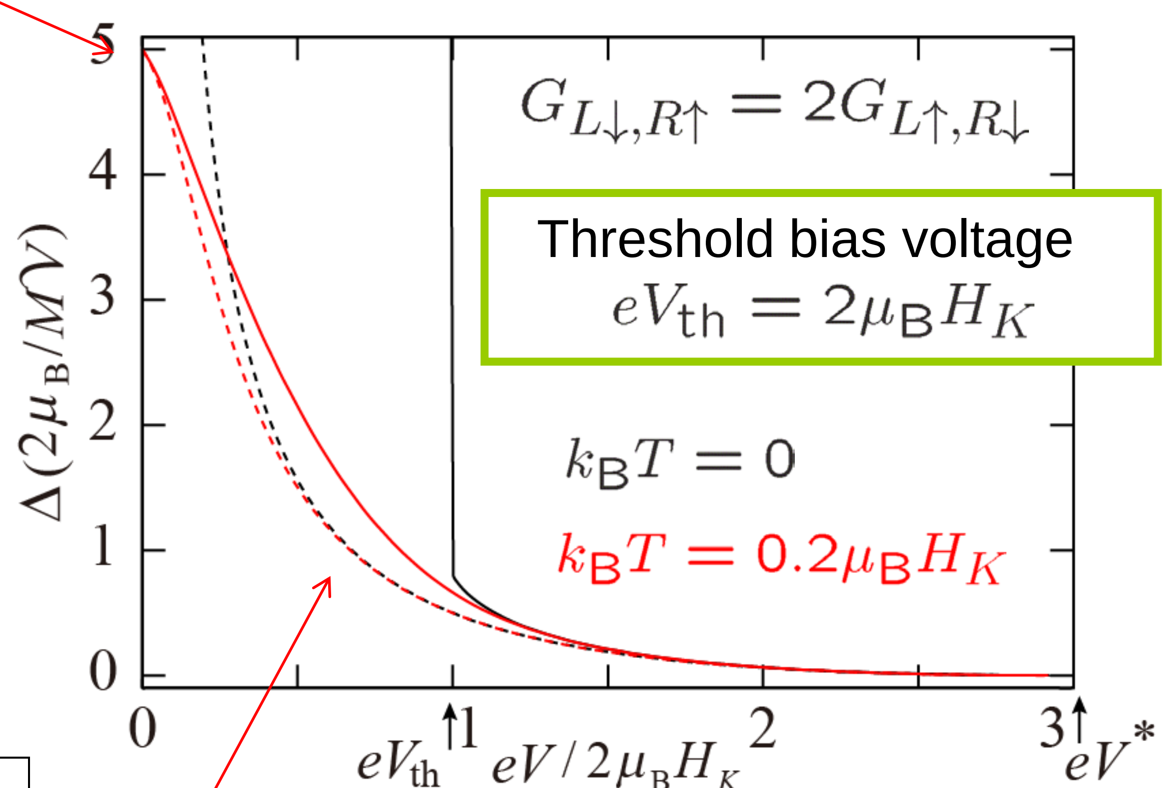
$$iS = -\int_0^\tau dt \left[i\xi(t)\dot{E}(t) - \mathcal{F}_G(\lambda=0, \xi\hbar\Omega(E(t)); \Omega(E(t))) \right]$$

Switching exponent

Switching probability $P \approx e^{-\Delta}$

$$\Delta = \frac{M H_K \mathcal{V}}{2 k_B T}$$

Arrhenius law



Arrhenius-like law

$$\Delta = \frac{M H_K \mathcal{V}}{2 k_B T_{eff}} \left(1 - \frac{V}{V^*}\right)^2$$

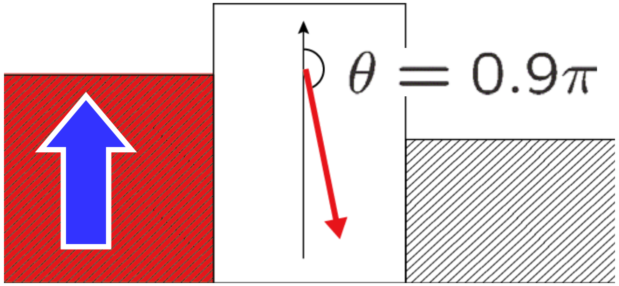
[Taniguchi, Imamura PRB 2011]

Spin-torque shot noise

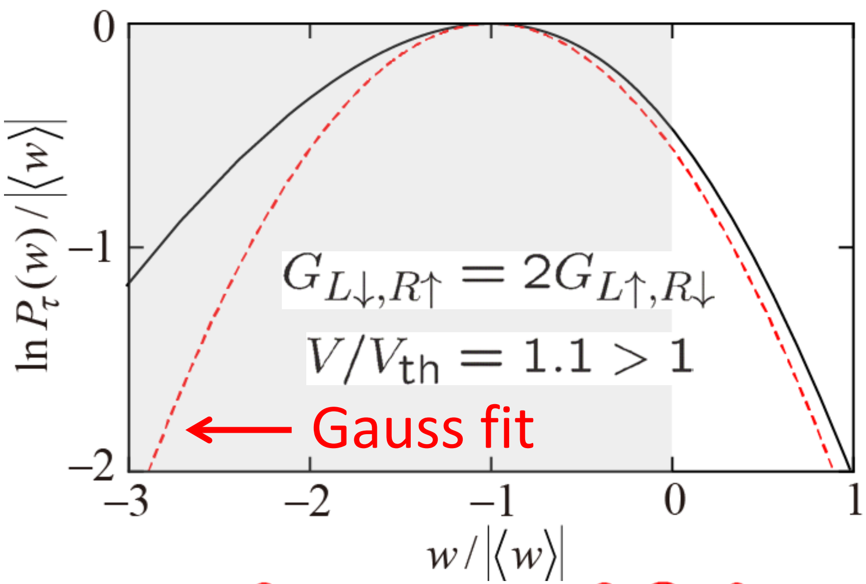
$$T_{eff} \approx \frac{2 G_{L\uparrow,R\downarrow} G_{L\downarrow,R\uparrow}}{(G_{L\uparrow,R\downarrow} + G_{L\downarrow,R\uparrow})^2} eV$$

[Nunez, Duine PRB 2008]

Probability distribution of work near anti-parallel alignment



$eV_{\text{th}} < eV < eV^*$

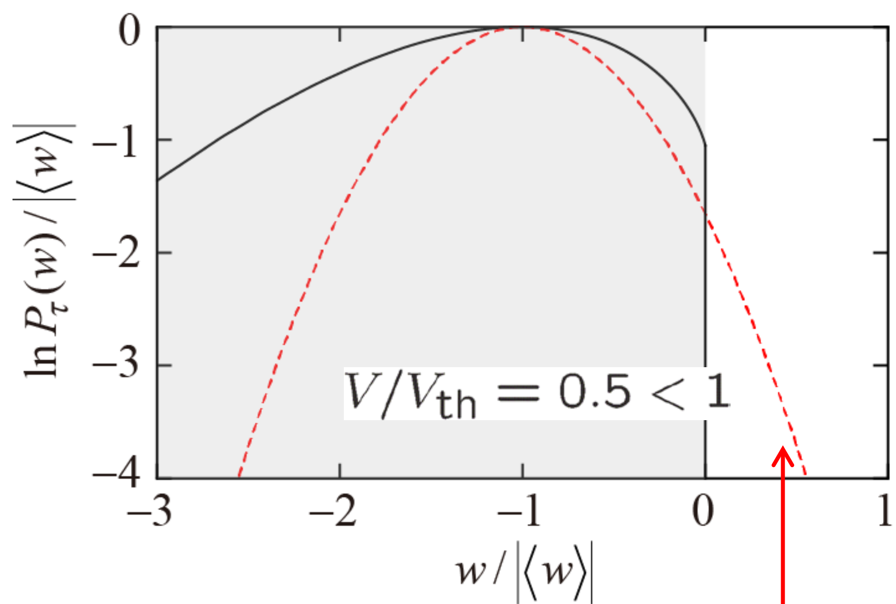


$$\langle \delta w^2\rangle_{\text{Gauss}} = \hbar\Omega^2\, \mathrm{I}_{S_z}^{\Omega=0}$$

[Brataas, Tserkovnyak, Bauer, PRL 2008]

$$\langle w\rangle = -0.21\tau G_{L\uparrow,R\downarrow}(2\mu_{\text{B}}H_K)^2$$

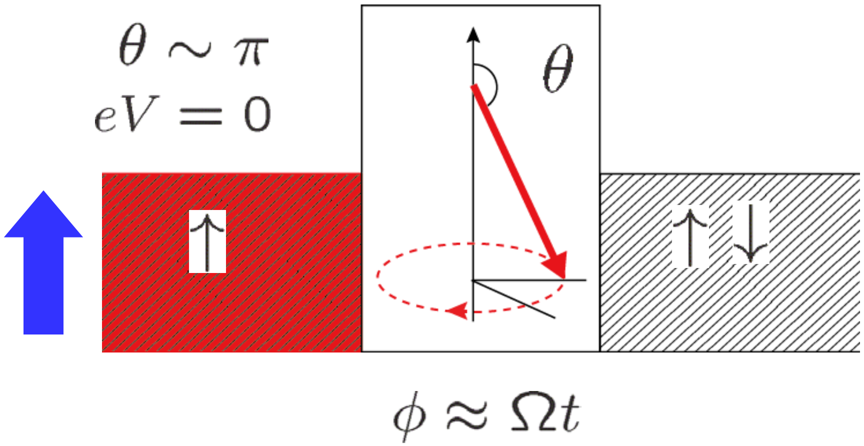
$eV < eV_{\text{th}} < eV^*$



Artificial switching events

$$\langle w\rangle = -0.15\tau G_{L\uparrow,R\downarrow}(2\mu_{\text{B}}H_K)^2$$

Backaction by the adiabatic spin-pumping

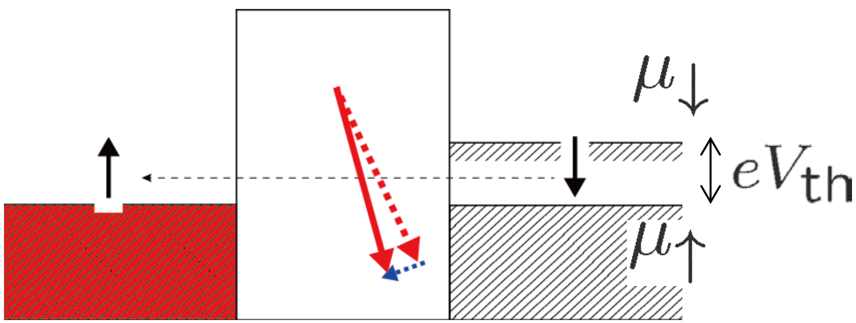


Wave functions in the leads on the rotating frame

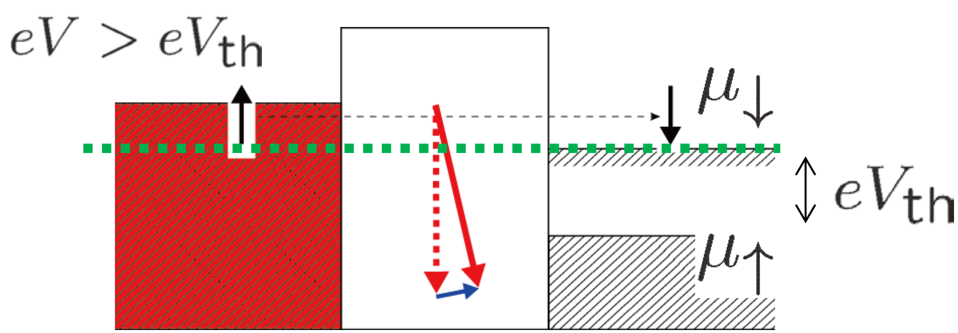
$$\begin{cases} \psi_{L,R\uparrow} \to e^{+i\Omega t/2}\psi_{L,R\uparrow} \\ \psi_{L,R\downarrow} \to e^{-i\Omega t/2}\psi_{L,R\downarrow} \end{cases}$$

$$\mu_\downarrow - \mu_\uparrow = |\hbar\Omega| = eV_{\mathrm{th}} = 2\mu_{\mathrm{B}}H_K$$

Spin splitting of chemical potential



Damping



Spin-torque

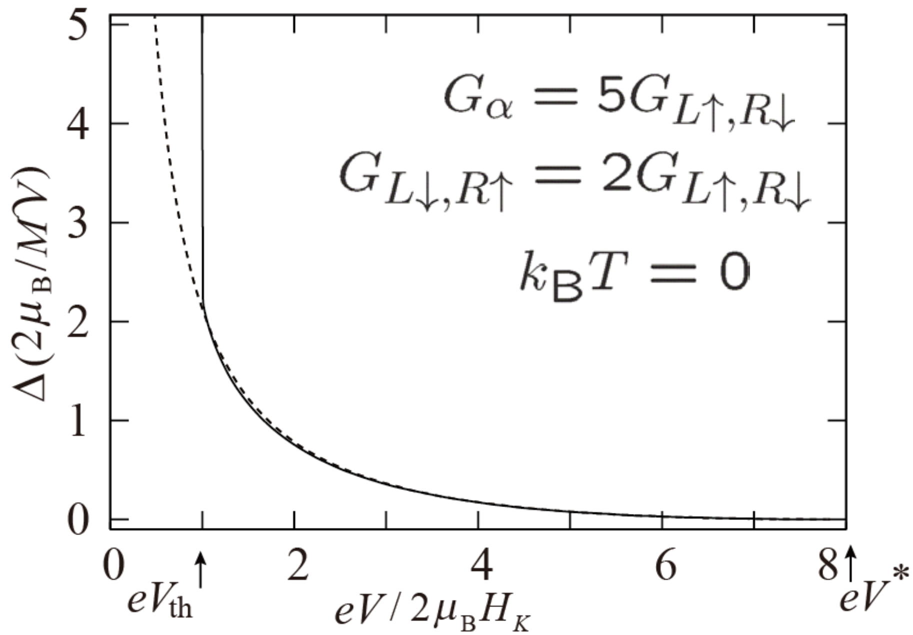
✓ Backaction (spin-pumping) is crucial to maintain the consistency with FT

Switching exponent with finite Gilbert damping & thermal noise

$$\overline{E}(t+\Delta t) - \overline{E}(t) = w - \int_t^{t+\Delta t} dt' \overline{p}_\alpha(t')$$

$$\langle \overline{p_\alpha} \rangle = G_\alpha (\hbar\Omega)^2 \sin^2\theta \qquad G_\alpha = \pi\alpha M\mathcal{V}/h\mu_{\mathrm{B}}$$

$$\langle \delta\overline{p_\alpha(t)}\,\delta\overline{p_\alpha(t')} \rangle = 2k_{\mathrm{B}}T \langle \overline{p_\alpha} \rangle \delta(t-t')$$ → Fluctuation dissipation theorem



$$\frac{eV^*}{2\mu_{\mathrm{B}}H_K} = \frac{G_{L\downarrow,R\uparrow} + G_{L\uparrow,R\downarrow} + G_\alpha}{|G_{L\downarrow,R\uparrow} - G_{L\uparrow,R\downarrow}|}$$

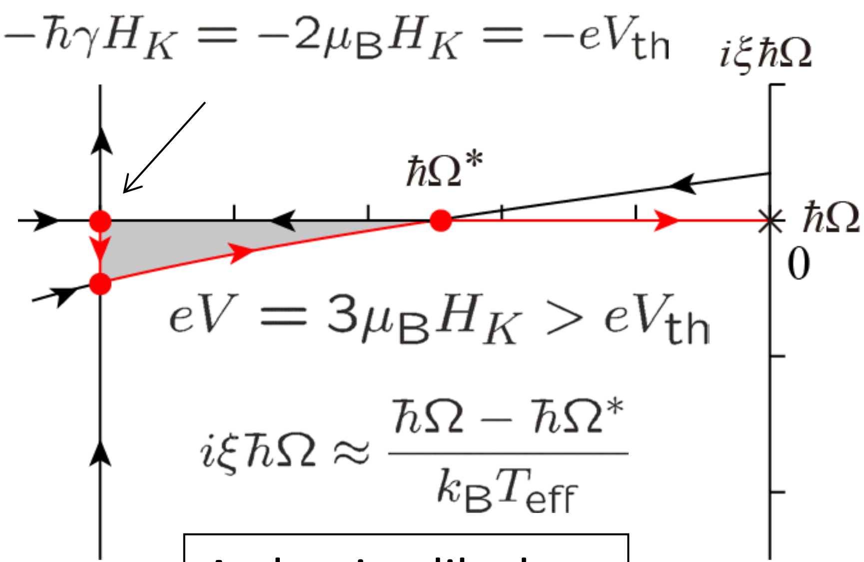
Optimal-path approximation

$$P = \int \mathcal{D}\xi \int_{E(0)}^{E(\tau)} \mathcal{D}E\, e^{i\mathcal{S}} \approx e^{-\Delta}$$

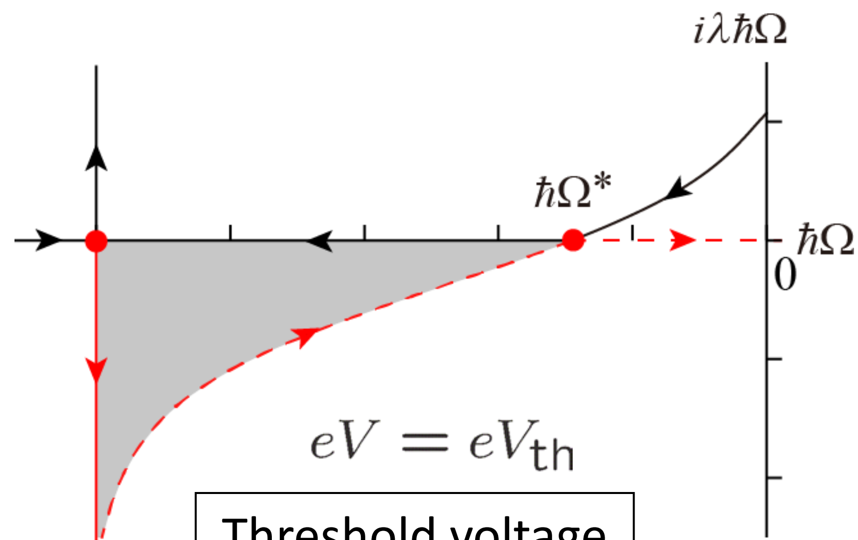
$$i\mathcal{S} = -\int_0^{\tau} dt \left[i\xi(t)\dot{E}(t) - \overbrace{\mathcal{F}_G(\lambda = 0, \xi\hbar\Omega(E(t)); \Omega(E(t)))}^{\text{“Hamiltonian”}} \right]$$

“canonical momentum” “coordinate”

$$\Omega(E) = -\sqrt{-2\gamma^2 H_K E/(M\mathcal{V})}$$



Arrhenius-like law



Threshold voltage

Action along the optimal path

$$\Delta = -i\mathcal{S}^* = \frac{M\mathcal{V}}{2\mu_B\hbar\gamma H_K} \times \text{(area of shaded region)}$$

Summary

1. We calculated the switching probability induced by fluctuating non-conservative & non-equilibrium spin-torque.
2. The backaction (spin pumping) is crucial to maintain the consistency with the fluctuation theorem for a small engine.
3. We found the threshold voltage, which is the onset of probabilistic switching events by spin-torque shot noise

YU, Tomohiro Taniguchi, PRL **114**, 186601, (2015)