Kashiwa 2015

T. Martin

Centre de Physique Théorique
Aix Marseille Université

Josephson like effect and Cooper pair transfer in multi-terminal superconducting

PRB 83, 125421 (2011)
PRB 85, 035419 (2012)
PRB 87, 214501 (2013)
PRB 90, 075419 (2014)

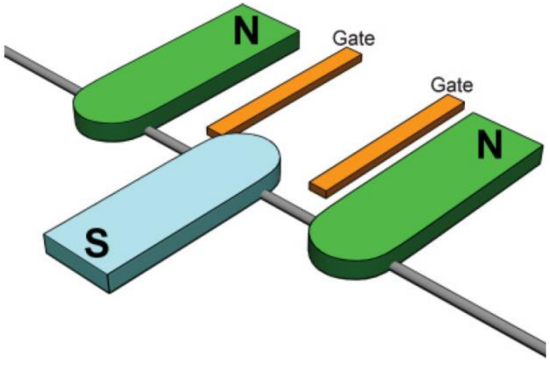


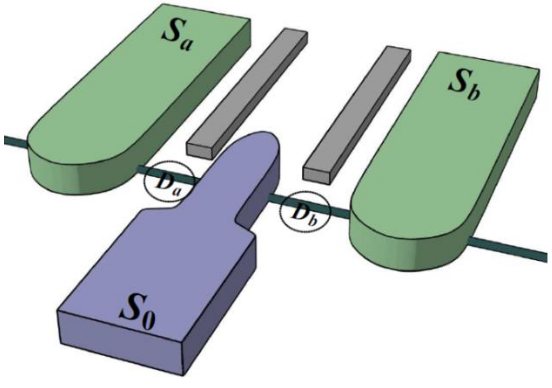

OUTLINE

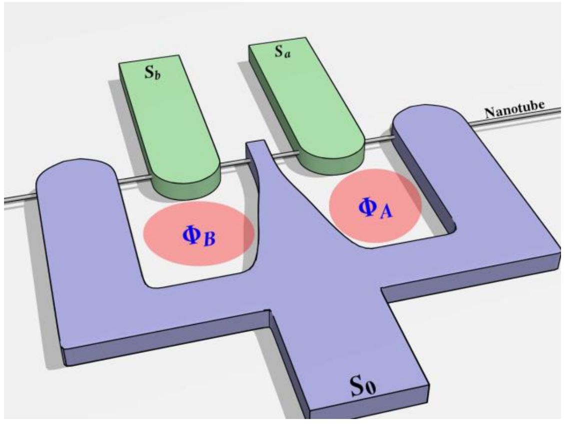
Crossed Andreev Reflection in NSN probed by noise correlations

Multipair DC Josephson resonances probed with comensurate voltages applied to the superconducting leads

Equilibrium setup for probing multipair resonances by Fourier analysis of the the critical current.



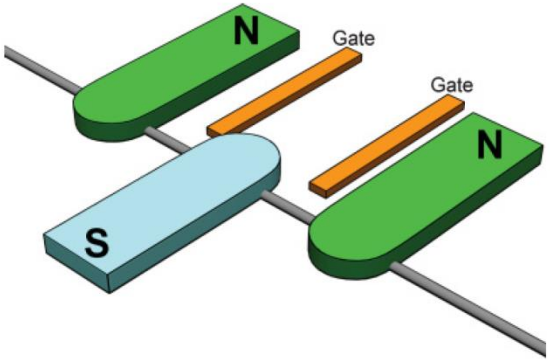


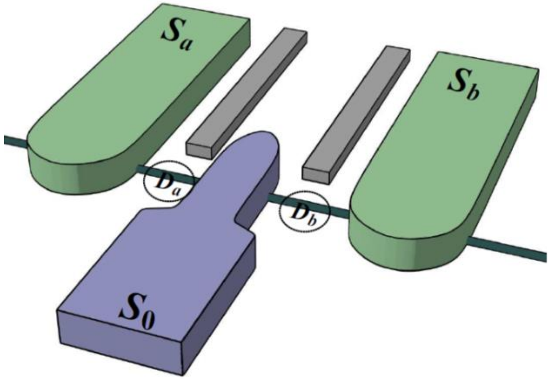


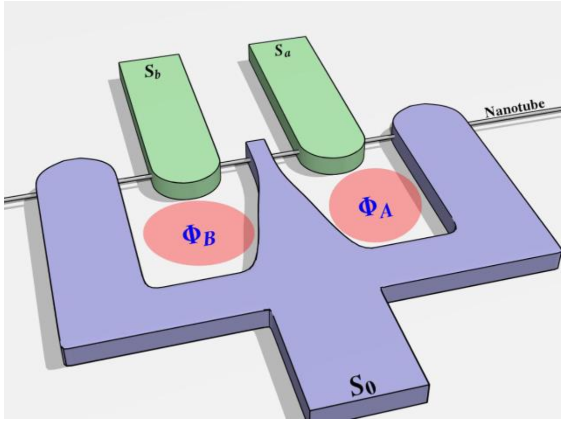
Collaborators:

•T. Jonckheere, J. Rech, D. Chevallier
(Superconducting beam splitter with dots, with(out) interactions)

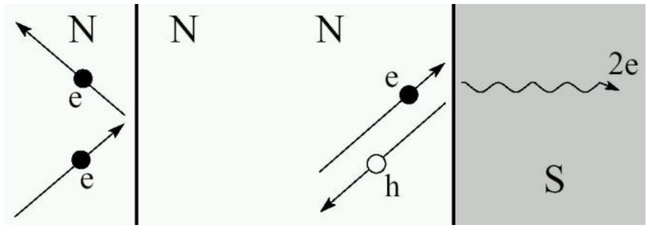
• D. Feinberg, R. Melin, D. Douçot, T. Jonckheere, J. Rech (3 superconductors , quartets, equilibrium…)

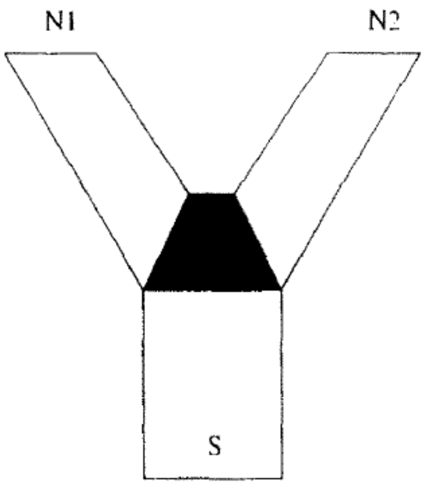




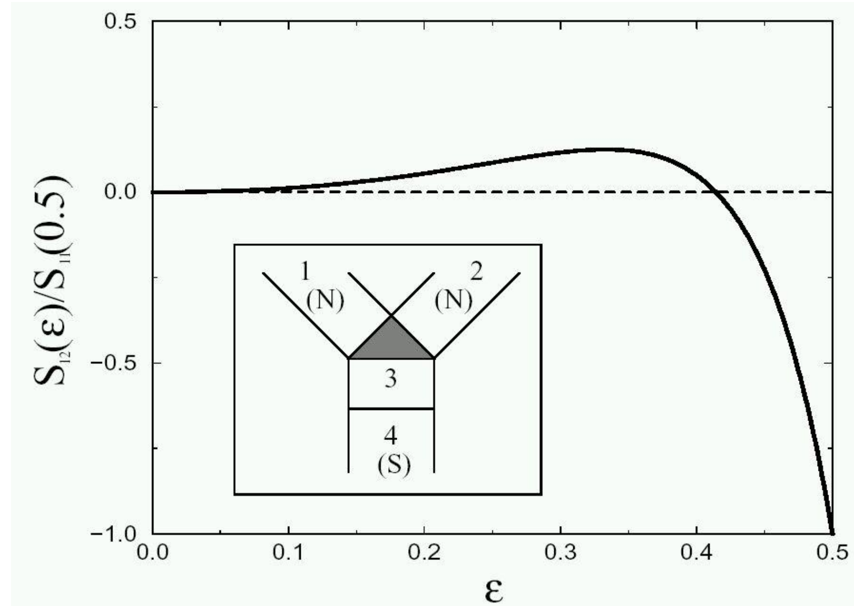


Normal metal/superconducting junctions

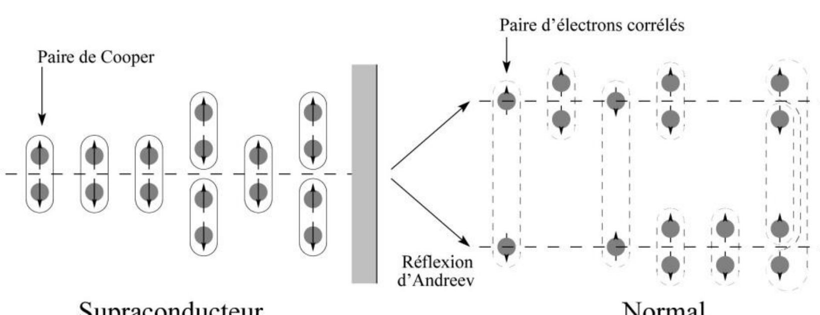




Martin, Phys Lett. A 1996



Torrès Martin EPJB 99



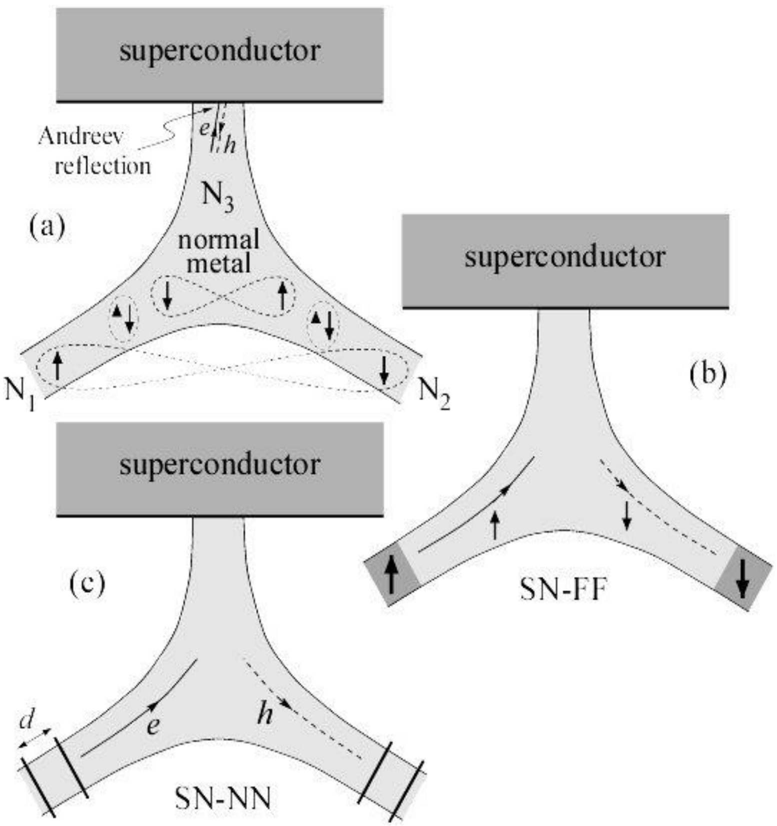
scattering theory suggests that noise crossed correlations can be positive

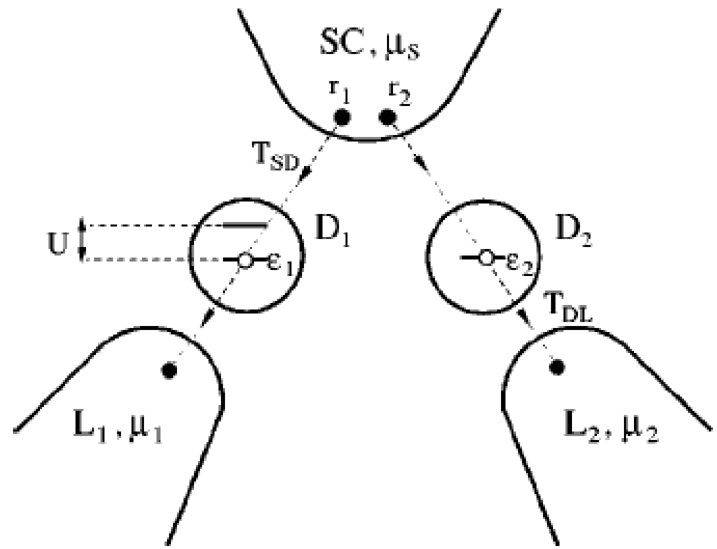
Superconductor: a source of entangled electrons

Lesovik Martin Blatter EPJB 2001

Recher, Sukorukov, Loss PRB 2001

Chtchelkatchev et al. PRB 2002





T matrix calculation of the current: shows singlet state on dots

Positive noise cross correlations for energy filters or spin filters

Borling, Belzig, Bruder PRL 02 **FCS !!!**
Samuelsson Buttiker Chaotic 02

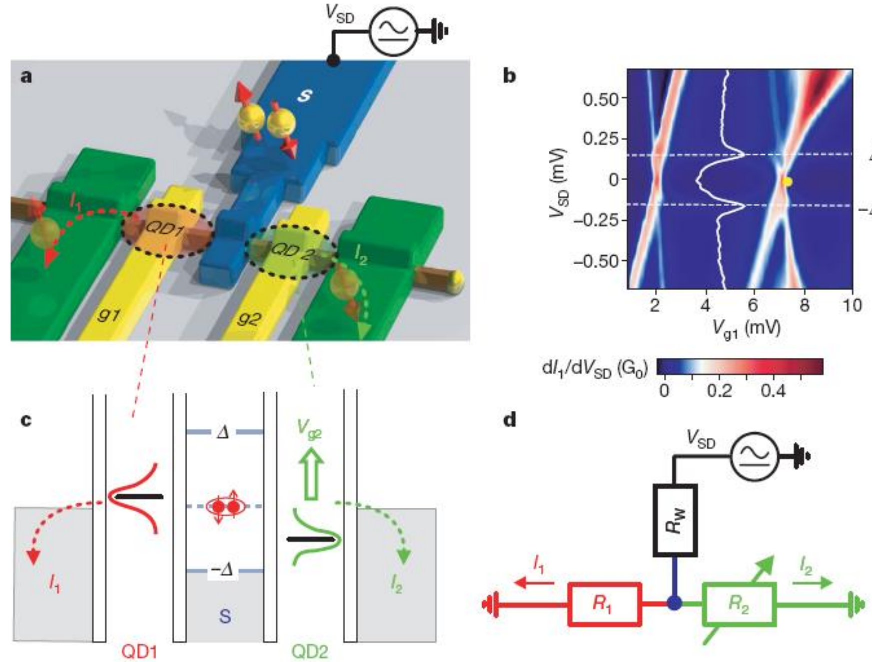
Current measurements showing non local effects

nature Vol 461|15 October 2009|doi:10.1038/nature08432

LETTERS

Cooper pair splitter realized in a two-quantum-dot Y-junction

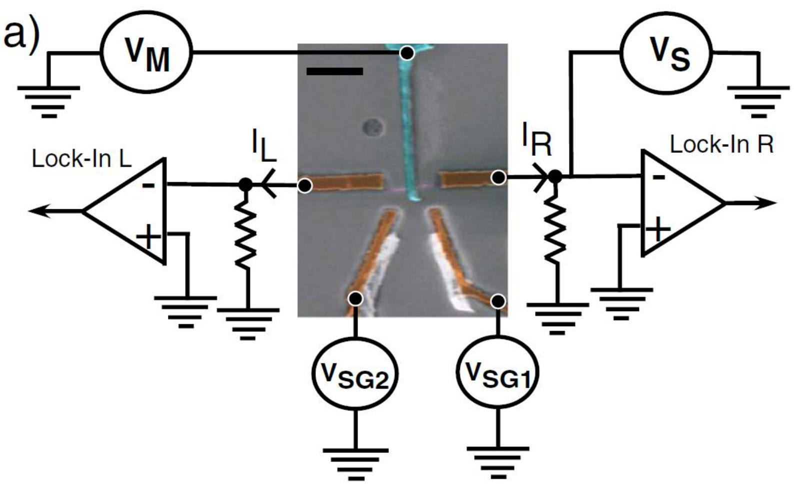
L. Hofstetter[1]*, S. Csonka[1,2]*, J. Nygård[3] & C. Schönenberger[1]

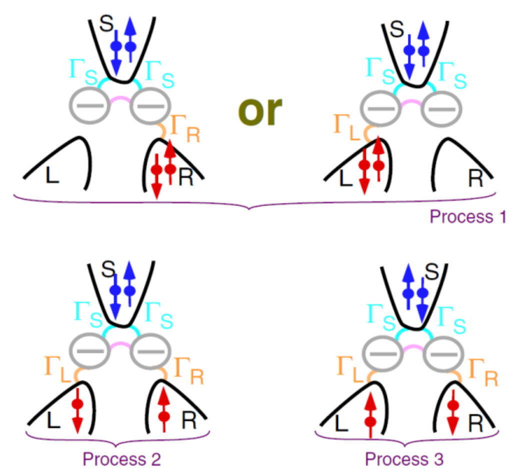


PRL 104, 026801 (2010) PHYSICAL REVIEW LETTERS week ending 15 JANUARY 2010

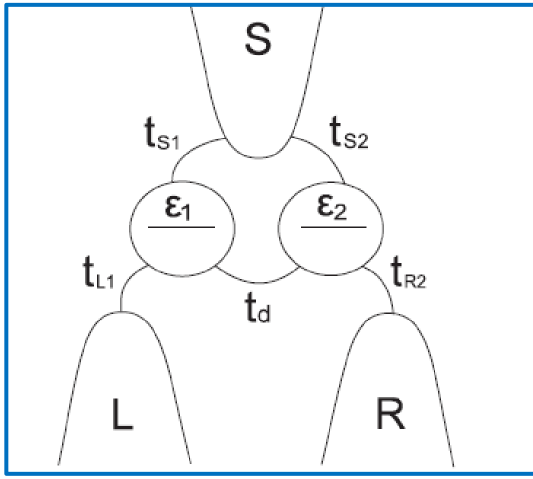
Carbon Nanotubes as Cooper-Pair Beam Splitters

L. G. Herrmann,[1,2,5] F. Portier,[3] P. Roche,[3] A. Levy Yeyati,[4] T. Kontos,[1,2,*] and C. Strunk[5]





Model: BCS + dots + metallic/ BCS leads



$$H_{D_\alpha} = \epsilon_\alpha \sum_{\sigma=\uparrow,\downarrow} d^\dagger_{\alpha\sigma} d_{\alpha\sigma}$$

$$H_{D_1 D_2} = t_d d_1^\dagger \sigma_z d_2 + \text{h.c.}$$

hopping

$$\mathcal{T}_{j\alpha}(t) = t_{j\alpha} \sigma_z \, e^{i\sigma_z \int V_j dt}$$

$$H_{T_\alpha}(t) = \sum_{jk} \Psi^\dagger_{jk} \mathcal{T}_{j\alpha}(t) d_\alpha + \text{h.c.}$$

$$d = \begin{pmatrix} d_\uparrow \\ d^\dagger_\downarrow \end{pmatrix}$$

$$H_j = \sum_k \Psi^\dagger_{jk} (\xi_k \sigma_z + \Delta_j \sigma_x) \Psi_{jk}$$

Nambu spinors

$$\Psi_{jk} = \begin{pmatrix} \psi_{jk,\uparrow} \\ \psi^\dagger_{j(-k),\downarrow} \end{pmatrix}$$

1 particle Green's function of the dot

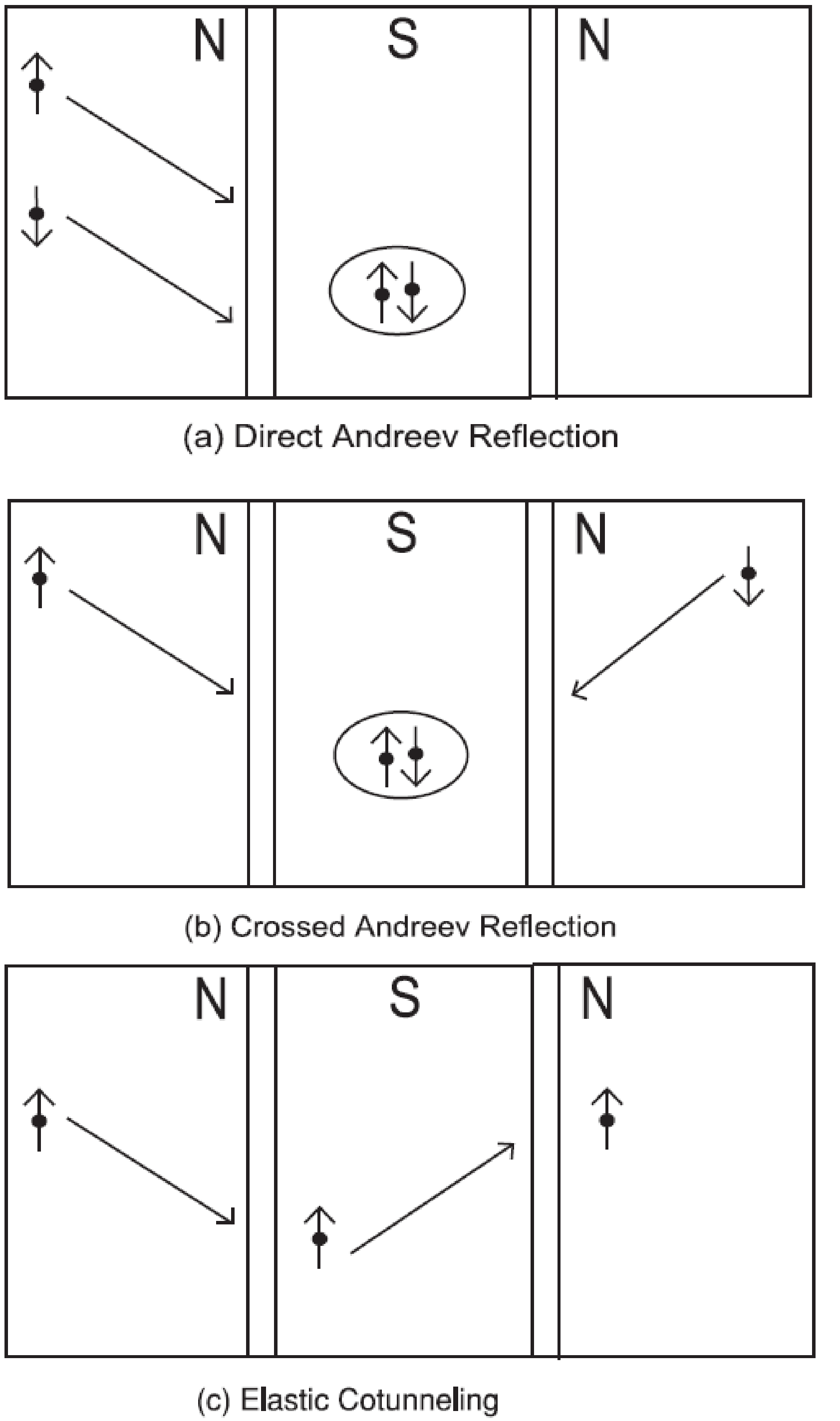
$$\check{G}^{ss'}_{\alpha\alpha'}(t,t') = -i \left\langle T_C \left\{ d^s_\alpha(t) d^{\dagger s'}_{\alpha'}(t') \right\} \right\rangle$$

3 competing Processes:

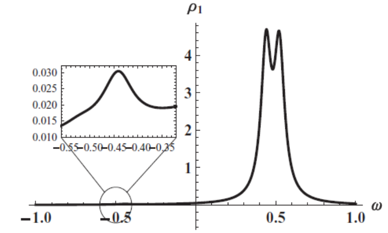
Direct Andreev reflection DAR

Crossed Andreev Reflection CAR

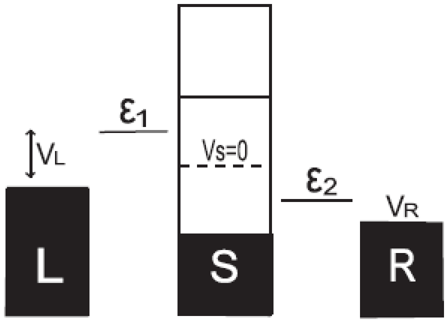
Electron cotunelling through S, EC



NO INTERACTION ON THE DOTS:



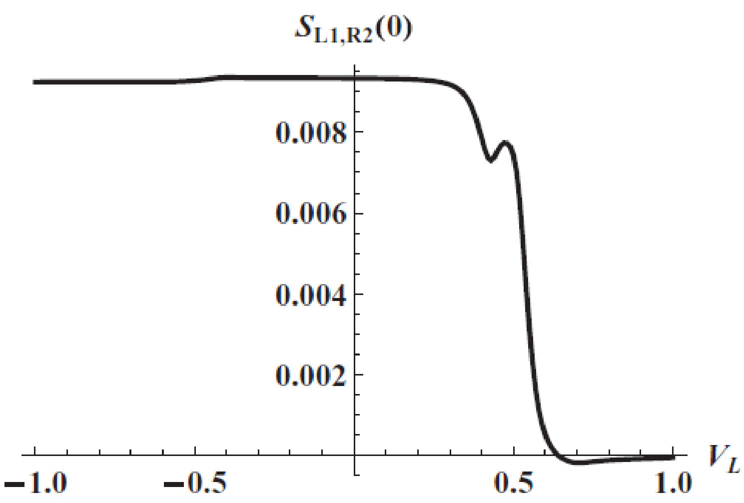
- Novel features in density of states of the dots



- CAR processes optimized by antisymmetric configuration

- DAR contributes to the current, but not to the noise cross correlation signal

- (Direct tunneling between the dots « spoils » the positive correlation signal)

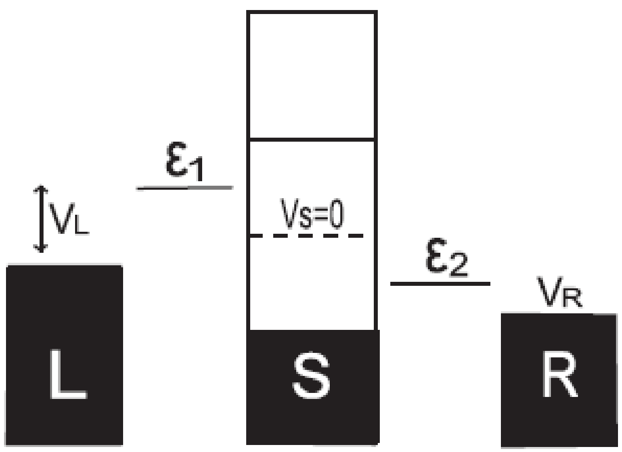


What about interactions ?
include them perturbatively
(current conserving scheme)

Interactions in the Cooper Pair beam Splitter reduce the current

$$U_\alpha n_{\alpha\uparrow} n_{\alpha\downarrow}$$

Antisymmetric case



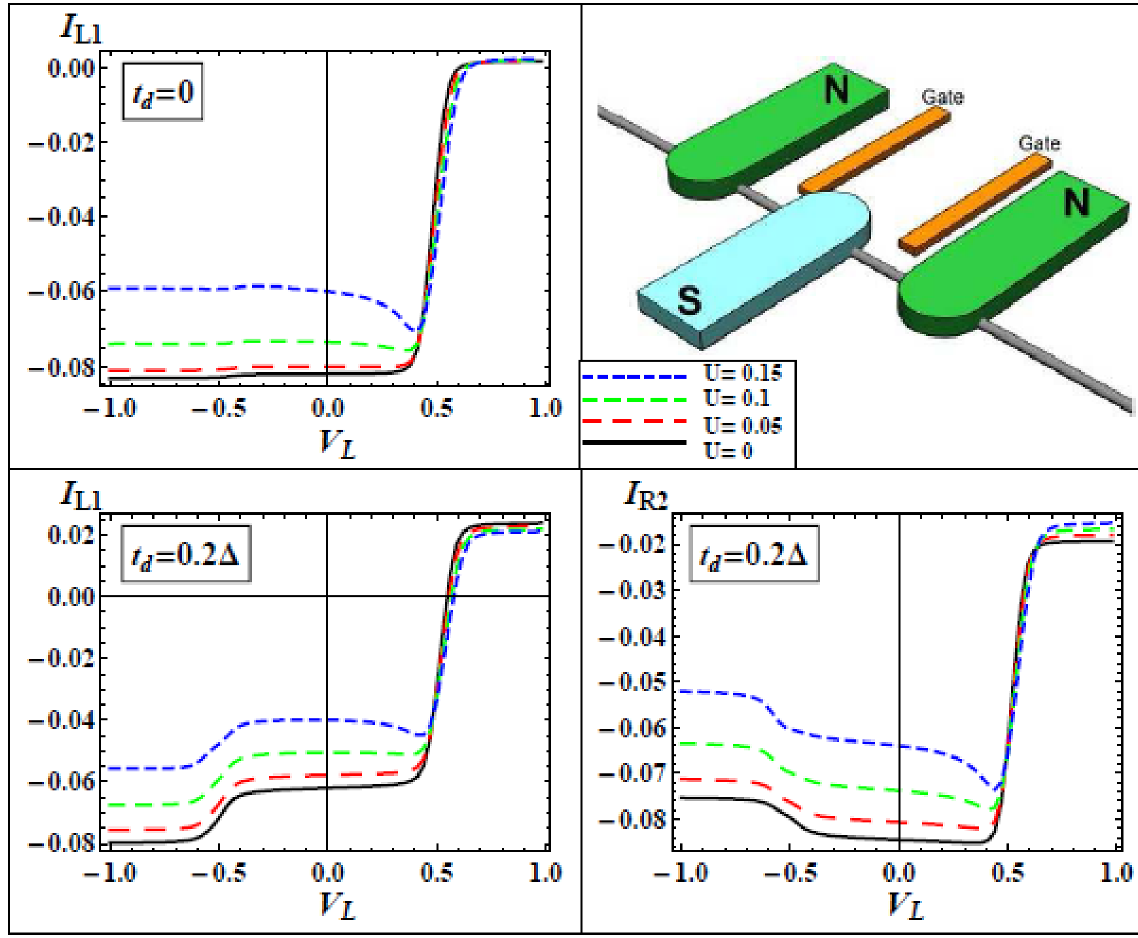


FIG. 2: Currents I_{L1} and I_{R2} (arbitrary units) as a function of V_L, for different values of the interaction $U = 0, 0.05, 0.1, 0.15\Delta$, and the parameters (in units of Δ) $\beta = 100$, $\epsilon_1 = 0.5$, $\epsilon_2 = -0.5$, $V_R = -0.7$ and $t_{L1} = t_{S1} = t_{S2} = t_{R2} = 0.2$. The upper left panel corresponds to $t_d = 0$, while the lower panels are computed for $t_d = 0.2\Delta$. The upper right panel shows a representation of the setup.

No tunneling between the dots: **increase positive Xcorr** !

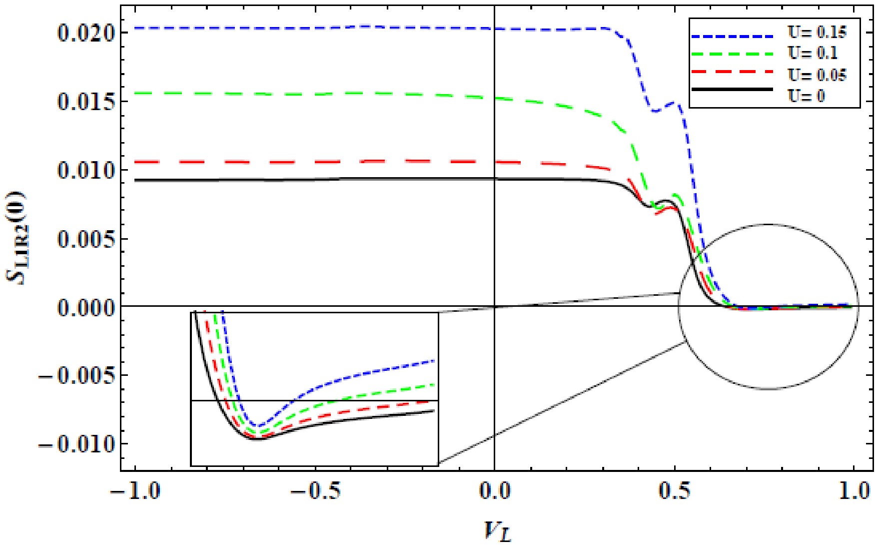


FIG. 3: Current cross-correlations (arbitrary units) as a function of V_L, for $U = 0, 0.05, 0.1, 0.15\Delta$, and the same parameters as in Fig. 2, in the absence of direct tunneling.

LARGE REGIMES WHERE NOISE CROSS CORRELATIONS ARE POSITIVE

INTERACTIONS HELP

With tunneling between the dots: **positive correlations still robust**

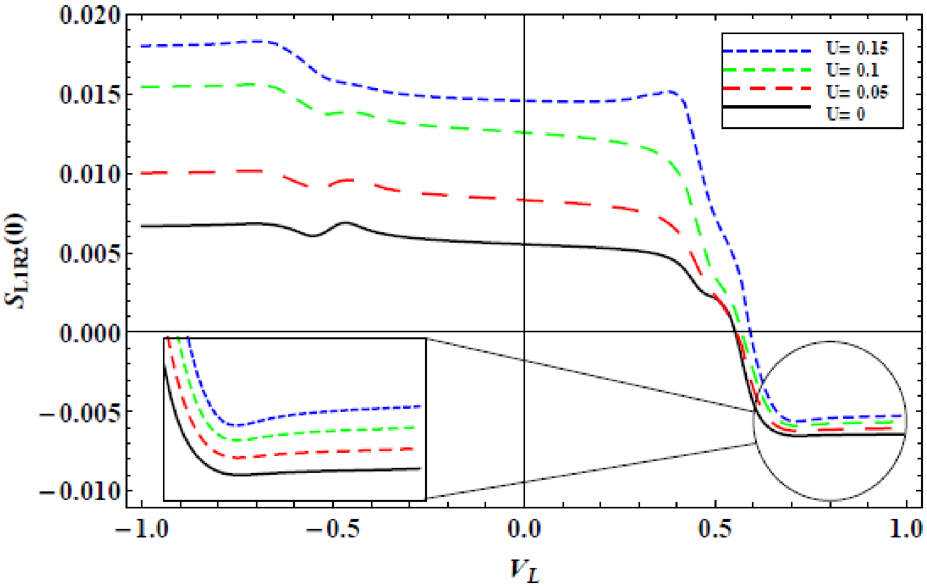
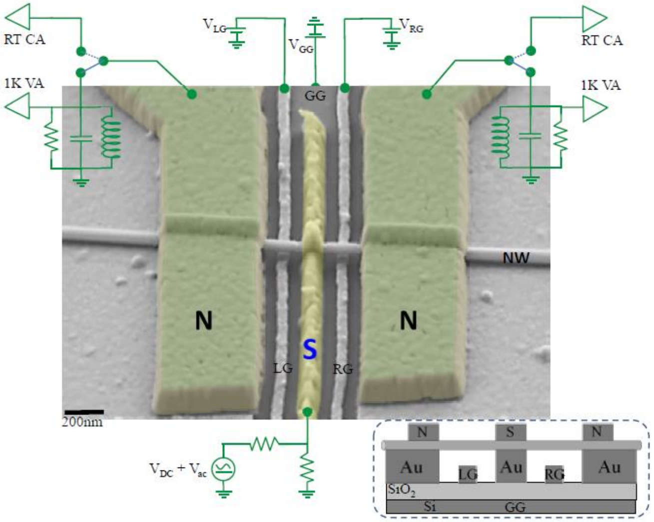


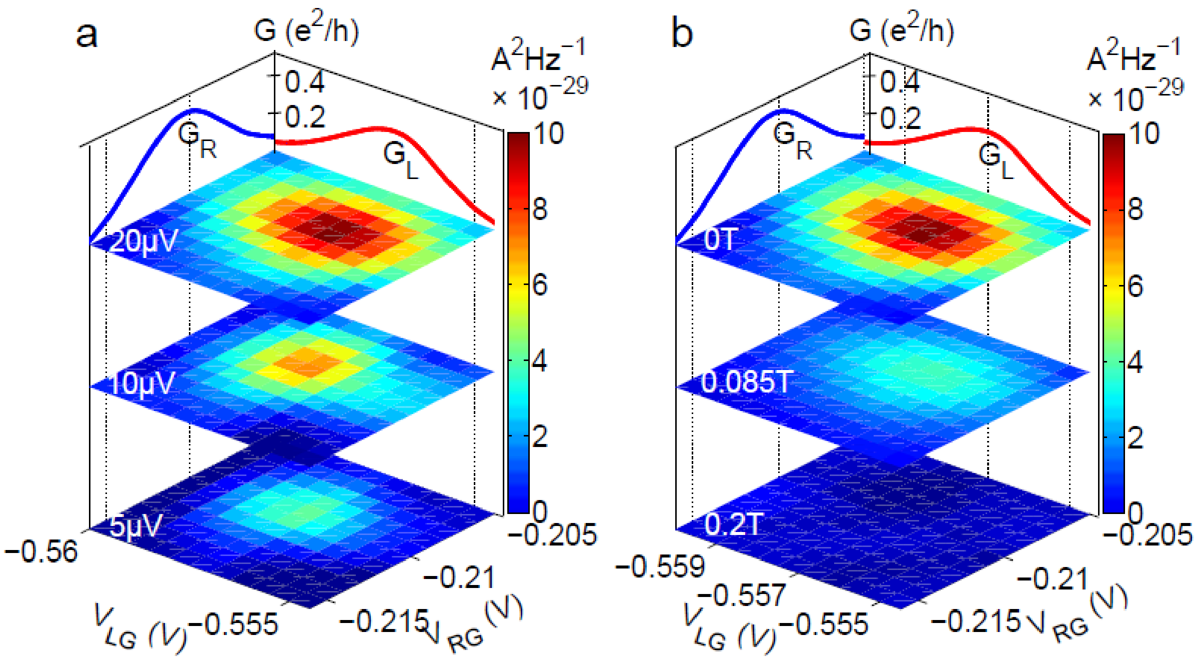
FIG. 4: Current cross-correlations (arbitrary units) as a function of V_L, for $U = 0, 0.05, 0.1, 0.15\Delta$, and the same parameters as in Fig. 2, in the presence of direct tunneling, $t_d = 0.2\Delta$.

$$U_\alpha n_{\alpha\uparrow} n_{\alpha\downarrow}$$

Evidence of entangled electrons born from Cooper pairs splitting via current and noise correlations

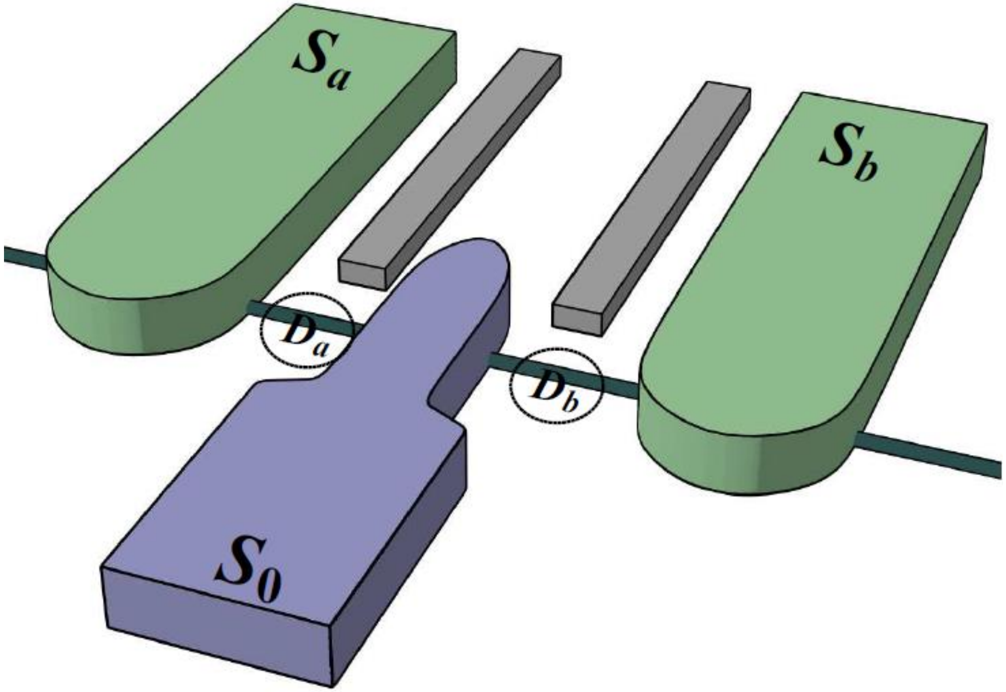
Anindya Das, Yuval Ronen, Moty Heiblum,*
Diana Mahalu, Andrey V. Kretinin and Hadas Shtrikman





Positive noise cross correlations

3 superconductor bi-junction off equilibrium



Superconductors separated by quantum dots
Different voltages on each of them
Dots (generated by nanowires) between 2 Superconductors (S)
Phases applied on each S
Same Hamiltonian as before, except for gaps and phases

2 superconductors

DC Josephson effect: phase dependent current at equilibrium

AC Josephson effect: DC bias imposed → time oscillations at the Josephson frequency

DC bias imposed + (small) AC Bias → Shapiro steps = DC current at specific voltages, phase dependent.

3 superconductors

1 biased junction can generate an AC drive which affects the other biased junction and vice versa

Here: use a « thin » central superconductor where
CROSSED ANDREEV REFLECTION operates

Main messages

At commensurate voltages $nV_a + mV_b = 0$ → synchronization of 2 AC Josephson effects → **DC Josephson resonances**

Pi shift for « quartet » resonances n=m=1 at low bias

$$I_Q = I_{Q0} \sin(\varphi_a + \varphi_b - 2\varphi_0)$$

Tunability (enhancement of DC resonances) when gates tune the position of dot levels

Other processes, such as DC quasiparticle-pair interference effects, also contribute → phase dependent MAR

Ingredients

(Nambu spinor notation)

$$\hat{\mathcal{H}}_T(t) = \sum_{jk\alpha} \Psi^\dagger_{jk}\, t_{j\alpha} e^{i\sigma_z \varphi_j/2}\, \mathbf{d}_\alpha + \mathrm{h.c.}$$

$$\varphi_j(t) = \varphi_j^{(0)} + 2eV_j t/\hbar$$

Current

$$\langle I_j(t)\rangle = -2\mathrm{Re}\left\{ \mathrm{tr}\left[\sigma_z \left(\hat{\Sigma}_j \circ \hat{G}\right)^{+-}(t,t)\right]\right\}$$

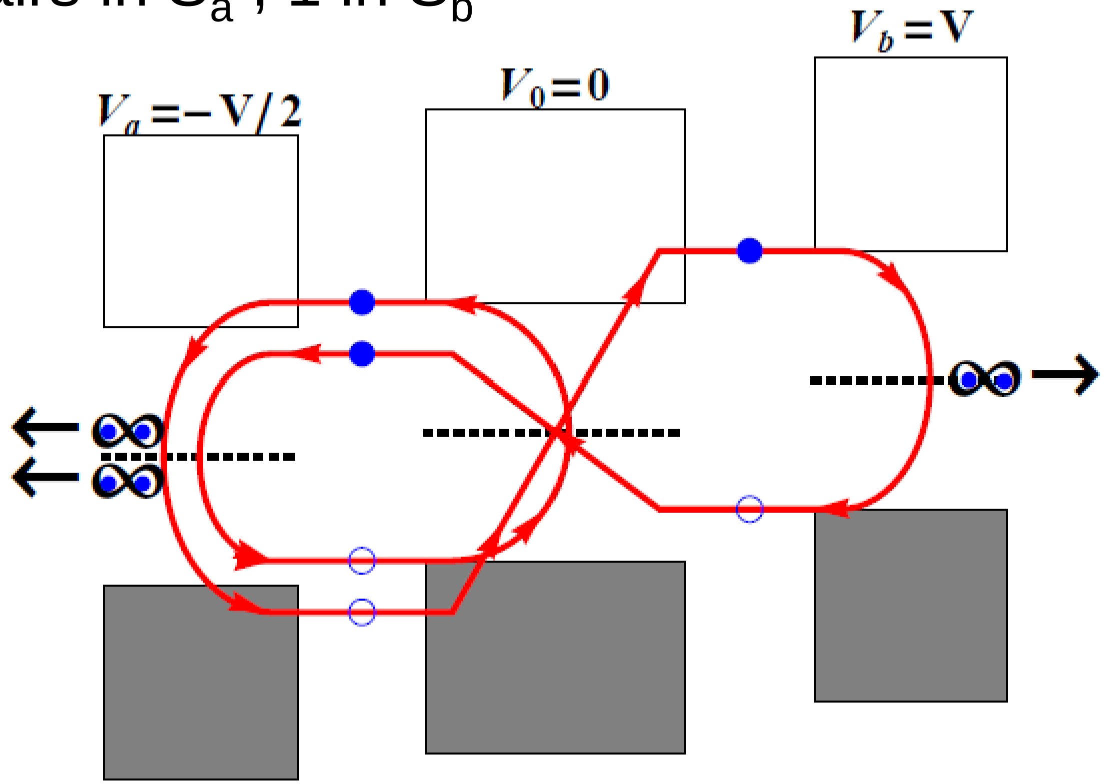
« Meir Wingreen » formula with dot Greens function

WHEN VOLTAGES ARE COMMENSURATE, DC JOSEPHSON

Example: « sextet » current for $V_a = -V_b/2$

2 Crossed Andreev Reflections

2 Cooper pairs in S_a , 1 in S_b

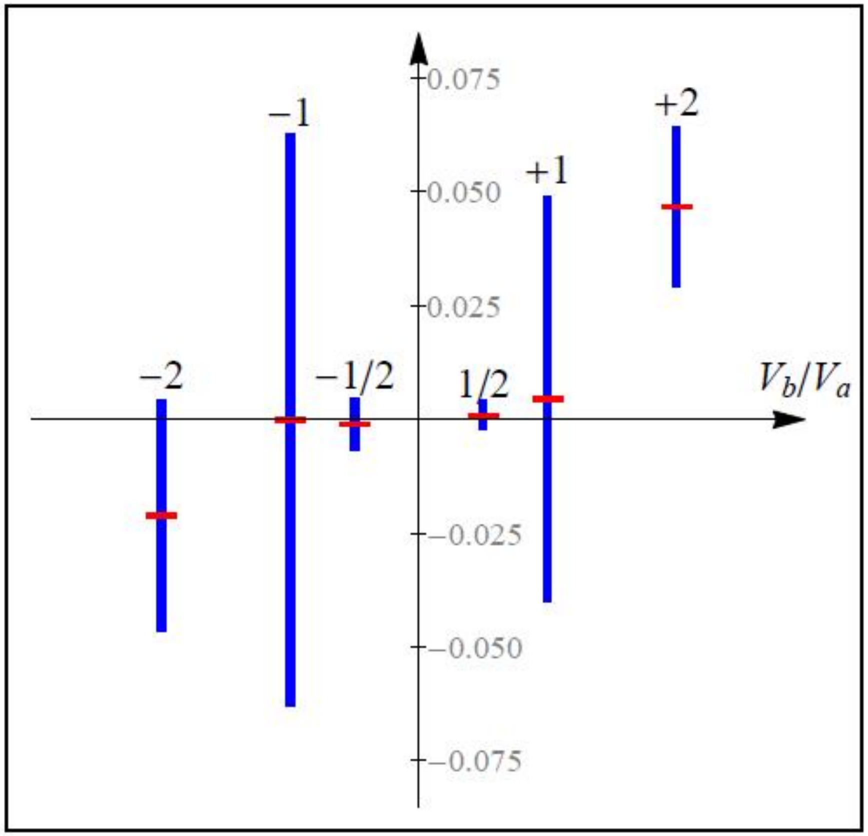


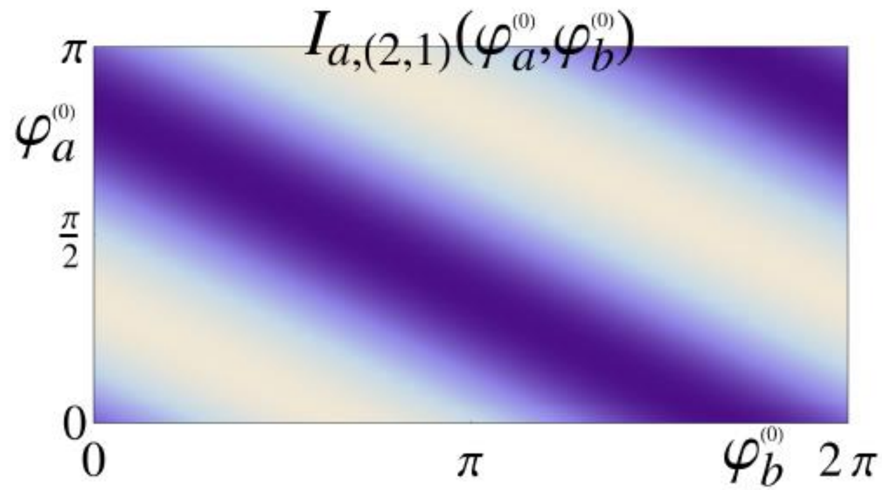
A) « Metallic » regime

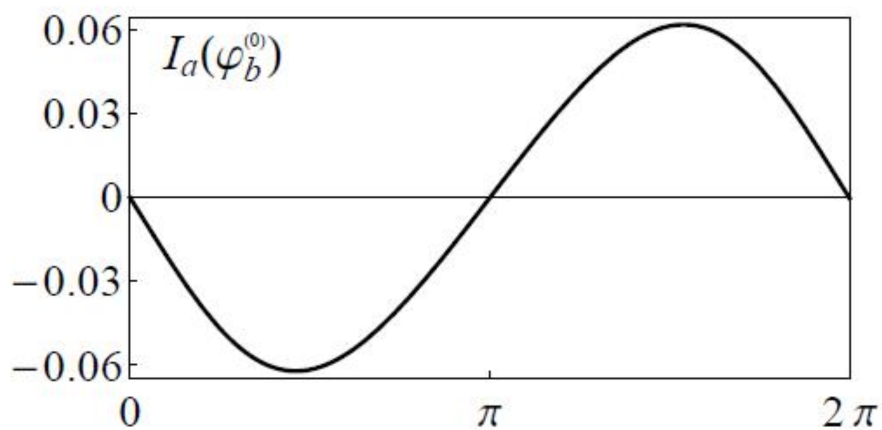
Dot levels wider than the gaps, placed in the quasiparticle spectrum

$$I_{a/b} = \sum_{n,m} I_{a/b,(n,m)} \sin(n\varphi_a + m\varphi_b)$$ Phase dependance

Critical currents



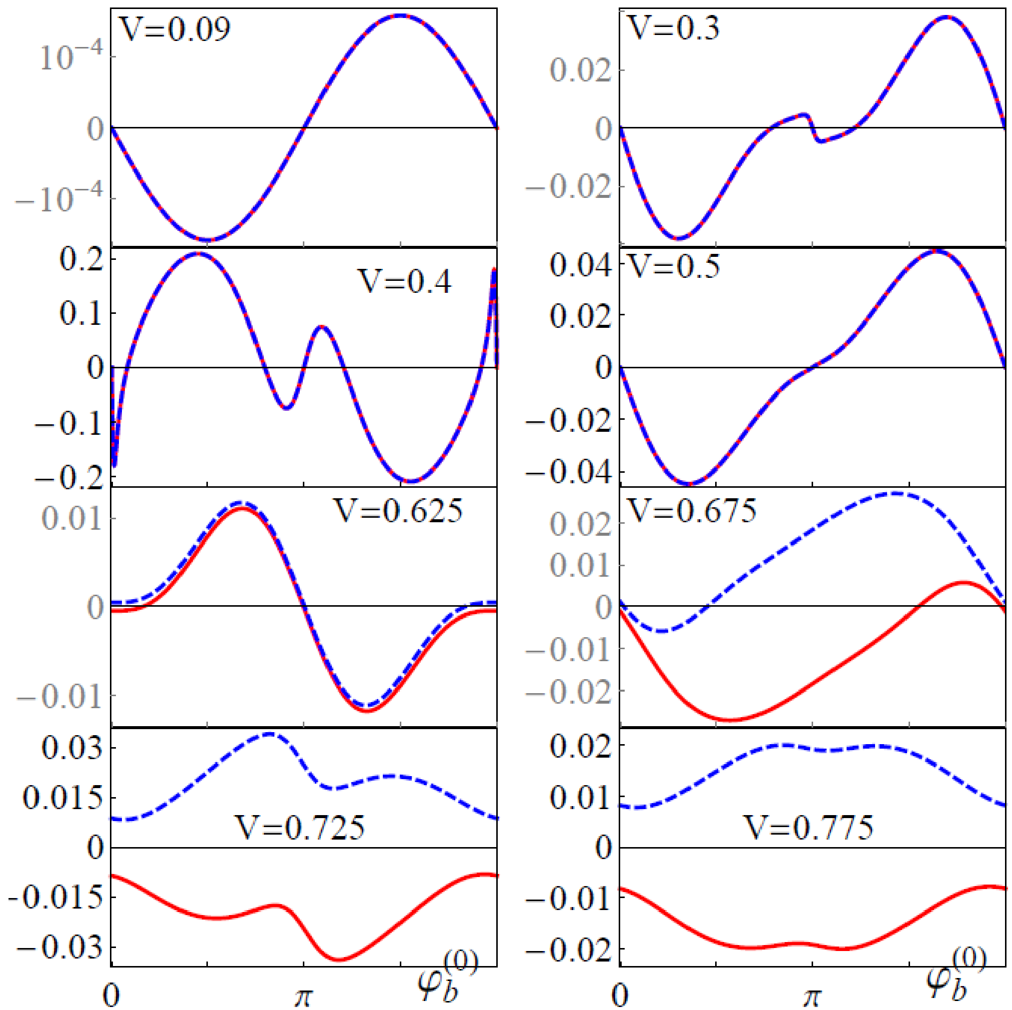




(Quartet and cotunneling signals)

Pi shift

Current phase relations (quartet regime) to Sa and Sb

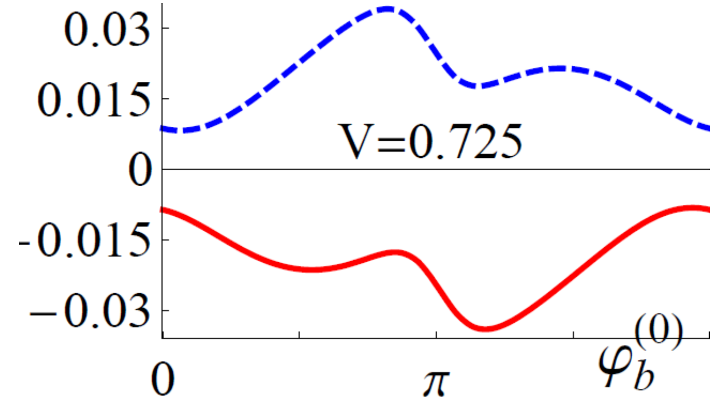


at low voltages, the current to « Sa » and « Sb » are equal

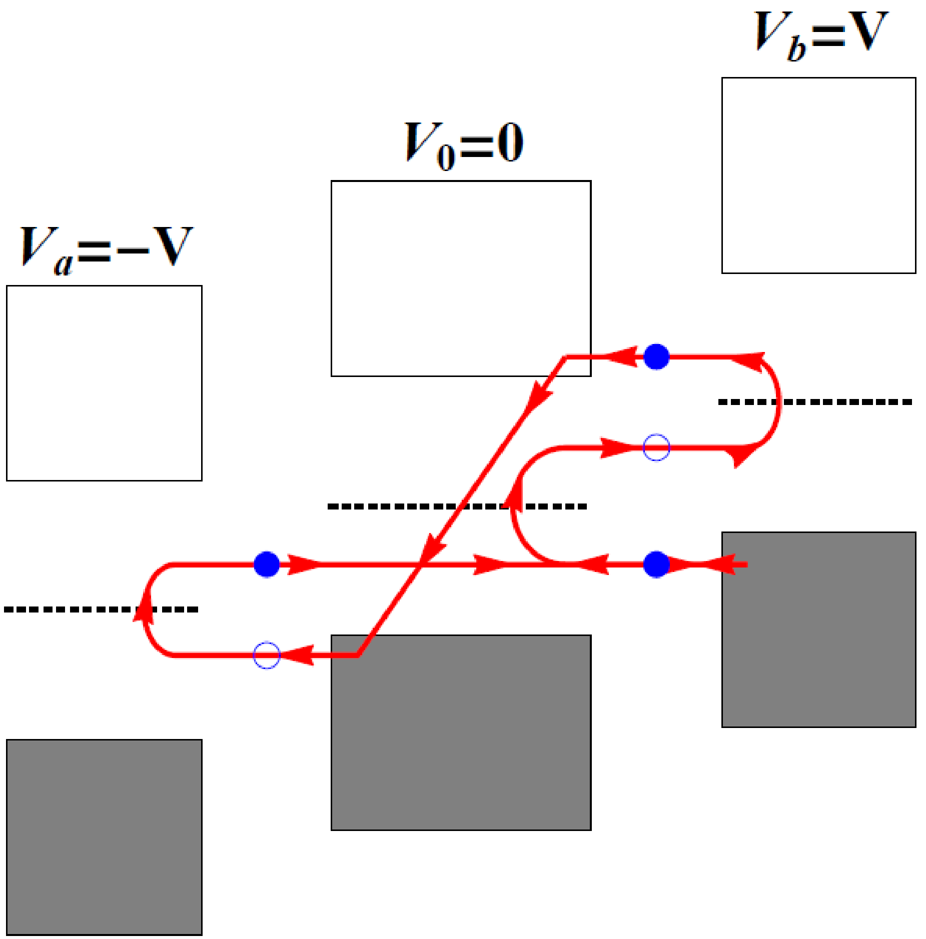
Close to the fist MAR onset, they start to differ

WHY ?

Why do the two currents deviate at high voltage ?

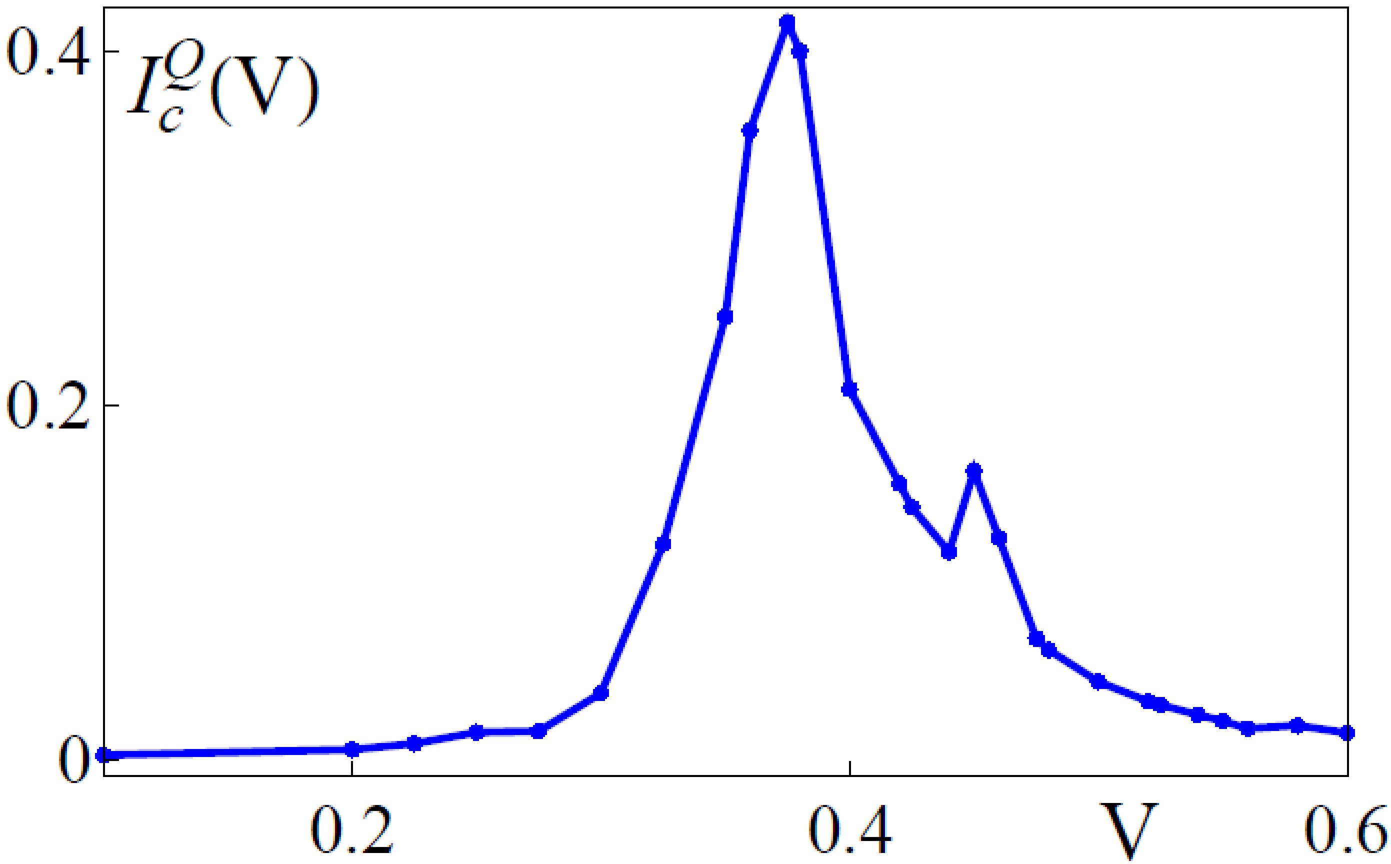


New phase sensitive process:
« Phase sensistive MAR »

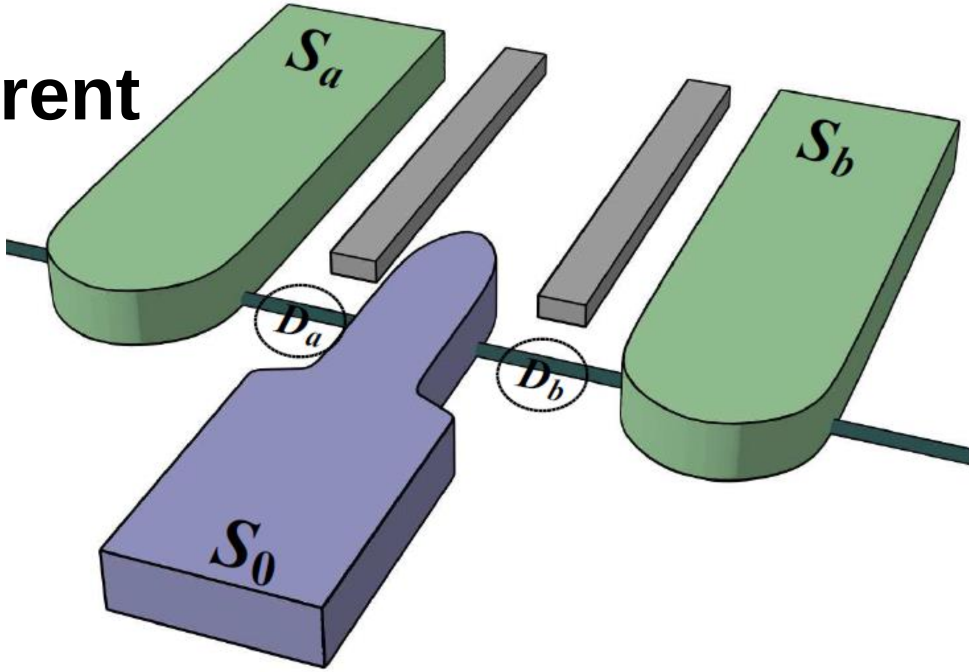


Why use dots in between the superconductors ?

Enhancement of the multipair signal by tuning the dot levels
In resonance with the lead bias



3 contributions to the current



$$\bar{I} = (\langle I_a \rangle, \langle I_b \rangle)$$

$$\bar{V} = (V_a, V_b)$$

$$\bar{\varphi} = (\varphi_a, \varphi_b)$$

$$\bar{\epsilon} = (\epsilon_a, \epsilon_b)$$

$$\boxed{\bar{I}(\bar{\varphi}, \bar{V}, \bar{\epsilon}) = \bar{I}^{MP}(\bar{\varphi}, \bar{V}, \bar{\epsilon}) + \bar{I}^{phMAR}(\bar{\varphi}, \bar{V}, \bar{\epsilon}) + \bar{I}^{qp}(\bar{V}, \bar{\epsilon})}$$

Assuming electron hole symmetry to hold

$$\bar{I}(\bar{\varphi}^{(0)}, \bar{V}, \bar{\epsilon}) = -\bar{I}(-\bar{\varphi}^{(0)}, -\bar{V}, -\bar{\epsilon})$$

Observed symmetries

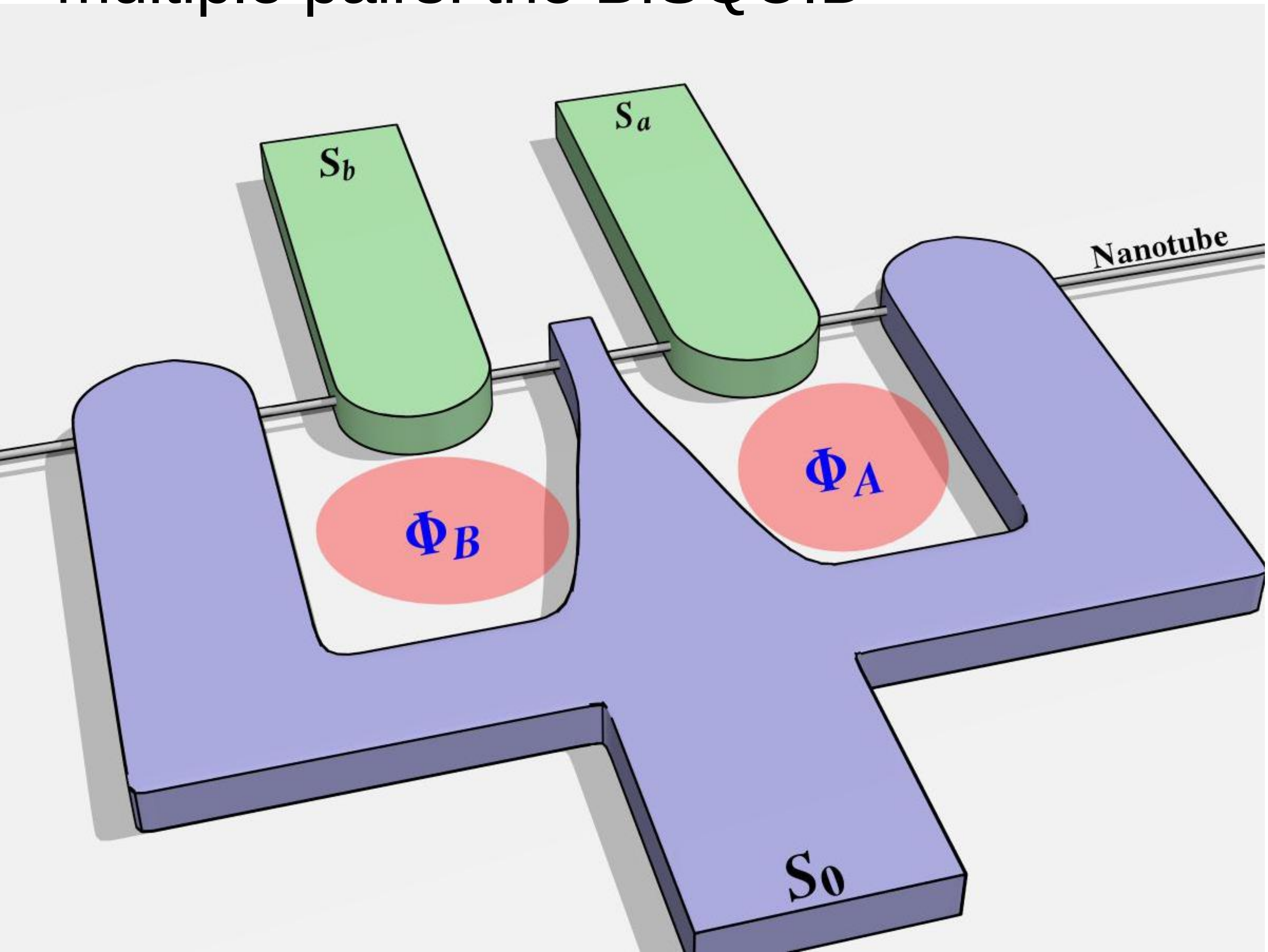
$\bar{I}^{qp}$ Phase insensitive, odd in V

$\bar{I}^{MP}$ function of $(n\ \varphi_a + m\ \varphi_b)$, odd in phases and even in V

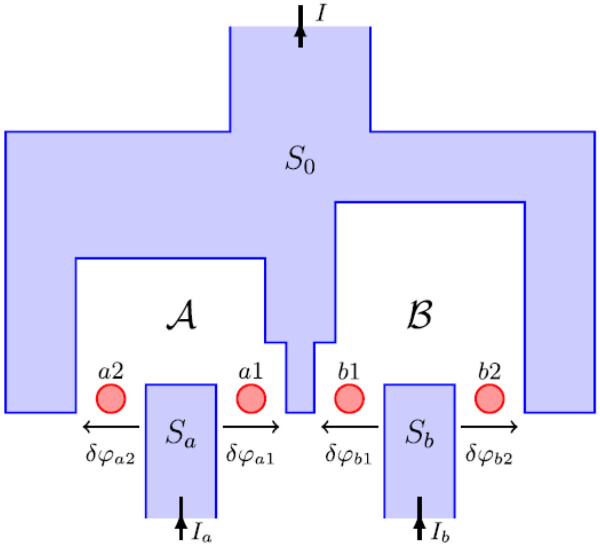
$$m\langle I_a\rangle = n\langle I_b\rangle$$

$\bar{I}^{phMAR}$ function of $n\ \varphi_a + m\ \varphi_b$, even in phases and odd in V

Equilibrium evidence of entangled multiple pairs: the BISQUID



PHENOMENOLOGY



$$\delta\varphi_{a1} = \varphi_s - \varphi_a,$$

$$\delta\varphi_{b1} = \varphi_s - \varphi_b,$$

$$\delta\varphi_{a2} = \varphi_s - \varphi_a - 2\pi\frac{\Phi_{\mathcal{A}}}{\Phi_0}$$

$$\delta\varphi_{b2} = \varphi_s - \varphi_b + 2\pi\frac{\Phi_{\mathcal{B}}}{\Phi_0}$$

1 pair

$$I_{1\mathrm{P}} = I_{\mathrm{J}}[\sin\delta\varphi_{a1} + \sin\delta\varphi_{b1} + \sin\delta\varphi_{a2} + \sin\delta\varphi_{b2}]$$

$$I_{2\mathrm{P,local}} = I_{\mathrm{JJ}}[\sin(2\delta\varphi_{a1}) + \sin(2\delta\varphi_{b1}) + \sin(2\delta\varphi_{a2}) + \sin(2\delta\varphi_{b2})]$$

2 pairs in 1 junction

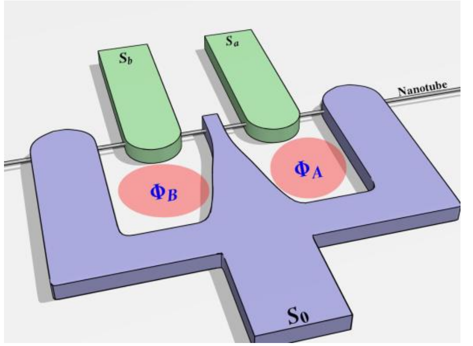
2 pairs, CAR involved

$$I_{2\mathrm{P,quartet}} = I_{\mathrm{Q}}\sin(\delta\varphi_{a1} + \delta\varphi_{b1})$$

$$I_{2\mathrm{P,pair\ cotunneling}} = I_{\mathrm{PC}}\sin(\delta\varphi_{a1} - \delta\varphi_{b1})$$

2 pairs in EC involved

Microscopic calculation



$$I_\alpha = -\frac{2e}{\beta\hbar}\frac{\partial \log Z_{\text{eff}}}{\partial\delta\varphi_\alpha}$$
$$= -\frac{2e}{\beta\hbar}\frac{\partial}{\partial\delta\varphi_\alpha}\sum_n \log[\det\hat{G}^{-1}(i\omega_n)]$$

Dot Green's function

$$\bar{I}_{\text{tot}} = \frac{1}{2\lambda_F}\int_{-\lambda_F}^{\lambda_F} dr\, \frac{-2e}{\beta\hbar}\sum_n\sum_\alpha \frac{\partial \log[\det\hat{G}^{-1}(i\omega_n)]}{\partial\delta\varphi_\alpha}$$

Probe critical current

$$I_c(\Phi_{\mathcal{A}},\Phi_{\mathcal{B}}) = \max_{\varphi_a,\varphi_b}|\bar{I}_{\text{tot}}(\delta\varphi_{a1},\delta\varphi_{b1},\delta\varphi_{a2},\delta\varphi_{b2})|$$

Average magnetic flux

$$\Phi = \frac{\Phi_{\mathcal{A}} + \Phi_{\mathcal{B}}}{2}$$

Assymmetry

$$\eta = \frac{\mathcal{S}_{\mathcal{B}} - \mathcal{S}_{\mathcal{A}}}{\mathcal{S}_{\mathcal{B}} + \mathcal{S}_{\mathcal{A}}}.$$

$$I_c\left(\Phi_{\mathcal{A}}, \Phi_{\mathcal{B}}\right) = \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} I_{n,m} e^{i2n\pi \frac{\Phi_{\mathcal{A}}}{\Phi_0}} e^{i2m\pi \frac{\Phi_{\mathcal{B}}}{\Phi_0}}$$

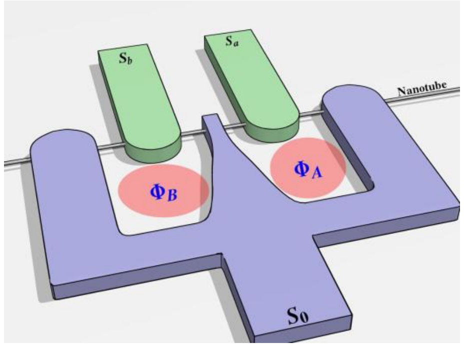
$$I_c(\Phi) = \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} I_{n,m} e^{2i\pi \frac{\Phi}{\Phi_0}[n(1-\eta)+m(1+\eta)]}$$

$$\tilde{I}_c(\mathcal{N}) = \frac{1}{\Phi_0} \int d\Phi \, I_c(\Phi) e^{-2i\pi\mathcal{N} \frac{\Phi}{\Phi_0}}$$

$$= \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} I_{n,m} \delta(n(1-\eta) + m(1+\eta) - \mathcal{N})$$

This has peaks at

$$\mathcal{N} = m + n + (m - n)\eta$$



Corresponding to the (n, m) harmonics of the critical current

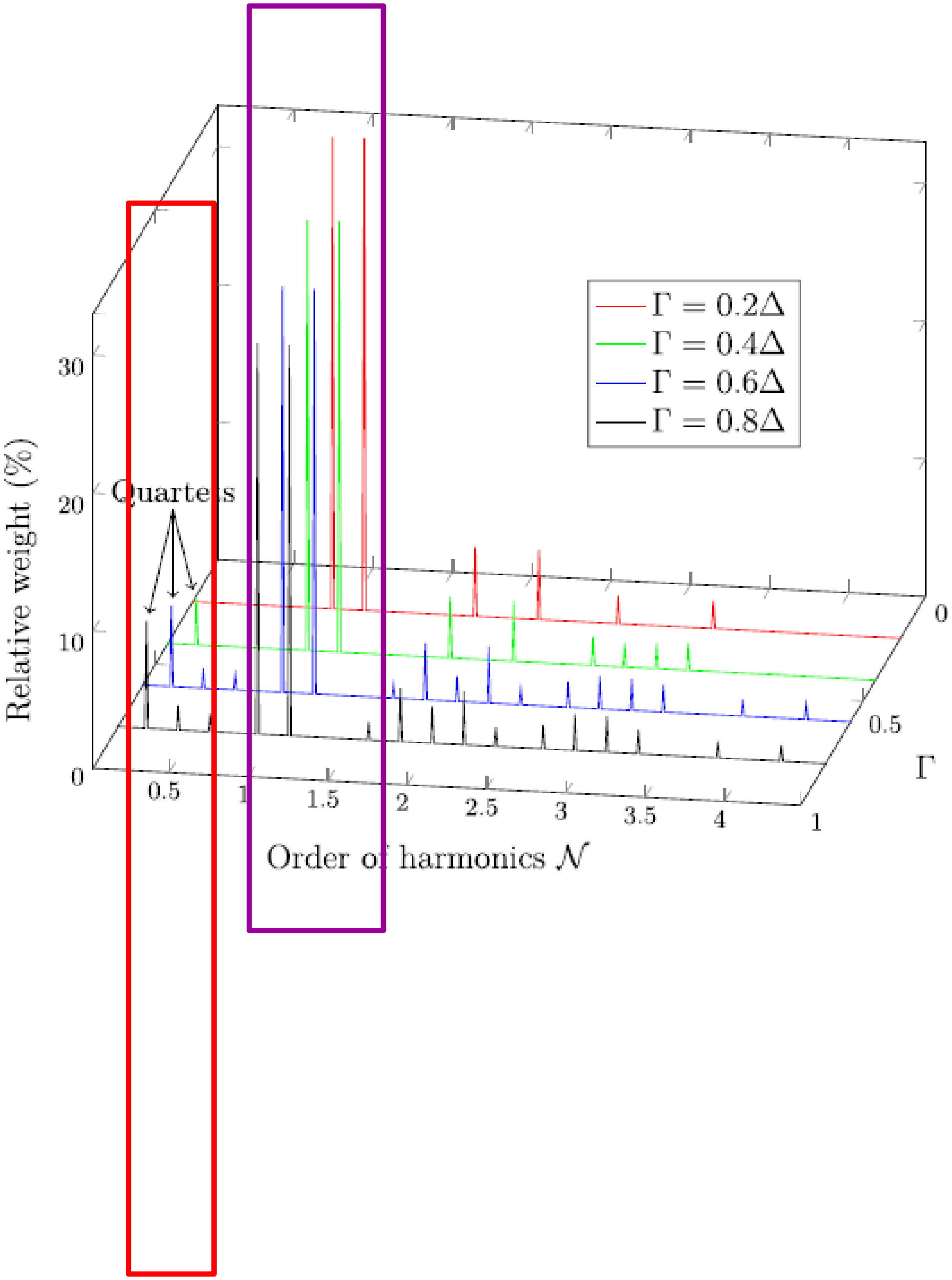
Technical details

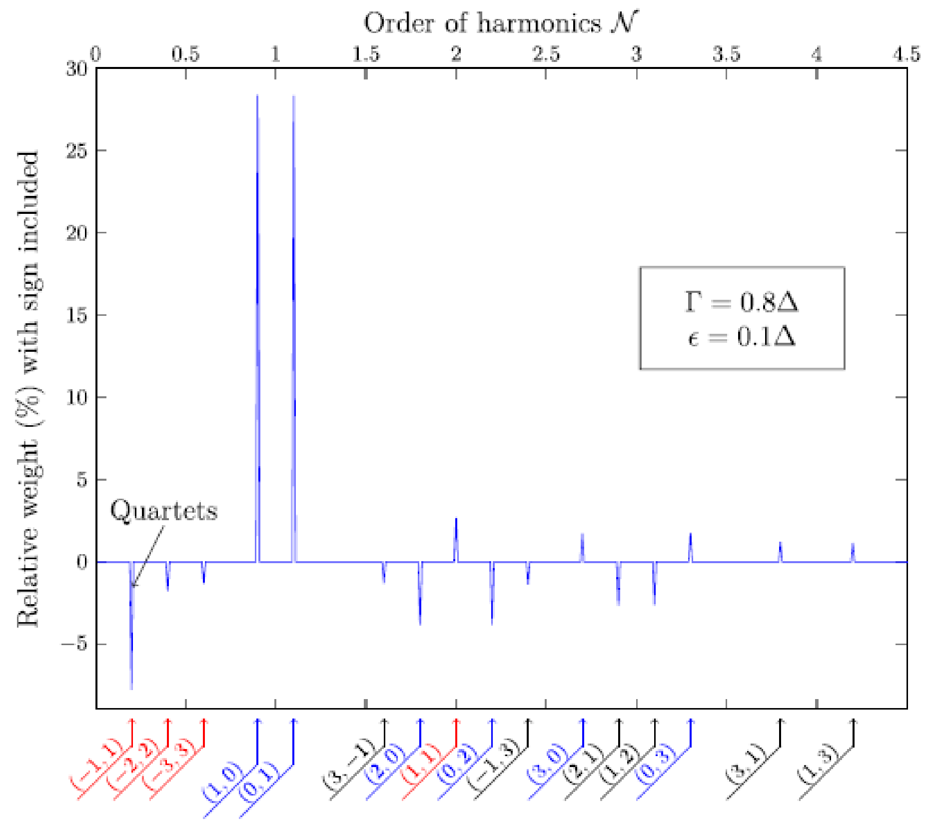
Here, finite separation for CAR process is taken into account.

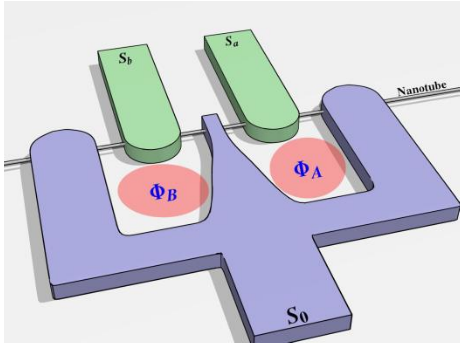
Average over separation distance is performed

Dot position/symmetry does not matter

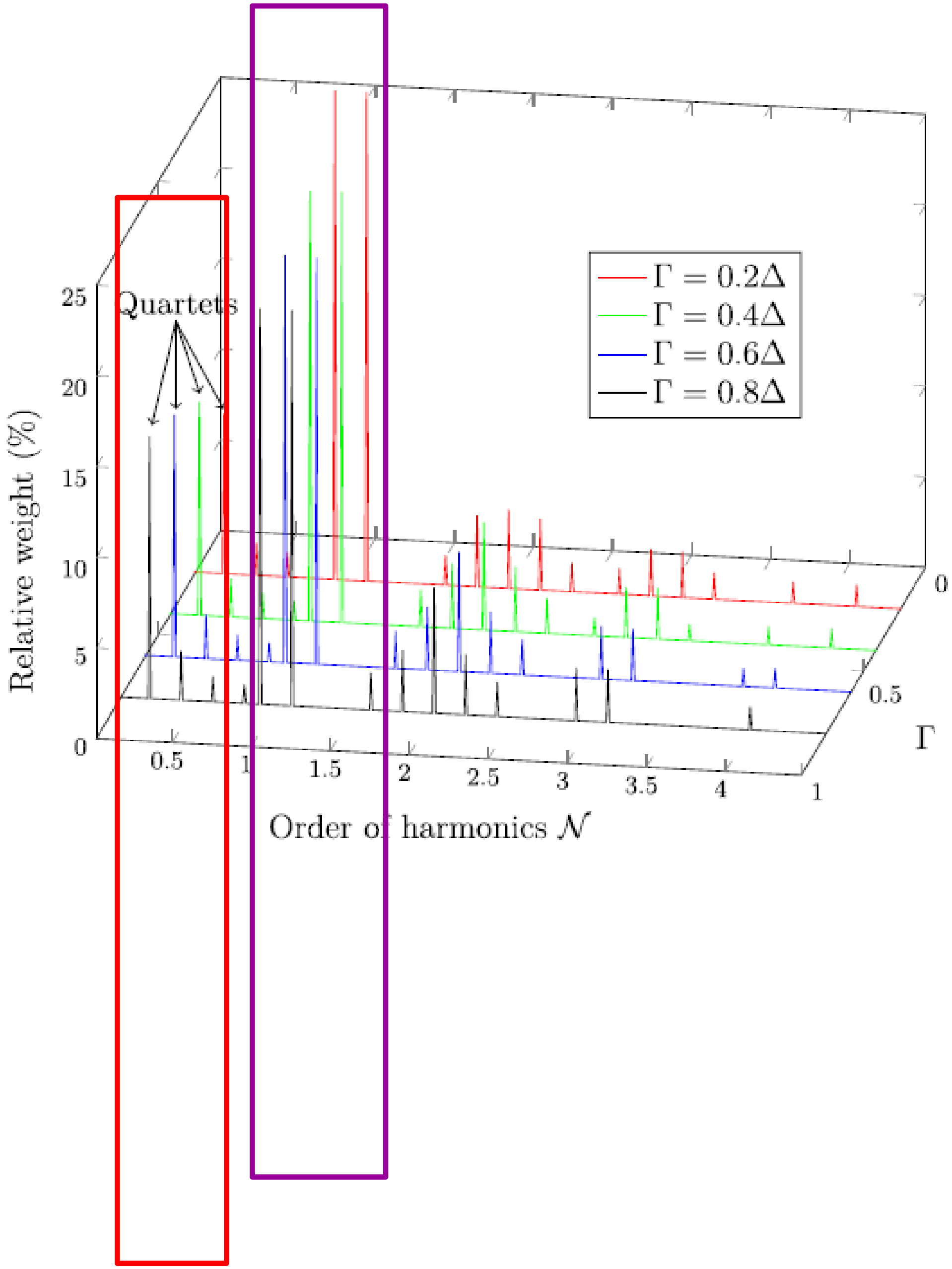
Non-Resonant case

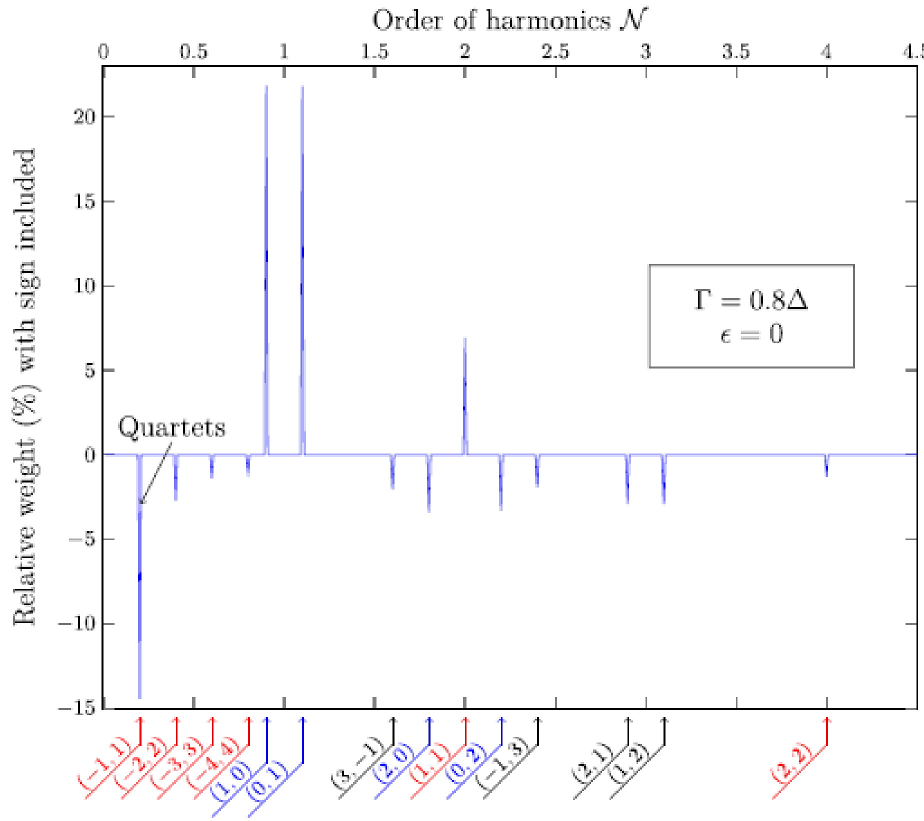


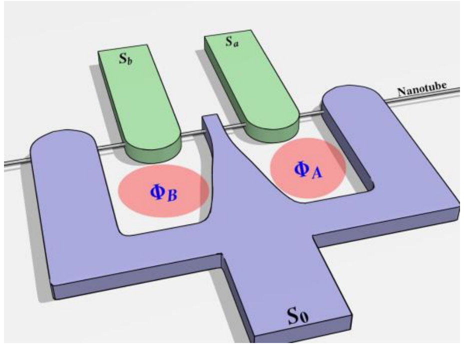




Resonant case

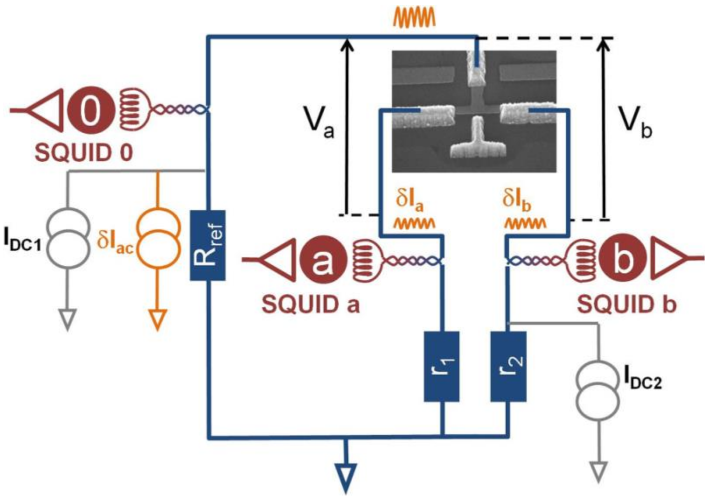


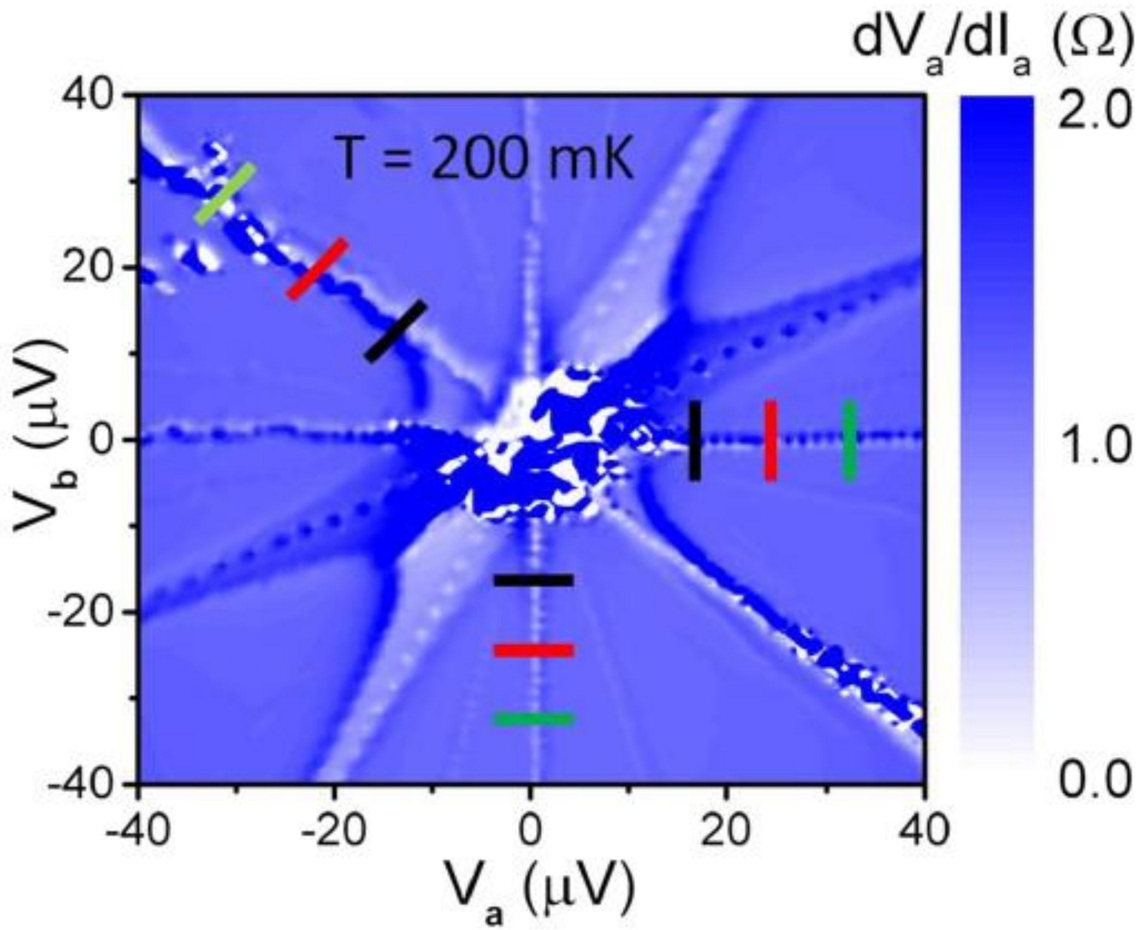




Possible first evidence of multiple pair resonances

Subgap structure in the conductance of a three-terminal Josephson junction
A. H. Pfeffer, J. E. Duvauchelle,H. Courtois, R. M´elin, D. Feinberg, F. Lefloch
PRB **90**, 075401 (2014)





CONCLUSION:

• NS forks to split Cooper pairs: antisymmetric level configuration favors positive correlations

•Weak Coulomb interactions enhance positive crossedcorrelations

• Multipair DC resonances with 3 superconductors although out of equilibrium

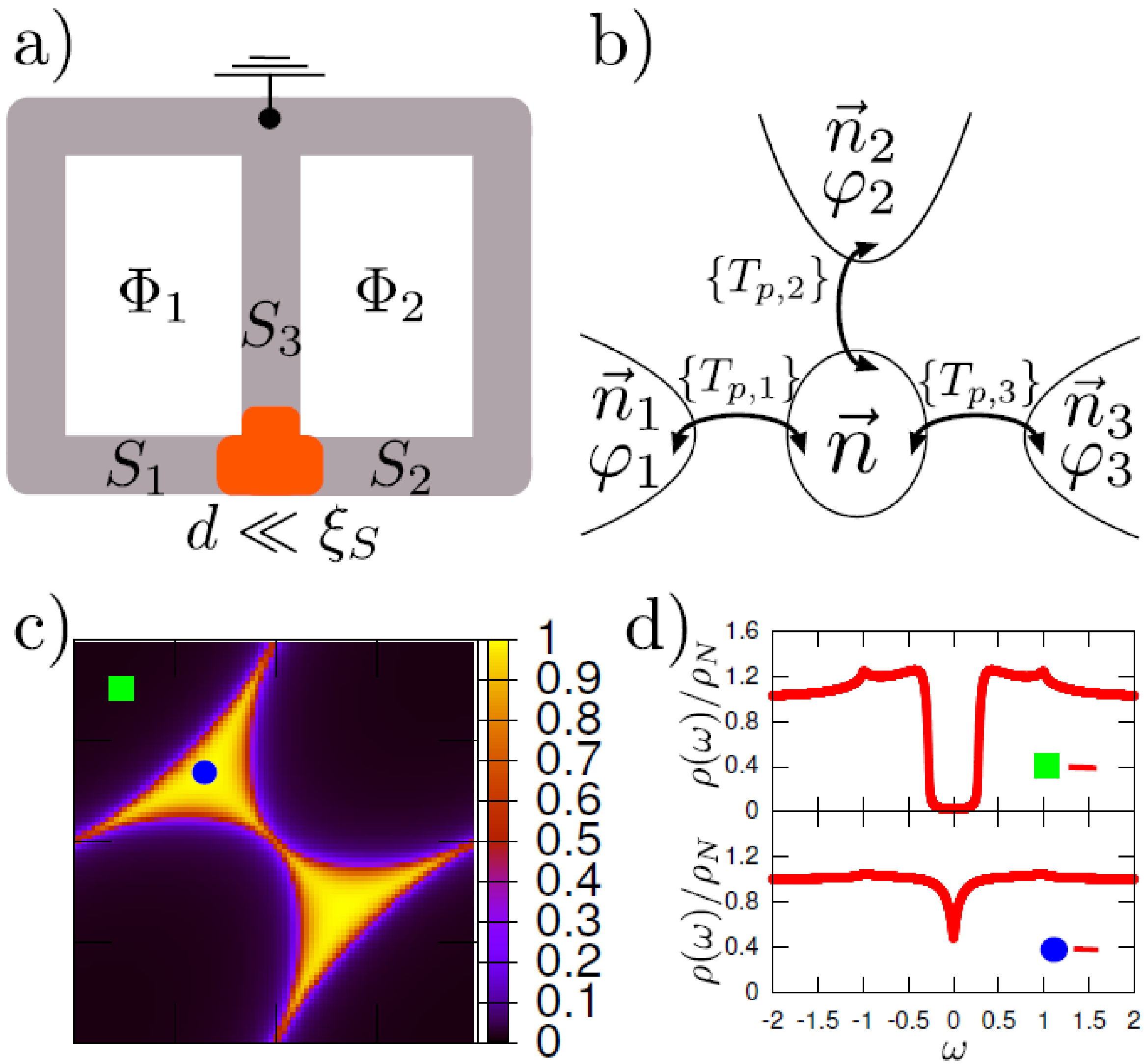
• Equilibrium setup: the BISQUID

• Perspectives: incomensurate voltages (Noise) interactions on dots, circuit theory to treat metallized dots

PRB 83, 125421 (2011)
PRB 85, 035419 (2012)
PRB 87, 214501 (2013)
PRB 90, 075419 (2014)

Closing the proximity gap in a three-terminal superconducting junction

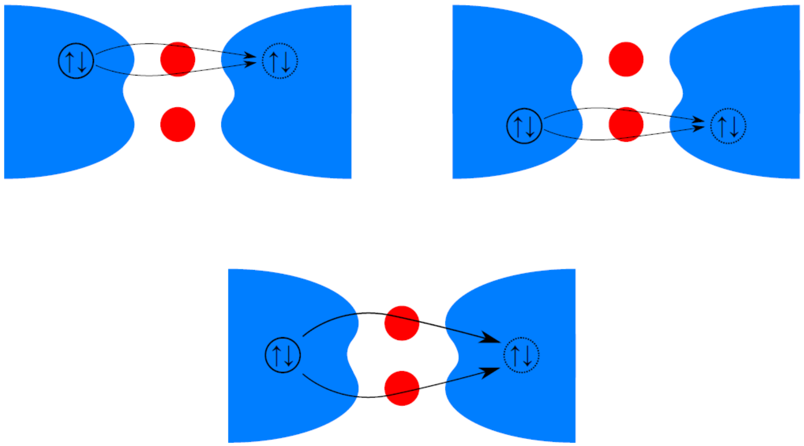
C. Padurariu,[1,2] T. Jonckheere,[3] J. Rech,[4] R. Melin,[4] D. Feinberg,[4] T. Martin,[4] and Yu. V. Nazarov[4]

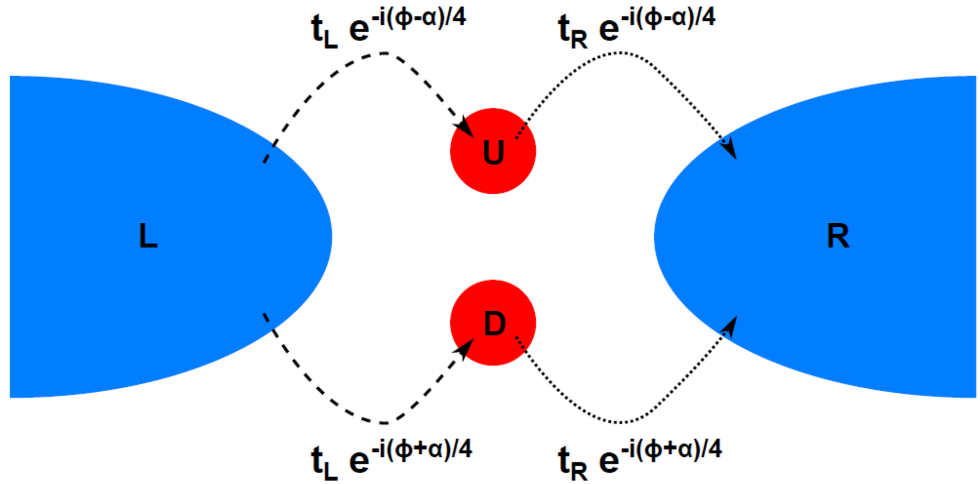


Circuit theory

Cooper pair splitting in a nanoSQUID geometry at high transparency

R. Jacquet,[1] J. Rech,[1] T. Jonckheere,[1] A. Zazunov,[2] and T. Martin[1]





$$J(\phi,\alpha) = I_1 \sin(\phi+\alpha) + I_2 \sin(\phi-\alpha) + I_{\mathrm{CAR}} \sin\phi$$