New Perspectives in Spintronic and Mesoscopic Physics

Singularity of the spectrum of Andreev levels in multi-terminal Josephson junction

Tomohiro Yokoyama

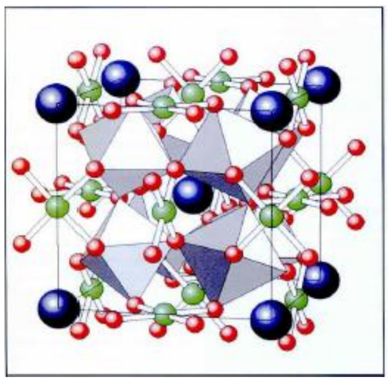
&

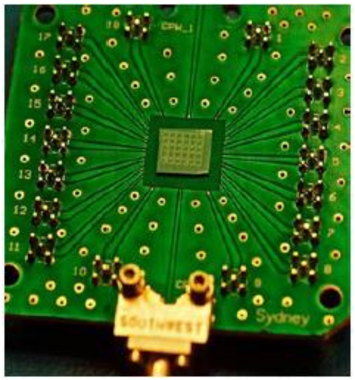
Yuli V. Nazarov

Idea of this study

- Proposal for ‘artificial material’

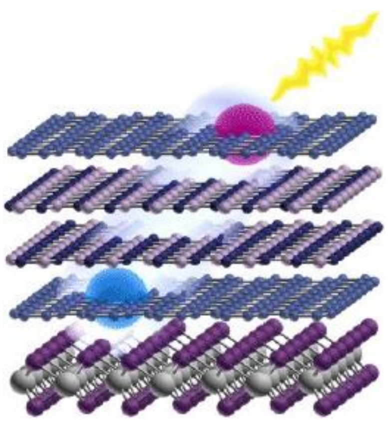
material





device

Artificial material

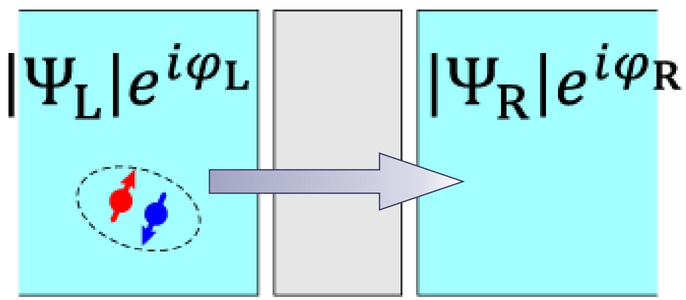


Nanodevice

Introduction

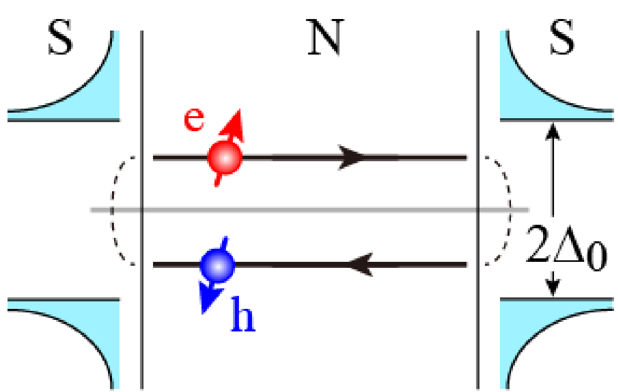
Introduction

Conventional 2-terminal Josephson junction



Tunneling of Cooper pair through an insulating or a normal region

- *Andreev bound state* (ABS)



Quasiparticle state formed by the Andreev reflection in S/N/S junctions

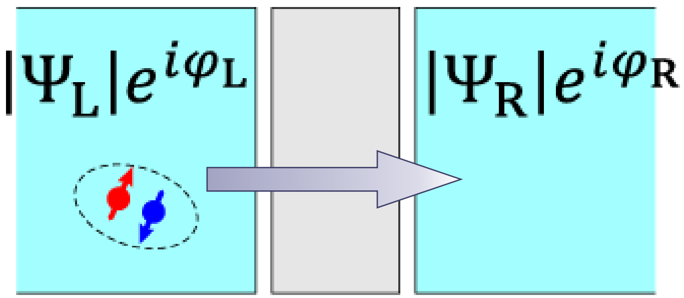
$$E_n(\varphi) = \Delta_0\sqrt{1 - T_n \sin^2(\varphi/2)}$$

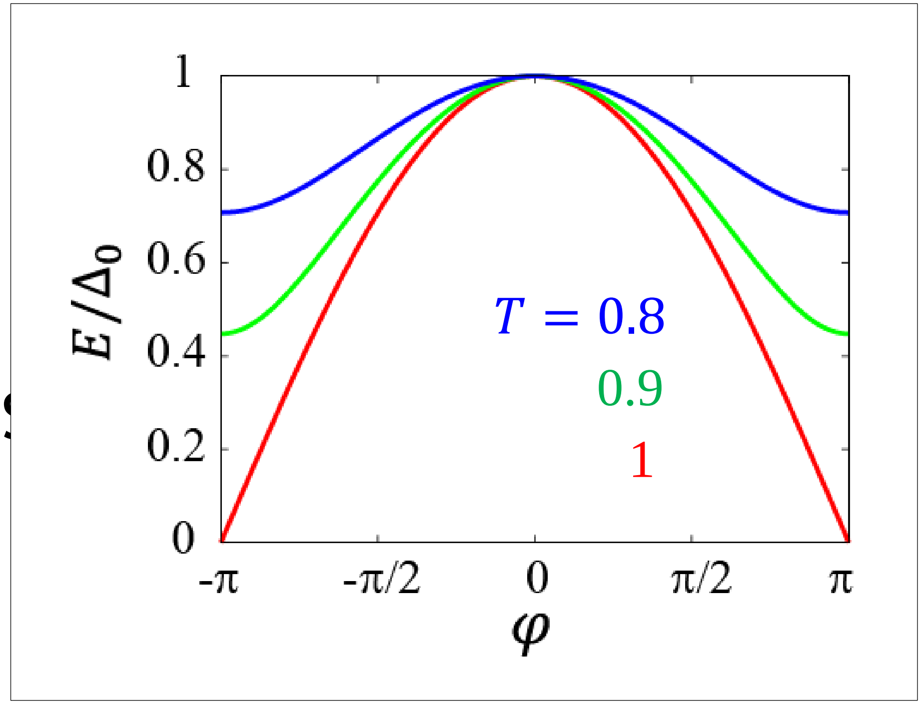
$$\varphi \equiv \varphi_L - \varphi_R$$

- the Cooper pair transports via the ABSs

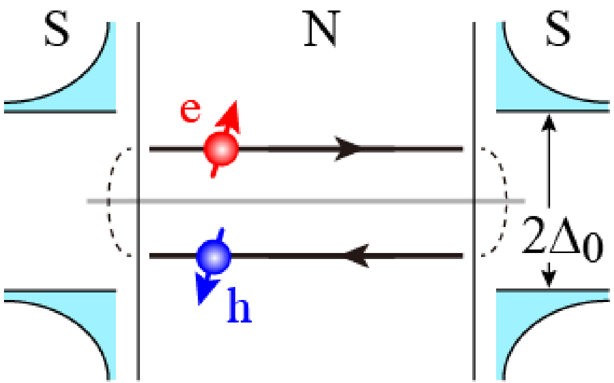
Introduction

Conventional 2-terminal Josephson junction





- *Andreev bound state* (ABS



$$\boxed{E_n(\varphi) = \Delta_0\sqrt{1 - T_n \sin^2(\varphi/2)}}$$

$$\varphi \equiv \varphi_{\mathrm{L}} - \varphi_{\mathrm{R}}$$

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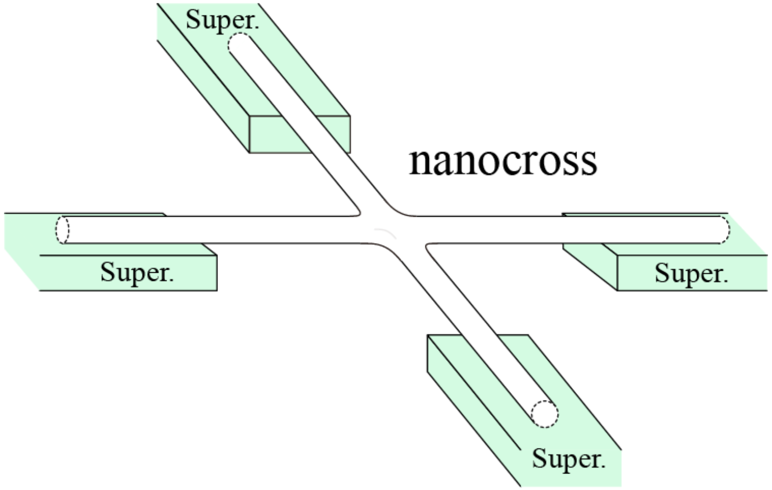
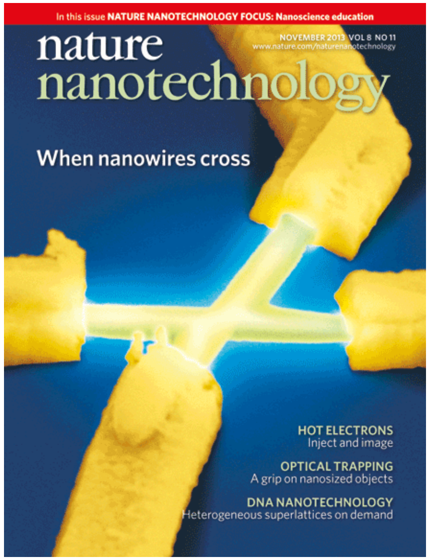
Introduction

Multi-terminal Josephson junction





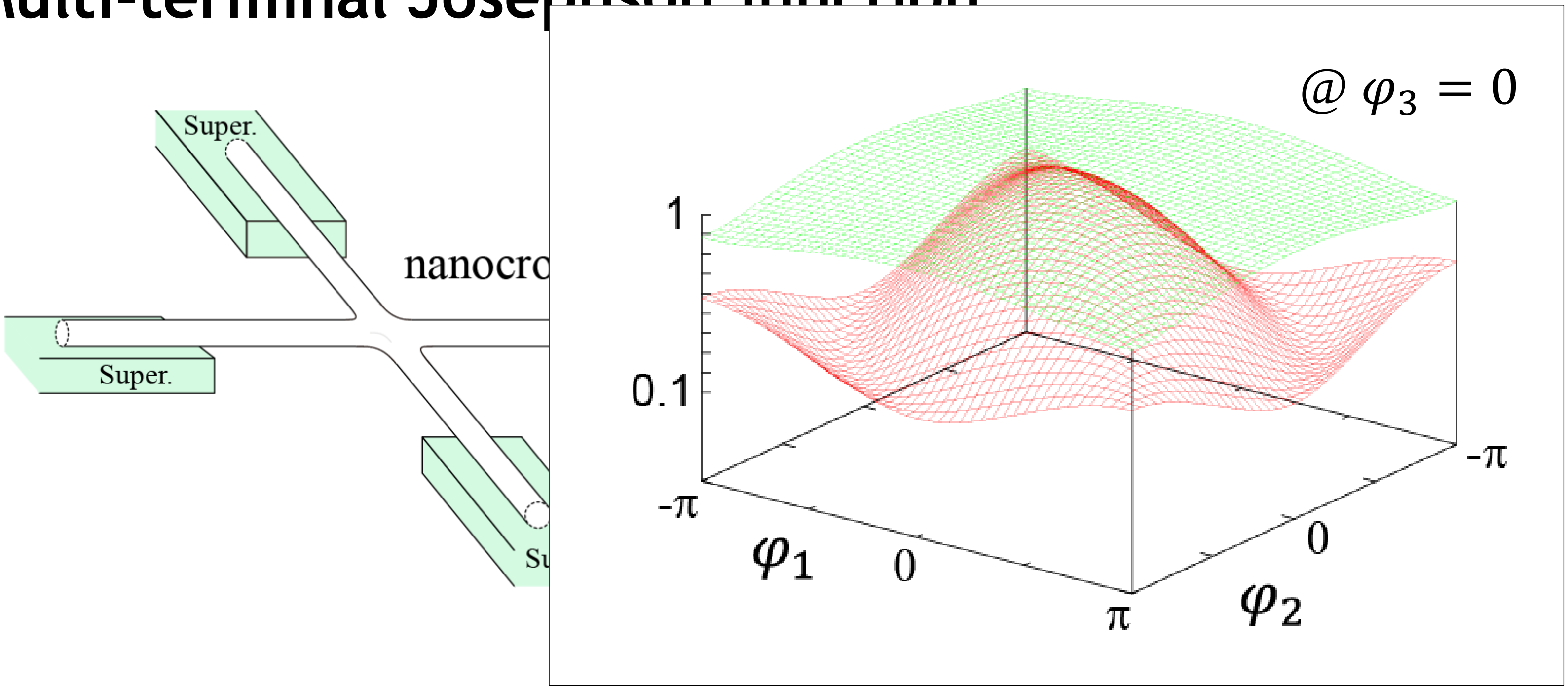
Plissard *et al.*,
Nature Nanotech. **8**, 859 (2014).

- *N*-1 independent phase differences
- The ABS energies E_n are 2π periodic in all phases

➡ We can regard as **a band structure**, $E_n(\varphi_1, \cdots, \varphi_{N-1})$.

Introduction

Multi-terminal Josephson junction



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- The ABS energies E_n are 2π periodic in all phases

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Introduction

Topological materials

- Band structure: eigenenergy $E(q_x, q_y, q_z)$
- Berry curvature field: from eigenstates $\Psi(\vec{q})$

$$B_z(\vec{q}) = i\left(\left\langle \frac{\partial\Psi}{\partial q_x} \middle| \frac{\partial\Psi}{\partial q_y} \right\rangle - \left\langle \frac{\partial\Psi}{\partial q_y} \middle| \frac{\partial\Psi}{\partial q_x} \right\rangle\right)$$

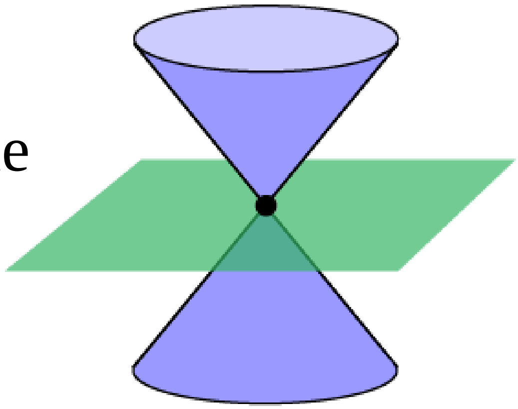
- 2D topological invariant

$$C = \frac{1}{2\pi}\int dq_x dq_y B_z(\vec{q})$$

- Quantum Hall effect ⇨ quantized transconductance
- Topological physics (**Weyl semimetal**, spin Hall insulator, surface states, Majorana fermion, etc)

Introduction

Weyl singularity in band structure

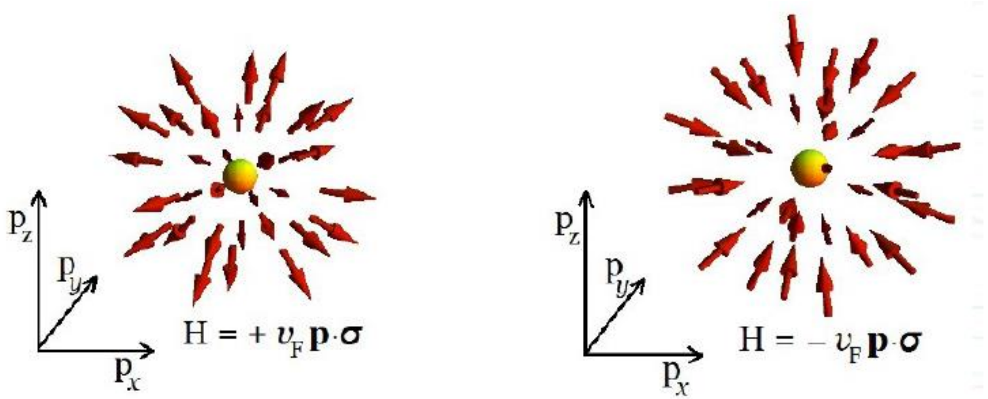


- Weyl fermion: massless relativistic particle
- Conical spectrum
- In the vicinity of certain $\vec{q}$,

$$H = \vec{\sigma} \cdot (\delta \vec{q})$$

- Theoretically predicted in materials
- The points work as monopoles of the Berry curvature field

X. Wan, PR **83**, 201202 (2011).



This talk

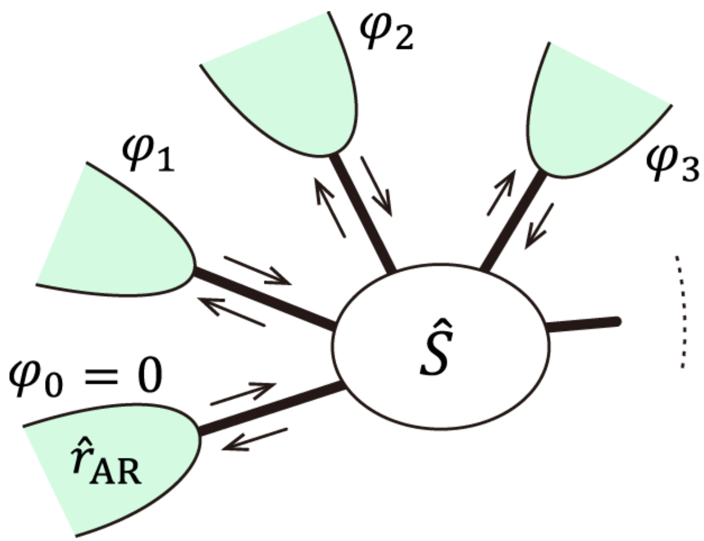
- Multi-terminal junction as *topological material*
- Peculiarities and singularities of the ABS energy in multi-terminal junctions
- Weyl singularity in Andreev spectrum
- Spin-orbit (SO) interaction
- Analogy of the ABS between $E = 0$ and $E = \Delta$
- Superconducting gap edge touching

Model & formulation

Model & formulation

Beenakker's formulation

Beenakker, PRL **67**, 3836 (1991).



$$\boxed{\hat{r}_{\mathrm{eh}}\hat{s}_{\mathrm{h}}\hat{r}_{\mathrm{he}}\hat{s}_{\mathrm{e}}\boldsymbol{\psi} = \boldsymbol{\psi}}$$

$$s_{\mathrm{h}}(-E) = -\hat{g}s_{\mathrm{e}}^{*}(E)\hat{g}$$

$$\hat{r}_{\mathrm{he}} = e^{-i\hat{\varphi}}e^{i\chi} \qquad \boxed{\hat{g} = -i\hat{\sigma}_y}$$

$$e^{-i\hat{\varphi}} = \mathrm{diag}\left(e^{-i\varphi_0}, e^{-i\varphi_1}, \cdots, e^{-i\varphi_{N-1}}\right)$$

$$\chi = \arccos(E/\Delta)$$

$$\boxed{\det(e^{i2\chi} - S(\boldsymbol{\varphi})) = 0}$$

$$S(\boldsymbol{\varphi}) = -\hat{g}s^{*}(\boldsymbol{\varphi}, -E)\hat{g}s(\boldsymbol{\varphi}, E)$$

$$s(\boldsymbol{\varphi}, E) \equiv e^{-i\hat{\varphi}/2}s_{\mathrm{e}}(E)e^{+i\hat{\varphi}/2}$$

*The phase breaks time-reversal invariance: $s(\boldsymbol{\varphi}, -E) \neq -\hat{g}s^{*}(\boldsymbol{\varphi}, E)\hat{g}$

Model & formulation

Scattering matrix with/without SO interaction

$s_\mathrm{e} = U \Lambda U^\dagger$: given by random matrix

$S(\boldsymbol{\varphi}) = -\hat{g} s^* \hat{g} s$

$\hat{g} = -i \hat{\sigma}_y$

- no SO interaction

  U: orthogonal

  $-\hat{g} s \hat{g} = s$

  $S(\boldsymbol{\varphi}) = s^* s$

- with SO interaction

  U: symplectic

  $u \equiv \hat{g} s$

  $S(\boldsymbol{\varphi}) = -u^* u$

Single parameter p_SO

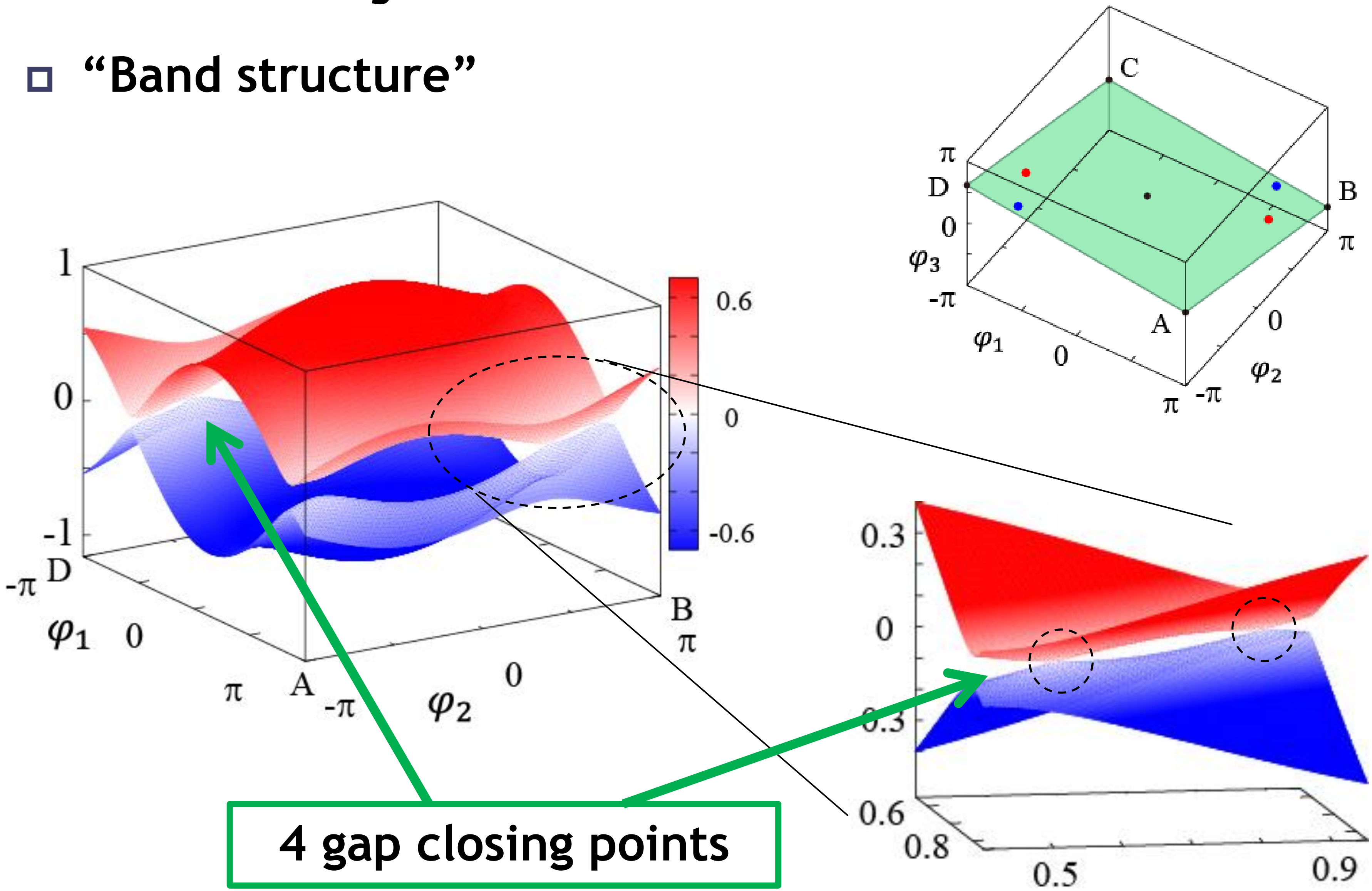
$(0 \leq p_\mathrm{SO} \leq 1)$

Yokoyama, Eto, and Nazarov, JPSJ **82**, 054703 (2013).

Band gap closing point

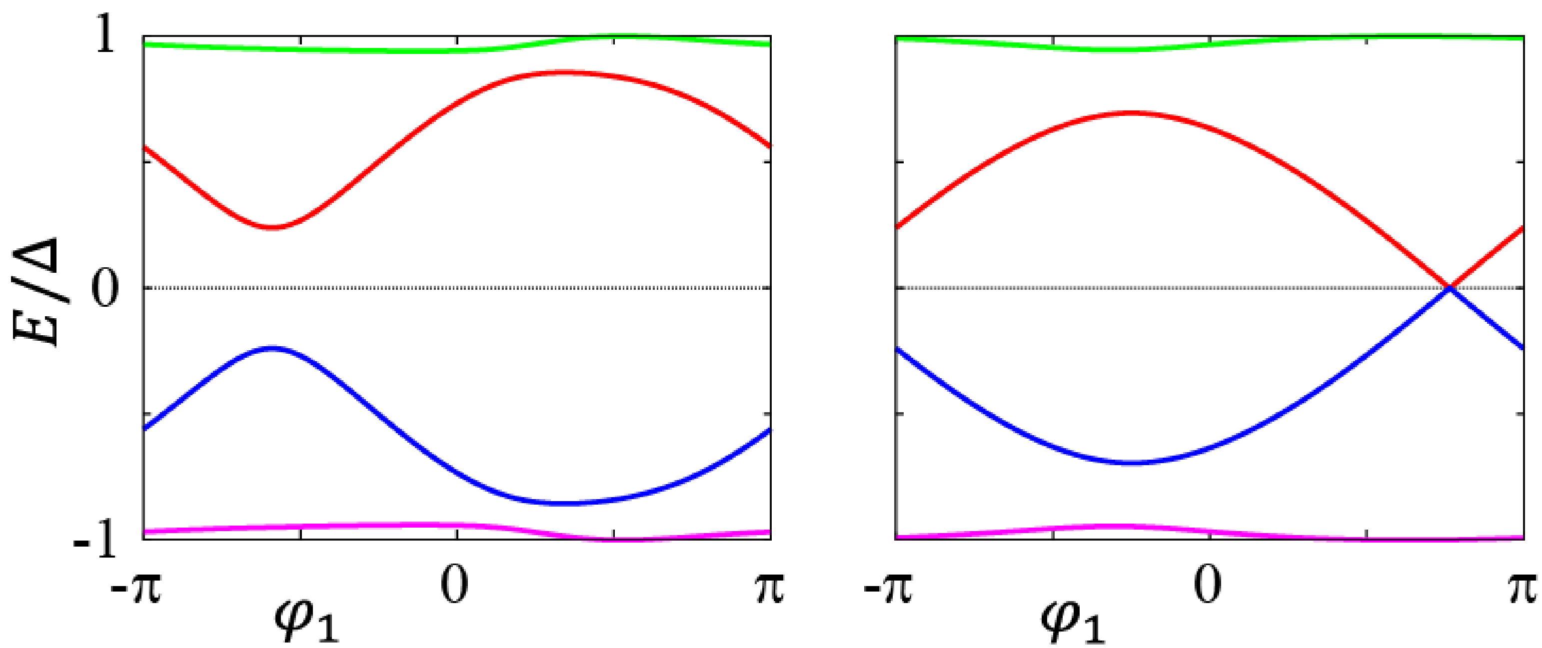
4-terminal junction

- “Band structure”



Property of Weyl point

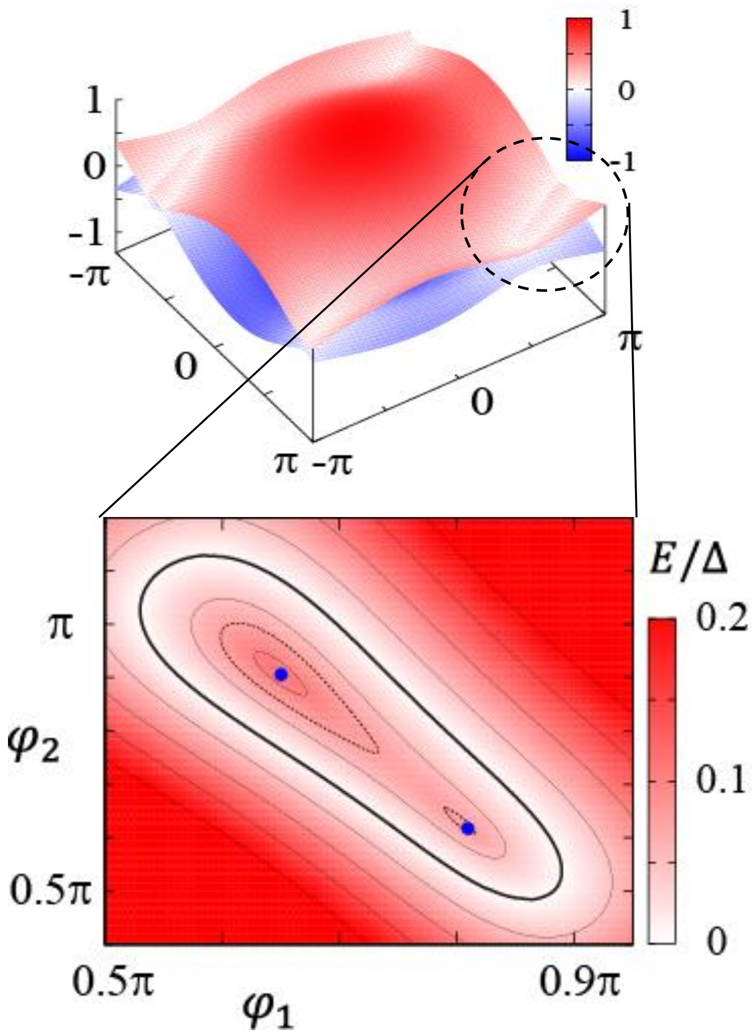
Conical spectrum of the ABS

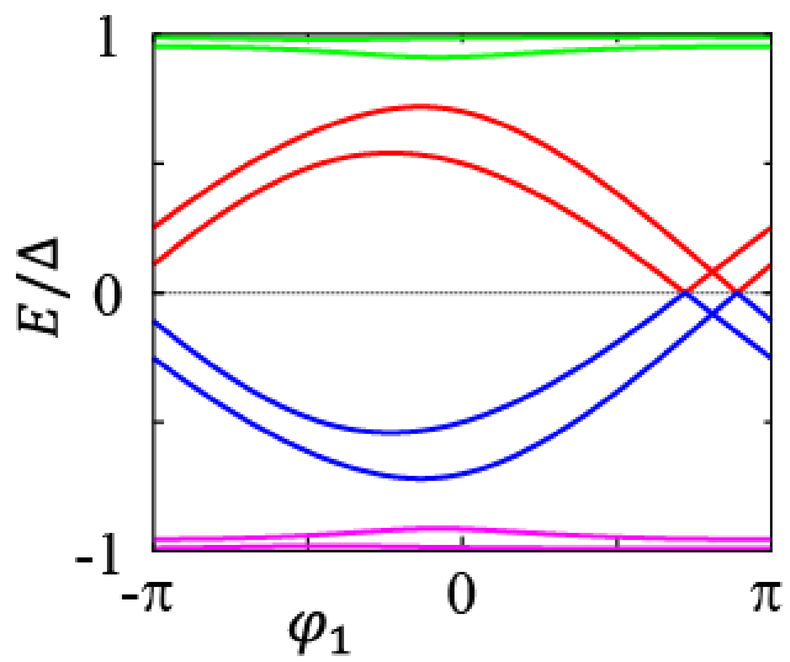


- φ_2 and φ_3 are fixed.
- Time-reversal invariant guarantees $E_n(\boldsymbol{\varphi}) = E_n(-\boldsymbol{\varphi})$.
- The conical points come in group of 4.

Property of Weyl point

- Split by SO interaction

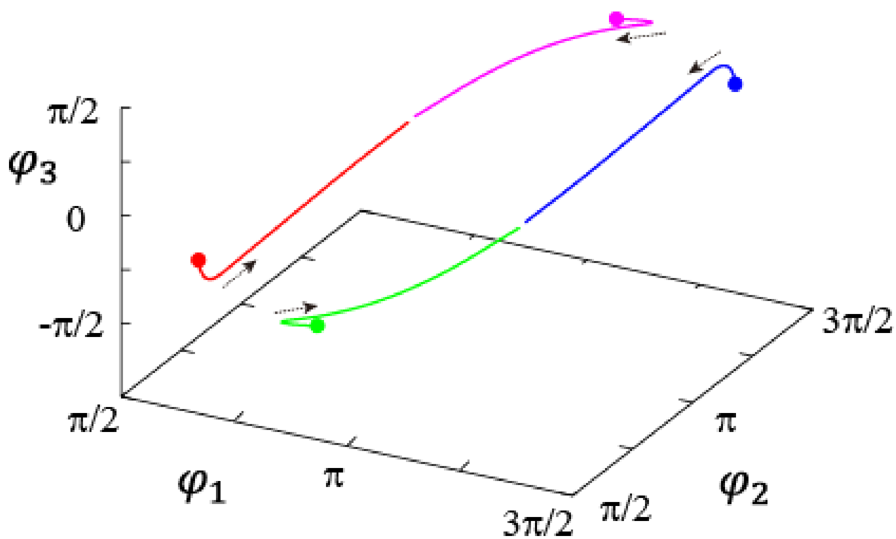




- Split of conical point in spin
- 2D surface of zero energy

Property of Weyl point

Topological protection and pair annihilation



topological protection: **Weyl fermion**

- ✓ The Weyl point is present even in the absence of SO interaction!
- ✓ No exotic materials!

Berry curvature

$|\psi_n(\boldsymbol{\varphi})\rangle$: Wavefunction of the ABS

$$B_{n,3}(\varphi_1,\varphi_2,\varphi_3) \equiv -2\mathrm{Im}\left\langle \frac{\partial\psi_n}{\partial\varphi_1} \middle| \frac{\partial\psi_n}{\partial\varphi_2} \right\rangle$$

- Suppose fixed φ_3

$$\iint d\varphi_1 d\varphi_2 B_3 = 2\pi C$$

$B_3 = \sum_k (n_k - 1/2) B_{k,3}$

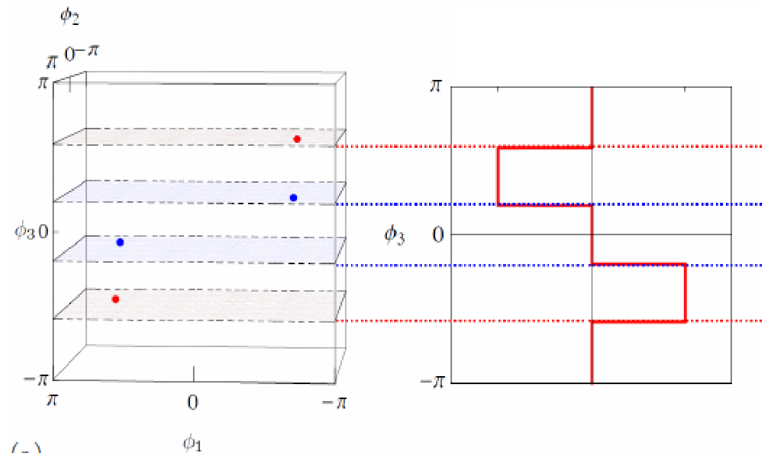
C : Chern number

if the band is trivial,

$C = 0$ for all φ_3

if the Weyl point is presesnt,

$C \neq 0$ between two points

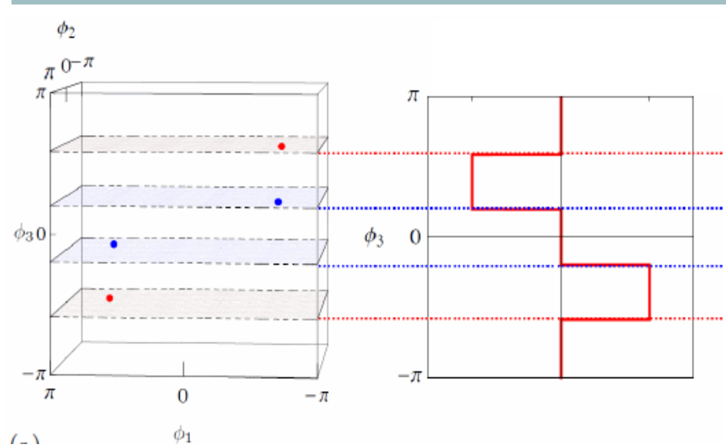


Riwar *et al.*, arXiv: 1503.06862

Transconductance

- Phase sweep by small voltage

$$\dot{\varphi}_1 = 2eV_1/\hbar$$
$$\dot{\varphi}_2 = 2eV_2/\hbar$$



$$\Rightarrow \quad \bar{B}_3 = \frac{1}{(2\pi)^2} \iint d\varphi_1 d\varphi_2 B_3 = C/(2\pi)$$

- Current correction

$$I_1/(2e) = \frac{\partial E}{\partial \varphi_1} - \dot{\varphi}_2 \bar{B}_3 \qquad I_2/(2e) = \frac{\partial E}{\partial \varphi_2} - \dot{\varphi}_1 \bar{B}_3$$

  - Transconductance

$$\Rightarrow \quad \sigma_{12} = -(2e)^2 \bar{B}_3/\hbar = -\frac{2e^2}{\pi\hbar} C$$

Riwar *et al.*, arXiv: 1503.06862

Gap edge touching point

What's expected?

Mathematical analogy

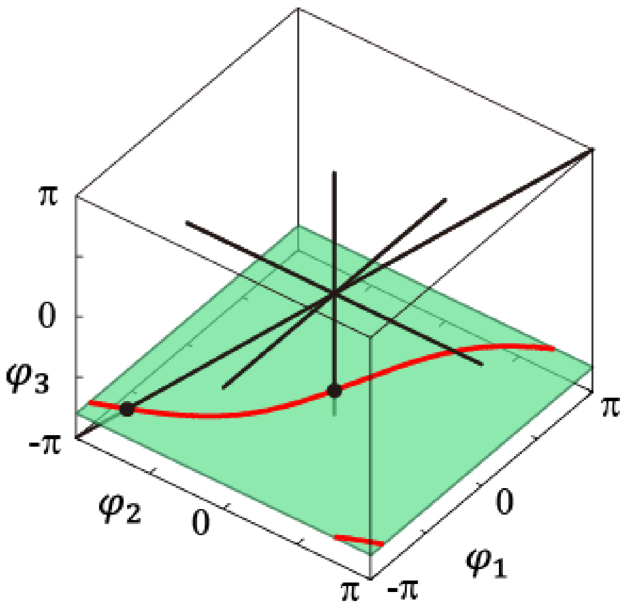
$$\boxed{\det(e^{i2\chi} - S(\boldsymbol{\varphi})) = 0}$$

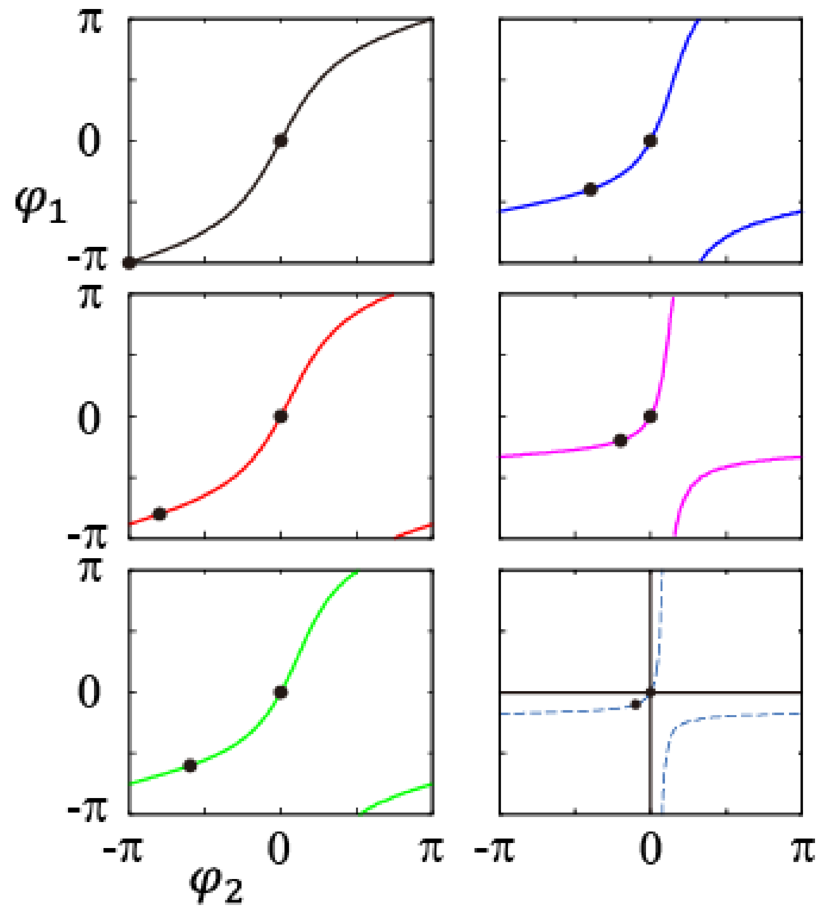
$$S(\boldsymbol{\varphi}) = -\hat{g}s^*\hat{g}s$$

	• Zero energy	• Super. gap edge
	$e^{i2\chi} = -1$	$e^{i2\chi} = +1$
no SO	$-\psi = s^*s\psi$	$\psi = s^*s\psi$
	Weyl point	2D surface ?
with SO	$-\psi = -u^*u\psi$	$\psi = -u^*u\psi$
$u = \hat{g}s$	2D surface at $E = 0$	Another singularity?

Super. gap edge

- 2D surface of gap edge touching

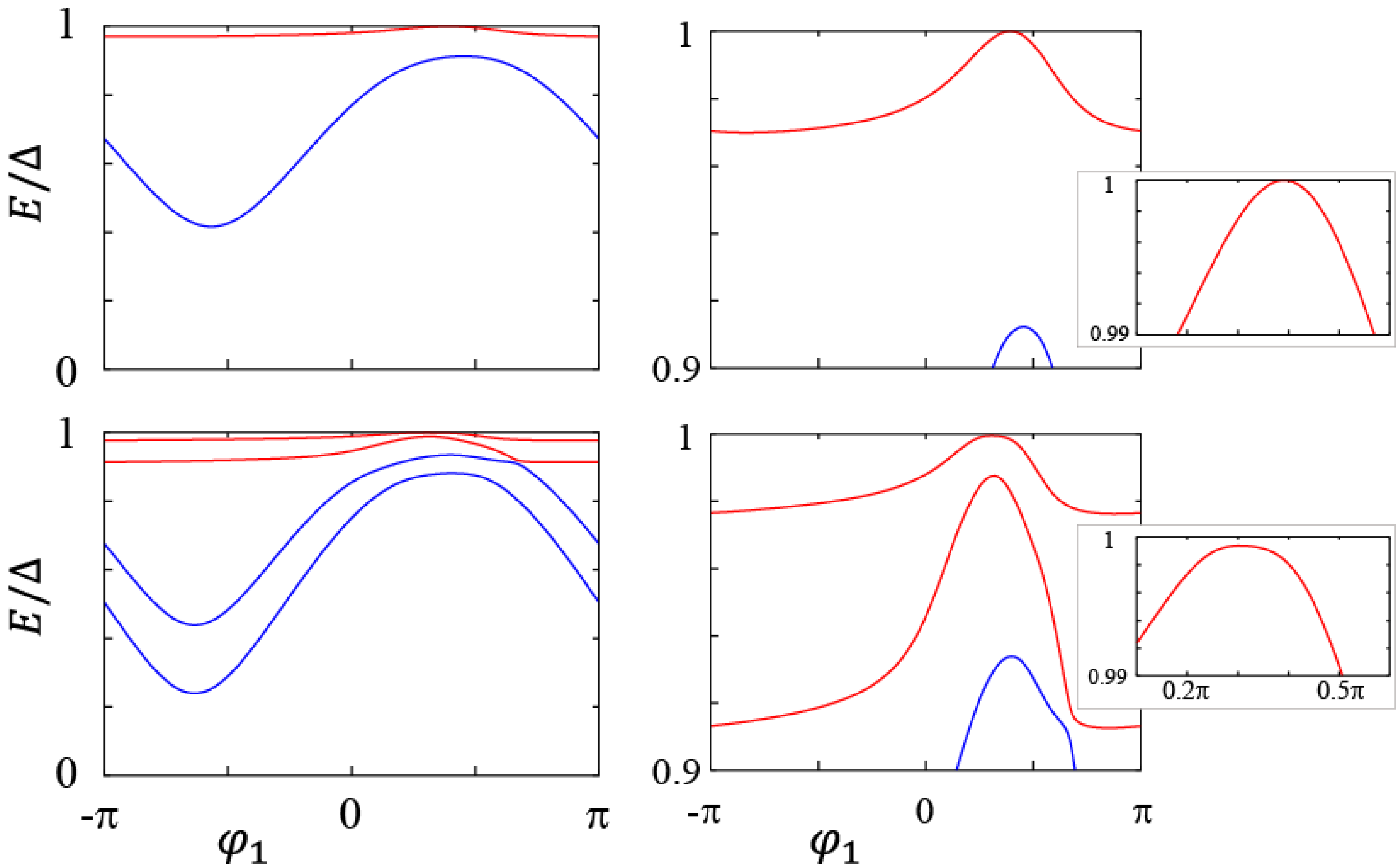




When the SO interaction is present, only 4 lines remain

→ symmetry line

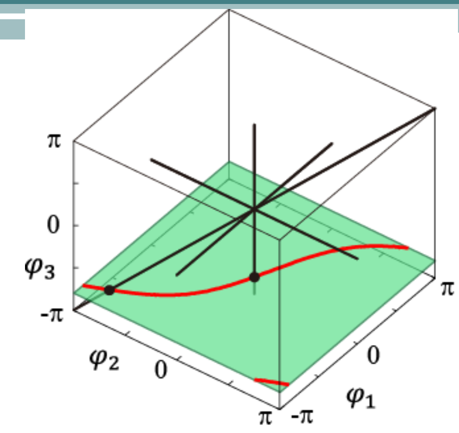
- **Super. gap edge**
  - Removing by SO interaction

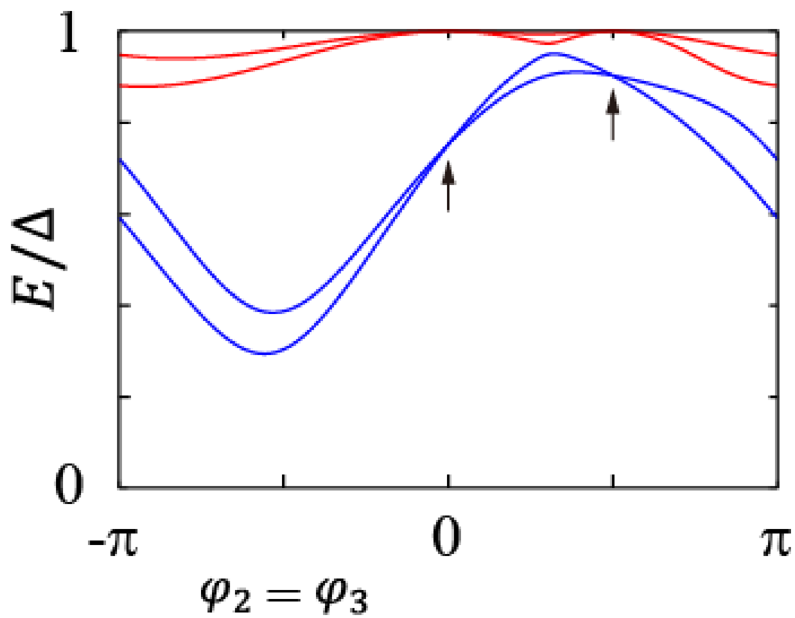


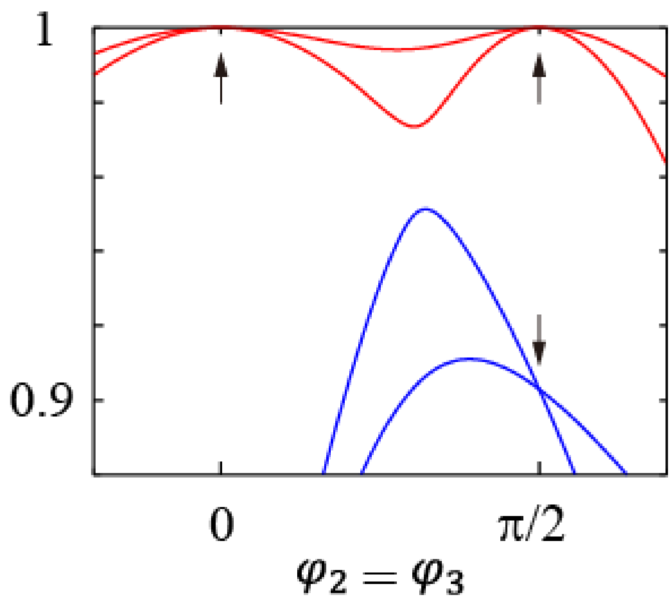
Super. gap edge

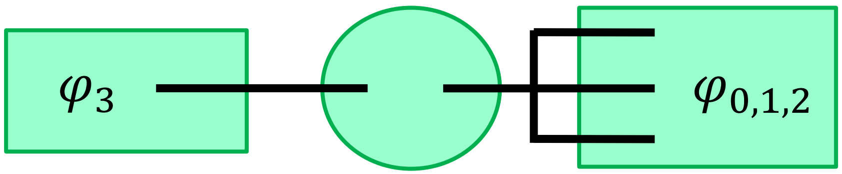
Symmetry line

Three of the four phase are equal,
$\varphi_0 = \varphi_1 = \varphi_2 \neq \varphi_3$.



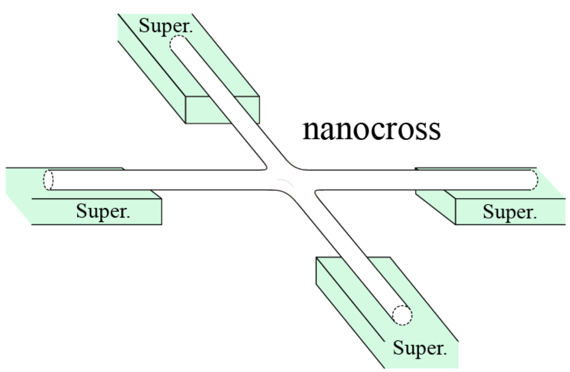


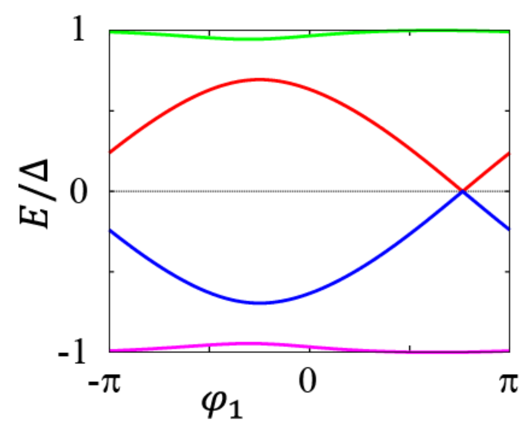




Conclusions

- Multi-terminal Josephson junction
- Band structure of the Andreev spectrum
- Weyl singularity even without SO interaction
- Peculiarity of gap edge touching
- Higher dimensional physics





Yokoyama and Nazarov, (in preparation)

Effective Hamiltonian

Effective Hamiltonian (1)

$$\boxed{\det(e^{i2\chi} - S(\boldsymbol{\varphi})) = 0}$$

- Two states at Weyl points: $s^*(\boldsymbol{\varphi}_\mathrm{c})s(\boldsymbol{\varphi}_\mathrm{c})|a,b\rangle = -|a,b\rangle$

$$e^{i2\chi} \approx -1 + i2E/\Delta$$

$$s(\boldsymbol{\varphi}) = s(\boldsymbol{\varphi}_\mathrm{c})e^{iX(\delta\boldsymbol{\varphi})} \approx s(\boldsymbol{\varphi}_\mathrm{c})(1 + iX)$$

$$s(\boldsymbol{\varphi}_\mathrm{c}, \boldsymbol{K}) = s(\boldsymbol{\varphi}_\mathrm{c})e^{i\hat{\boldsymbol{\sigma}}\cdot\boldsymbol{K}} \approx s(\boldsymbol{\varphi}_\mathrm{c})(1 + i\hat{\boldsymbol{\sigma}}\cdot\boldsymbol{K})$$

$$\boxed{H_{E=0} = \Delta\sum_{j=13} X_j\check{\Sigma}_j + \Delta\hat{\boldsymbol{\sigma}}\cdot\boldsymbol{K}_0}$$

$$X_1 + iX_2 = -\langle b|X|a\rangle$$

$$X_3 = (\langle b|X|b\rangle - \langle a|X|a\rangle)/2$$

$$\boldsymbol{K}_0 = (\langle b|\boldsymbol{K}|b\rangle - \langle a|\boldsymbol{K}|a\rangle)/2$$

Effective Hamiltonian (2)

- Two states at gap edge: $s^*(\boldsymbol{\varphi}_\mathrm{c})s(\boldsymbol{\varphi}_\mathrm{c})|a\uparrow, b\downarrow\rangle = |a\uparrow, b\downarrow\rangle$

$$e^{i2\chi} \approx 1 + i\sqrt{8\epsilon} \qquad \epsilon = 1 - E/\Delta$$

$$s(\boldsymbol{\varphi}_\mathrm{c}, d) = s(\boldsymbol{\varphi}_\mathrm{c})e^{iEd} \approx s(\boldsymbol{\varphi}_\mathrm{c})(1 + i\Delta d)$$

$$H_{E=\Delta}|\Psi\rangle = \sqrt{2\epsilon}|\Psi\rangle$$

$$\boxed{H_{E=\Delta} = X_0\check{\Sigma}_3 + \sum_{j=0}^{2} \hat{\boldsymbol{\sigma}} \cdot \boldsymbol{K}_j\check{\Sigma}_j + \sum_{j=0}^{2} D_j\check{\Sigma}_j}$$

$$X_0 = (\langle a|X|a\rangle - \langle b|X|b\rangle)/2$$

$$\boldsymbol{K}_0 = (\langle a|\boldsymbol{K}|a\rangle + \langle b|\boldsymbol{K}|b\rangle)/2$$

$$\boldsymbol{K}_1 + i\boldsymbol{K}_2 = \langle b|\boldsymbol{K}|a\rangle$$

$$D_0 = (\langle a|D|a\rangle + \langle b|D|b\rangle)/2$$

$$D_1 + iD_2 = \langle b|D|a\rangle$$

causality: $(D_0 > \sqrt{D_1^2 + D_2^2})$

Effective Hamiltonian (2)

- Eigenenergies of $H_{E=\Delta} = X_0$

$$\lambda_\pm = \sqrt{X_0^2 + L(\boldsymbol{K}_j) \pm 2M(\boldsymbol{K}_j, X_0)}$$

$$L(\boldsymbol{K}_j) = \boldsymbol{K}_0 \cdot \boldsymbol{K}_0 + \boldsymbol{K}_1 \cdot \boldsymbol{K}_1 + \boldsymbol{K}_2 \cdot \boldsymbol{K}_2$$

$$M(\boldsymbol{K}_j, X_0) = \sqrt{(\boldsymbol{K}_0 \cdot \boldsymbol{K}_1)^2 + (\boldsymbol{K}_0 \cdot \boldsymbol{K}_2)^2 + (X_0\boldsymbol{K}_0 - \boldsymbol{K}_1 \times \boldsymbol{K}_2)^2}$$

λ_- can be zero when the SO interaction $\boldsymbol{K}_{j=0,1,2}$ satisfies some conditions in 6D $(\boldsymbol{K}_j, X_0)$ parameter space.

$$27b^4 + 2ab^2(a^2 - 9c) - c(a^2 - c)^2 = 0$$

$$a = -K_0^2 + K_1^2 + K_2^2 \qquad b = -2\boldsymbol{K}_0 \cdot (\boldsymbol{K}_1 \times \boldsymbol{K}_2)$$

$$c = (K_0^2 + K_1^2 + K_2^2)^2 - 4\{(\boldsymbol{K}_0 \cdot \boldsymbol{K}_1)^2 + (\boldsymbol{K}_0 \cdot \boldsymbol{K}_2)^2\} - 4K_1^2K_2^2 + 4(\boldsymbol{K}_1 \cdot \boldsymbol{K}_1)^2$$

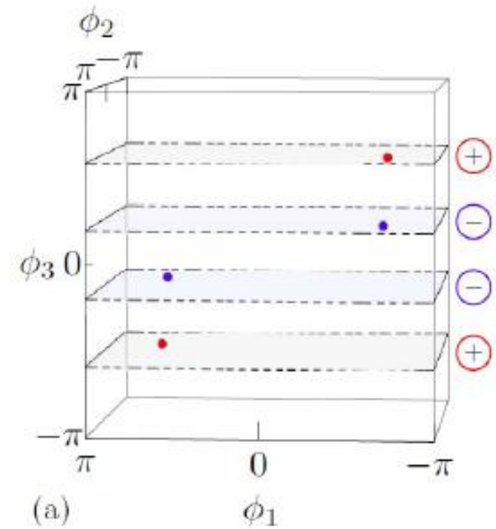
Transconductance

R.-P. Riwar, M. Houzet, J. S. Meyer, and Yu. V. Nazarov,
arXiv: 1503.06862

Berry curvature

$|\psi_n(\boldsymbol{\varphi})\rangle$: Wavefunction of the ABS

$$B_{n,3}(\varphi_1, \varphi_2, \varphi_3) \equiv -2\mathrm{Im}\left\langle \frac{\partial \psi_n}{\partial \varphi_1} \middle| \frac{\partial \psi_n}{\partial \varphi_2} \right\rangle$$



- Suppose fixed φ_3

$$\iint d\varphi_1 d\varphi_2 B_3 = 2\pi C$$

$B_3 = \sum_k (n_k - 1/2) B_{k,3}$

C : Chern number

if the band is trivial,

$C = 0$ for all φ_3

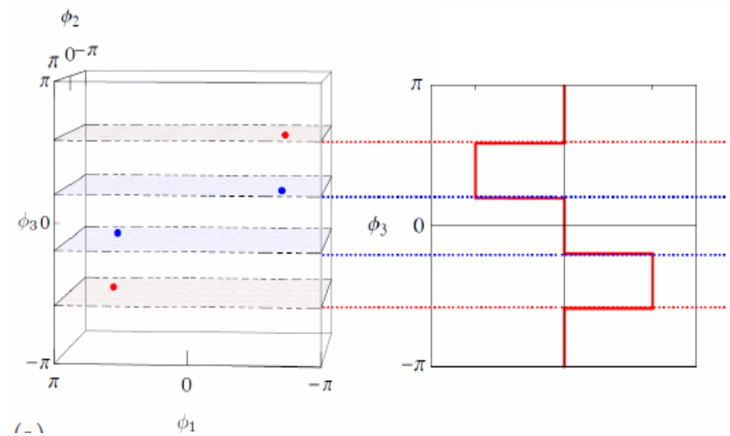
if the Weyl point is presesnt,

$C \neq 0$ on the $\varphi_1\varphi_2$ plane → Weyl point

Transconductance

Phase sweep by small voltage

$$\dot{\varphi}_1 = 2eV_1/\hbar$$
$$\dot{\varphi}_2 = 2eV_2/\hbar$$



$$\bar{B}_3 = \frac{1}{(2\pi)^2}\iint d\varphi_1 d\varphi_2 B_3 = C/(2\pi)$$

Current correction

Riwar *et al.*, arXiv: 1503.06862

$$I_1/(2e) = \frac{\partial E}{\partial \varphi_1} - \dot{\varphi}_2\bar{B}_3 \qquad I_2/(2e) = \frac{\partial E}{\partial \varphi_2} - \dot{\varphi}_1\bar{B}_3$$

- Transconductance

$$\sigma_{12} = -(2e)^2\bar{B}_3/\hbar = -\frac{2e^2}{\pi\hbar}C$$

Formulation

Scattering matrix with/without SO interaction

$s_e = U \Lambda U^\dagger$: given by random matrix

$S(\boldsymbol{\varphi}) = -\hat{g} s^* \hat{g} s$

$\hat{g} = -i\hat{\sigma}_y$

- Absence: circular **orthogonal** ensemble

  U: real unitary

$$S(\boldsymbol{\varphi}) = s^* s$$

$$-\hat{g} s \hat{g} = s$$

- Presence: circular **symplectic** ensemble

  U: complex unitary with time-reversed eigenstates

$$S(\boldsymbol{\varphi}) = -u^* u$$

$$u = \hat{g} s$$

Formulation

- Tuning parameter p_{SO} of scattering matrix

$$U = (\psi_1, \hat{g}\psi_1^*, \cdots, \psi_M, \hat{g}\psi_M^*)$$

$$\psi_j = \begin{pmatrix} u_j \\ d_j \end{pmatrix} \xrightarrow[\text{orthonormalization for } u_j]{} \begin{pmatrix} \tilde{u}_j \\ 0 \end{pmatrix}$$

$$\boxed{\tilde{\psi}_j = \mathrm{Re}\begin{pmatrix} \tilde{u}_j \\ 0 \end{pmatrix} + p_{\mathrm{SO}}\left[\psi_j - \mathrm{Re}\begin{pmatrix} \tilde{u}_j \\ 0 \end{pmatrix}\right]}$$

Orthonormalize $(\tilde{\psi}_j, \hat{g}\tilde{\psi}_j^*)$ again