

TOPOLOGICAL VALLEY CURRENTS IN GAPPED GRAPHENE

Leonid Levitov (MIT)


- **Berry phase in gapped graphene**
- **Valley Hall effect without edge states**
- **Detecting valley currents in G/hBN superlattices: an all-electrical approach**

NPSMP2015 symposium, ISSP, Tokyo University

Berry curvature and topological currents

Electrons in crystals have charge, energy, momentum and Berry's curvature (**built-in vorticity**)

Semiclassical
eqs of motion:

$$\mathbf{v}_{\mathbf{k}} = \frac{1}{\hbar} \frac{\partial \epsilon_{\mathbf{k}}}{\partial \mathbf{k}} + \dot{\mathbf{k}} \times \Omega(\mathbf{k})$$


$$\dot{\mathbf{k}} = e\mathbf{E} + e\mathbf{v}_{\mathbf{k}} \times \mathbf{B}$$

Berry curvature and topological currents

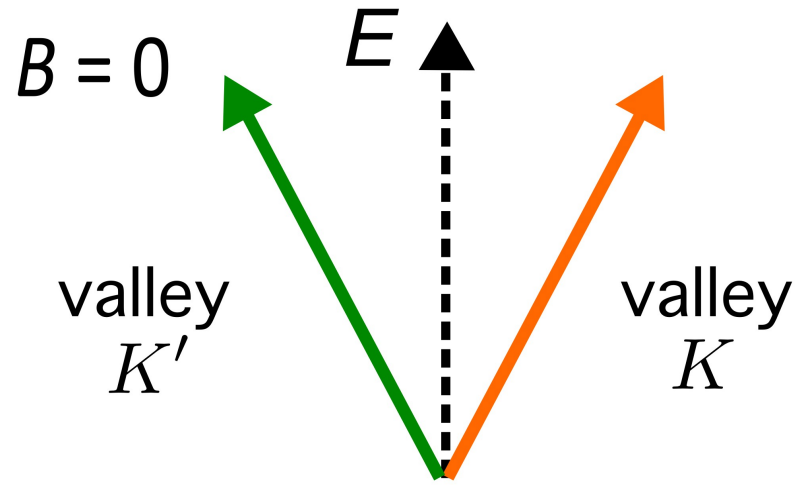
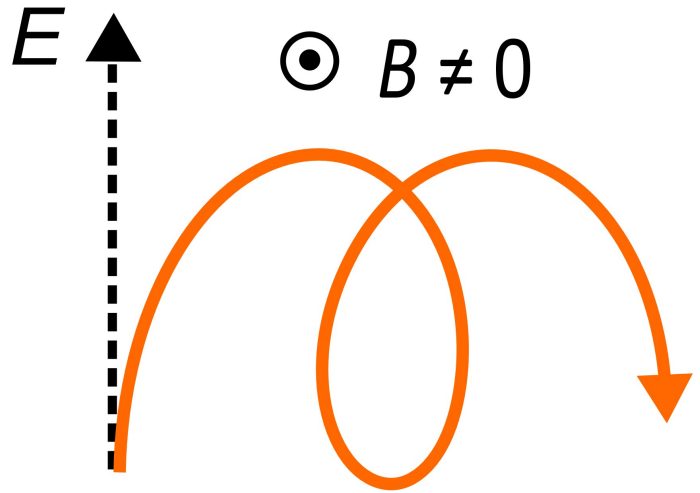
Electrons in crystals have charge, energy, momentum and Berry's curvature (**built-in vorticity**)

Semiclassical
eqs of motion:

$$\mathbf{v}_{\mathbf{k}} = \frac{1}{\hbar} \frac{\partial \epsilon_{\mathbf{k}}}{\partial \mathbf{k}} + \dot{\mathbf{k}} \times \Omega(\mathbf{k})$$

$$\dot{\mathbf{k}} = e\mathbf{E} + e\mathbf{v}_{\mathbf{k}} \times \mathbf{B}$$

Hall currents at $B=0$



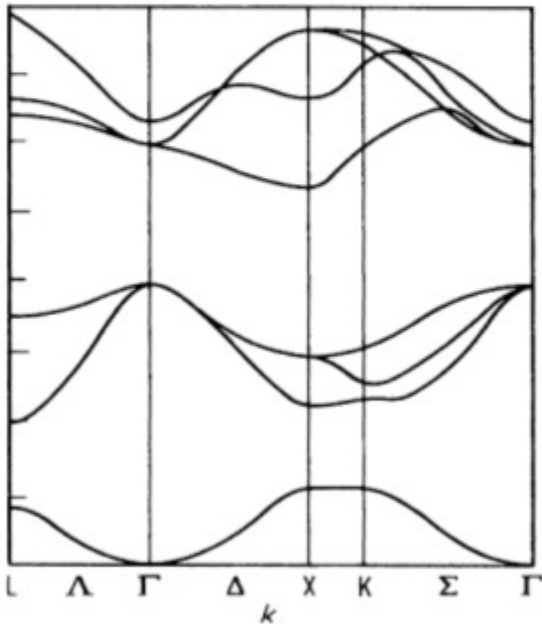
History and context

Karplus and Luttinger (1954), Blount (1955):
"anomalous velocity"

**Semiclassical Eq.
of motion:**

$$\mathbf{v}_{\mathbf{k}} = \frac{1}{\hbar} \frac{\partial \epsilon_{\mathbf{k}}}{\partial \mathbf{k}} + \dot{\mathbf{k}} \times \Omega(\mathbf{k})$$

$$\dot{\mathbf{k}} = e\mathbf{E} + e\mathbf{v}_{\mathbf{k}} \times \mathbf{B}$$



Confine particle to single band (no interband transitions)

Price of projection (constraint):
Anomalous Velocity and Berry phase

Modern Theory: Chang & Niu (1996), Sundaram & Niu (1999), Haldane (2002), Jungworth, Niu, MacDonald (2002), see also review by Nagaosa (2009) and references therein

Berry connection/Berry's vector potential

$$\Omega_n(\mathbf{k}) = \nabla_{\mathbf{k}} \times \mathbf{A}_n(\mathbf{k}), \quad \mathbf{A}_n(\mathbf{k}) = i \langle u_n(\mathbf{k}) | \nabla_{\mathbf{k}} | u_n(\mathbf{k}) \rangle.$$

Berry curvature

Bloch wavefunction

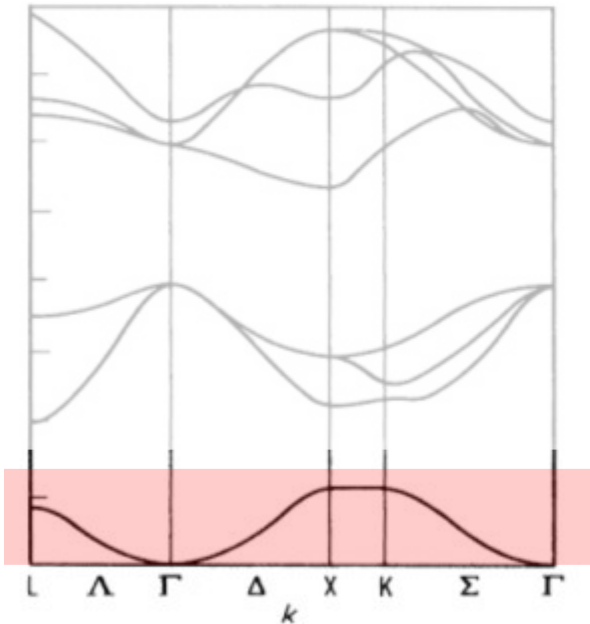
History and context

Karplus and Luttinger (1954), Blount (1955):
"anomalous velocity"

Semiclassical Eq. of motion:

$$\mathbf{v}_{\mathbf{k}} = \frac{1}{\hbar} \frac{\partial \epsilon_{\mathbf{k}}}{\partial \mathbf{k}} + \dot{\mathbf{k}} \times \Omega(\mathbf{k})$$

$$\dot{\mathbf{k}} = e\mathbf{E} + e\mathbf{v}_{\mathbf{k}} \times \mathbf{B}$$



Confine particle to single band (no interband transitions)

Price of projection (constraint):
Anomalous Velocity and Berry phase

Modern Theory: Chang & Niu (1996), Sundaram & Niu (1999), Haldane (2002), Jungworth, Niu, MacDonald (2002), see also review by Nagaosa (2009) and references therein

Berry connection/Berry's vector potential

$$\Omega_n(\mathbf{k}) = \nabla_{\mathbf{k}} \times \mathbf{A}_n(\mathbf{k}), \quad \mathbf{A}_n(\mathbf{k}) = i \langle u_n(\mathbf{k}) | \nabla_{\mathbf{k}} | u_n(\mathbf{k}) \rangle.$$

Berry curvature

Bloch wavefunction

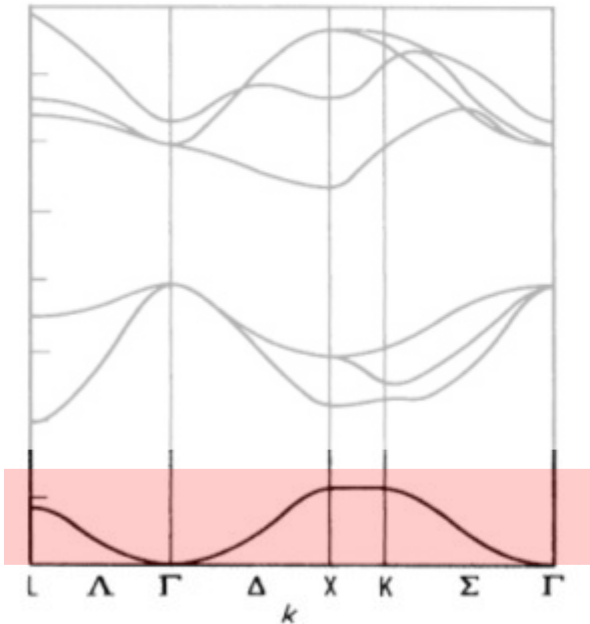
History and context

Karplus and Luttinger (1954), Blount (1955):
"anomalous velocity"

Semiclassical Eq. of motion:

$$\mathbf{v}_{\mathbf{k}} = \frac{1}{\hbar} \frac{\partial \epsilon_{\mathbf{k}}}{\partial \mathbf{k}} + \dot{\mathbf{k}} \times \Omega(\mathbf{k})$$

$$\dot{\mathbf{k}} = e\mathbf{E} + e\mathbf{v}_{\mathbf{k}} \times \mathbf{B}$$



Confine particle to single band (no interband transitions)

Price of projection (constraint):
Anomalous Velocity and Berry phase

Modern Theory: Chang & Niu (1996), Sundaram & Niu (1999), Haldane (2002), Jungworth, Niu, MacDonald (2002), see also review by Nagaosa (2009) and references therein

Berry connection/Berry's vector potential

$$\Omega_n(\mathbf{k}) = \nabla_{\mathbf{k}} \times \mathbf{A}_n(\mathbf{k}), \quad \mathbf{A}_n(\mathbf{k}) = i \langle u_n(\mathbf{k}) | \nabla_{\mathbf{k}} | u_n(\mathbf{k}) \rangle.$$

Berry curvature

Bloch wavefunction

Schematic derivation of anomalous velocity

$$H = V(\mathbf{R}) + \epsilon_n(\mathbf{k}),$$

Evolution of wavefunction picks up a Berry phase

$$\langle \mathbf{r} | \psi \rangle = u_{n\mathbf{k}}(\mathbf{r}) e^{-i\chi(\mathbf{k})} \quad \text{with} \quad \chi(\mathbf{k}) = - \int_C^{\mathbf{k}} d\mathbf{k}' \cdot \overset{\text{Berry connection}}{\mathbf{A}(\mathbf{k}')}$$

Gauge this phase away: $\mathbf{x} = \mathbf{R} + \mathbf{A}(\mathbf{k})$

$$H' = V(i\nabla_{\mathbf{k}} + \mathbf{A}) + \epsilon_n(\mathbf{k}) \quad \rightarrow \quad [x_i, x_j] = i\epsilon^{ijk} \Omega_k.$$

Equation of motion:

$$\hbar \mathbf{v} = -i[\mathbf{x}, H'] = \nabla_{\mathbf{k}} \epsilon_n(\mathbf{k}) + \left(\frac{\partial V}{\partial \mathbf{x}} \right) \times \Omega(\mathbf{k}).$$

$$\Omega_n(\mathbf{k}) = \nabla_{\mathbf{k}} \times \mathbf{A}_n(\mathbf{k}),$$

Berry curvature

Analogy w/ Magnus effect

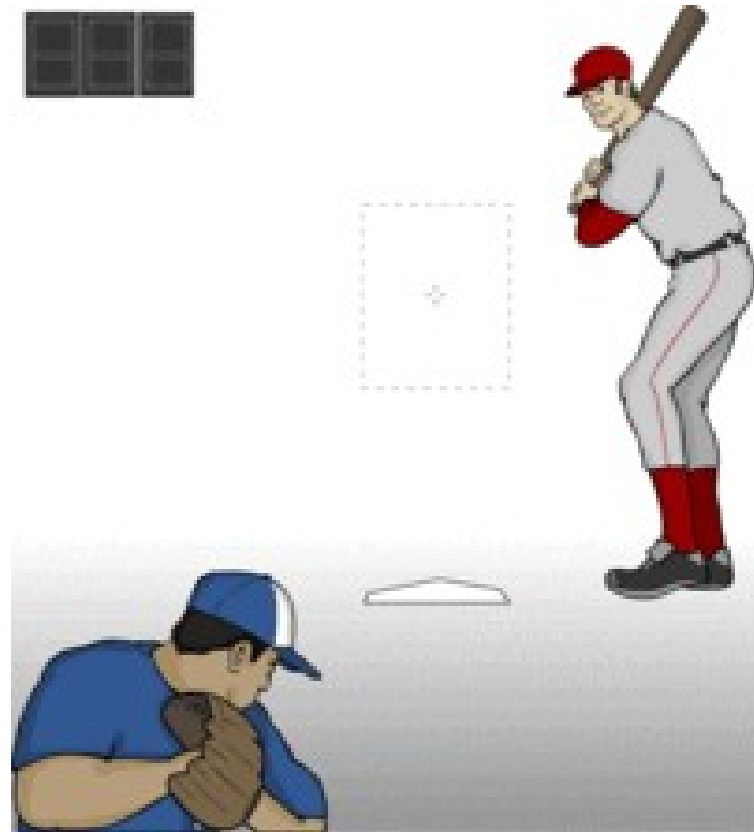
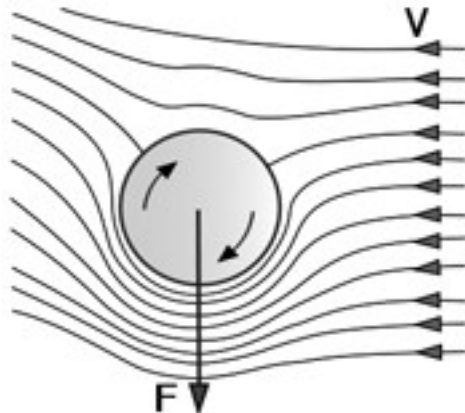
THE MAGNUS EFFECT

FASTER AIR



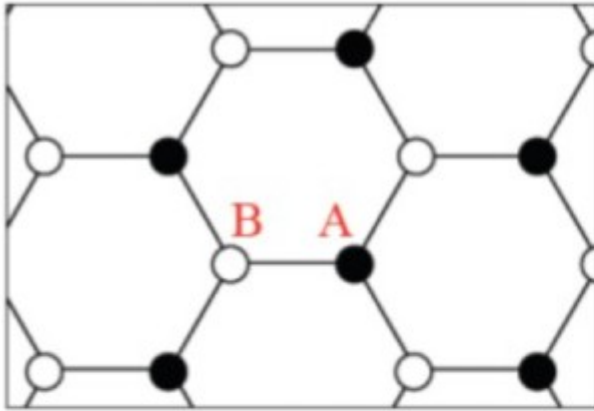
SLOWER AIR

Fastball - Pitcher's Perspective



Berry phase, Berry curvature and gap opening in graphene

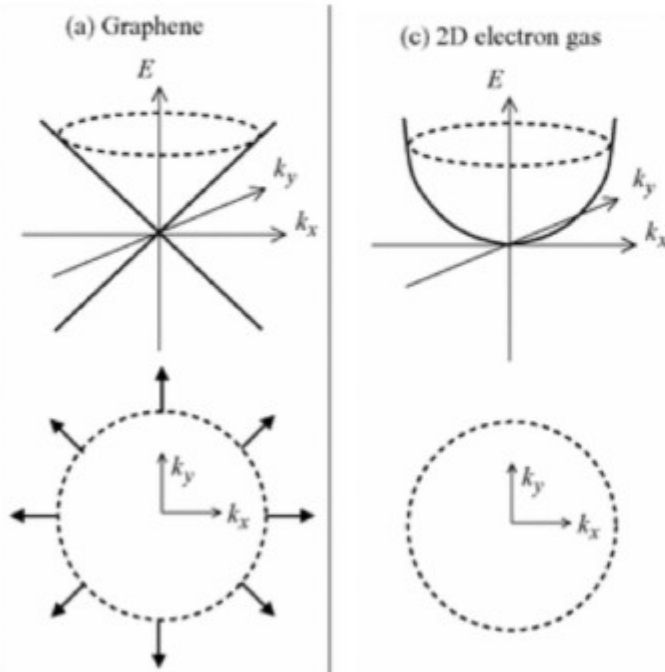
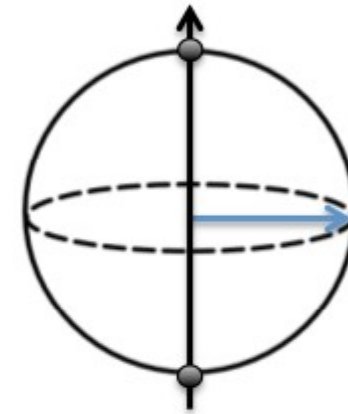
Berry phase in hexagonal lattice



Spinor-type wavefunction:

$$\psi_{\pm, \mathbf{K}}(\mathbf{k}) = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\theta_{\mathbf{k}}/2} \\ \pm e^{i\theta_{\mathbf{k}}/2} \end{pmatrix}$$

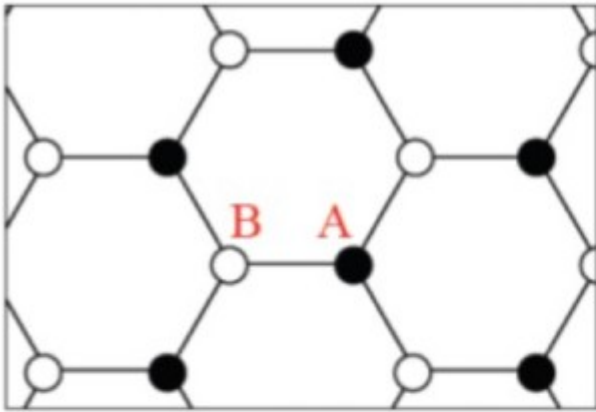
Map onto Bloch Sphere:



- Eigenvectors in x-y plane
- Berry's phase (full rotation)

$$\int_C \langle \psi_{\mathbf{k}} | \partial_{\mathbf{k}} | \psi_{\mathbf{k}} \rangle d\mathbf{k} = \pi$$

Massive/gapped Dirac particles



$$\Omega_n(\mathbf{k}) = \nabla_{\mathbf{k}} \times \mathbf{A}_n(\mathbf{k}), \quad \mathbf{A}_n(\mathbf{k}) = i \langle u_n(\mathbf{k}) | \nabla_{\mathbf{k}} | u_n(\mathbf{k}) \rangle.$$

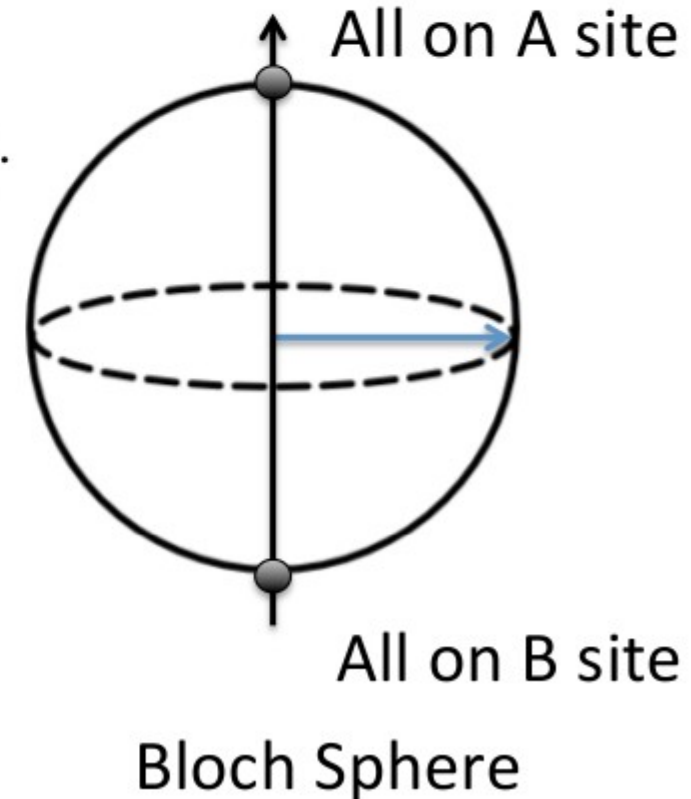
Finite $\Omega_n(\mathbf{k})$ can occur when either is broken

Time-reversal symmetry

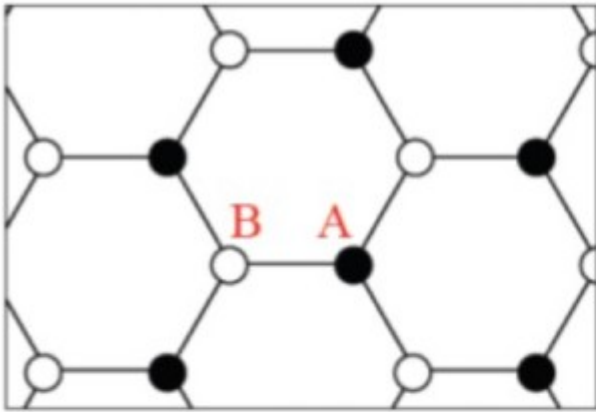
$$\Omega_n(-\mathbf{k}) = -\Omega_n(\mathbf{k})$$

Inversion symmetry

$$\Omega_n(-\mathbf{k}) = \Omega_n(\mathbf{k})$$



Massive/gapped Dirac particles



$$\Omega_n(\mathbf{k}) = \nabla_{\mathbf{k}} \times \mathbf{A}_n(\mathbf{k}), \quad \mathbf{A}_n(\mathbf{k}) = i \langle u_n(\mathbf{k}) | \nabla_{\mathbf{k}} | u_n(\mathbf{k}) \rangle.$$

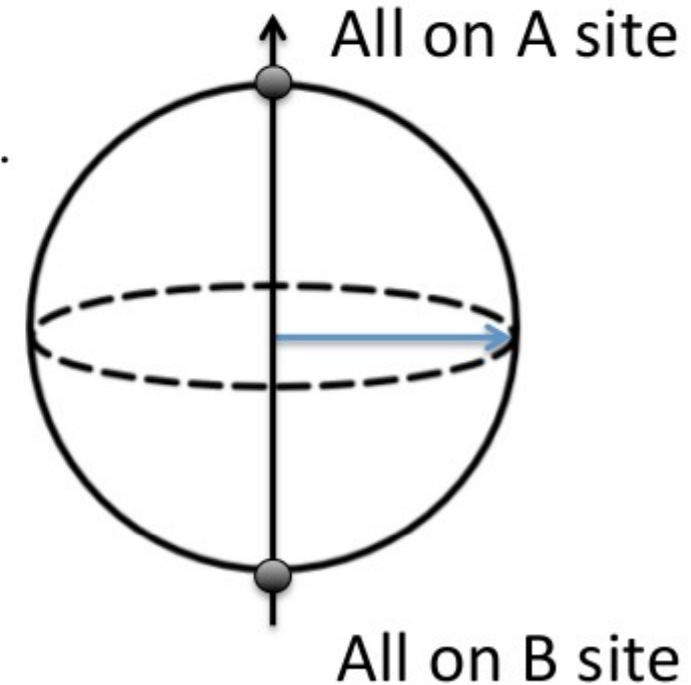
Finite $\Omega_n(\mathbf{k})$ can occur when either is broken

Time-reversal symmetry

$$\Omega_n(-\mathbf{k}) = -\Omega_n(\mathbf{k})$$

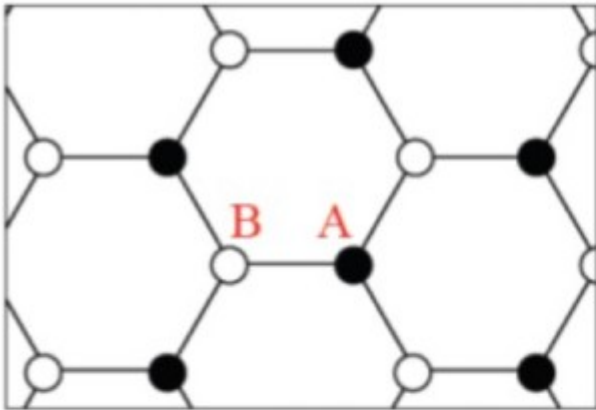
Inversion symmetry

$$\Omega_n(-\mathbf{k}) = \Omega_n(\mathbf{k})$$



Bloch Sphere

Massive/gapped Dirac particles



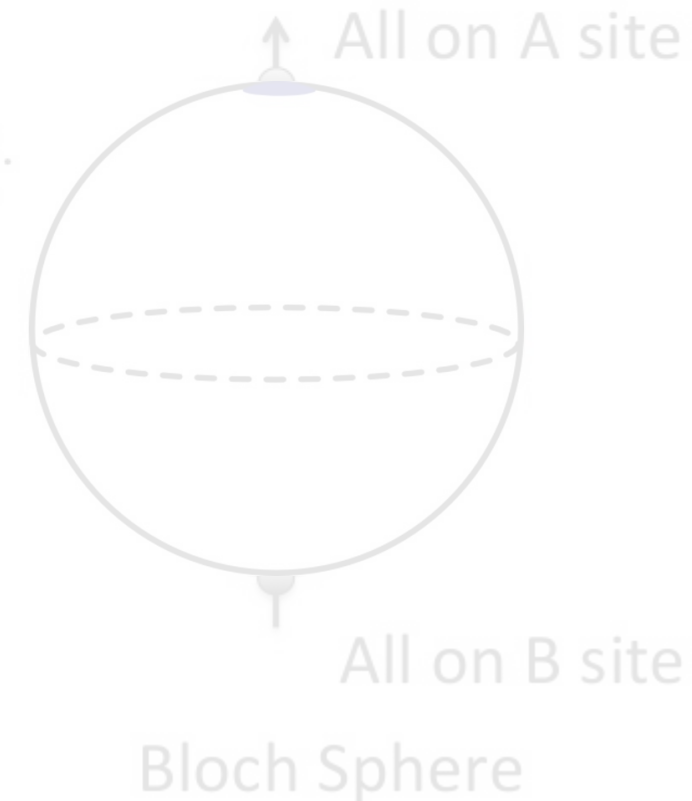
$$\Omega_n(\mathbf{k}) = \nabla_{\mathbf{k}} \times \mathbf{A}_n(\mathbf{k}), \quad \mathbf{A}_n(\mathbf{k}) = i \langle u_n(\mathbf{k}) | \nabla_{\mathbf{k}} | u_n(\mathbf{k}) \rangle.$$

Finite $\Omega_n(\mathbf{k})$ can occur when either is broken

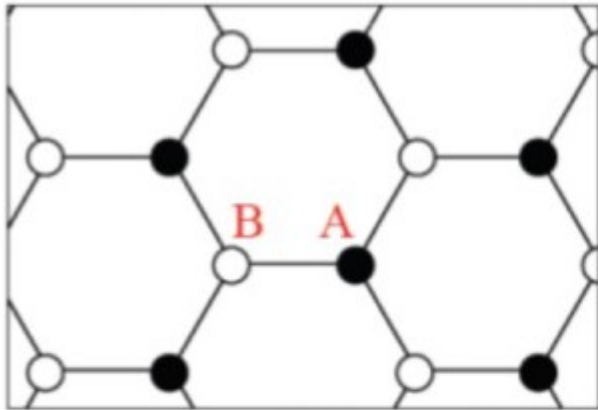
Time-reversal symmetry
 ~~$\Omega_n(-\mathbf{k}) = -\Omega_n(\mathbf{k})$~~

Inversion symmetry
 $\Omega_n(-\mathbf{k}) = \Omega_n(\mathbf{k})$

Magnetic AHE



Massive/gapped Dirac particles



A/B sublattice asymmetry a gap-opening perturbation

$$\Omega_n(\mathbf{k}) = \nabla_{\mathbf{k}} \times \mathbf{A}_n(\mathbf{k}), \quad \mathbf{A}_n(\mathbf{k}) = i \langle u_n(\mathbf{k}) | \nabla_{\mathbf{k}} | u_n(\mathbf{k}) \rangle.$$

Finite $\Omega_n(\mathbf{k})$ can occur when either is broken

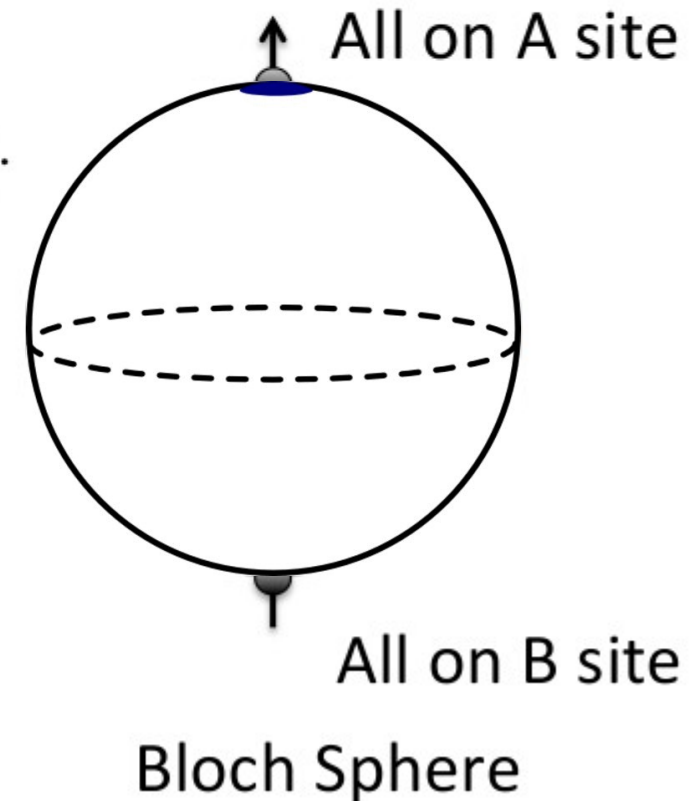
Time-reversal symmetry

$$\Omega_n(-\mathbf{k}) = -\Omega_n(\mathbf{k})$$

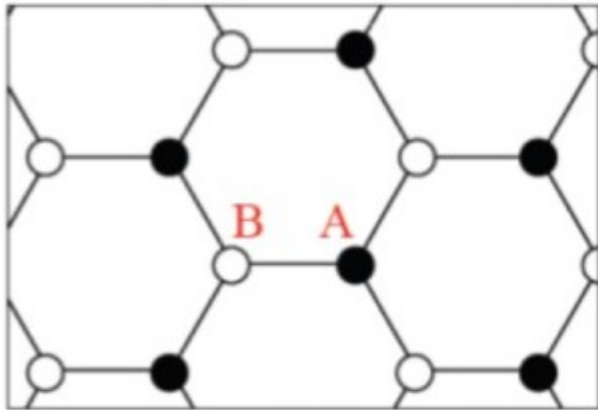
~~Inversion symmetry~~

~~$$\Omega_n(-\mathbf{k}) = \Omega_n(\mathbf{k})$$~~

Valley or spin
AHE



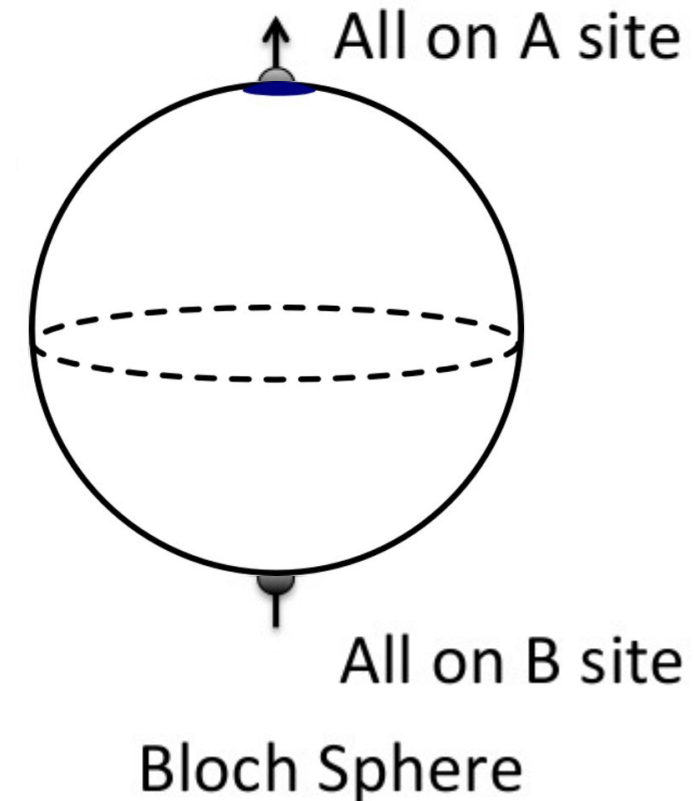
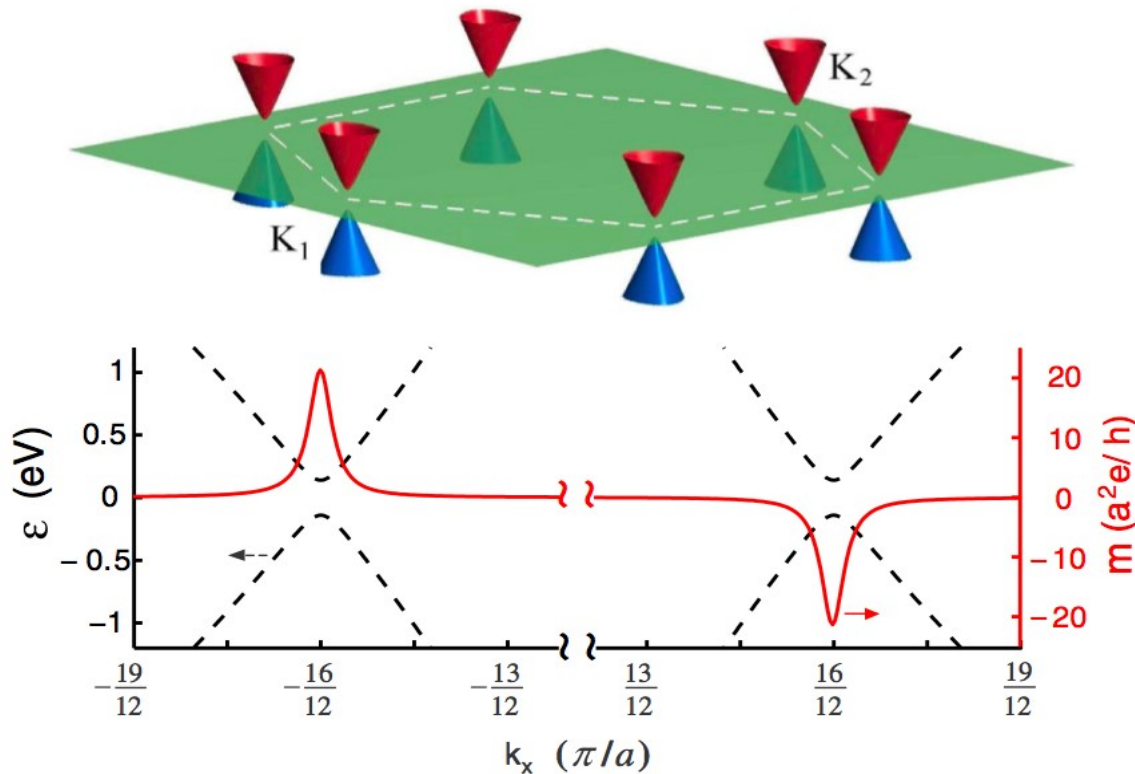
Massive/gapped Dirac particles



A/B sublattice asymmetry a gap-opening perturbation

Berry curvature hot spots above and below the gap

D. Xiao, W. Yao, and Q. Niu, PRL 99, 236809 (2007)



Create topological bands in
graphene?
(and play curveball)



Justin Song



Polnop Samutpraphoot



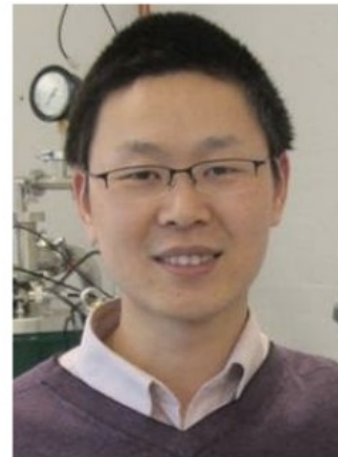
Yuri Lensky



Andrey Shytov



Andre Geim



Geliang Yu



Roman Gorbachev

Song, Shytov, LL PRL 111, 266801 (2013)

Song, Samutpraphoot, LL arXiv:1404.4019 (2014)

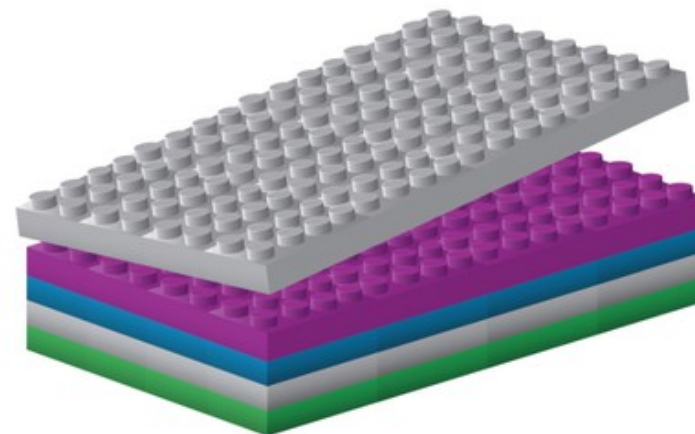
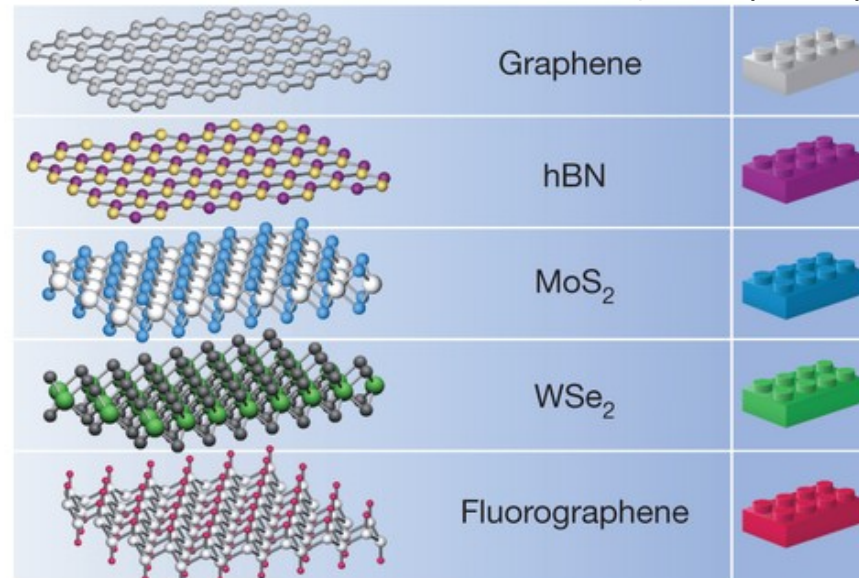
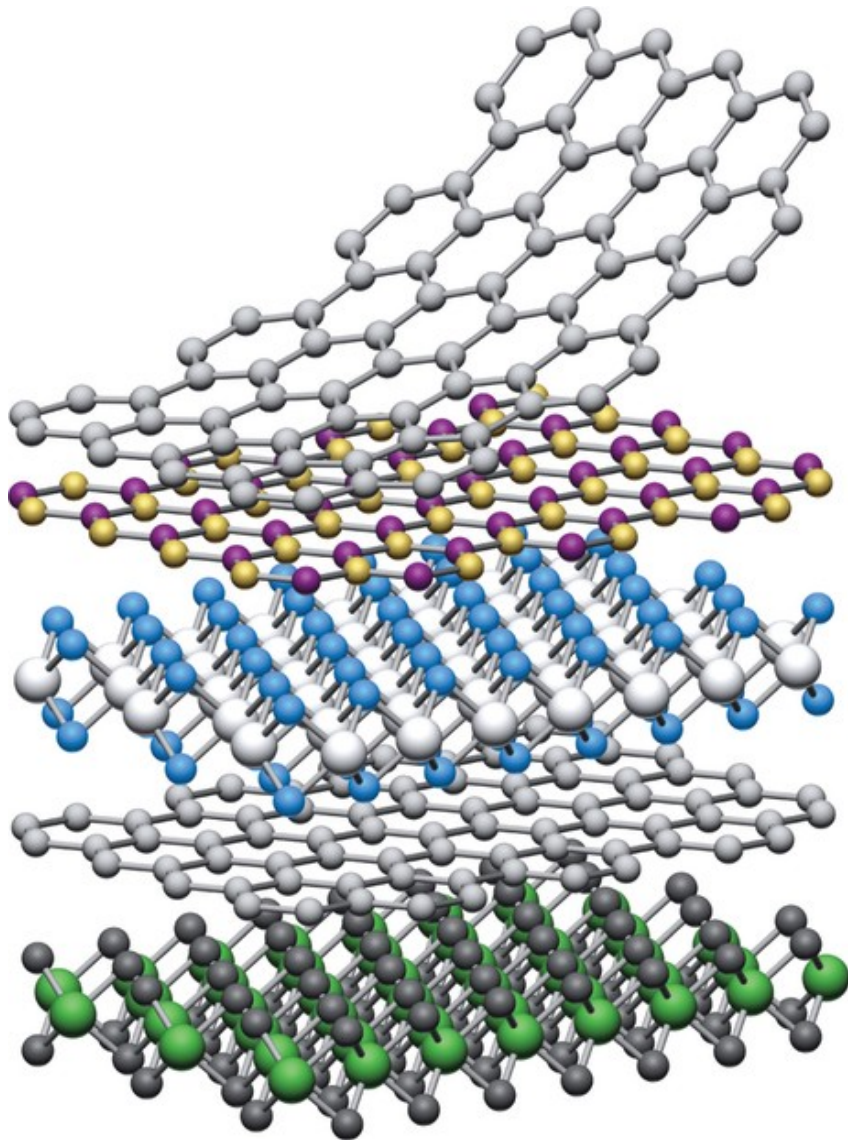
Gorbachev, Song et al arXiv:1409.0113 (2014)

Lensky, Song, Samuthrapoot, LL, arXiv:1412.1808 (2014)

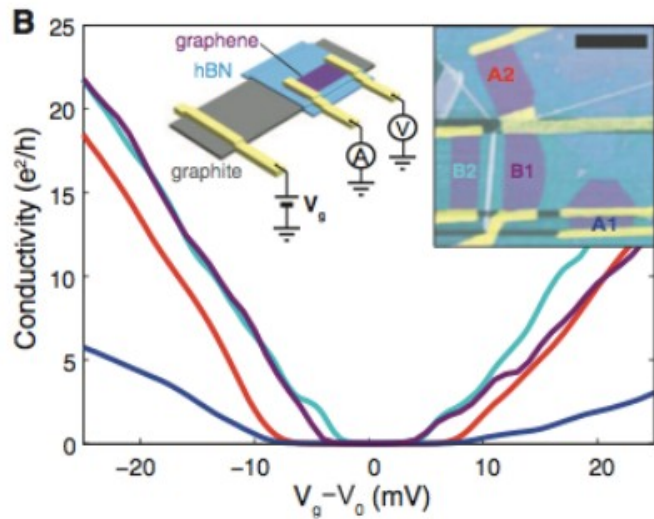
Stacked Van der Waals heterostructures

Stacked atomically thin layers: van der Waals crystals, atomic precision, axes alignment

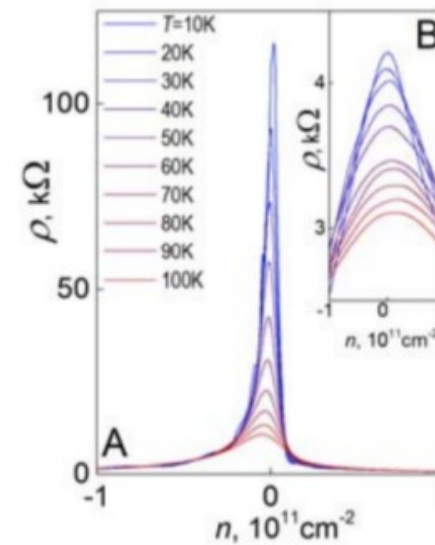
Image from: Geim & Grigorieva, Nature 499, 419 (2013)



Gap opening for G/hBN

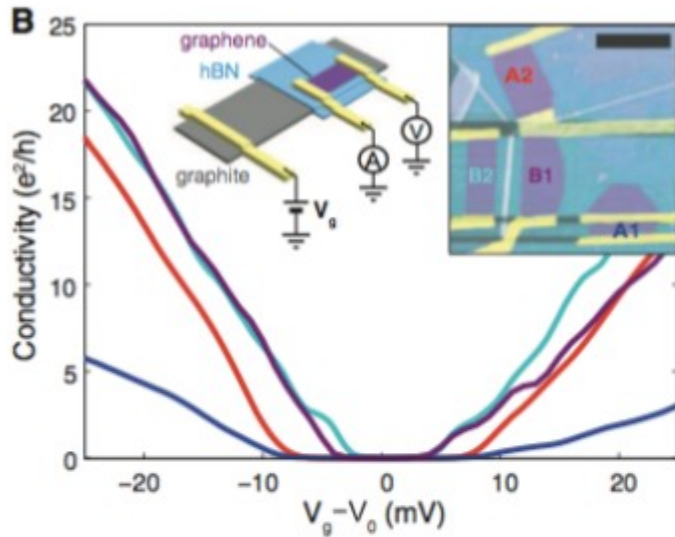


B. Hunt, et. al., *Science*, 340, 1430 (2013) (MIT Group)
See also stanford and columbia groups

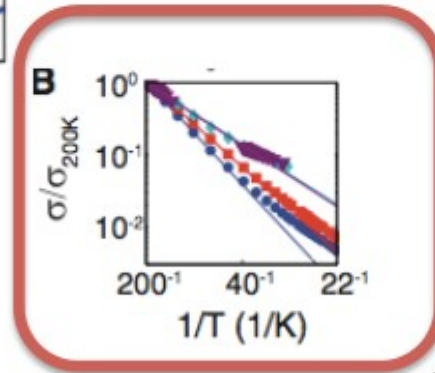


CR Woods, et.al. *Nat. Phys* (2014) (Manchester Group)

Gap opening for G/hBN



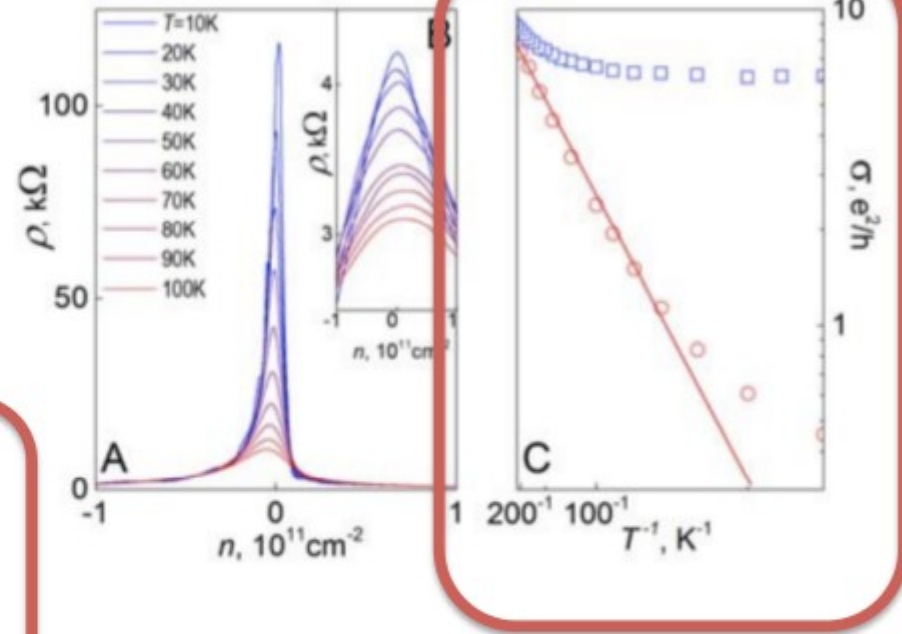
Arhenius plot



Activated behaviour

$$\sigma \propto e^{-\Delta/k_B T}$$

Activated behaviour

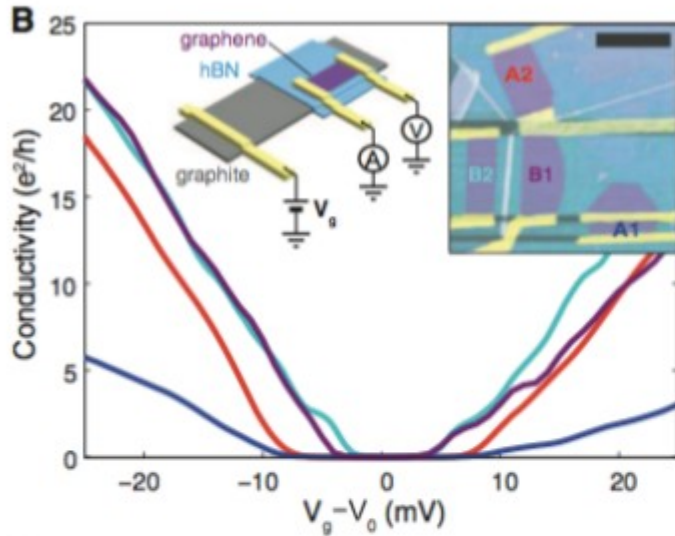


B. Hunt, et. al., *Science*, 340, 1430 (2013) (MIT Group)
See also stanford and columbia groups

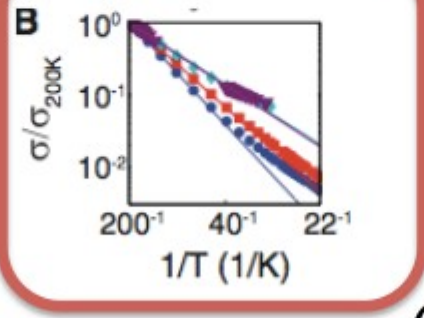
CR Woods, et.al. *Nat. Phys* (2014) (Manchester Group)

Activated behavior: gap $\Delta \sim 200-400$ K

Gap opening for G/hBN



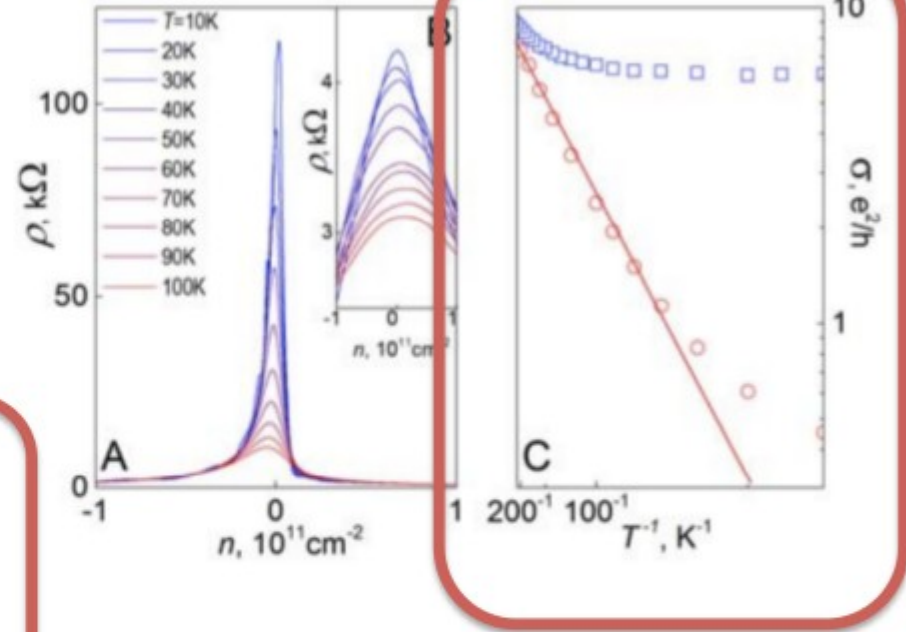
Arhenius plot



Activated behaviour

$$\sigma \propto e^{-\Delta/k_B T}$$

Activated behaviour



B. Hunt, et. al., *Science*, 340, 1430 (2013) (MIT Group)
See also stanford and columbia groups

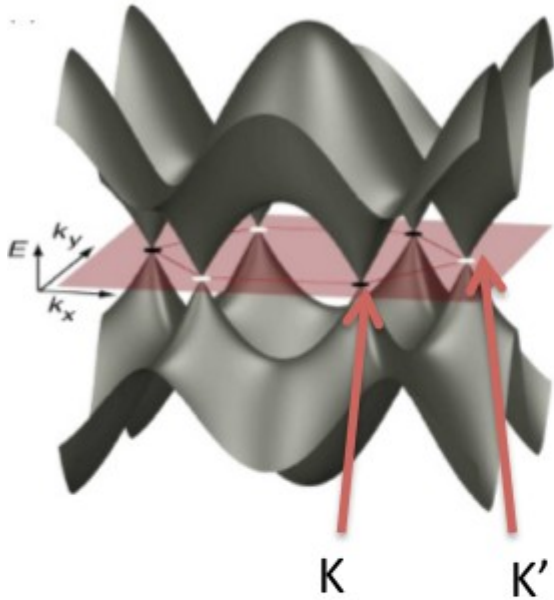
CR Woods, et.al. *Nat. Phys* (2014) (Manchester Group)

Activated behavior: gap $\Delta \sim 200-400$ K

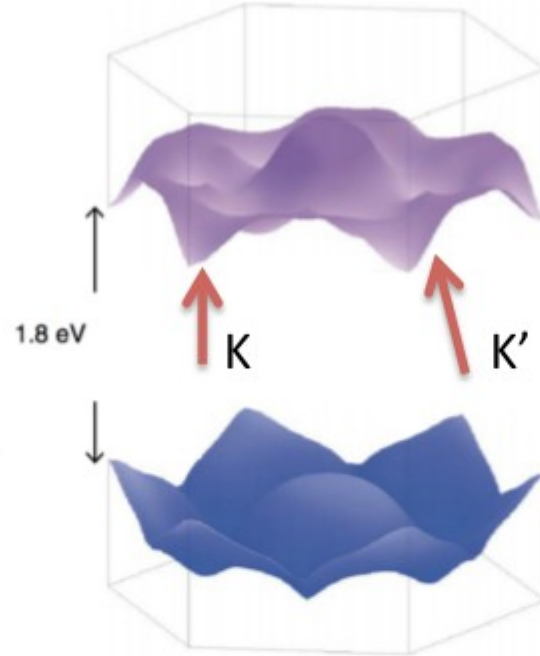
11.06.2015 Valley currents: Gorbachev, Song, et. al. , *Science* (2014)

Valley index

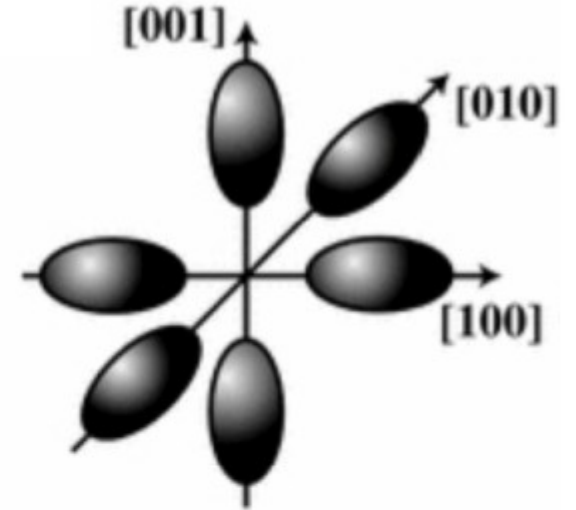
Valleys in Graphene



Valleys in MoS₂



Valleys in Bulk Si

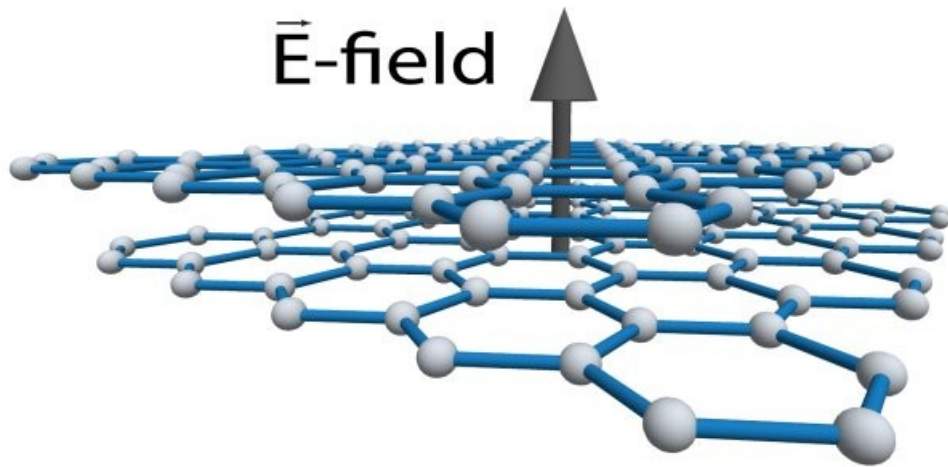


Internal degree of freedom

Long-lived, inter valley scattering \approx Hundreds of ps

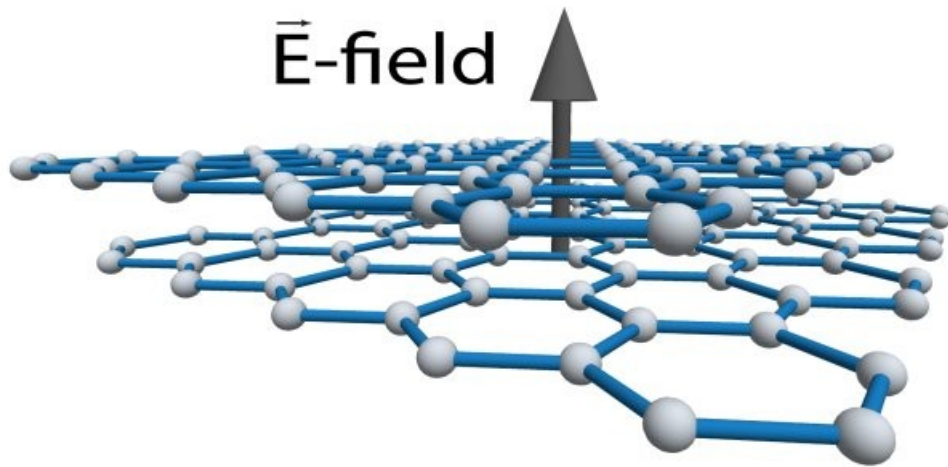
Valley current, $J_K - J_{K'}$

Dual gated bilayer graphene: field-tunable gap and Berry curvature



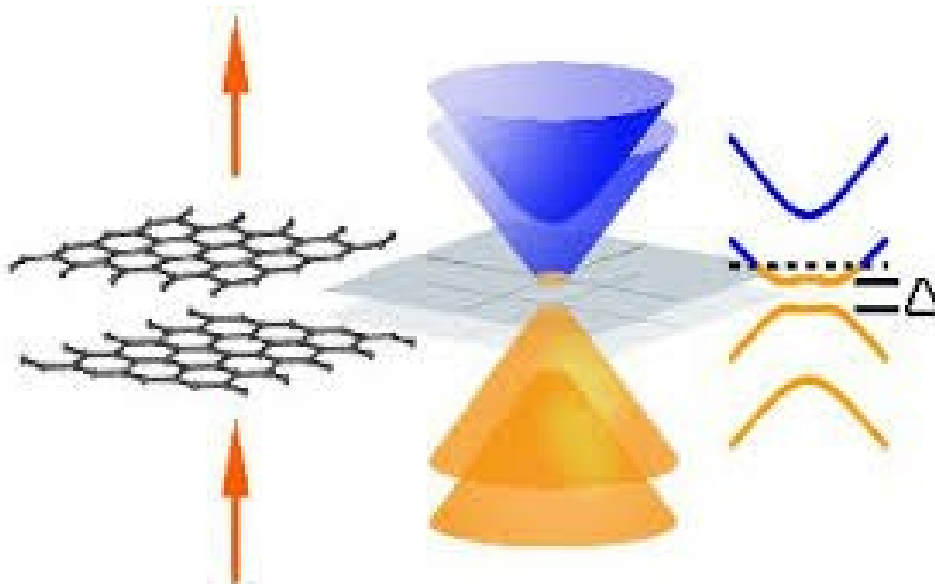
Broken A/B sublattice symmetry

Dual gated bilayer graphene: field-tunable gap and Berry curvature



Broken A/B sublattice symmetry

Berry curvature hot spots above and below the gap

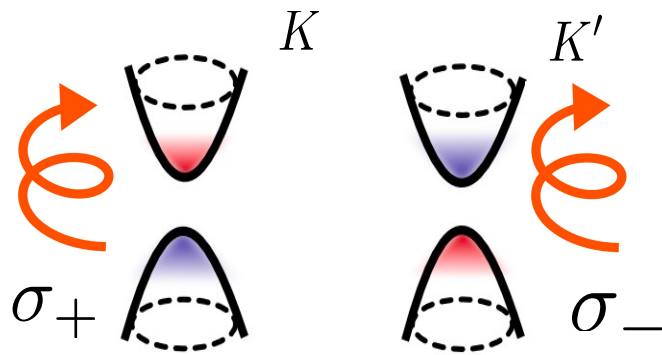


Valley currents:

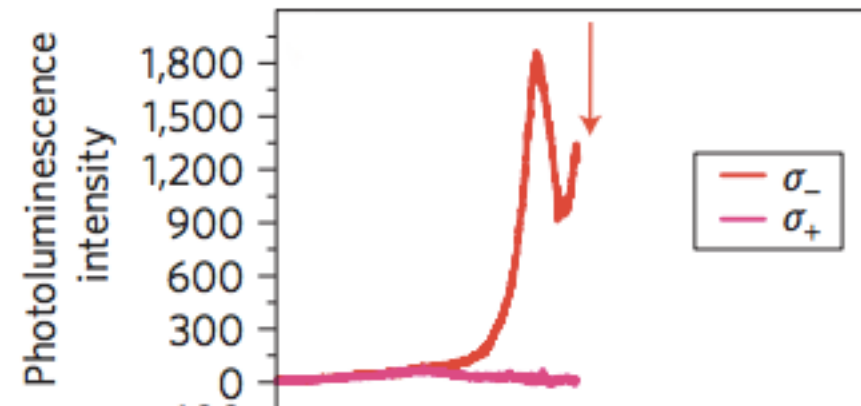
Shimazaki et al.
arxiv:1501.04776 (2015)

Optical control of valleys

Optical selection rules: addressing valleys individually

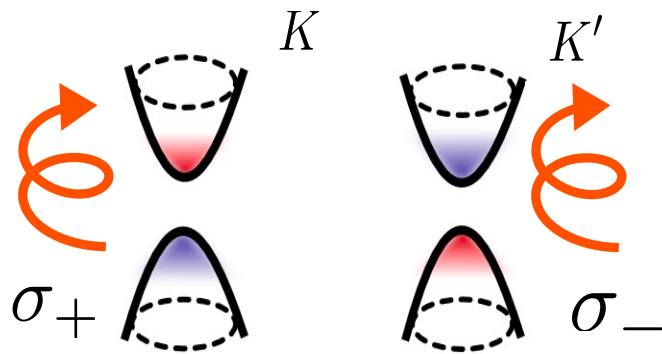


PL (MoS₂) after shining σ_-

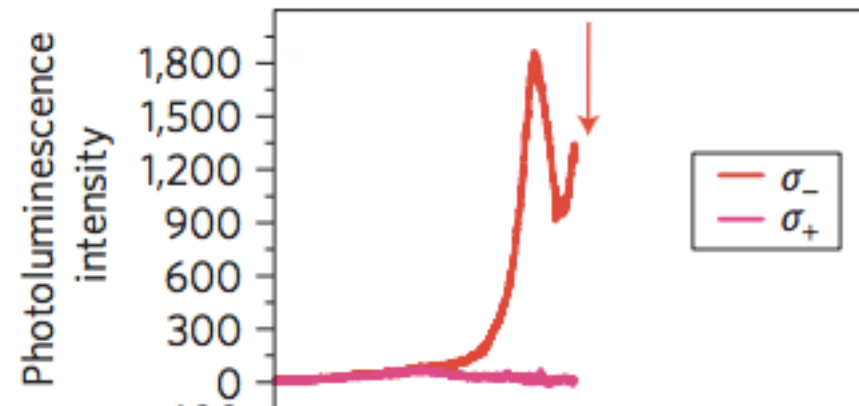


Optical control of valleys

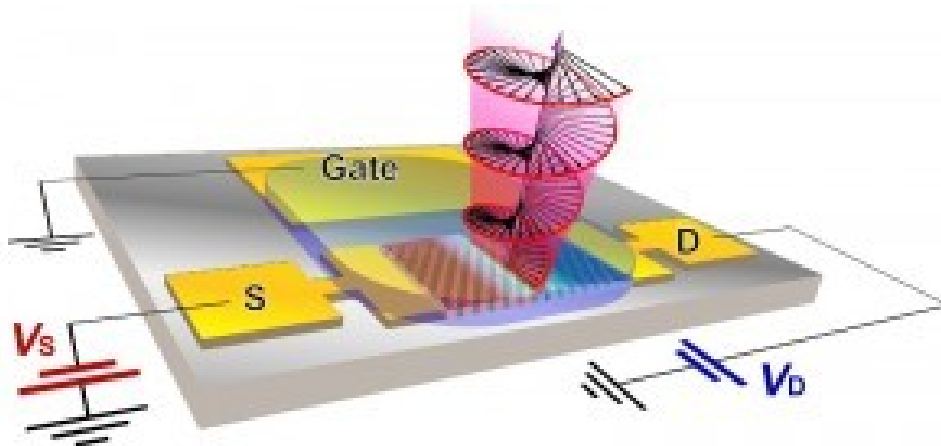
Optical selection rules: addressing valleys individually



PL (MoS₂) after shining σ_-

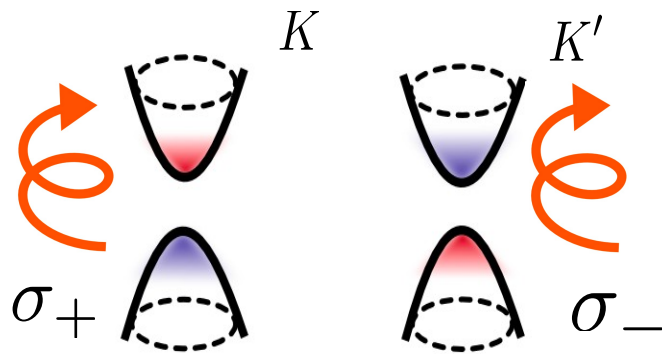


Electrically switchable chiral light-emitting transistor

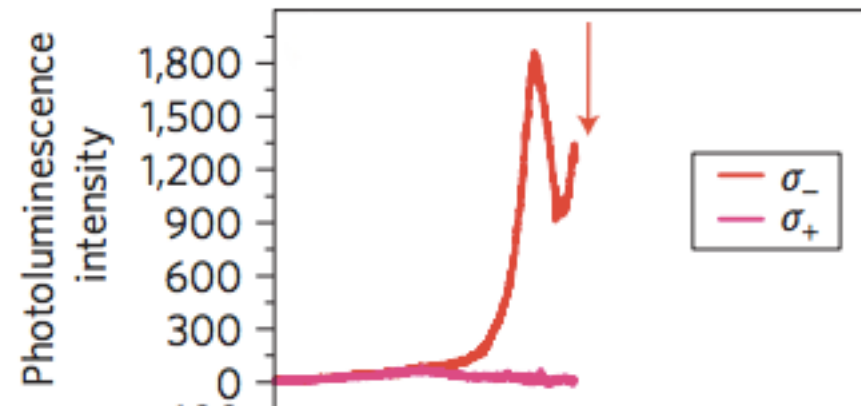


Optical control of valleys

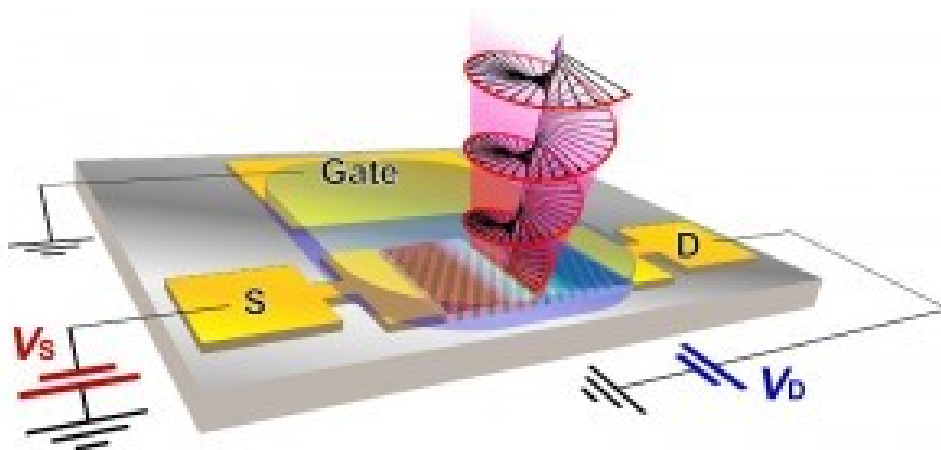
Optical selection rules: addressing valleys individually



PL (MoS₂) after shining σ^-

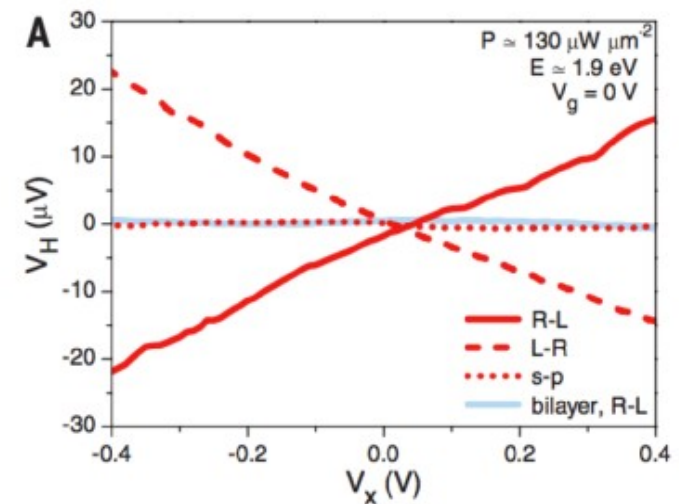
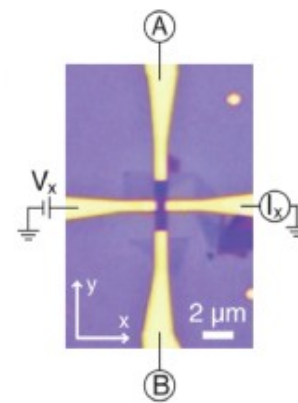


Electrically switchable chiral light-emitting transistor



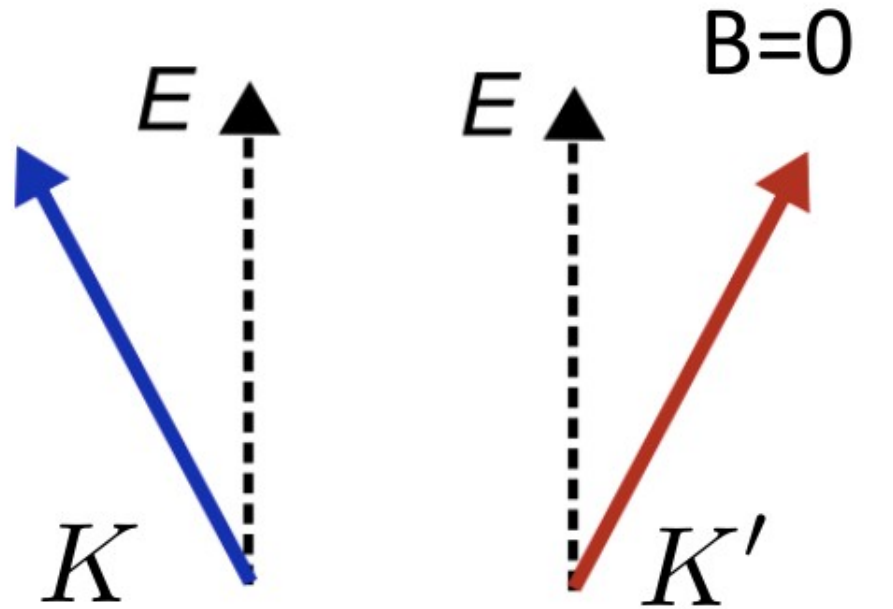
Zhang et. al. , Science (2014)

Valley Selective Hall Effect in MoS₂



Mak, McGill, Park, McEuen, Science (2014)

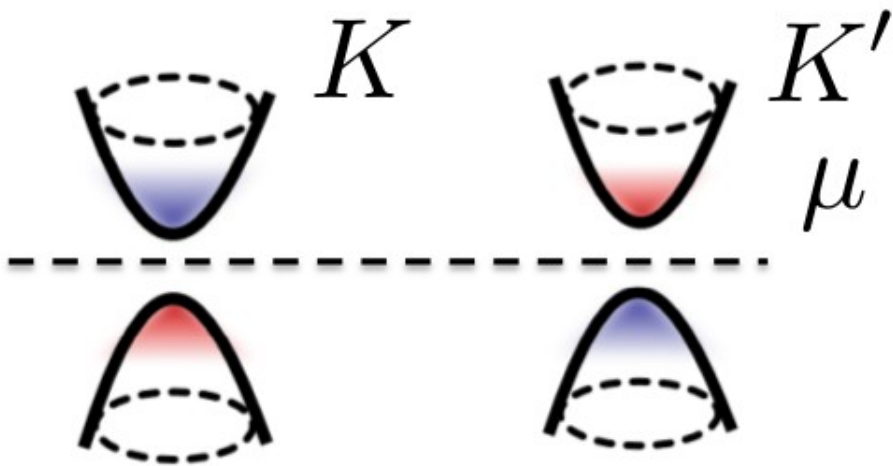
Valley transport in the gap?



- **No edge states** demanded by symmetry or topology + rough boundaries will scatter between valleys

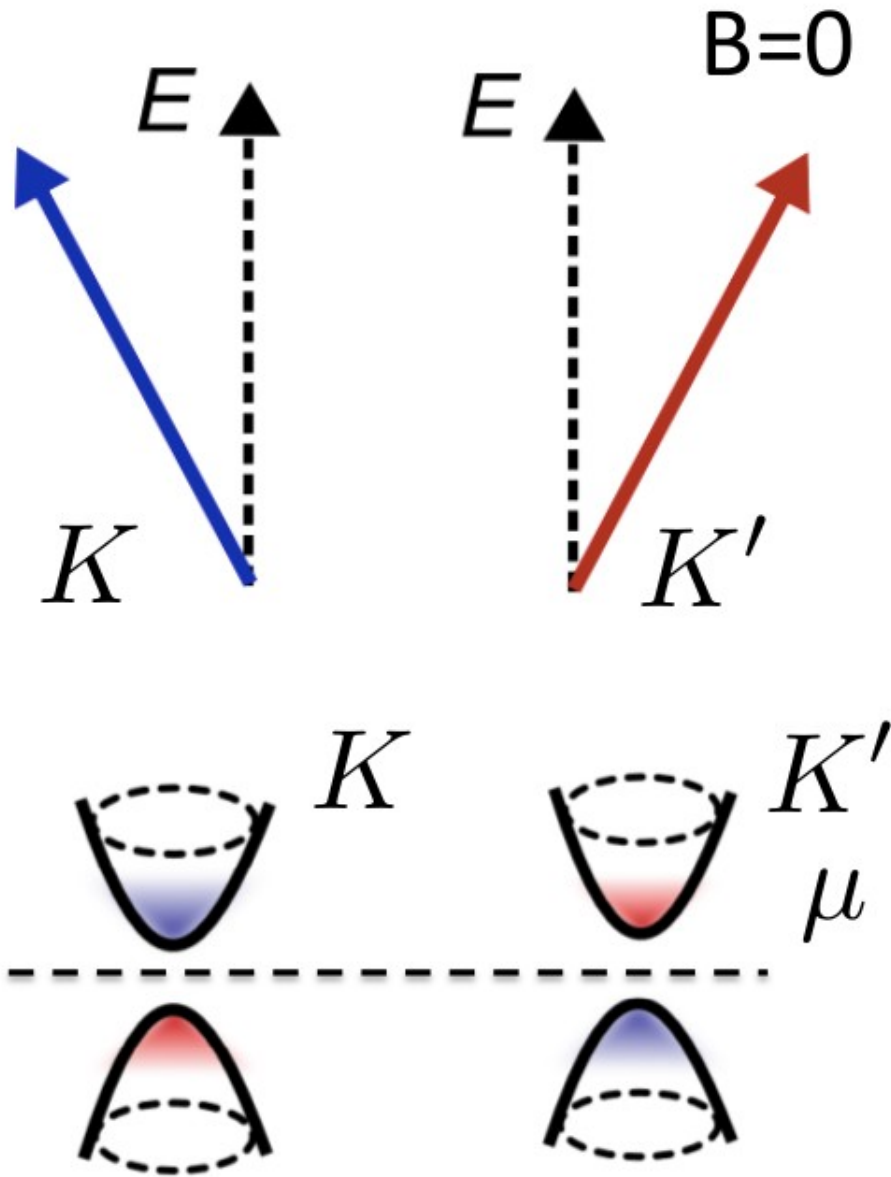
No valley current?

$$\mathbf{J}_v = 0?$$



Conventional wisdom:
Valley Hall Conductivity vanishes in the gap
Xiao, Yao, Niu, PRL 2007, PRL 2012

Valley transport in the gap?



- **No edge states** demanded by symmetry or topology + rough boundaries will scatter between valleys

No valley current?

$$\mathbf{J}_v = 0?$$

Conventional wisdom:

Valley Hall Conductivity vanishes in the gap

Xiao, Yao, Niu, PRL 2007, PRL 2012

revisit

Claim:

$\mathbf{J}_v \neq 0$ even when in the gap

Currents for an edgeless setting

Lensky, JS, Samutpraphoot, Levitov, arxiv: 1412:1808 (2014)

$$H = \begin{pmatrix} \Delta & vp_- \\ vp_+ & -\Delta \end{pmatrix} - eEx, \quad p_{\pm} = p_1 \pm ip_2,$$

Berry curvature

$$\mathbf{v} = \frac{\partial \epsilon}{\partial \mathbf{p}} + \dot{\mathbf{p}} \times \Omega(\mathbf{k})$$

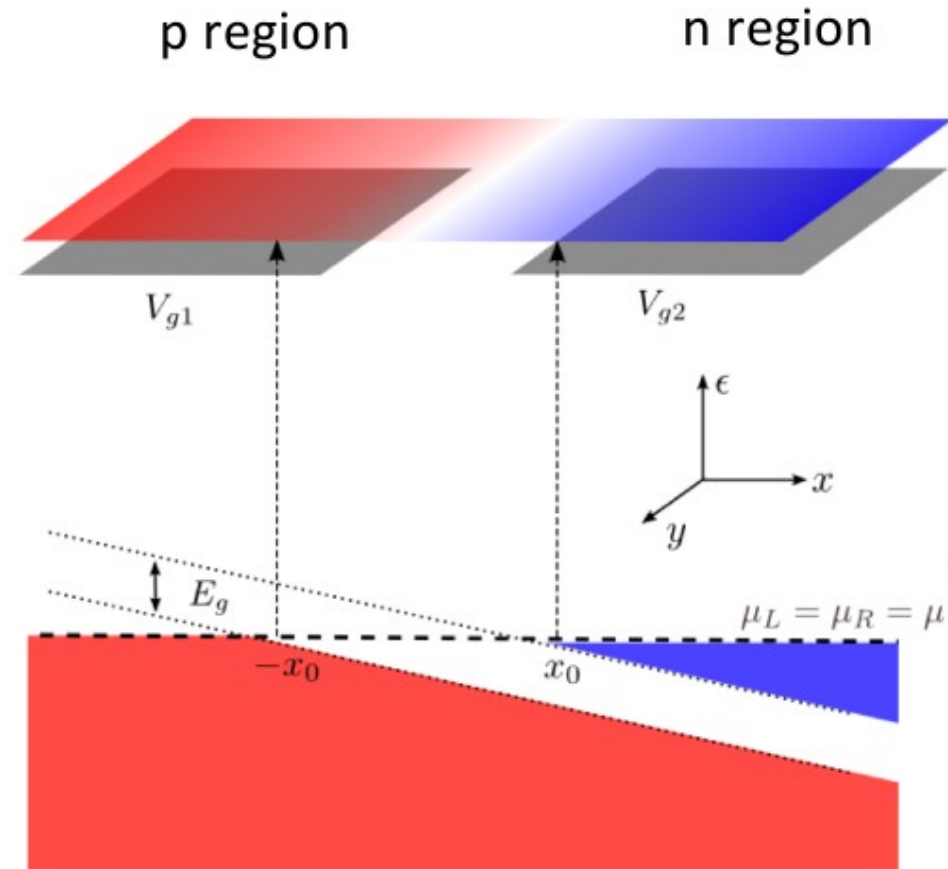
$$\dot{\mathbf{p}} = e\mathbf{E}, \quad \mathbf{p} = \hbar\mathbf{k}$$

Solving for velocity in y-direction:

$$\dot{y} = \frac{\text{Group vel: } vp_y}{\sqrt{v^2|\mathbf{p}|^2 + \Delta^2}} + \text{Anomalous vel: } \Omega(\mathbf{p})eE$$

Summing to get currents:

$$\mathbf{j} = \sum_{\mathbf{p}, \pm} e\dot{\mathbf{r}} f(\mathbf{p})$$

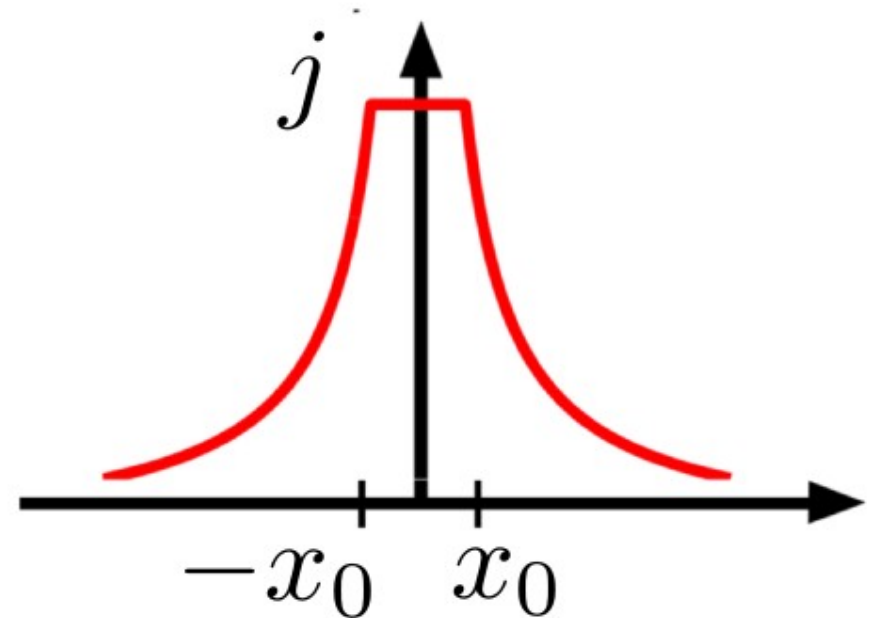
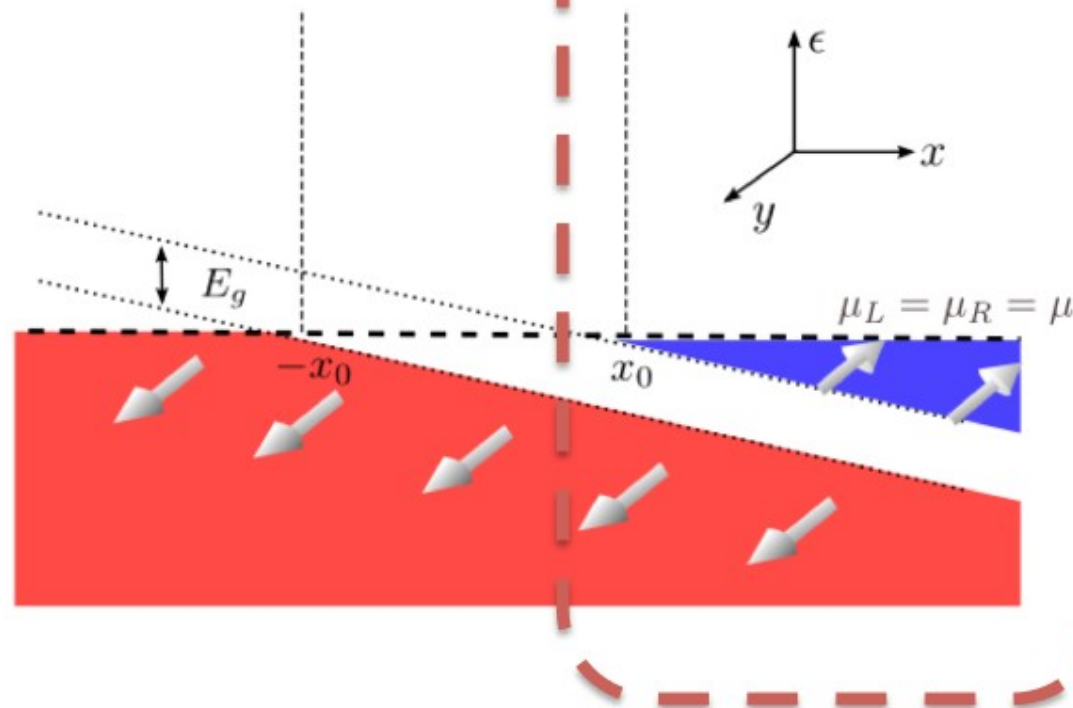
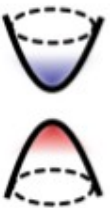


Currents peak in the gap

Partially cancels

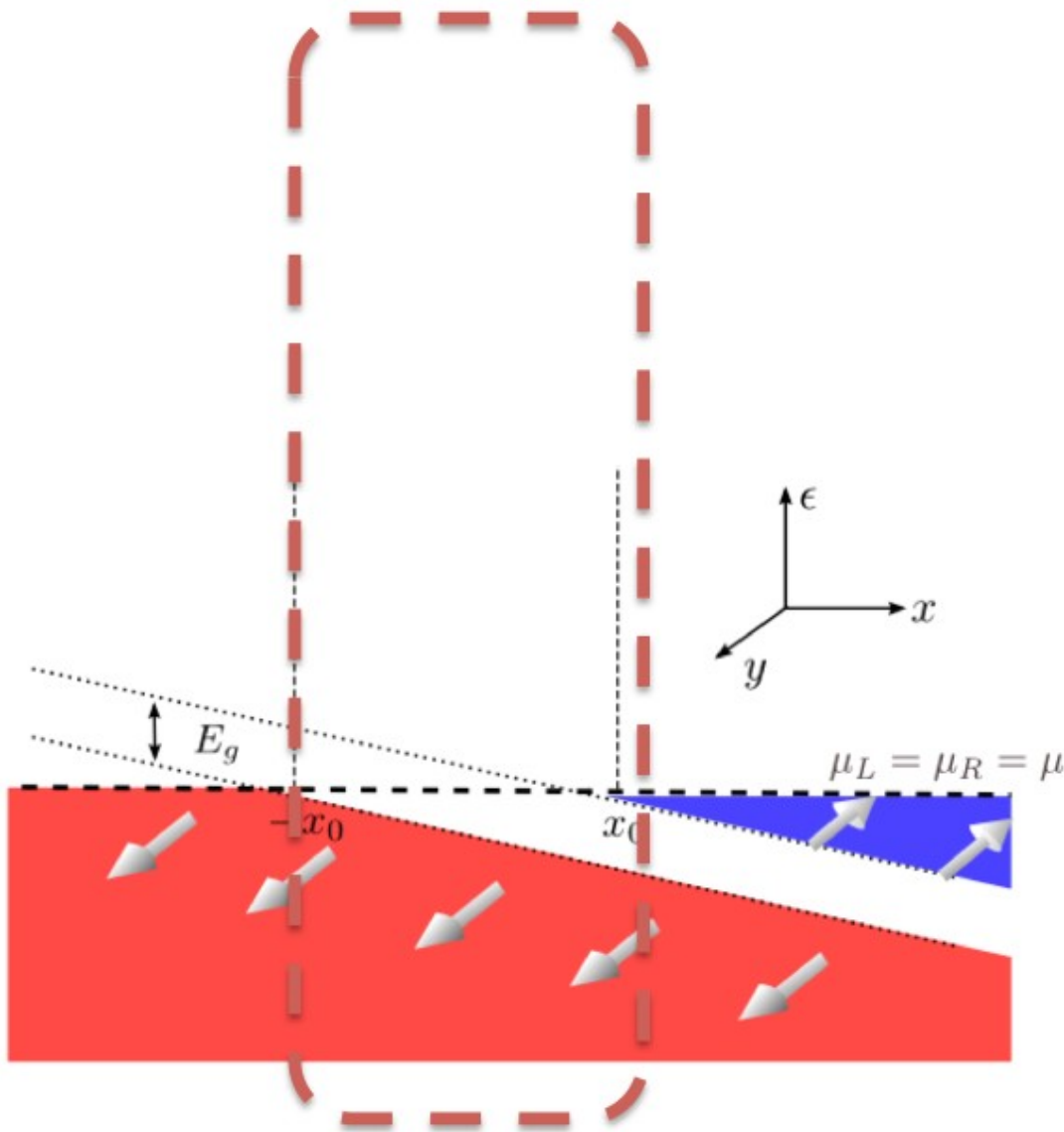
Berry curvature in \pm bands:

$$\Omega_{\pm}(\mathbf{p}) = \frac{v^2 \Delta}{\pm 2(v^2 |\mathbf{p}|^2 + \Delta^2)^{3/2}}$$



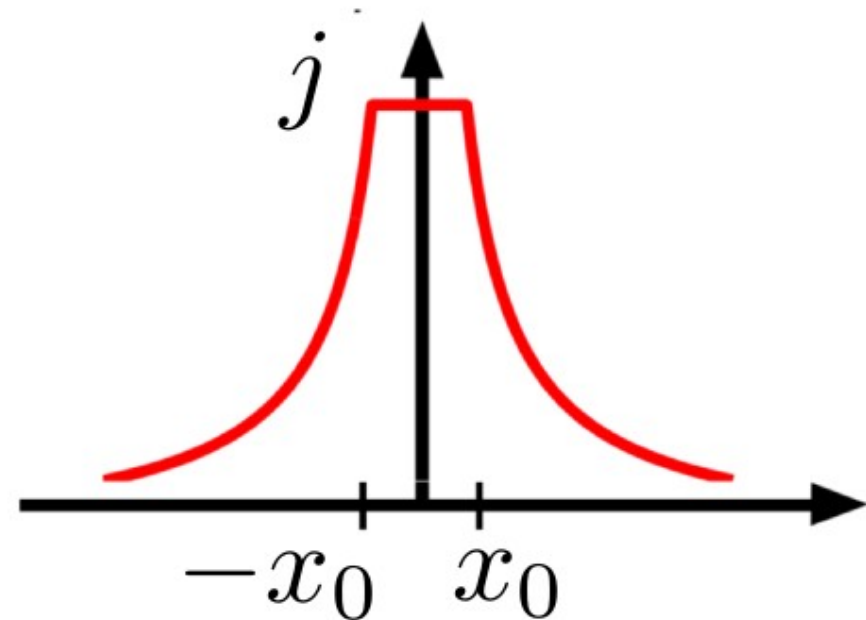
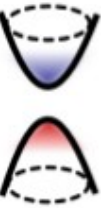
Currents peak in the gap

Maximum valley current

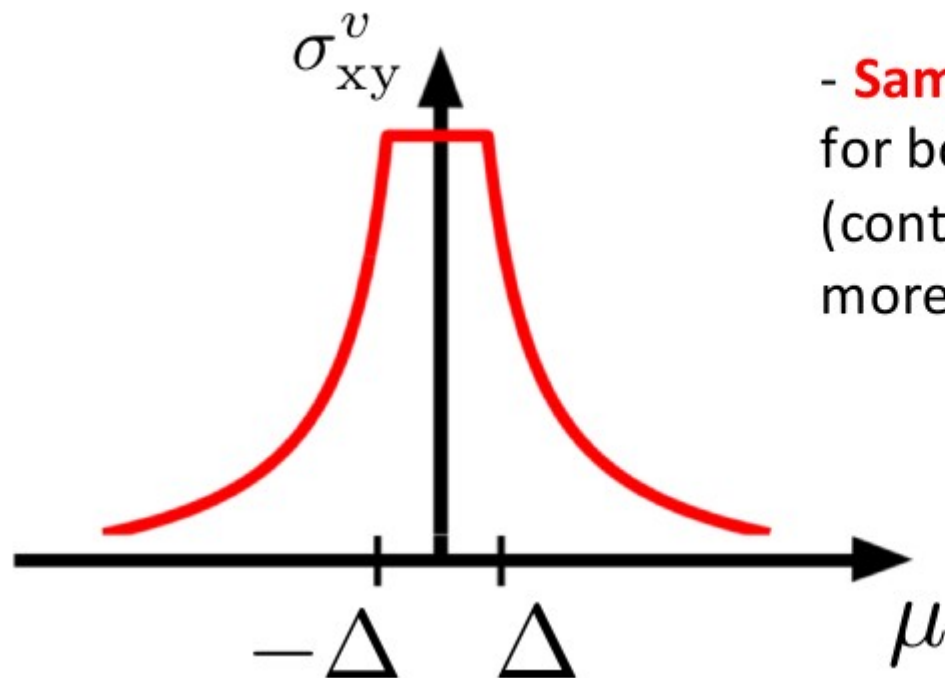


Berry curvature in \pm bands:

$$\Omega_{\pm}(\mathbf{p}) = \frac{v^2 \Delta}{\pm 2(v^2 |\mathbf{p}|^2 + \Delta^2)^{3/2}}$$



Valley Hall conductivity



- Peaks in the gapped region
- **Same sign** of valley hall conductivity for both p and n doping (contrast with ordinary hall effect; more robust against density inhomogeneity)

Microscopic particle trajectories

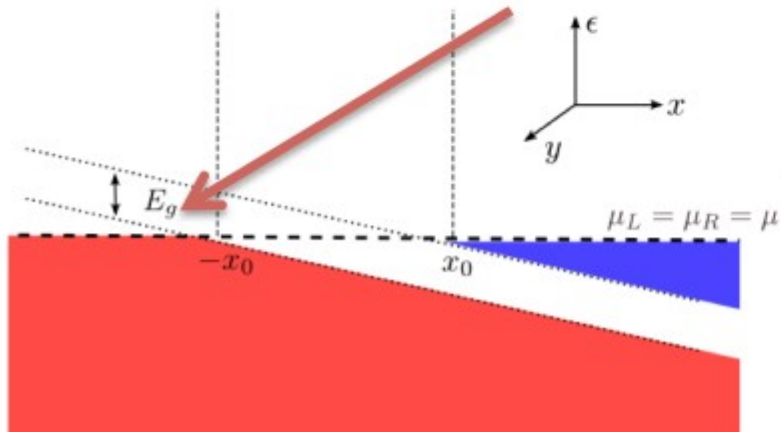
$$\mathbf{v} = \frac{\partial \epsilon}{\partial \mathbf{p}} + \dot{\mathbf{p}} \times \Omega(\mathbf{k})$$

$$\dot{\mathbf{p}} = e\mathbf{E}, \quad \mathbf{p} = \hbar\mathbf{k}$$

Particle trajectory for $p_y = 0$:

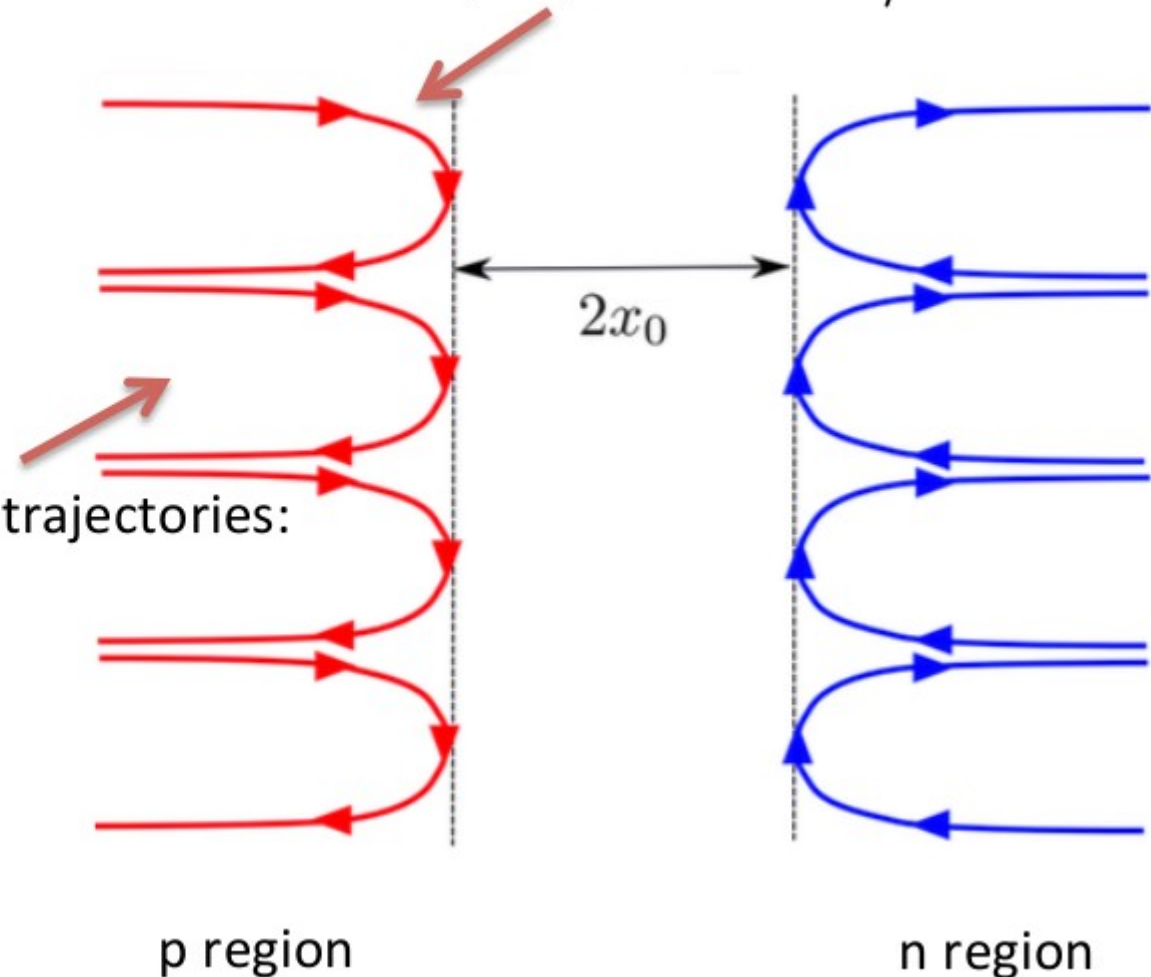
$$\left(\frac{2(y - y_0)}{\ell}\right)^2 + \left(\frac{x_0}{x}\right)^2 = 1$$

Focus on $\epsilon = 0$ trajectories:



Trajectories close to gap region

Side jump, $\ell = \hbar v / \Delta$



Fully gapped system under bias

$$L < 2x_0 = 2\Delta/eE$$



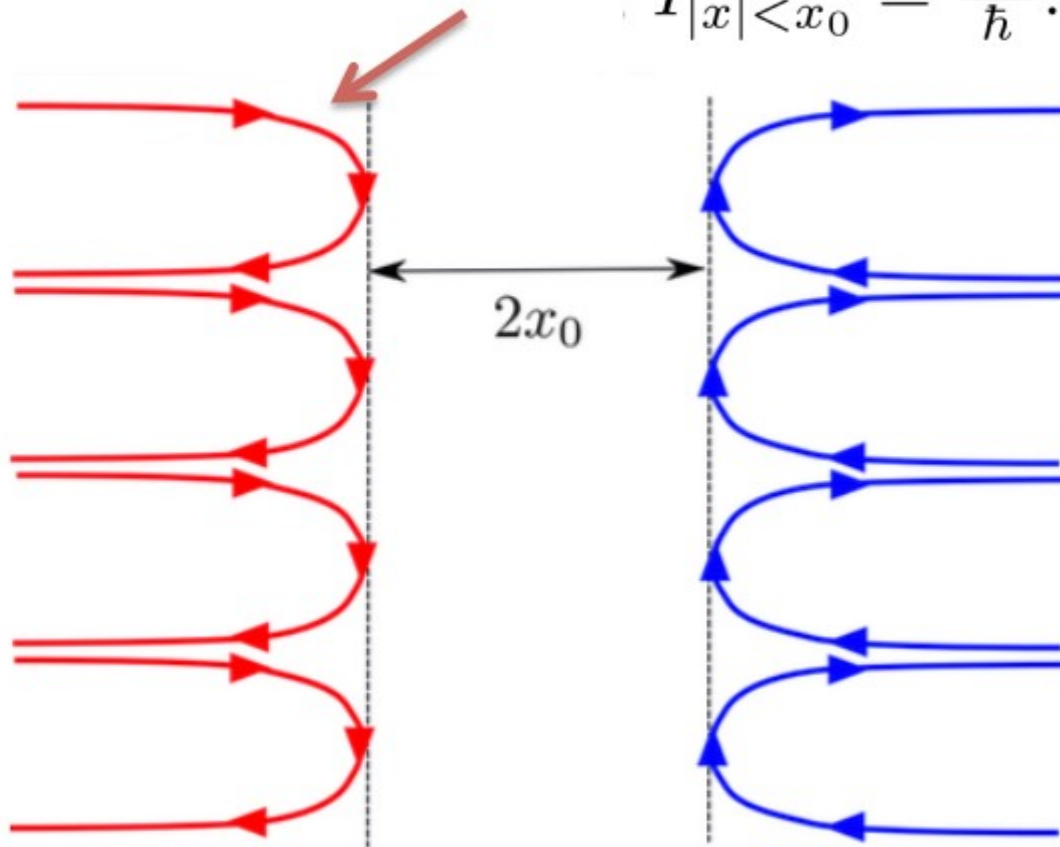
$$j = \sigma_{xy}^v E$$

Universal valley hall conductivity:

$$\sigma_{xy}^v = \frac{e^2}{2h} \text{ per valley/ flavor}$$

Side jump, $\ell = \hbar v / \Delta$

- Side jump is small
- E independent
- $I_{|x| < x_0} = \frac{e\Delta}{\hbar}$.



p region

n region

Valley transport in a fully gapped system

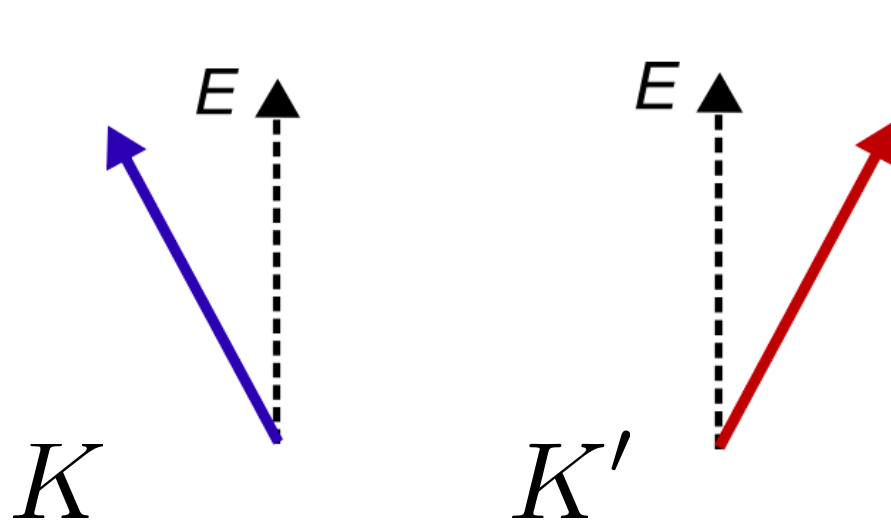
$\mathbf{J}_v \neq 0$ even when in the gap without edge states (current flows in system bulk);

Valley current peaks in the gap region; outside gap region partial cancellation; σ_{xy}^v has same sign for p & n regions

In fully gapped samples, valley currents transmitted by under gap states: “dissipationless”.

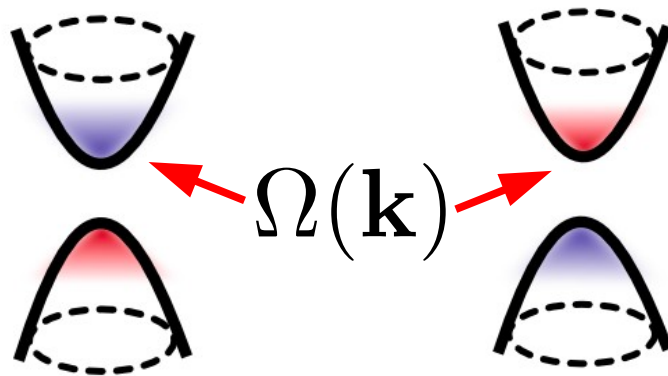
Systems: graphene/hBN and bilayer graphene:
Gorbachev, Song, et. al. , Science (2014)
Shimazaki et al. Arxiv:1501.04776 (2015)

Use Berry curvature to electrically manipulate valleys

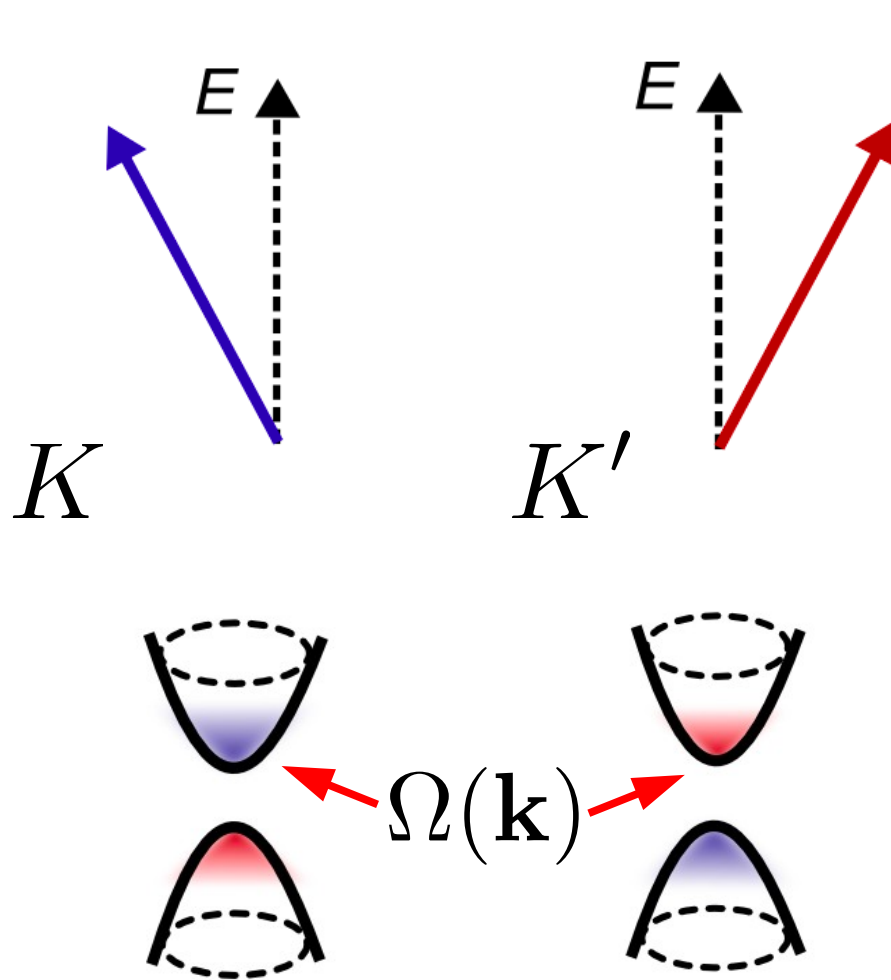


$$\mathbf{v}_{\mathbf{k}} = \frac{1}{\hbar} \frac{\partial \epsilon_{\mathbf{k}}}{\partial \mathbf{k}} + \dot{\mathbf{k}} \times \Omega(\mathbf{k})$$

$$\dot{\mathbf{k}} = e\mathbf{E} + e\mathbf{v}_{\mathbf{k}} \times \mathbf{B}$$



Use Berry curvature to electrically manipulate valleys



$$\mathbf{v}_{\mathbf{k}} = \frac{1}{\hbar} \frac{\partial \epsilon_{\mathbf{k}}}{\partial \mathbf{k}} + \dot{\mathbf{k}} \times \Omega(\mathbf{k})$$

$$\dot{\mathbf{k}} = e\mathbf{E} + e\mathbf{v}_{\mathbf{k}} \times \mathbf{B}$$

Valley Hall effect:

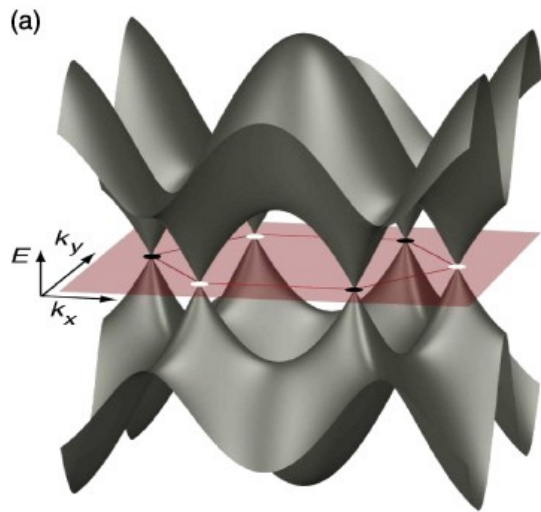
Transverse charge-neutral currents

$$\vec{J}_v = \vec{J}_K - \vec{J}_{K'}$$

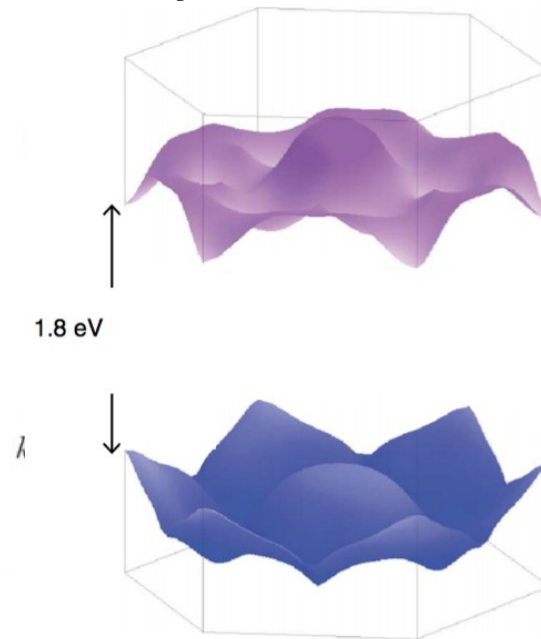
$$\vec{J}_v = \sigma_{xy}^v \vec{z} \times \vec{E}$$

Valley currents

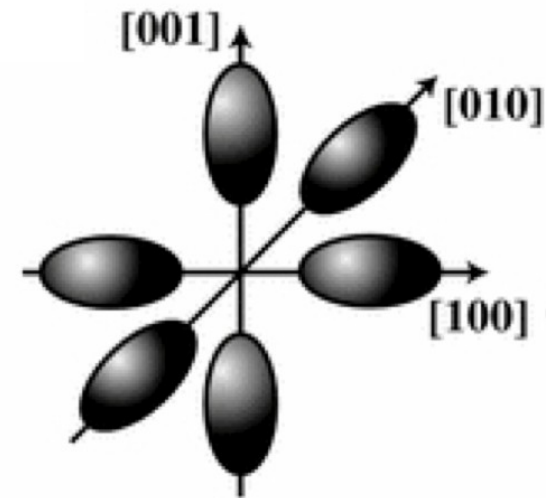
Valleys in Graphene



Valleys in MoS2



Valleys in Bulk Si



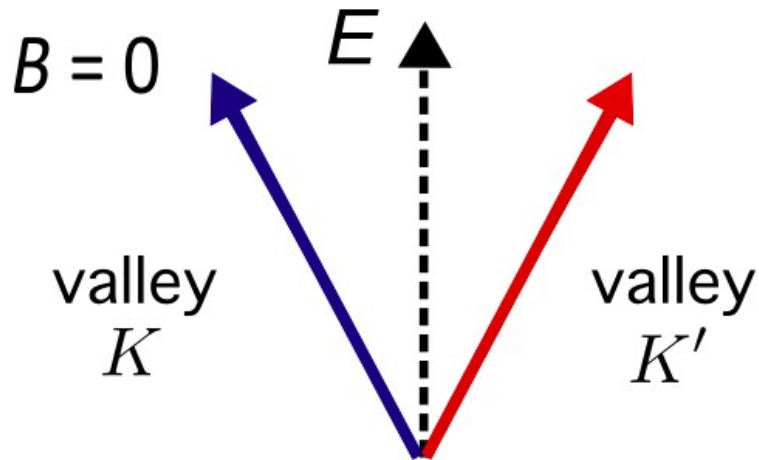
Berry curvature

$$\sigma_{xy}^v \neq 0$$

No Berry curvature

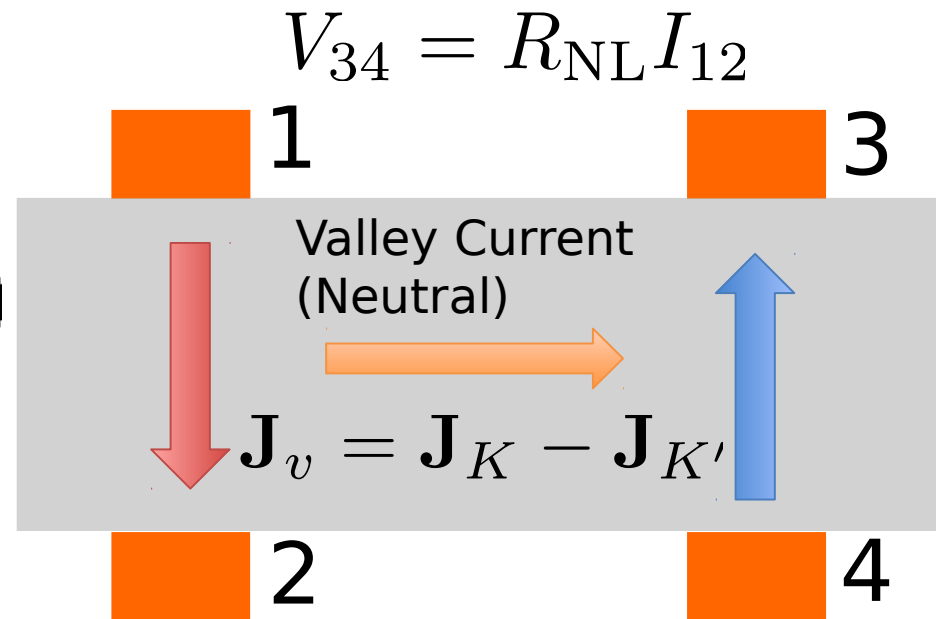
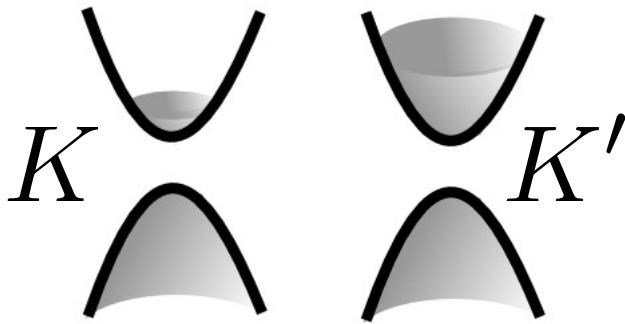
$$\sigma_{xy}^v = 0$$

Detecting valley currents

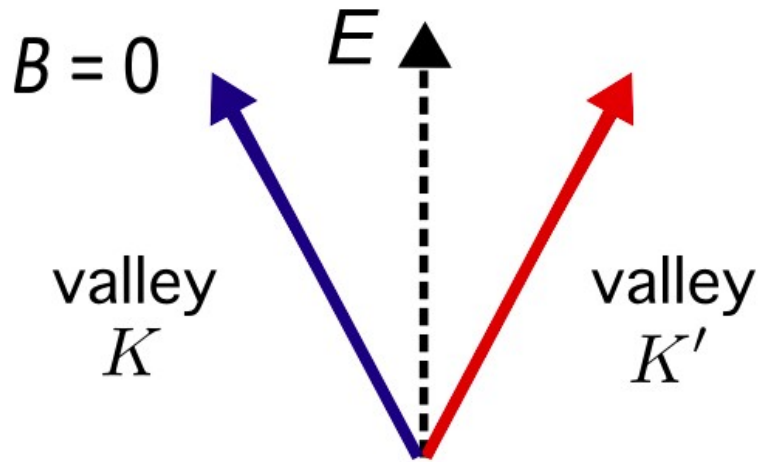


$$\sigma_{xy}^v = N \frac{e^2}{h} \int d^2 k \Omega(k) f(k)$$

Pump valley imbalance

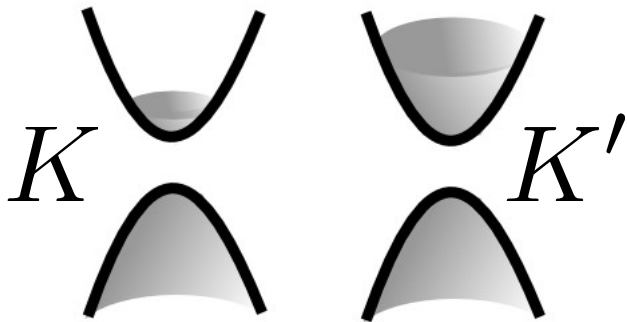


Detecting valley currents

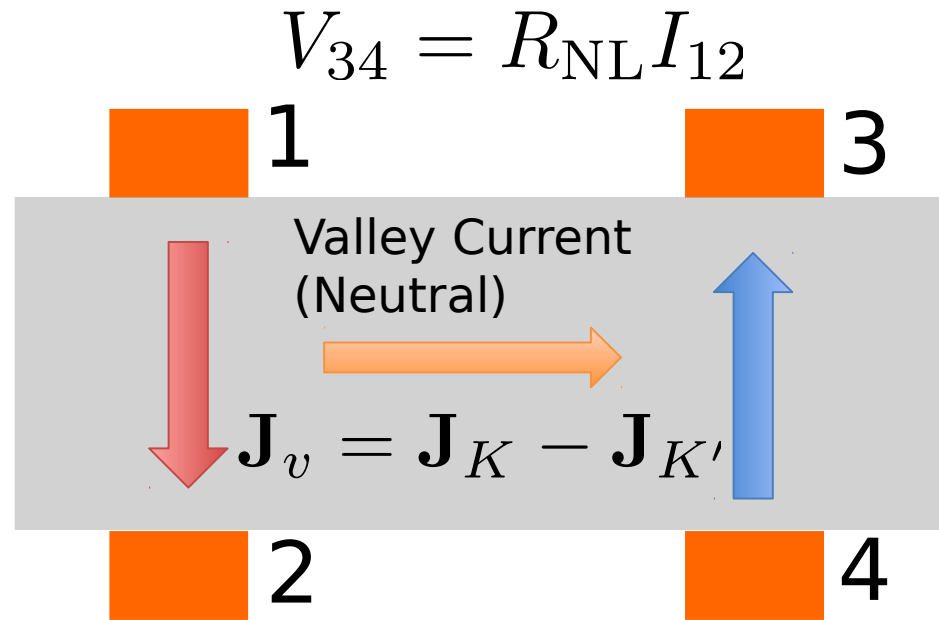


$$\sigma_{xy}^v = N \frac{e^2}{h} \int d^2 k \Omega(k) f(k)$$

Pump valley imbalance



11.06.2015



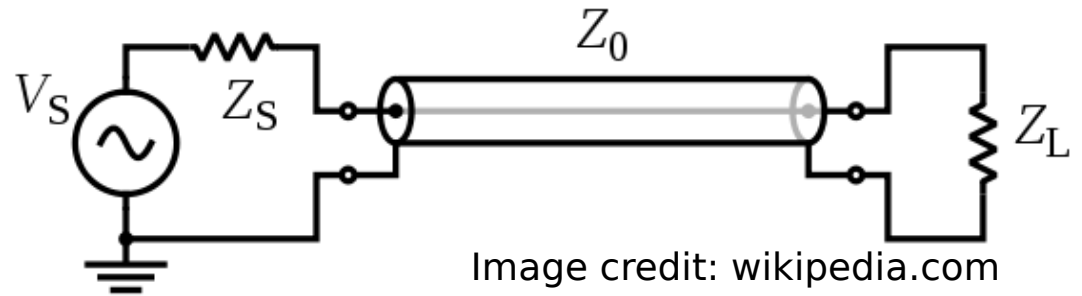
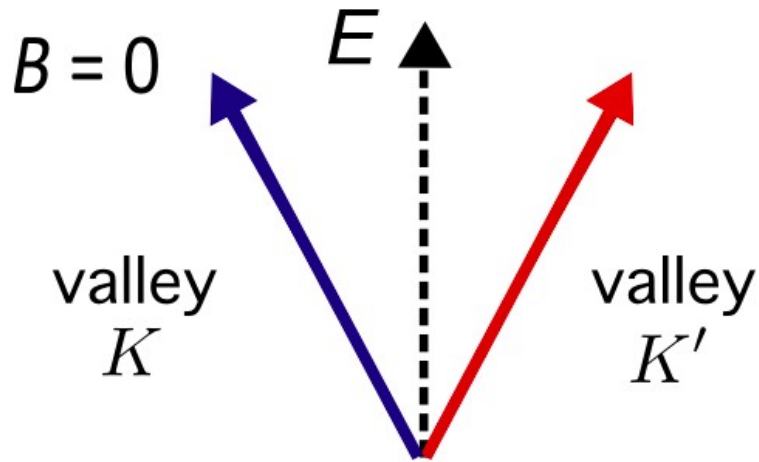
Valley Hall Effect (VHE):

$$\mathbf{J}_v = \frac{\sigma_{xy}^v}{\sigma} \mathbf{j} \times \hat{\mathbf{z}}$$

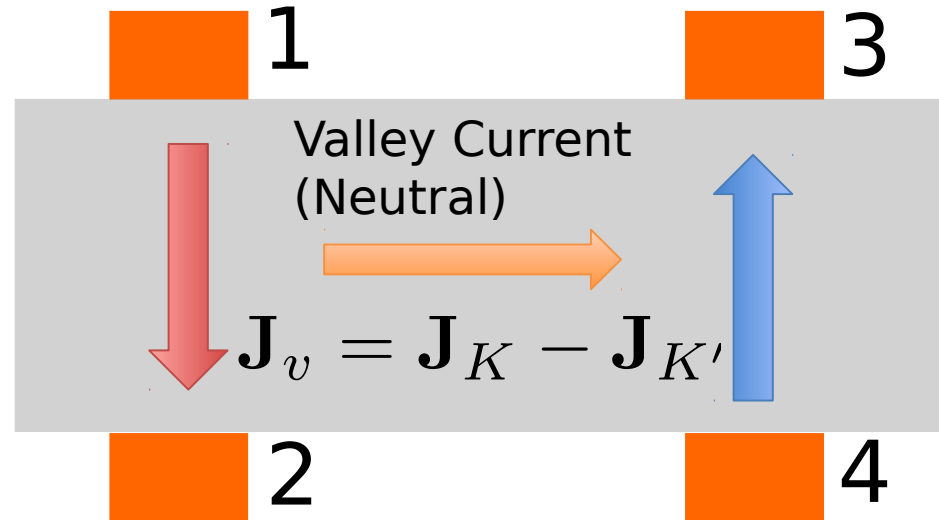
Reverse Valley Hall Effect (RVHE):

$$\mathbf{E} = -\frac{\sigma_{xy}^v}{\sigma^2} \mathbf{J}_v \times \hat{\mathbf{z}}$$

Detecting valley currents

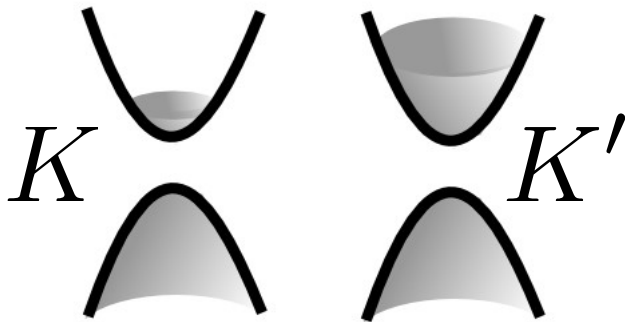


$$V_{34} = R_{NL} I_{12}$$



$$\sigma_{xy}^v = N \frac{e^2}{h} \int d^2 k \Omega(k) f(k)$$

Pump valley imbalance



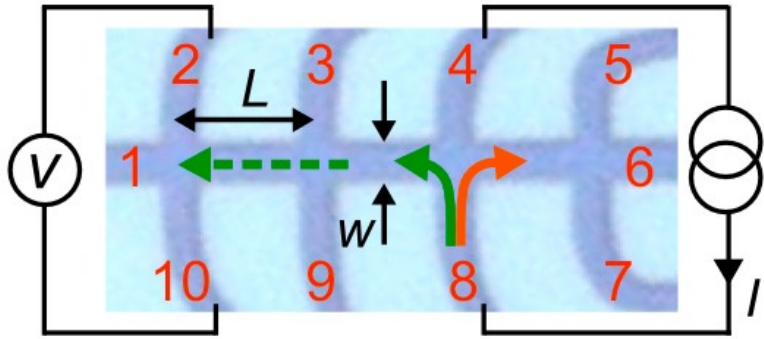
Valley Hall Effect (VHE):

$$\mathbf{J}_v = \frac{\sigma_{xy}^v}{\sigma} \mathbf{j} \times \hat{\mathbf{z}}$$

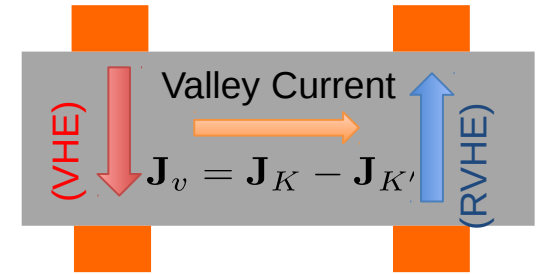
Reverse Valley Hall Effect (RVHE):

$$\mathbf{E} = -\frac{\sigma_{xy}^v}{\sigma^2} \mathbf{J}_v \times \hat{\mathbf{z}}$$

Nonlocal response in aligned G/hBN



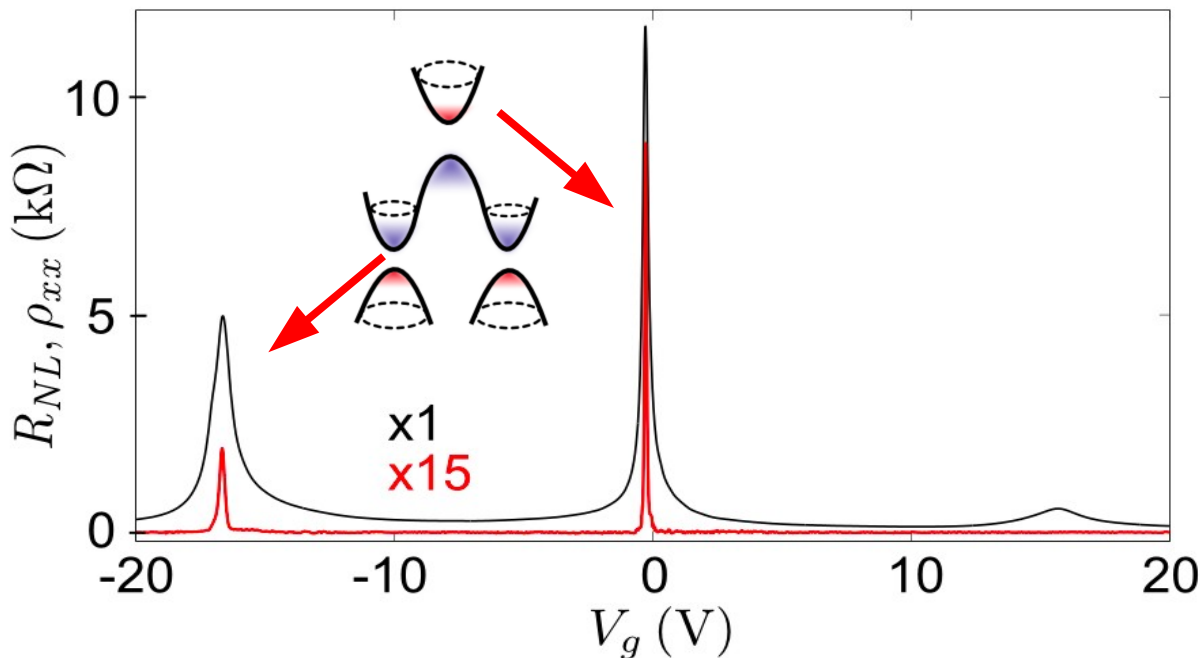
Collaboration:
U Manchester



$$V_{2,10} = R_{NL} I_{4,8}$$

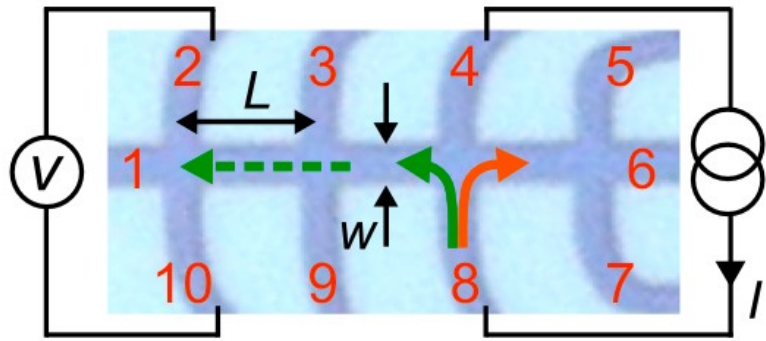
Van der Pauw bound: $R_{NL}^{VdP} \approx \rho_{xx} e^{-\pi L/w}$

Berry hot spots

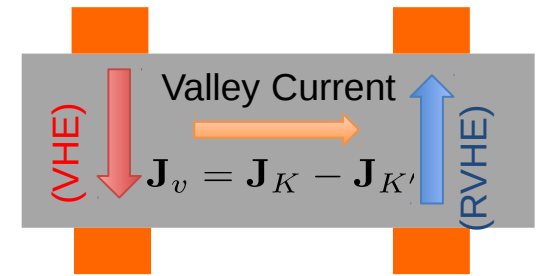


Gorbachev, Song, et. al. , Science (2014)

Nonlocal response in aligned G/hBN



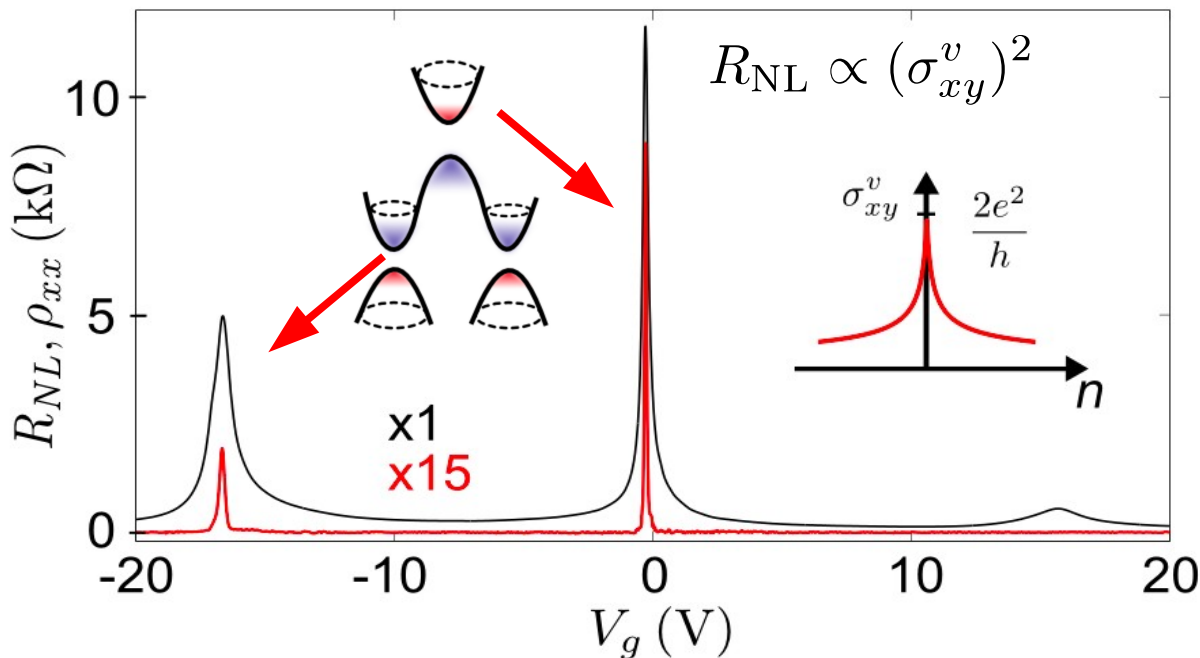
Collaboration:
U Manchester



$$V_{2,10} = R_{\text{NL}} I_{4,8}$$

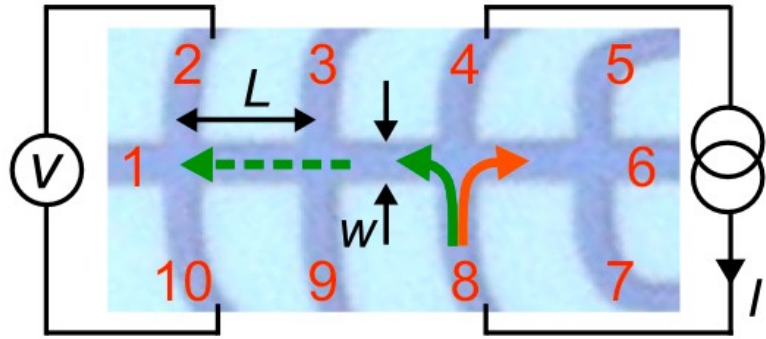
Van der Pauw bound: $R_{\text{NL}}^{\text{VdP}} \approx \rho_{xx} e^{-\pi L/w}$

Berry hot spots

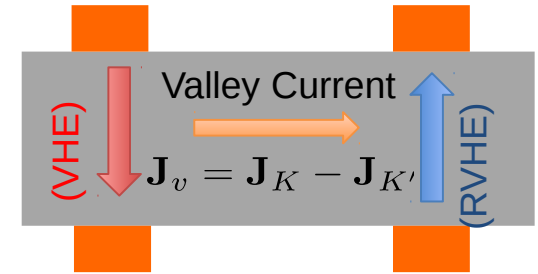


Gorbachev, Song, et. al. , Science (2014)

Nonlocal response in aligned G/hBN



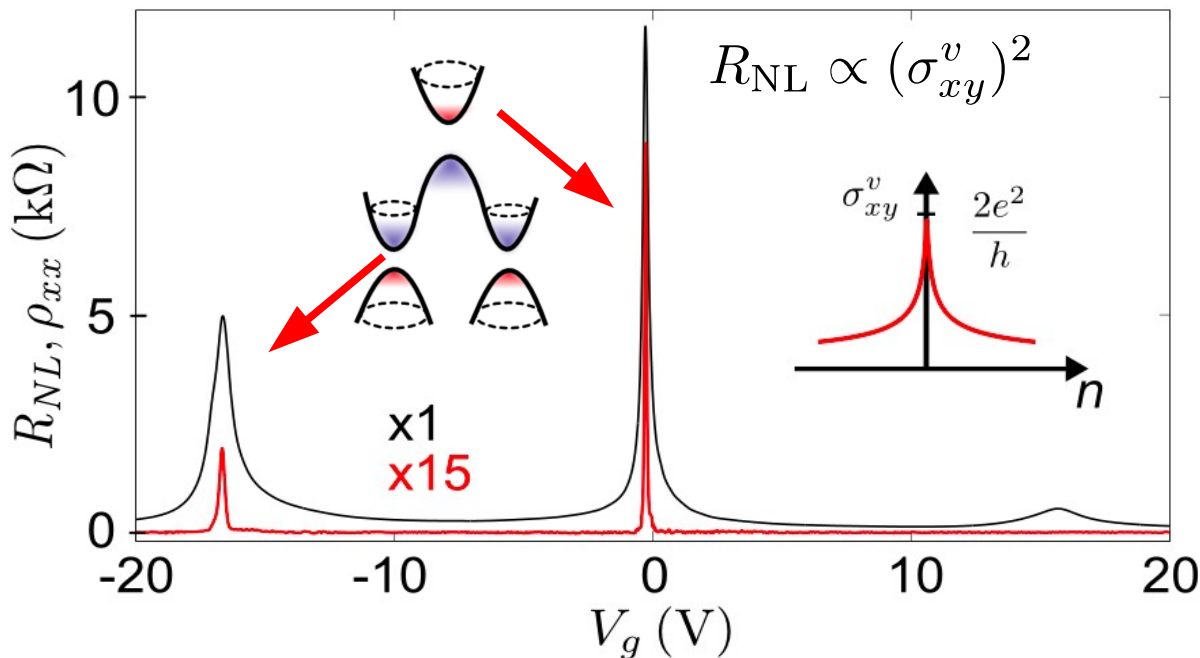
Collaboration:
U Manchester



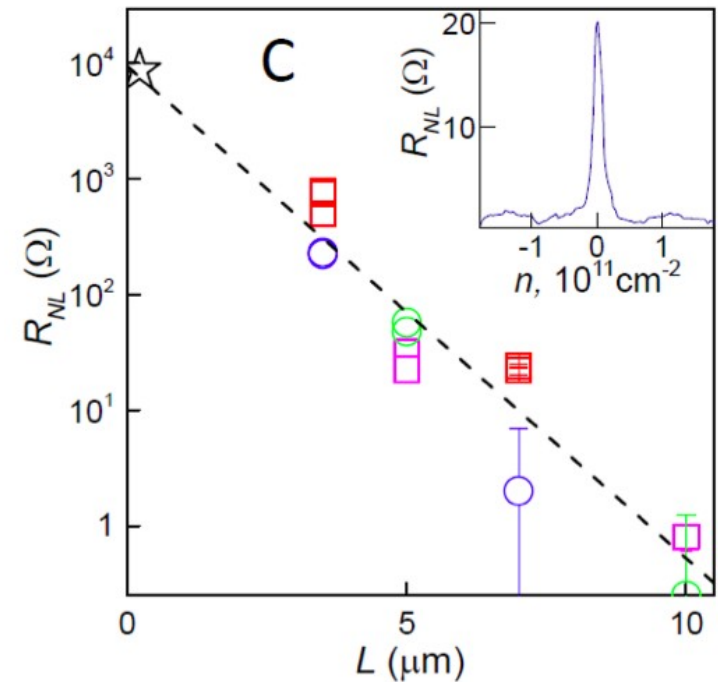
$$V_{2,10} = R_{\text{NL}} I_{4,8}$$

Van der Pauw bound: $R_{\text{NL}}^{\text{VdP}} \approx \rho_{xx} e^{-\pi L/w}$

Berry hot spots

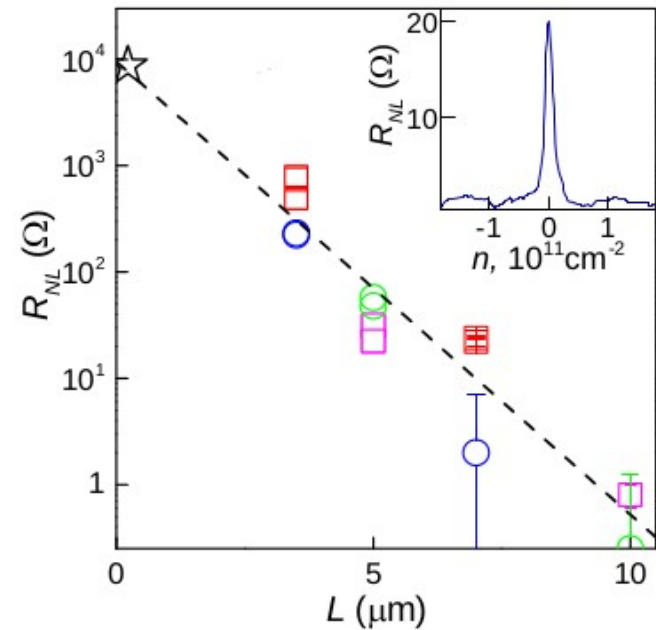
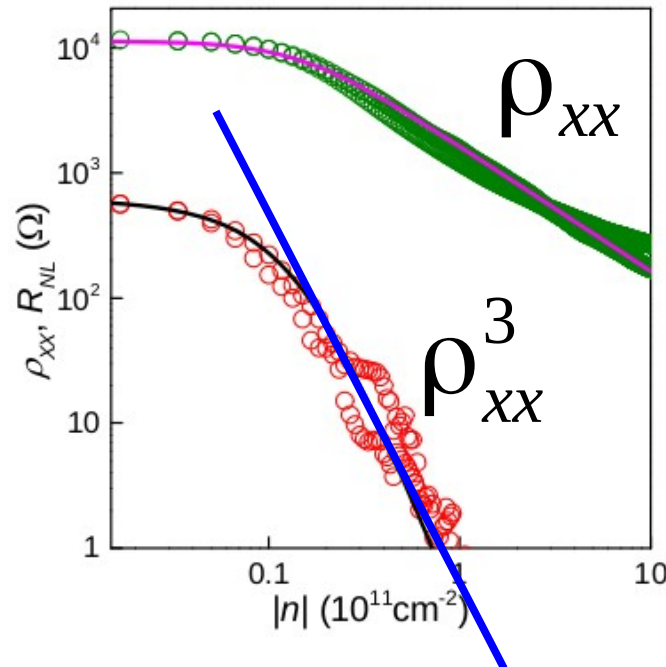
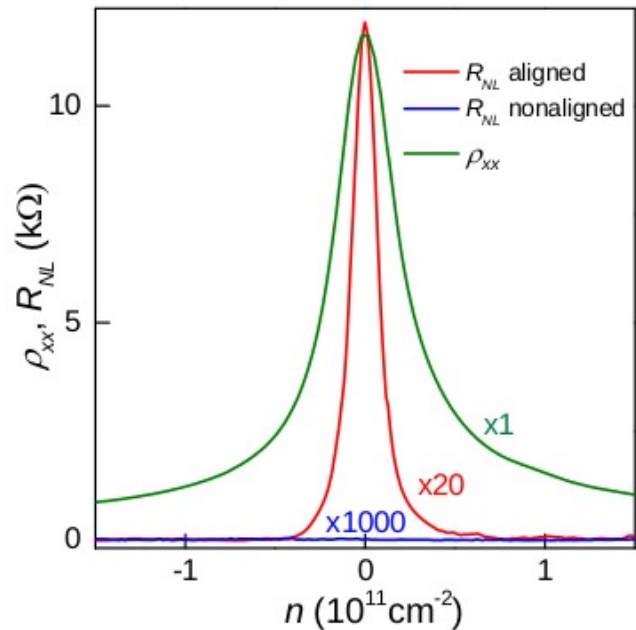


Distance dependence



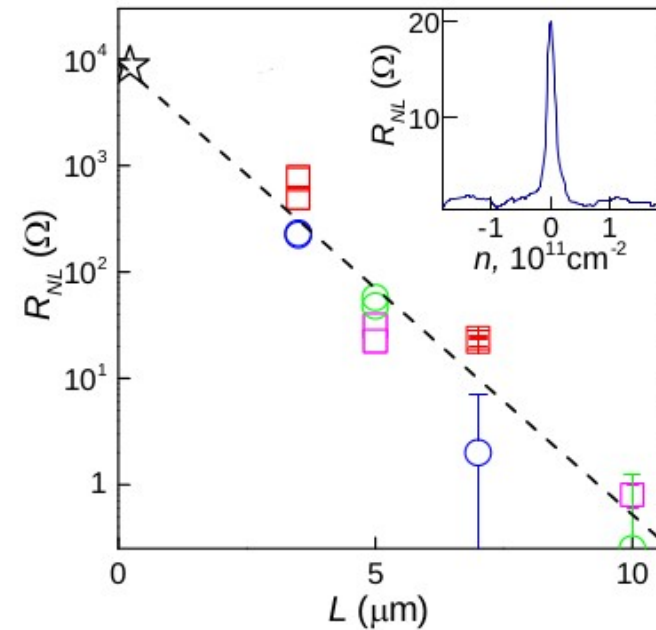
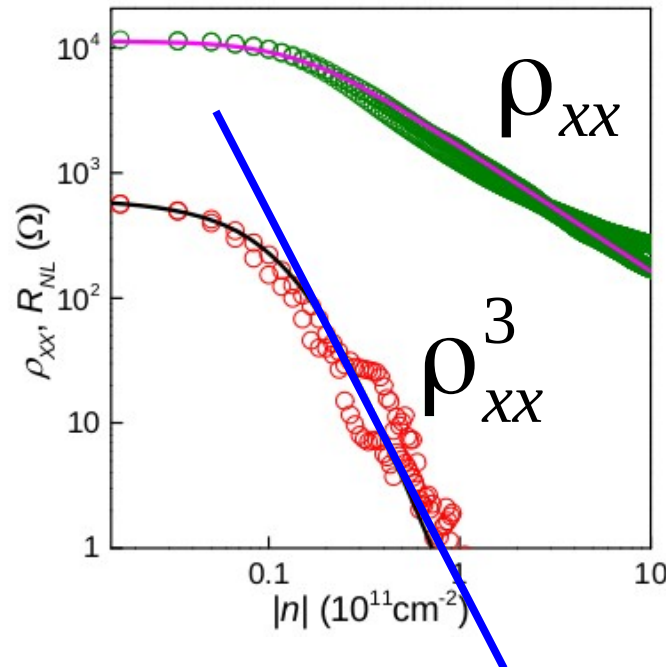
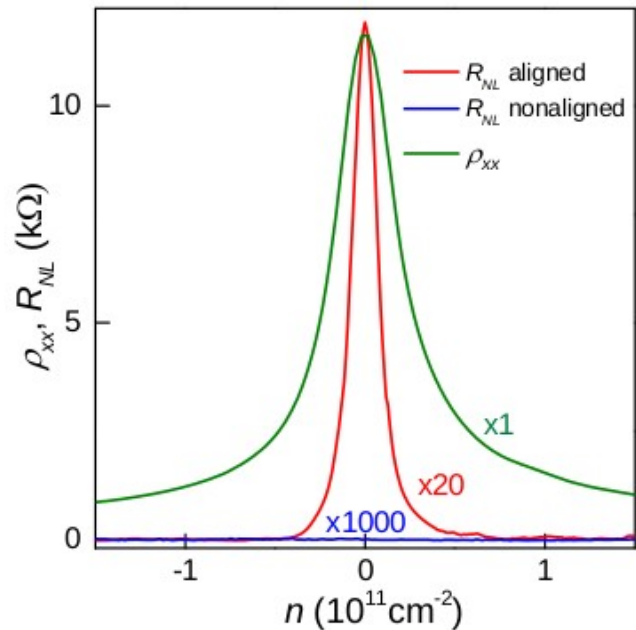
Checklist

- ✓ 1) Non-ohmic: stray charge currents too small, super-sharp density dependence; mediated by long-range neutral currents
- 2) Observed at $B=0$, excludes energy and spin (prev work)
- 3) Good quantitative agreement w/ topo valley currents for Berry curvature induced by gap opening
- 4) Seen in aligned G/hBN devices, never in nonaligned devices
- 5) Scales as cube of ρ_{xx} as expected for valley currents



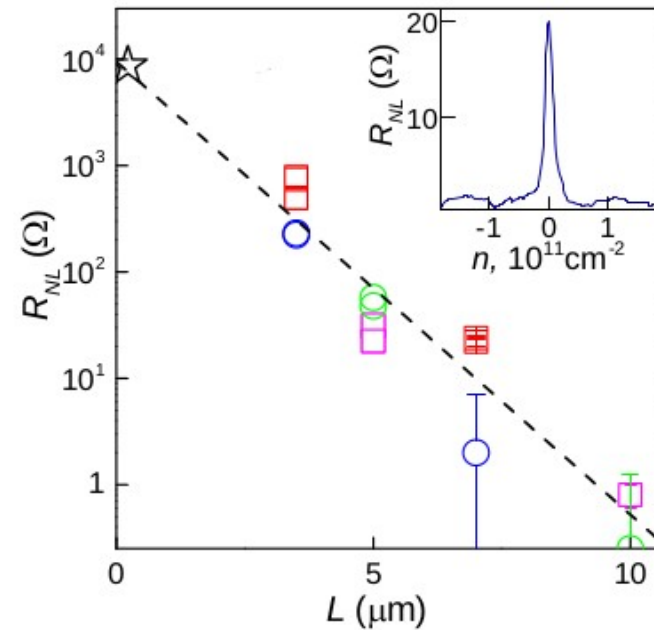
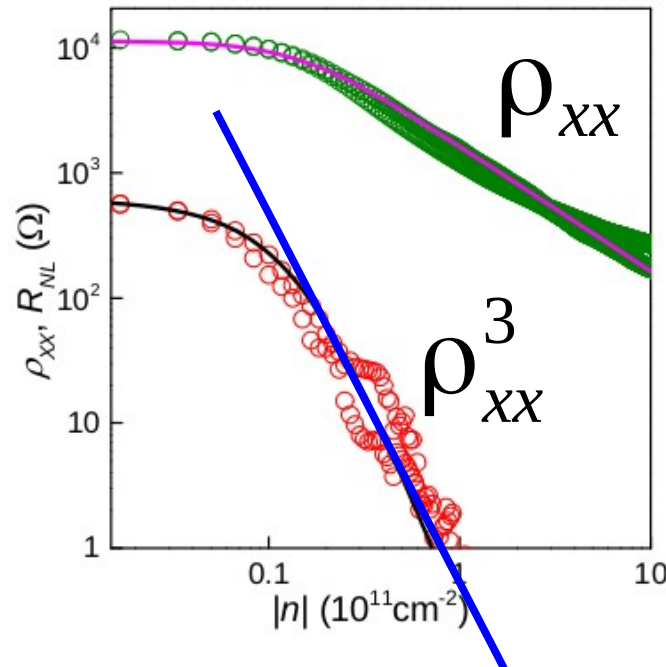
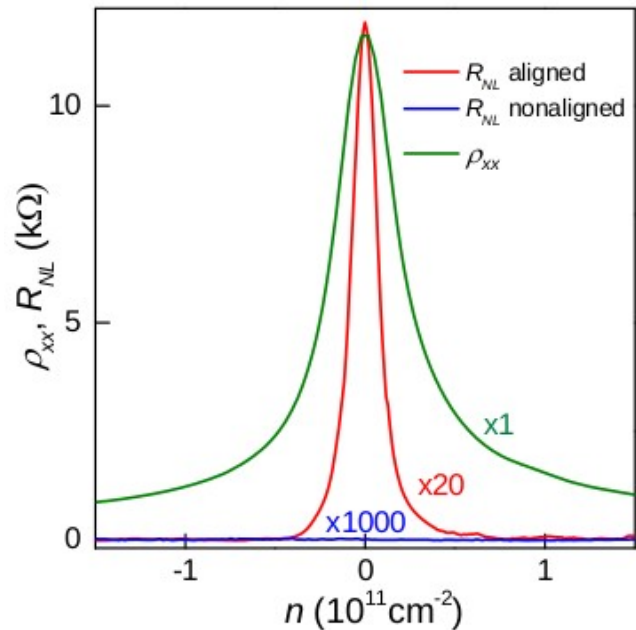
Checklist

- ✓ 1) Non-ohmic: stray charge currents too small, super-sharp density dependence; mediated by long-range neutral currents
- ✓ 2) Observed at $B=0$, excludes energy and spin (prev work)
- ✓ 3) Good quantitative agreement w/ topo valley currents for Berry curvature induced by gap opening
- 4) Seen in aligned G/hBN devices, never in nonaligned devices
- 5) Scales as cube of ρ_{xx} as expected for valley currents



Checklist

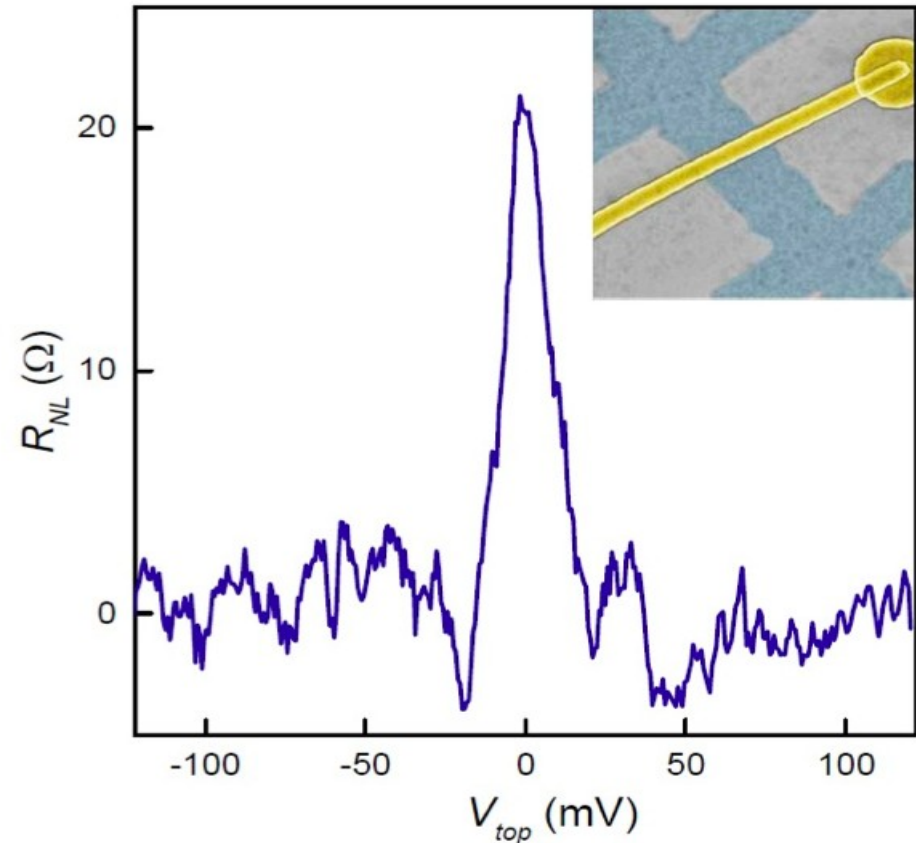
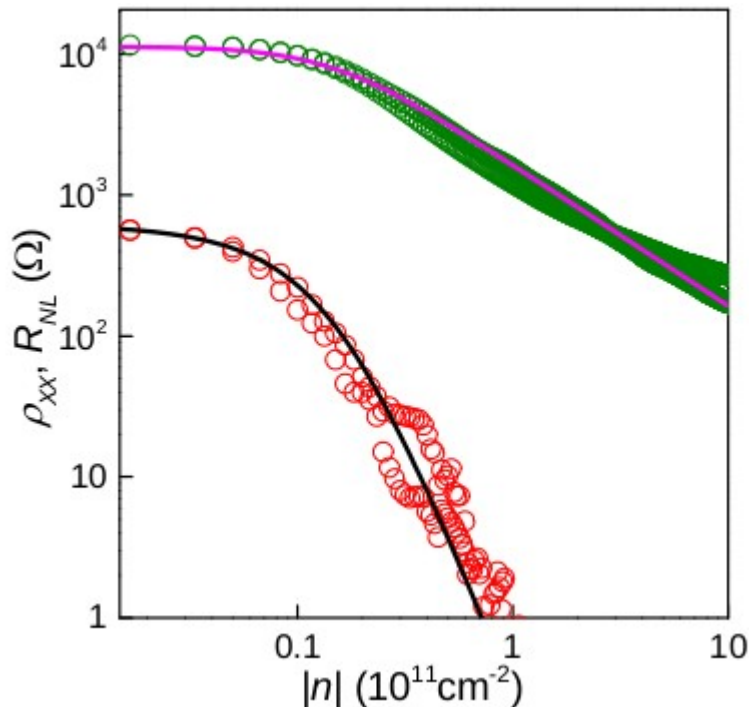
- ✓ 1) Non-ohmic: stray charge currents too small, super-sharp density dependence; mediated by long-range neutral currents
- ✓ 2) Observed at $B=0$, excludes energy and spin (prev work)
- ✓ 3) Good quantitative agreement w/ topo valley currents for Berry curvature induced by gap opening
- ✓ 4) Seen in aligned G/hBN devices, never in nonaligned devices
- ✓ 5) Scales as cube of ρ_{xx} as expected for valley currents



Valley transistor: proof of concept

- 1) Full separation of valley and charge current
- 2) ~ 140 mV/decade
- 3) Gate-tunable valley current

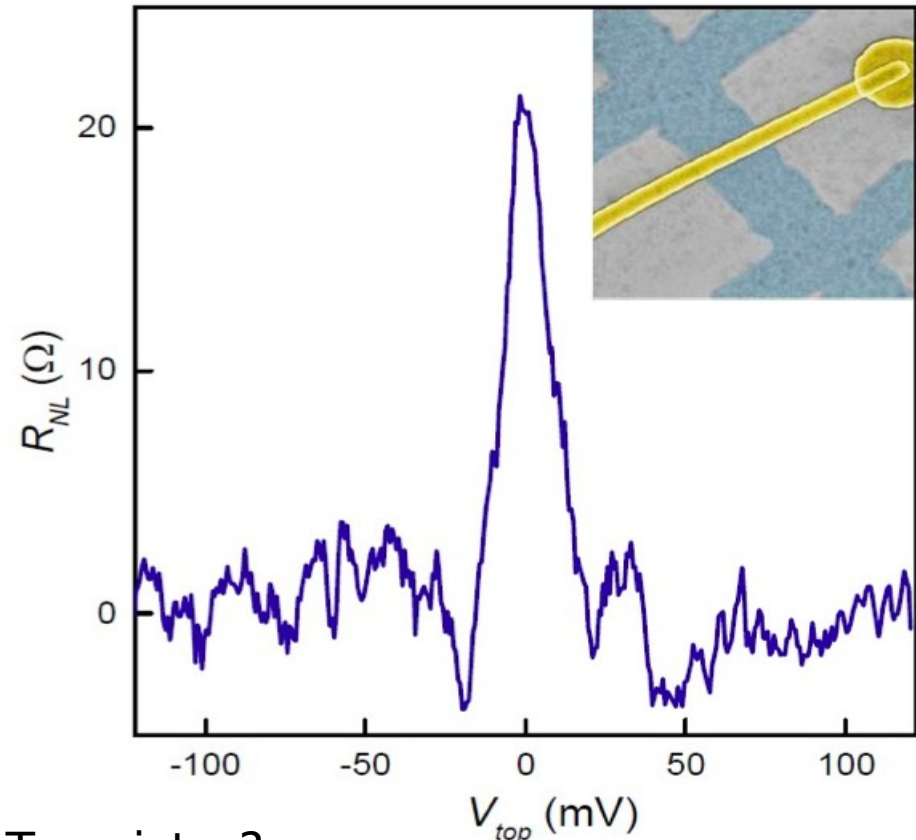
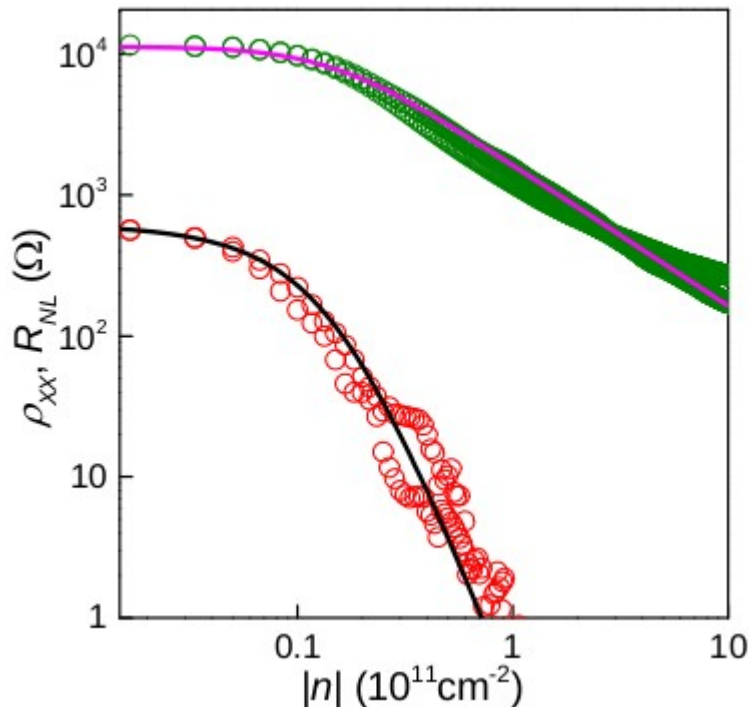
Modulation > 100 fold



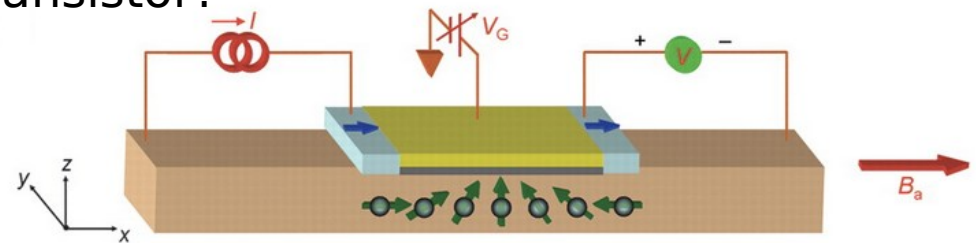
Valley transistor: proof of concept

- 1) Full separation of valley and charge current
- 2) ~ 140 mV/decade
- 3) Gate-tunable valley current

Modulation > 100 fold



Spin Transistor?



Koo, et. al., Science (2009), see also

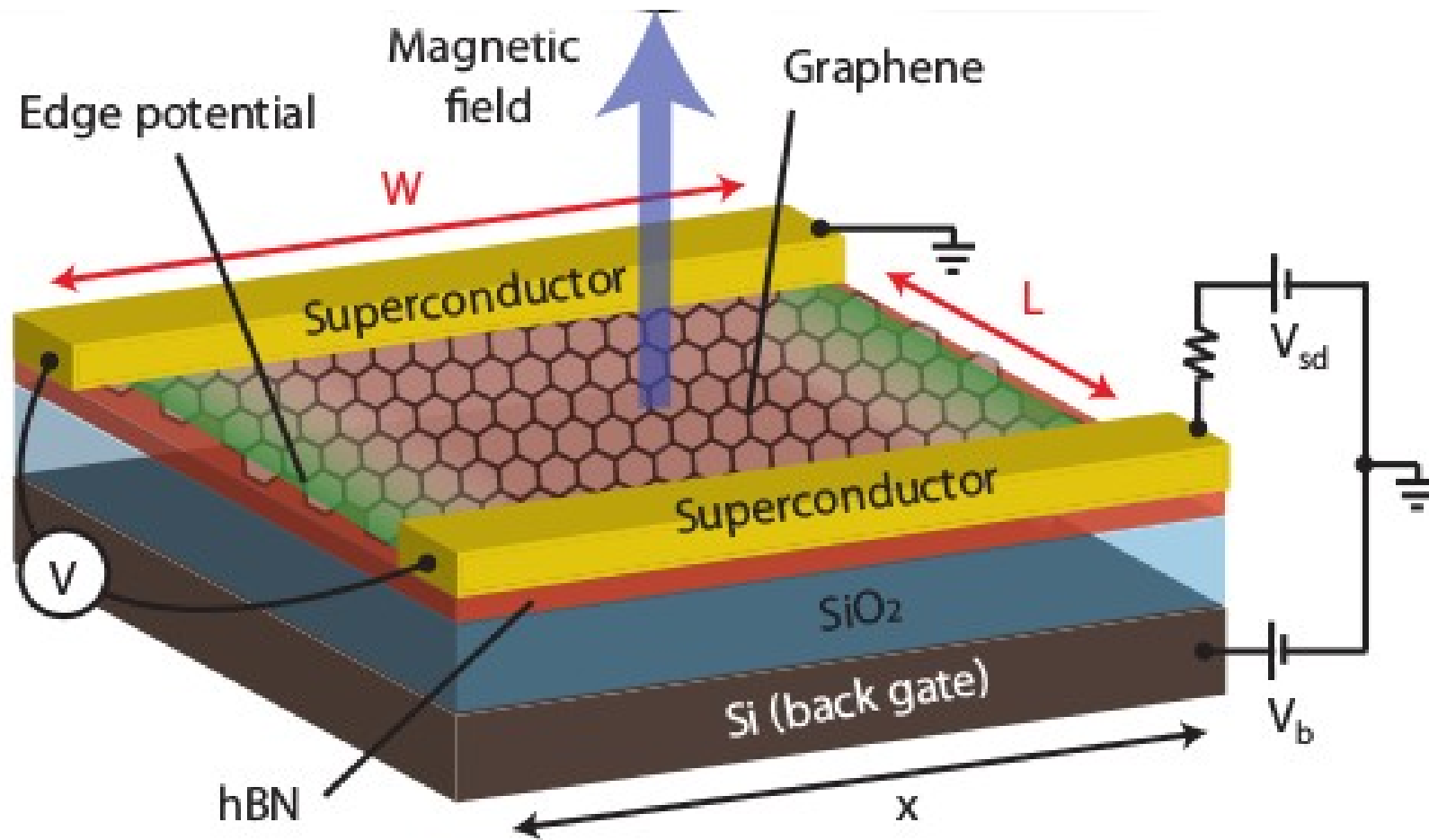
Gorbachev, Song, et. al., Science (2014) Wunderlich, et. al., Science (2010)

Original Proposal: Datta, Das, APL (1990)

Future

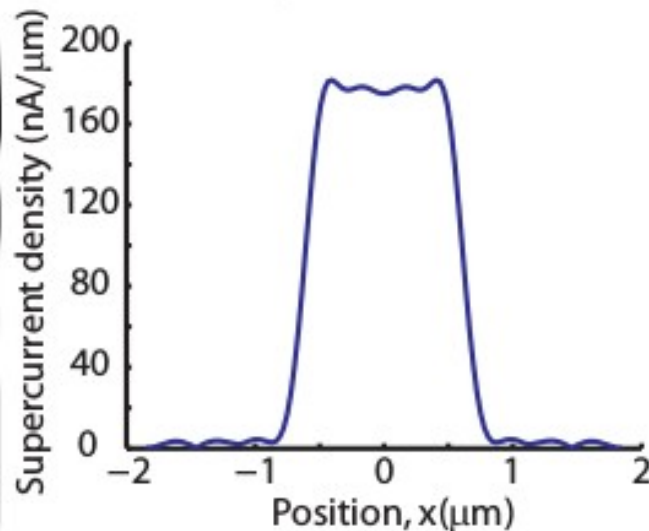
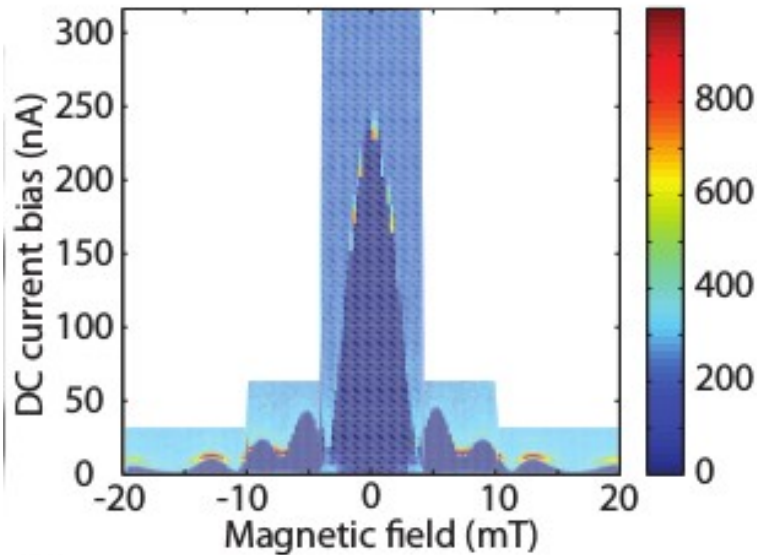
- **Chargeless long-range currents: Dissipationless transport?**
- **Berry curvature spectroscopy (signs, Chern numbers)**
- **Waveguides for valley currents**
- **Valley currents in 1D channels (graphene edge, BLG domain walls, p-n junctions)**

Edge currents from Josephson interferometry (Yacoby group 2015)

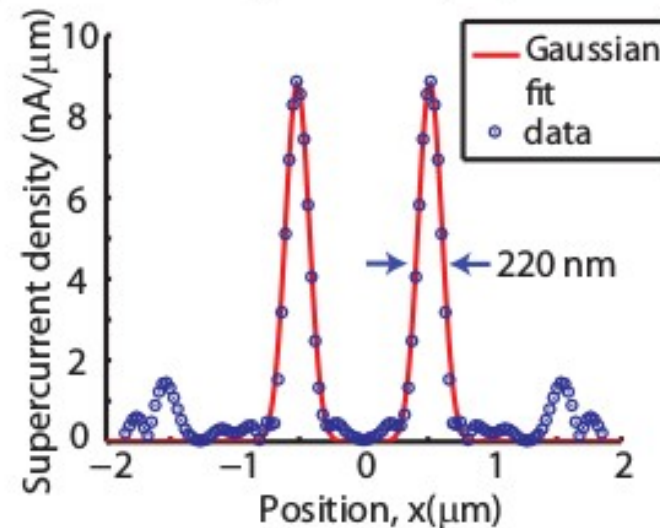
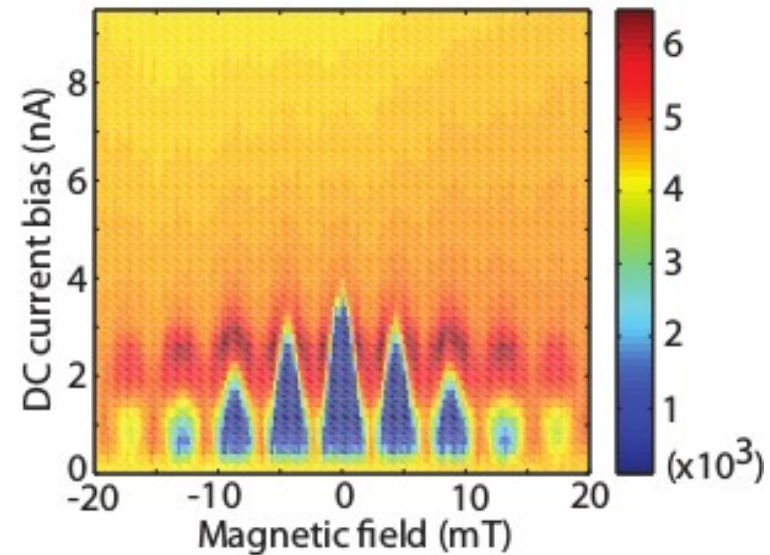


Edge currents from Josephson interferometry (Yacoby group 2015)

Away from DP



Near DP



Edge currents from Josephson interferometry (Yacoby group 2015)

Evidence for guided-wave states at the edge

